Relative Theory of National Production

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ABSTRACT: In this paper, we at first give a critical review to classical GDP accounting theory on three questions, i.e. the productive issue, the market price issue and welfare issue. In the following parts, we put forward a relative production theory. Our theory think that the material production is the foundation of mankind society, and there is a developing series from material production to political and cultural production; in an economic system as a whole, every sector set gives a certain volume of net final products to the other sectors' set. For different investigation purposes, different production notions could be chosen.

KEYWORDS: National production accounting, GDP, input-output accounts, relative production theory

1. Introduction

In 1950, R.Stone and J.Utting linked input-output (IO) Table and National Economic Accounting System first in the First International Input-Output Techniques Conference. In 1968, IO table was introduced into the System of National Accounts (SNA). Since then, the input-output techniques have produced rich data for various economic analyses.

But there are some essential differences between Leontief input-output economics and traditional mainstream economics. One is that Leontief input-output models always include intermediate input in them, and intermediate direct input coefficient matrices are the central and remarkable characters of them. However, the structure of the input-output table designed in SNA is obedient to the classical concept of GDP in traditionally mainstream macroeconomics.

Economists know that *production* is a very important notion in *Capital* written by K. Marx, and it has been disputed for so many years. Definition of production is also fundamental in SNA for calculating GDP; however it has been criticized at many aspects (Greg Ogle 2000). For example, Against calculating GDP in market prices, someone have pointed out that, "The reason why market prices should, in one form or other, be taken as basis of valuation is that they can be expected to be proportional to the direct or indirect relative marginal utilities derived from products. The existence of taxes and subsidies on production, market imbalances and the fact that all the products are not sold in the market makes it however impossible to find a set of prices that would meet this condition. Just because there are no pure market economies, there is a formidable insoluble problem of valuation even in the business sector."(Aulin-Ahmavaara, 1992) Due to the defects in the GDP definition of SNA, there is not an absolute standard formula to compare the results of the production process in different periods and in different national economies. The input-output table provides a good frame to clear this issue in some sense. The other parts will propose a relative theory of national production to treat this problem.

The questions to modern GDP accounting theory in essence are three aspects. The first is how to distinguish production from consumption or final products from intermediate products. The second is whether it is reasonable and scientific to calculate GDP by adding various products in market

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prices, or what are the scientific meanings of GDP formed from various products in market prices[①]. The third is about the relationship between GDP and welfare[②]. The last two are more qualitative and may not be discussed in an input-output frame, so that only the first question would be dealt with in this paper in a way.

Though SNA has given a definition for production, the calculation of GDP is incompletely reasonable yet. The paradox in it has shown clearly through the compiling principle of input-output table. According the IO table of US 1990, producers of government services only input value added (VA), and did not input any intermediate cost, and all the commodities and services used by government activities were accounted in final use. But according the IO table of China 1992, public administration as a production sector not only input VA but also input various intermediate costs. The product of public administration was all finally used by total institutional consumption (i.e. public consumption). Though the two treatments on government services have the equal value theoretically in calculating GDP, but it exposes the shortcomings of production concept. The definition of production in SNA is not clear, so that the GDP accounting theory is not reasonable enough.

In fact, if the meanings of production are applied thoroughly, the products of government will be split into two parts in which one serves production sectors, and the other serves resident sector[③]. Thus, the cost of the first part for production should be accounted into intermediate use that should not go into GDP, and the part for resident should be accounted into final use that should go into GDP. Then by the aspect of accounting, the tax from intermediate sectors should be moved from the primitive input to intermediate input, and the tax from household consumption should be laid in the same line under household consumption in final use. In this principle, Chinese government is more productive than US government, because the former joins more deeply in the management process of commodity production, and so the government should get bigger percentage of the tax.

We should notice one fact that the traditional input-output accounts in SNA is designed to serve the accounting theory of GDP, so that the government has been given a special treatment: as an intermediate production sector and as well as a final consumption sector. Consequently, it has revealed the contradictions in modern national income accounting theory. If we consider this kind of problem in general circulation of an economy, the resident sector can also be thought as a production sector. Then, how would we account GDP? What are the final products of production except export and investment?

