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Linkages, Impact & Feedback

in Light of Linear Similarity

Nikolaos K. Adamou, Ph.D.[1]

In memory of ***Carlton Lemke*** & ***Romesh Diwan***[2]

In honor of ***Mária Augusztinovics*** & ***Carl Carlucci***[3]

Abstract: This paper uses the linear similarity between production & allocation activities in order to explore linkages & impact between final demand & value added. Augusztinovics' notion of *final structure matrix* (FSM) as the functional form that links final demand to value added via interrelations of production & allocation is extended by an introduction of two-dimensional distributions. Production and allocation based *Final Structure Matrices* emerging from two-dimensional causal distributions are the same, an expected outcome, since production and allocation are presented by similar matrices. The link between production and allocation *Final Structure Matrices* provide either a Markov or symmetric feedback matrices. The sectoral view transmitting the impact from final demand to value added and vice versa is provided by final demand – value added multipliers.

A critical view to traditional multiplier and linkage analyses reveals that *Power & Sensitivity of Dispersion* coefficients deliver the same information as to that of a row and column summation of the Leontief inverse. In contrast to inconclusive implications of the traditional multipliers, it is shown that total gross output weighted multiplier of the Leontief model is the same as the analogously constructed multiplier derived from Ghosh's model. Although, as will be shown the decomposition of the value added driven multiplier differentiates from the decomposition of the final demand driven multiplier.

Japanese input-output tables from 1995 and 2000 are utilized in order to provide empirical validation to the proposed developments.

Key Words: Input-Output, Linkages, Impact, Similarity, Markov, 2-D Distributions, Feedback

JEL Classification: C67, D57, R15

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1 Department of Business Management, Borough of Manhattan Community College / The City University of New York, 199 Chambers Street, New York City, NY 1007-1097, office S657, phone +1 212 220 8220, fax +1 212 2201281, nadamou@bmcc.cuny.edu, adamou_nikolaos@yahoo.com

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2 Carlton Lemke was my Professor of Mathematics, and Romesh Diwan was my Professor of Economics & Econometrics at Rensselaer Polytechnic Institute. They were also my mentors. I was using mathematics before I met Carlton, but he taught me how to think using matrices. Romesh taught me that all tools we use in analysis are as powerful as knives, and if we use them carelessly we injure ourselves. Both departed from this world recently. Let their memory be eternal.

3 Mária Augustinovics inspired most of my work in input-output economics. Carl Carlucci, as the secretary of the Ways & Means Committee of the New York State Assembly, questioned the practical validity of the traditional multipliers and encouraged me to look at this issue before the Ways & Means Committee of the New York State initiated any specific policies based on implications of the traditional multipliers.

1 *Linkages & Impact in light of linear similarity*

Ghosh (1958) presented an interindustry model as an *alternative* to Leontief's model that may be used in *allocation decisions* for *planned* economies. Yamada (1961) discussed similarity, although not in strictly linear algebra formation. In Yamada's work the allocation model was not an alternative to that of Leontief's, but a parallel one. Yamada discussed production and allocation processes as two sides of the same activity. Yamada's work did not have any significant impact on the subject in western literature. On the contrary, Augusztinovics' work, presented in 1968 and published in 1970, was referenced in almost every single presented and published work about the "*supply driven I-O models.*" Augusztinovics provided a clear and sound methodology linking final demand & value added through the production and allocation processes. Augusztinovics' work was *not taken into serious consideration* in the debate with respect to relevance, meaning or implausibility of the *"supply"* model *despite the fact that she was always referenced*. She was the first scholar that related explicitly the final demand items (that she appropriately called *final use*[4]) to the items of value added, and used Ghosh's model not as an alternative but in association with Leontief's model. The contribution she provided is significant, and her theoretical analysis was supported by empirical evidence.

Adamou & Gowdy (1990) provided clarifications and extensions to Augusztinovics' work. In this work, Augusztinovics' *complex coefficients*, the matrices that are post and pre multiplied to Leontief inverse and its similar matrix were characterized as "*inner*" structures because they provide two different aspects of the esoteric information of the interdependencies of the system, and were clarified either as *weighted multipliers* or *distribution matrices*. Inner structures define in essence correctly the backward and forward linkages for both production and allocation processes. These Augusztinovics' *final structure matrices* relate final use and value added and vice versa, and were combined to define *feedback structure matrices* distributing final use and value added to each other.

Gowdy, (1991) in an empirical examination interpreted as a multiplier the *inner structure* what is indeed a Markov distribution. This point was noted in the literature.[5] Adamou (1995) clarified various fallacies about the "supply" driven models and discussed structural aspects of similarity and symmetry relating the "supply" & "demand" driven models. Although similarity was mentioned as a relationship between the two models by Miller & Blair (1985), similarity it is not taken into analytical consideration. Adamou & Şenesen (2001) show similarity's application in sectoral productivity studies, where one may identify where productivity is generated and where the benefits from the generated productivity are paid off.