2. A Complete I-O Table

In order to describe the principle of production accounting clearly, a kind of IO table is designed here, and we call it *complete IO table*(Liu Xinjian, 1996).

An economic system can be divided into seven types of sectors:

1). Productive enterprises units except commerce enterprises and financial enterprises;

2). Commerce enterprise units;

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① Because today's commodities are more and more multifunctional and evolve frequently, the traditional price indices are losing their rationality, especially for an economy as a whole.

② Many academic and official organizations have started to research and implement hedonic priced or green GDP accounting programs.

③ Similar though less complicated thing happens for financial intermediation services, see (Nadim Ahmad,et al., 2003)

3). Financial enterprise units;

4). Public affair units (like health, education, etc.);

5). Government and other political units;

6). Fiscal ministry;

7). Residents.

Here, fiscal ministry is not a real institutional body with human being, it just a tax collecting and paying-out machine. The real fiscal ministry with human being activities is included in government units. Besides import and export, i.e. foreign units are not considered here.

In extensive production view, or from the view of economic circulation, the all of above units are both productive and consumption sectors. Their products in a certain period of time are consumed or become the basement of next production period--fixed assets or inventory. The activity with the latter is called as capital formation or investment. The production of labour force as a special reproductive activity is not given special treatment in such a simple input-output accounting system in this paper.

Based on the above notes about an economic system, a complete input-output table is as following:

	Consumption Using							Capital Formation							Total Output
	1	2	3	4	5	6	7	1	2	3	4	5	6	7	
1 2 3 4 5 6 7	x_{ij}							c_{ij}							x_i
Surplus															
Total Input															

Generally, Input-Output Table is a kind of transaction account table, i.e., except fiscal income and transfer payments, Sector *j*, for example, must get certain products or service from Sector *i* when it pays Sector *i* certain money.

3. Input-Output Accounting System

3.1. Basic accounting identical equations

As we know, national income accounting identities are the basic relationship in System of National Accounts. This kind of relations is set up in a more general meaning in input-output accounting system.

Let $\{x_{kj},\ j=1,2,\cdots,n\}$ represents the output distribution sequence of sector *k*, $\{x_{ik},\ i=1,2,\cdots,n\}$ represents the input sequence for sector *k* from other sectors. Among them, x_{kn} represents the capital formation, and x_{nk} indicates the operational surplus. Let $N=\{1,2,\cdots,n\}$, according to the relations of input-output balances, there must be

$$\sum_{i\in N} x_{ik} = \sum_{j\in N} x_{kj} = x_k \tag{1}$$

where x_k is the gross output of sector k($k \neq n$). For $k=n$, x_n can be thought as the gross stock of an economy (including fixed capitals and inventory), and $x_{nn} = 0$.

Formula (1) is called as the basic identity of input-output accounting. If x_{kk} is subtracted from each side of Formula (1), then [①]

$$\sum_{\substack{i \in N \\ i \neq k}} x_{ik} = \sum_{\substack{j \in N \\ j \neq k}} x_{kj} = Y_k \tag{2}$$

The meaning of Formula (2) is that the aggregated input from the other sector to sector *k* always equals to the aggregated output from sector *k* to the other sectors.

If the increased inventory in capital formation is a rational need level in a productive process, then the economic meanings of Formula (2) can be thought as that (a) the market is clear up in an economy and stays at a equilibrium state, (b) the products of sector *k* left a surplus except that was used in its inside and the surplus was sold to the other sectors, and the return was used to buy, from other sectors, what was needed in its production processes.

3.2. Material production accounting

We insist on that only material production sectors are the real production sectors of a society and other service sectors are the extension or socialization of consumption, so that the material production accounting is an accounting to the production capacity of an economic system.

Let M represents the set of material production sectors, which generally includes agriculture, industry, construction, freight transport, commerce and other sectors that provide technical services (technology and management) for the proceeding four kinds of sectors. It can be got from Eq. (2) that

$$\sum_{k \in M} \sum_{\substack{i \in N \\ i \notin M}} x_{ik} = \sum_{k \in M} \sum_{\substack{j \in N \\ j \notin M}} x_{kj} = Y_M \tag{3}$$

where $Y_M = \sum_{k \in M} Y_k$.

The Quantities in the sides of Eq.(3) are just the net output values. In fact, Eq.(3) has the same economic meanings with Eq.(2) when the set of material production sectors was thought as a single sector.

For a closed economic system, any social progress was impossible without the full development of material production. Above all, the development of material production provides other consumption and service production sectors surplus products and surplus labour.

3.3. Market production accounting

The sector whose main volume of products (services included) is traded directly through market is called *market production sector*.

Market producers are the main economic body of a nation with a market economy. Generally, this kind of sector sets includes such sectors as commodity enterprise sectors, commercial enterprise

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[①] When the prices are equal to the market equilibrium prices, the ratios of Y_k to labour number represents the net labour productivity.

sectors, and monetary enterprise sectors (except national central bank).
Let *S* represents the set of market production sectors, then

$$\sum_{k\in S}\sum_{\substack{i\in N\\ i\notin S}} x_{ik} = \sum_{k\in S}\sum_{\substack{j\in N\\ j\notin S}} x_{kj} = Y_S \tag{4}$$

where $Y_S = \sum_{k\in S} Y_k$.

It is the same reason that Y_S in Eq.(4) is the net social production of market production to other sector sets.

3.4. General Production Accounting

In market economic systems, we define a general production activity in which people use their brain and physical energy to serve other people directly. (about feminism)
Let G represents the set of general production sectors, it includes the proceeding five sectors of the seven sectors in § 2, then

$$\sum_{k\in G}\sum_{\substack{i\in N\\ i\notin G}} x_{ik} = \sum_{k\in G}\sum_{\substack{j\in N\\ j\notin G}} x_{kj} = Y_G \tag{5}$$

where $Y_G = \sum_{k\in G} Y_k$.

Here Y_G is the contribution to resident consumption and accumulation invested by all workers in the society, and it can also represent the general labour output made by the whole society. Certainly, this kind of representation has only relative meanings, i.e., the meanings exist just as comparing with time series for the same society. Because the output of each sector is not only the results by work but also the results of distribution by some special rights (including reasonable or unreasonable, e.g. monopoly), it is impossible to compare Y_G especially in absolute volume.

3.5. Narrow CAA (Consumption-Accumulation Accounting)

Narrow CAA is the distribution accounting of Y_G between consumption and accumulation. By basic balance equation of I-O table, there is

$$\sum_{i=1}^{n-1} x_{in} = \sum_{j=1}^{n-1} x_{nj} = IN \tag{6}$$

Here IN is the total social accumulation or extensive capital formation. "Extensive" means that it includes family accumulation that is consumed directly by family.
Like in classical SNA, Eq.(6) means

Total Saving ≡ Total Investment.

Let $$C_1 = Y_G - IN \tag{7}$$

then C_1 represents the resident consumption.

Let $$\alpha_1 = \frac{IN}{C_1} \tag{8}$$

then α_1 represents the n*arrow accumulation - consumption ratio.*

When calculating material productive accumulation, *IN* should be divided into IN_1 and IN_2, i.e., productive accumulation and non-productive accumulation. According to that, IN_1/C and IN_2/C can be calculated.

3.6. Classical GDP Accounting

In Classical GDP Accounting，the 4^{th}~7^{th} sectors (see §2) all belong to final using sectors. Here we use complete I-O table to give a resemble GDP of the classical one. In fact, every country in the world could have different regulations in GDP accounts.