As was noted by Dietzenbacher[6] (2002) *"there is not a relationship between multipliers and linkages to mathematical similarity"* of the Leontief Inverse **L** and its similar matrix **G**.[7]

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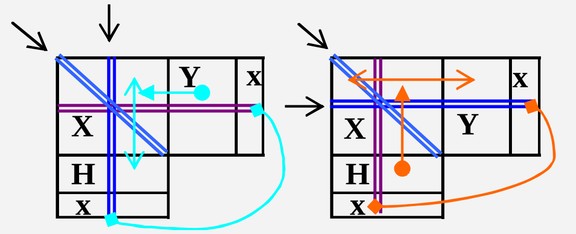
4 This is so because "demand" is a term that relates prices to quantities, while "use" is a term that indicates the usage of items that have given value.

5 Iris Claus, 2002, *Inter industry linkages in New Zealand*, NEW ZEALAND TREASURY, WORKING PAPER 02/09, JUNE/2002

6 "As one of the referees remarked, the matrices **A** and **B** and the inverses **L** and **G** are mathematically similar, i.e. $\mathbf{A} = \hat{\mathbf{x}}\mathbf{B}\hat{\mathbf{x}}^{-1}$ and $\mathbf{L} = \hat{\mathbf{x}}\mathbf{G}\hat{\mathbf{x}}^{-1}$. Unfortunately, this relationship does not in general induce a simple relationship between multipliers or linkages. To my knowledge, the only exception is when the dominant eigenvalue of **A** (respectively **B**) is used as a weighted

Schema 1
Views of the circular flows in input – output accounting as the basis of linear similarity



Final demand generates the total production (gross output in value terms) that is in balance with the total sectoral demand. This process is becoming feasible due to the interrelationship of the processing sectors. Value added generates the value of total sectoral demand that is in balance with the value of total production, (gross output). This is workable via the interrelationship of the allocation process among industrial sectors.

Causes for the generation of output are *both* final demand and value added. Production will cease if there is no demand for it, or if there is not income to pay for it.

The **consequence** of the dual causal relationship is the **production** and **allocation** of output. As output is produced, its value is compared (ratios) to both final demand and value added. These ratios are the **structural outcome** of the process. For every ¥ value of production they provide its proportion to the associated final demand and value added.

The matrix of interindustry transactions **X** f or any given industrial sector presents: *diagonally* the sales of the industry to itself, *column wise* purchases from other industries, and value added requirements **H**, and *row wises* sales to other industries and to final demand **Y**. The relationship of matrix **X** to vector **x** provides the analytical base for the two similar models.

The production based – final demand driven model has the final demand matrix Y to interact to interrelated production sectors as the base for the multiplier and the value added in association with the interrelated production, which provides a Markov distribution matrix. In an analogous way, value added in its association with interrelated allocation sectors, provides the basis for a multiplier matrix, while final demand and interrelated allocation yields another Markov distribution.

Oosterhaven (1988 & 1989) and Miller (1989) wrote on the implausibility of the similarity to the Leontief model. The literature on this subject did not take linear similarity into consideration. On the contrary, this paper is based on the use of mathematical similarity.

While economists focused on the dominant eigenvalue, similarity indicates that all eigenvalues of similar matrices are the same – not only the dominant eigenvalue. An appropriate treatment of the similar matrices **Z** and $\tilde{\mathbf{Z}}$ points out that, the weighted gross output multiplier identified by both similar matrices is exactly the same. Following Augusztinovics' (1997) views on circularity, similarity allows one to view Leontief and Ghosh approaches as complementary and not contradicting, or as alternatives to each other (Schema 1).

One of the reasons for the misconceptions about using the *supply driven model* is that the interindustry model is taken to be somewhat different than what it really is, an **accounting model** of circular flow. Accounting models are given in three steps: (a) an accounting identity[8], (b) an analytical assumption[9], and (c) the resulted model derived from the identity incor-

average of the backward (respectively forward) linkages, measured as the column (respectively row) sums of **A** (respectively **B**). Due to the similarity of **A** and **B**, the average backward linkages are equal to the average forward linkages. Including indirect effects and taking the column sums of **L** and the row sums of **G** yields the same result for their weighted averages (see also DIETZENBACHER, 1992)."
Endnote 3 on page 135 Erik Dietzenbacher "Interregional Multipliers: Looking Backward, Looking Forward" *Regional Studies*, Vol. 36.2, pp 125–136, 2002

7 The notation used for the Leontief inverse in this paper is **Z** and not **L**, and instead of **G**, as for Ghoshian inverse, the adapted here notation is $\tilde{\mathbf{Z}}$ taken from linear algebra to indicate that a matrix is similar to **Z**.

8 Augusztinovics assumption about homogeneity (1995, p. 272) refers to the construction of the table, while the assumption of linearity refers to the construction of the simple static model.