Let T represents the set of proceeding three sectors defined in §2, then

$$\sum_{k\in T}\sum_{\substack{i\in N\\ i\notin T}} x_{ik} = \sum_{k\in T}\sum_{\substack{j\in N\\ j\notin T}} x_{kj} = Y_T = GDP \tag{9}$$

Let $$C_2 = Y_T - IN \tag{10}$$

then C_2 is the classical total consumption, including resident consumption inside family and socialized consumption.

To do the same as above, we have *classical accumulation-consumption ratio* as

$$\alpha_2 = \frac{IN}{C_2} \tag{11}$$

3.7． Extensive CAA (Consumption-Accumulation Accounting)

Our extensive CAA is based on narrow production accounting. Because the consumption here includes the materials consumed by some producers who serve resident living, it is "extensive".

Let $$C_3 = Y_M - IN \tag{12}$$

then C_3 is extensive consumption, and

$$\alpha_3 = \frac{IN}{C_3} \tag{13}$$

is the *extensive accumulation-consumption ratio*.

4. Input-Output Analysis

The input-output accounting system mentioned above is based on the balance relation of input-output. The following input-output analysis is based on the basic input-output relations through input-output coefficients.

4.1. Input-output coefficients

Let

$$a_{ij} = \frac{x_{ij}}{x_j}, \quad i, j = 1, 2, \cdots, \ n \tag{14}$$

then, a_{ij} is the basic input-output coefficient—direct input coefficient.

4.2. Basic equation system of Input-output analysis

Defining a matrix A_H, column vectors x_H and y_H. Here H is a set of some sectors, $A_H = (a_{ij}, i, j \in H)$, $x_H = (x_k, k \in H)$, $y_H = (y_k, k \in H)$, and

$$y_k = \sum_{\substack{j \in N \\ j \notin H}} x_{kj} \ ,$$

then, it must be

$$A_H x_H + y_H = x_H \tag{15}$$

When $H \subset N$, it must be

$$x_H = (I - A_H)^{-1} y_H \tag{16}$$

Applying the relations above to material production accounting, market production accounting and general production accounting, we get

$$A_M x_M + y_M = x_M = (I - A_M)^{-1} y_M \tag{17}$$

$$A_S x_S + y_S = x_S = (I - A_S)^{-1} y_S \tag{18}$$

$$A_G x_G + y_G = x_G = (I - A_G)^{-1} y_G \tag{19}$$

If we divide general production sectors into material production sectors, market non-material production sectors and non-market production sectors, and mark their variables in M, $\overline{S}$ and $\overline{G}$ separately, then Eq.(19) may be separated as

$$\begin{pmatrix} A_M & A_{M\overline{S}} & A_{M\overline{G}} \\ A_{\overline{S}M} & A_{\overline{S}} & A_{\overline{S}\overline{G}} \\ A_{\overline{G}M} & A_{\overline{G}\overline{S}} & A_{\overline{G}} \end{pmatrix} \begin{pmatrix} x_M \\ x_{\overline{S}} \\ x_{\overline{G}} \end{pmatrix} + \begin{pmatrix} y_M \\ y_{\overline{S}} \\ y_{\overline{G}} \end{pmatrix} = \begin{pmatrix} x_M \\ x_{\overline{S}} \\ x_{\overline{G}} \end{pmatrix}$$

i.e.

$$\begin{cases} A_M x_M + A_{M\overline{S}} x_{\overline{S}} + A_{M\overline{G}} x_{\overline{G}} + y_M = x_M \\ A_{\overline{S}M} x_M + A_{\overline{S}} x_{\overline{S}} + A_{\overline{S}\overline{G}} x_{\overline{G}} + y_{\overline{S}} = x_{\overline{S}} \\ A_{\overline{G}M} x_M + A_{\overline{G}\overline{S}} x_{\overline{S}} + A_{\overline{G}} x_{\overline{G}} + y_{\overline{G}} = x_{\overline{G}} \end{cases} \tag{20}$$