9 The assumption needed is the linear relationship between gross sectoral output and the input requirements or similarly the sales made by each sector.

porating the assumption. The interindustry model is based on the ***accounting identity***. This identity provides gross output <u>either</u> from *column addition*[10] *of the interindustry transactions* ($\mathbf{X}$) *to the value added* ($\mathbf{H}$), ($\mathbf{i}^{\mathbf{T}}\mathbf{X}+\mathbf{i}^{\mathbf{T}}\mathbf{H}=\mathbf{x}^{\mathbf{T}}$), <u>or</u> as *row addition of the interindustry transactions and final demand* ($\mathbf{Y}$), and provide the same gross output ($\mathbf{x}$), ($\mathbf{Xi}+\mathbf{Yi}=\mathbf{x}$).

These two different aspects of identity are the source of similarity. The accounting identity $\mathbf{Xi}+\mathbf{Yi}=\mathbf{x}$ evolves into the Leontief model, where primary input are implicitly given in the model, while final demand plays the role of the exogenous causal building block of the model. Gross output is generated due to final demand and the linear interdependency of processing sectors with a given the assumption of proportional direct requirement of each sector to its gross product. The accounting identity $\mathbf{i}^{\mathbf{T}}\mathbf{X}+\mathbf{i}^{\mathbf{T}}\mathbf{H}=\mathbf{x}^{\mathbf{T}}$ serves as the basis of what is known as the Ghosh model. Primary inputs operate as the exogenous causal building block of the model. Final demand is implicitly given as the difference of the gross output to interindustry output. Total final demand equals in value total primary input. The Ghoshian model uses the proportional direct allocation of each sector to gross output as its analytical assumption.

The data for the model are given either in purchasing or producer prices, constant or current values, but always in ***value terms***.[11] There is no distinction between quantities and prices, and not an implicit or explicit assumption about any type of elasticity. Any price interpretation attempted so far has used additional assumptions along with the assumption of linearity. Published papers still present analyses based on interpretations of the *"supply"* & *"demand"* driven models without conclusive results due to the fact that linear similarity is not taken into account.[12] Linear similarity is the basis for the relationship between the allocation (*supply*[13] model) and the traditional production based interindustry model.

The only *assumption* **needed** is the linear relationship of gross output to interindustry transactions. The matrix of <u>direct input requirement coefficient</u>s $\mathbf{A}$ or its similar matrix of <u>direct sales allocation coefficients</u> $\widetilde{\mathbf{A}}$ (known in the literature as *output coefficients* and usually noted as $\mathbf{B}$) are defined as $\mathbf{A}=\mathbf{X}\left[diag(\mathbf{x})\right]^{-1}$ & $\widetilde{\mathbf{A}}=\left[diag(\mathbf{x})\right]^{-1}\mathbf{X}$. This indicates that both are similar matrices in a linear algebra sense and both are described based on the <u>same data</u> ($\mathbf{X}$, the value of interindustry transactions, and $\mathbf{x}$, the value of the gross sectoral output) and the <u>same assumption</u> of *proportional relationship of gross output to interindustry transactions*.[14]

The essence of similarity[15] transformation is the change of a basis of a matrix. One representation is that of production, and the other, the allocation activity. Production and allocation are the

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10 $\mathbf{i}$ indicates a vector of 1's

11 Japanese data are provided either in purchase or producer prices, but other statistical agencies provide data in basic prices, and others in current and constant prices.

12 Dietzenbacher, Erik & Gülay Günlük Şenesen (2003)

13 The term "*supply*" implies a relationship between prices and quantities in providing commodities, a relationship that is not presented or examined in inter-industry models.

14 $\mathbf{X}=\mathbf{A}\left[diag(\mathbf{x})\right]=\left[diag(\mathbf{x})\right]\widetilde{\mathbf{A}}$

15 Linear similarity between $\mathbf{A}$ and $\widetilde{\mathbf{A}}$ implies similarity for matrices $\left[\mathbf{I}-\mathbf{A}\right]$ & $\left[\mathbf{I}-\widetilde{\mathbf{A}}\right]$ as well as $\left[\mathbf{I}-\mathbf{A}\right]^{-1}$ & $\left[\mathbf{I}-\widetilde{\mathbf{A}}\right]^{-1}$. Matrices $\mathbf{A}$ and $\left[\mathbf{I}-\mathbf{A}\right]$ read column wise since the denominator of their fraction (sectoral gross output) is different from a column to a column. Matrices $\widetilde{\mathbf{A}}$ and $\left[\mathbf{I}-\widetilde{\mathbf{A}}\right]$ read row

two side vies of the same motion. Similar matrices have identical traces, determinants and characteristic values.

i. The same *trace,* $tr(\mathbf{I}-\mathbf{A}) = tr(\mathbf{I}-\tilde{\mathbf{A}})$, net purchases and sales of an industry to itself matching each other.

ii. The same *determinant,* $\det(\mathbf{I}-\mathbf{A}) = \det(\mathbf{I}-\tilde{\mathbf{A}})$, the relation function that combines all information given by a matrix into one single number, the determinant, indicates that similar matrices cover the same area, although located differently. The determinant of production or allocation provides the same portion of the net relative to gross production.

iii. Finally, the same *characteristic values*[16] provide exactly the same ***signals***, as purchases or sales.

As is noted, similar matrices provide *the same information* from *different points of view*, purchases and sales. Matrix $\mathbf{A}$ indicates what is purchased by an industry from all industries as a percentage of its gross output, and matrix $\tilde{\mathbf{A}}$ what is sold by an industry to all industries as a percentage of its gross output. It is obvious that *the selling pattern is not identical to the purchasing pattern in any industry*. It is also apparent that an industry, in order to be able to purchase from other industries, the value of sales provides the means allowing purchases to be paid. This is the logic relating these two similar matrices.

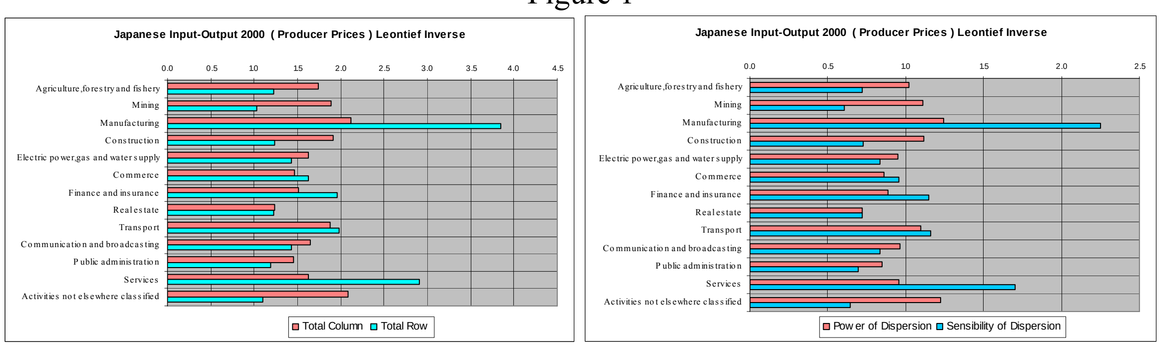
wise for the same reason. Matrices $\left[\,\mathbf{I}-\mathbf{A}\,\right]^{-1}$ and $\left[\,\mathbf{I}-\tilde{\mathbf{A}}\,\right]^{-1}$ have the same denominator, the $det\left[\,\mathbf{I}-\mathbf{A}\,\right] = det\left[\,\mathbf{I}-\tilde{\mathbf{A}}\,\right]$ and read column wise as well as row wise. The characteristic values of these matrices and the coefficients of the associated characteristic polynomials have linear relationships. Matrices $\mathbf{A}$, $\left[\,\mathbf{I}-\mathbf{A}\,\right]$ and $\left[\,\mathbf{I}-\mathbf{A}\,\right]^{-1}$ have the same set of characteristic vectors. While similar matrices have the same characteristic values, their characteristic vectors are not the same but related, as the similarity transformation implies, Needham (2001) p. 171

16 Adamou (1996)
There is a variation of opinions about the interpretation of the characteristic values. One interpretation of the dominant characteristic value is as if this represents a systemic growth factor. Another interpretation of all eigenvalues is that they play the role of a multiplier.
• If the dominant characteristic-value provides the systemic growth factor, then which matrix provides this factor, **A**, [**I**-**A**] or its inverse **Z**? Why does the actual data growth rate that is observable of any economy deviate so much from the appropriate evaluation of such dominant eigenvalues?
• If all characteristic values are multipliers, then to which sector are they associated, and by which causal change are they generated? A multiplier identifies the impact of a given exogenous variation (government consumption of the final products of various sectors has such a structure) to the over all outcome (what would be the value of the total domestic sectoral production). How does a given characteristic value not find association to a particular sector, like services or manufacturing?
Strang (1988), in his classical linear algebra text, explains characteristic values as a measurement of oscillation (p. 248-9), and suggests that they are the most important features of any dynamic system. He gives a bridge and stock-market example with respect to a marching army and a stock-broker's portfolio. In the first case, the marching unit does not want to have eigenvalues close to the structure of the bridge, while, in the second example, the stock-broker wants his portfolio to have eigenvalues as close to the market's as possible. We do not want the bridge to collapse, and we want our portfolios to behave like the market, in order to achieve successful investment outcomes. These cases do not look like multipliers or growth factors, but rather behavioral aspects.
The notion of a multiplier may be accepted as the diagonal matrix of eigenvalues when multiplied to the matrix of eigenvectors in matrix decomposition. Then the task is to provide an interpretation of the decomposition.