It can be got from above that

$$\begin{cases} x_M = B_M y_M + B_{M\bar{S}} y_{\bar{S}} + B_{M\bar{G}} y_{\bar{G}} \\ x_{\bar{S}} = B_{\bar{S}M} y_M + B_{\bar{S}} y_{\bar{S}} + B_{\bar{S}\bar{G}} y_{\bar{G}} \\ x_{\bar{G}} = B_{\bar{G}M} y_M + B_{\bar{G}\bar{S}} y_{\bar{S}} + B_{\bar{G}} y_{\bar{G}} \end{cases}$$

in which

$$B_G = \begin{pmatrix} A_M & A_{M\bar{S}} & A_{M\bar{G}} \\ A_{\bar{S}M} & A_{\bar{S}} & A_{\bar{S}\bar{G}} \\ A_{\bar{G}M} & A_{\bar{G}\bar{S}} & A_{\bar{G}} \end{pmatrix}^{-1} = (I - A_G)^{-1}$$

4.3. Total consumption analysis

In classical input-output analysis, there are total input coefficients and total need coefficients, and they are based on *final use* or *final demands*. Now, how can we know how much be needed when one unit product of sector *j* is produced in the whole economic system?

Here we have a closed square table--a complete input-output table, so we can write out

$$Ax = x \tag{22}$$

in which $A = (a_{ij})_{n\times n}, x = (x_k)_{n\times 1}$.

Let's consider the affection of k($k \neq n$) sector's variation in gross output. In fact, we can suppose k=1, then

$$\begin{pmatrix} A^{11} & A^{12} & A^{13} \\ A^{21} & A^{22} & A^{23} \\ A^{31} & A^{32} & A^{33} \end{pmatrix} \begin{pmatrix} X^1 \\ X^2 \\ X^3 \end{pmatrix} = \begin{pmatrix} X^1 \\ X^2 \\ X^3 \end{pmatrix}$$

in which $A^{11} = a_{11}$, $A^{13} = a_{1n}$, $A^{31} = a_{n1}$, $A^{33} = 0$, $X^1 = x_1$, $X^3 = x_n$,

i.e.

$$\begin{cases} A^{11}X^1 + A^{12}X^2 + A^{13}X^3 = X^1 \\ A^{21}X^1 + A^{22}X^2 + A^{23}X^3 = X^2 \\ A^{31}X^1 + A^{32}X^2 + A^{33}X^3 = X^3 \end{cases} \tag{23}$$

It can be got from Eq.(23) that

$$\begin{cases} X^2 = (I - A^{22} - A^{23}A^{32})^{-1}(A^{21} + A^{23}a_{n1})x_1 \\ X^3 = a_{n1}x_1 + A^{32}X^2 = x_n \end{cases} \tag{24}$$

The first formula of Eq.(24) indicates that X^2 needed by producing x_1 consists of two parts—the first is brought through intermediate consumption (A^{21}), the second is brought through the distribution of savings in investment sector. Here, we give four notes—

1).Certain savings must correspond with certain investment or certain money must correspond with certain materials;

2). There is not the second part for the sectors whose products cannot be used in capitals;

3).The distribution of investment (A^{23}) is a factor independent on production process and can be changed according to planning;

4).Because X^2 includes resident sector, x_1 also brings the direct and indirect increase of

resident income and causes demand to other production sectors again in certain proportions.
The second formula of Eq.(24) also consists of two parts, the first is brought directly by x_1, the second is induced through X^2.

5. Summary

We have a critical view to modern national production accounting in this paper and give an alternative system. Our theory can be called as ***relative production theory***. We see that the new accounting system does not give fundamental revision in quantity in calculating GDP, but we get an internally consistent system in national production accounting through a reasonable theory, and also we can get more information from a complete input-output table according to the new theory.

As to the last two questions mentioned in the first part, we don't think that there is any objective and final solution. Because of price and currency differences, only some kind of weighted average indicators can be used to make comparison between different economies. Because of the differences of subjective value views, it is difficult to say what kind of production should get into or out from GDP and how measuring it, and any attempt to give international and unique measurement of economic welfare or assets will be of no consequence and finally failed.

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