The analysis comparing the row and column summation of the non-weighted Leontief and Ghosh inverse matrices, $\mathbf{Z} = \left[\mathbf{I} - \mathbf{A}\right]^{-1}$ & $\widetilde{\mathbf{Z}} = \left[\mathbf{I} - \widetilde{\mathbf{A}}\right]^{-1}$, understandably provides inconclusive results. This is because the pattern of sales does not match the pattern of purchases in any industry. Matrices $\mathbf{Z}$ & $\widetilde{\mathbf{Z}}$ provide the total interrelationship among all processing sectors in purchasing and selling. The only common element is the intersectoral transactions, recorded in the main diagonal.

Row and the column summations of the element of the $\mathbf{Z}$ & $\widetilde{\mathbf{Z}}$ inverses were interpreted as different types of multipliers. The row summation of $\mathbf{Z}$ is interpreted as the outcome due to a demand equal to one ¥ in all sectors, while the column summation of $\mathbf{Z}$ is interpreted as the outcome due to a demand of one ¥ in a sector and zero to all other sectors. An analogous interpretation is given to multipliers based on the inverse matrix $\widetilde{\mathbf{Z}}$ with respect to value added. Any comparison of these four different multiplier concepts yields rightfully inconclusive results and different policy conclusions. The issue at the present moment is not the significant variation of the findings and their possible meaning, but the validity of the logical process that produces those results. Before this issue is addressed, it is meaningful to examine the row and column summations of the $\mathbf{Z}$ matrix to interrelationship measurements provided by Rasmussen (1956).

Figure 1

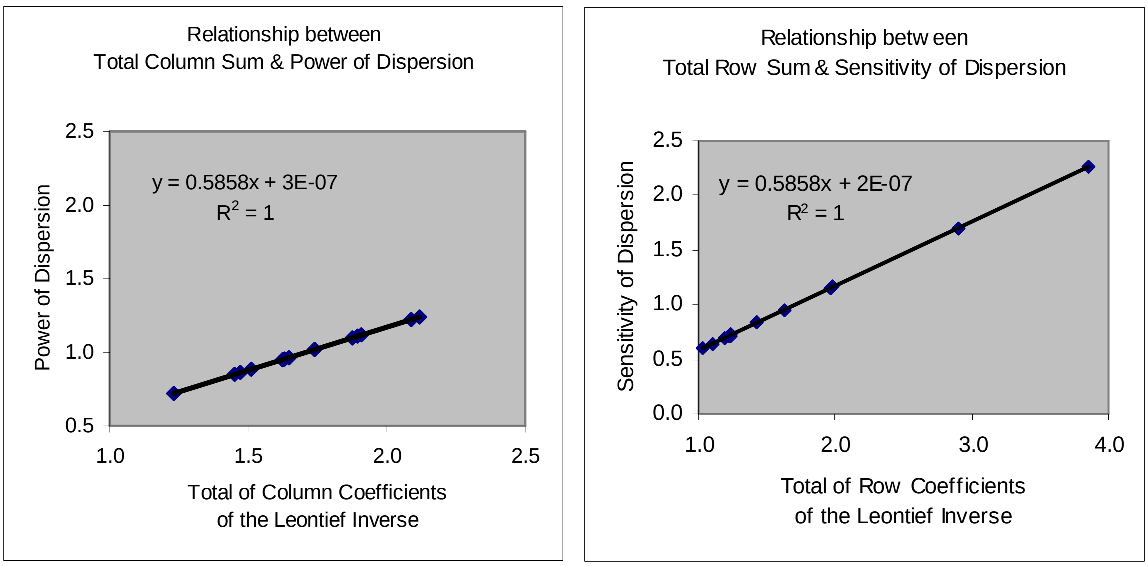


A careful observer may easily identify that the patterns of both graphs are exactly the same

The *Power of Dispersion* and the *Sensitivity of Dispersion* indices are assumed to indicate something different in essence than the row and column multipliers. A careful graphical examination of these two indices in comparison with row and column summation of the elements in the Leontief and Ghosh inverses indicates that the *Power of Dispersion* and the *Sensitivity of Dispersion* provide the *same pattern* in *different scales* as the row and column summations of the Leontief inverse (Figure 1). A standardized average of a vector's elements (which are these indices) provides comparable information with the summation of a vector's elements (which are the previously discussed multipliers). A change of scale though, does not alter the essence of the two concepts, which are analogously the same.

A further test of one to one correlation indicates that the column sum of the Leontief inverse and the power of dispersion coefficients are proportionately related, and *the exact same proportion* is the result of the relationship between the row sum of the Leontief inverse and the sensitivity of dispersion coefficients. Japanese input-output data published for 2000[17] illustrate that the proportionality factor is 58.58% (Figure 2). This means that power, as well as sensitivity of dispersion coefficients are *not different in essence* from the row and column summations of the Leontief inverse, but simply the same information on a different scale. Power of dispersion coefficients provide the same order as the (1, 0, …, 0) type multiplier and the sensitivity of dispersion coefficients indicate the equivalent relationship to (1, 1, …, 1) type multiplier. *The meaning of power and sensitivity of dispersion are NOT what we were trained to accept, but a transformation of the row and column summation of the Leontief inverse matrix to another scale.*

Figure 2



Although it is obvious that power and sensitivity of dispersion coefficients show the same information as the row and column summations of the two similar inverse matrices under investigation, the question of the nature of the sectoral economic multiplier still remains with inconclusive intriguing findings from the appropriate summations.


Minkowski

What is known to economists as a Leontief inverse matrix is known to all other scientists and engineers as a Minkowski inverse. We do have experiences and examples using Minkowski matrices that are helpful to draw comparisons. None of the other disciplines have used row and columns summations of the Minkowski inverse the way it is used in economics. Similar inverses $\mathbf{Z}$ and $\tilde{\mathbf{Z}}$ provide two different types of information as are pre-multiplied or post-multiplied by appropriate matrices or vectors. Matrix $\mathbf{ZY}$ allocates gross output to final demand, and matrix $\mathbf{H}\tilde{\mathbf{Z}}$ distributes gross output to value added. Given that final demand and value added are the two causal elements generating output, *final demand affects the rows of the Leontief inverse* (*NOT its*

[17] Thirteen sector aggregation data are used here.

columns), and *value added is engaged to the columns to its similar inverse* (*NOT its rows*). These two inverses **do not** have double entry causal links as the known backward and forward linkages imply (meaning one for sectors and another for the entire economy). Matrices $\mathbf{HZ}$ and $\tilde{\mathbf{Z}}\mathbf{Y}$ are *NOT* multiplier matrices at all. A multiplier transmits impact from a cause, demand, to a process, sectoral interdependency of production, yielding an outcome, a given value of production. A multiplier matrix $\mathbf{Z}$, post-multiplied by final demand $\mathbf{Y}$, yields the value sectoral output, $\mathbf{x}$. The demand is not the same in any sector or any type of final demand, and the value of produced output is not the same for all sectors.

Matrices $\mathbf{Z}$ and $\tilde{\mathbf{Z}}$ have two-way links, as Augusztinovics showed, but their meaning of the connection via their rows and the affiliation via their columns is different. Leontief inverse $\mathbf{Z}$ is connected to value added through its columns, comparable to the way that its similar inverse $\tilde{\mathbf{Z}}$ is related to final demand via its rows. The matrix presentation of final demand $\mathbf{Y}$, and value added $\mathbf{H}$, provides information for the entire economy as well as a specific sector as it is positioned in the economy.

A unit of sectoral gross output is distributed column wise, either to interindustry transactions and value added in the compound matrix $\begin{bmatrix} \mathbf{A} \\ \mathbf{H}_x \end{bmatrix}$, or, row wise to interindustry transactions and final demand, $\begin{bmatrix} \tilde{\mathbf{A}} & \mathbf{Y}_x \end{bmatrix}$.[18] These are Markov matrices; the summation of their columns and rows respectively is a unit (1). These linkages of the ***direct relations*** lead to equivalent linkages of the ***total interrelations*** of the industrial sectors, $\mathbf{H}_x\mathbf{Z}$ and $\tilde{\mathbf{Z}}\mathbf{Y}_x$ respectively, which are also Markov matrices. Although matrices $\mathbf{Z}$ and $\tilde{\mathbf{Z}}$ are multiplier matrices, and matrices $\mathbf{H}_x\mathbf{Z}$ and $\tilde{\mathbf{Z}}\mathbf{Y}_x$ are outcomes of matrix multiplication to a multiplier matrices, $\mathbf{H}_x\mathbf{Z}$ and $\tilde{\mathbf{Z}}\mathbf{Y}_x$ are not multipliers but Markov distributions. Matrices $\mathbf{H}_x$ and $\mathbf{Y}_x$ are parts of Markov matrices, and they do not indicate any causal effect. The sectoral outcome of the $\mathbf{H}_x\mathbf{Z}$ and $\tilde{\mathbf{Z}}\mathbf{Y}_x$ multiplication is the same, a unit, since the above multiplications indicate how a unit of sectoral output is produced and allocated given the interrelations of the production and allocation processes.

Linkages are defined by a matrix multiplication. Different matrices identify the nature of the distinct linkages. $\mathbf{Y}_m$ & $\mathbf{H}_m$ are column-wise and row-wise Markov matrices[19] underlying the reason for the economic activity of production. They show the distribution of a unit of each specific aspect of final demand and value added. Thus, these matrices indicate *equality* among the various aspects of final demand and value added.

$\mathbf{H}_x\mathbf{Z}$ & $\tilde{\mathbf{Z}}\mathbf{Y}_x$ are Markov matrices that characterize the ***total distributional outcome*** of production and allocation.

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18 $A_{ij} = \dfrac{X_{ij}}{x_j}$ $\mathbf{H_x} = \mathbf{H}\left[diag\left(\mathbf{x}^T\right)\right]^{-1}$, & $\tilde{A}_{ij} = \dfrac{X_{ij}}{x_i}$ $\mathbf{Y_x} = \left[diag\left(\mathbf{x}^T\right)\right]^{-1}\mathbf{Y}$

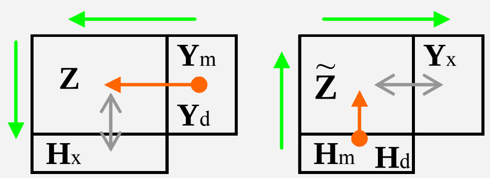
19 $\mathbf{Y}_m = \mathbf{Y}\left[diag\left(\mathbf{i}^T\mathbf{Y}\right)\right]^{-1}$ & $\mathbf{H}_m = \left[diag\left(\mathbf{H}\mathbf{i}\right)\right]^{-1}\mathbf{H}$

Row summation reveals causal relation from the right hand side of the Leontief inverse. Column summation discloses causal relation of the similar Leontief inverse from the left hand side. Leontief inverse has its causal linkage to final demand through its rows from its left, and its similar inverse has its causal linkage to value added though its columns from its right.

$\mathbf{ZY}_m$ is the weighted Leontief inverse Augusztinovics multiplier matrix. The row elements of the Leontief inverse are multiplied by the appropriate percentage of every type of final demand. Thus a unit of each final demand type distributed to various sectors has impact on associated processing sectors producing output. This is a much more acceptable case as a working hypothesis for an impact analysis. It has to be noted however, that not all types of final demand are equal to each other as the Markov matrix $\mathbf{Y}_\mathrm{m}$ implies. Usually, private consumption dominates the field; public spending varies in significance from one economy to another, exports counterbalance imports when there is stable trade.

Schema 2

Final Structure Matrices



Augusztinovics

$\mathbf{H}_x\mathbf{Z}\,\mathbf{Y}_m$ $\quad$ $\mathbf{H}_m\tilde{\mathbf{Z}}\mathbf{Y}_x$

Adamou

$\mathbf{H}_x\mathbf{Z}\,\mathbf{Y}_d$ $\quad$ $\mathbf{H}_d\tilde{\mathbf{Z}}\mathbf{Y}_x$

Production	Allocation
Final Demand Driven	Value Added Driven

Sectoral unit of output *distributed* to:

columns of production	rows of allocation
$\mathbf{H}_x\mathbf{Z}$	$\tilde{\mathbf{Z}}\mathbf{Y}_x$

Multiplier Matrices

Demand driven Production	Value added driven Allocation
$\mathbf{ZY}_m$	$\mathbf{H}_m\tilde{\mathbf{Z}}$
$\mathbf{ZY}_d$	$\mathbf{H}_d\tilde{\mathbf{Z}}$

An alternative to Augusztinovics Markov distribution of final demand is the two dimensional distribution of one unit of final demand each different category of final demand and processing sectors, $\mathbf{Y}_d$.[20] The weighted Leontief inverse multiplier matrix proposed is $\mathbf{ZY}_d$. This weighted inverse relates every row element of $\mathbf{Z}$ to the percentage of the total final demand. Matrix distribution $\mathbf{Y}_d$ captures the variations in the composition of final demand.

These two weighted multipliers capture different aspects of the same structure of final demand and may be used appropriately. Whenever one wishes to observe the impact of an equal amount that may be allocated for example to either private or public consumption or any other category of final demand, matrix $\mathbf{ZY}_m$ is utilized. If one wants to distinguish the impact of a unit of final demand with a particular distribution, $\mathbf{Y}_d$ may use $\mathbf{ZY}_d$.

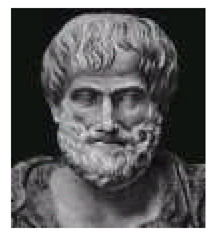
If instead of treating final demand & value added as matrices, one looks at the impact of the total final demand and value added, treating them as vectors, then one can see how this simplified case is based on the marginal distributions of the $\mathbf{Y}_\mathrm{d}$ and $\mathbf{H}_\mathrm{d}$ matrices. The unit distribution of the total final demand and total value added occurs. We know that gross output is given either as a post multiplication of a Leontief inverse to total final demand or as a pre-multiplication of a

20 $\mathbf{Y}_d = \mathbf{Y}\left[\mathbf{i}^T\mathbf{Y}\mathbf{i}\right]^{-1}$

Ghoshian inverse to total value added, $\mathbf{Zy} = \left(\mathbf{h}\widetilde{\mathbf{Z}}\right)^T$. In a comparable way, the distribution of a final demand unit post-multiplied to the Leontief inverse provides the same multiplier to the pre-multiplication of its similar inverse, $\mathbf{Zy_m} = \left(\mathbf{h}_m\widetilde{\mathbf{Z}}\right)^T$.[21] Similarity between $\mathbf{Z}$ & $\widetilde{\mathbf{Z}}$ implies that *weighed output multipliers* provided by both inverses are *identical*.

The column summation of the Leontief inverse multiplier indicates *demand exists only by a unit in one sector while there is no demand to any other sectors*. The row summation of the Leontief inverse multiplier assumes that *the demand is identical to a unit in all sectors*. Both approaches recall to memory the Hellenic myth about Theseus and Procrustes.[22] The *Procrustean action* to *stretch the exogenous impact of all sectors to a unit* or to *eliminate the impact of all sectors except one to zero and then stress the remaining to a unit* does not provide any pragmatic approach to reality, and its results seem irrelevant.

Aristotle taught the young Alexander that *assumptions should not be used* **except** whenever there is an *absolute necessity*, since **assumptions distort reality**, and ***each real case has its own virtues that one should explore***. Final demand and value added do not exist in any such pattern and fashion dictated by assumptions utilized in the traditional multiplier exercises. Final demand affects processing sectors in a particular distributional form. This distribution is important whenever we would like to evaluate the impact on gross output. We need to *liberate* our thinking process from our procrustean training if we wish to virtually and safely *travel* from a Doric, Spartan dominated *Peloponnesus,* to a completely different aesthetical, cultural and intellectual environment of *Athens*.



The multipliers we learned in our education are based on the *procrustean recipe* "*one size fits all*". Fortunately, currently provided data do not give just a final demand and value added vector

21 $$\frac{z_{11}y_1 + z_{12}y_2 + \cdots + z_{1n}y_n}{\sum_1^n y} = \frac{h_1\tilde{z}_{11} + h_2\tilde{z}_{21} + \cdots + h_n\tilde{z}}{\sum_1^n h}$$

22 «The most interesting of Theseus's challenges came in the form of an evildoer called Procrustes, whose name means "he who stretches." This Procrustes kept a house by the side of the road where he offered forcefully his hospitality to passing strangers. They were invited in for a pleasant meal and a night's rest in his very special bed. If the guest asked what was so special about it, Procrustes replied, "Why, **it has the amazing property that its length exactly matches whom so ever lies upon it**." What Procrustes didn't volunteer was the method by which this "one-size-fits-all" was achieved, namely as soon as the guest lay down Procrustes went to work upon him, stretching him on the rack if he was too short for the bed and chopping off his legs and head if he was too long. Theseus lived up to his do-unto-others credo, fatally adjusting Procrustes to fit his own bed».

in http://www.mythweb.com/heroes/theseus/theseus07.html
& http://www.sikyon.com/Athens/Theseus/theseus_eg01.html

but complete and detail matrices. The details of these matrices have very important information for policy makers. Those who are concerned about analyses applicable to policy decisions need to bring to their attention particular specifics. Otherwise, "academic" exercises will exist without considerable practical applicability. Final demand and value added matrices have more than just their total magnitude as a macroeconomic scalar. The relationship of the processing sectors to final demand and value added are given in a matrix form and we have to utilize them with full respect to all information these matrices provide, without any procrustean action.

Given the proportional relationship of primary inputs to gross output, $\mathbf{H_x}$, matrix $\mathbf{H_x Z}$ is a Markov distribution matrix of a unit of sectoral output to production requirement. This matrix involves the columns of the Leontief inverse. The incorporation of both column and rows of the Leontief inverse provides Augusztinovics' *final structure matrix* as $\mathbf{H_x Z Y}_m$, also a Markov distribution matrix. This matrix provides the way that a unit of each type of final demand affects the value added relationship to output via the total interrelationship of the production process. An alternative *final structure matrix* of the production approach proposed in this paper is a two-dimensional distribution matrix $\mathbf{H_x Z Y}_d$.

An analogous structural presentation of the allocation approach provides: (a) The sectoral distribution of gross output to final demand, $\mathbf{Y_x}$, that is a *portion* of a Markov matrix; (b) the distribution of a unit of each type of primary input, $\mathbf{H}_m$, a Markov matrix; and (c) the distribution of a primary input unit to industrial sectors and all components of value added, $\mathbf{H}_d$, a two dimensional distribution matrix. The interrelationship of allocation presented by the Leontief similar inverse are engaging their columns in the weighted multiplier matrices $\mathbf{H}_m\widetilde{\mathbf{Z}}$ and $\mathbf{H}_d\widetilde{\mathbf{Z}}$ as well as their rows in the Markov matrix $\widetilde{\mathbf{Z}}\mathbf{Y_x}$. Thus, two *final structure matrices* of the allocation approach are defined, Augusztinovics' Markov distribution matrix, $\mathbf{H}_m\widetilde{\mathbf{Z}}\mathbf{Y_x}$, and the two-way distribution matrix $\mathbf{H}_d\widetilde{\mathbf{Z}}\mathbf{Y_x}$ suggested in this paper.

It is noticeable that final structure matrices identifying the impact of a final demand and value added unit for production and allocation inverse matrices are equal, $\mathbf{H}_d\widetilde{\mathbf{Z}}\mathbf{Y_x} = \mathbf{H_x Z Y}_d$, as one may observe in Table 5. This is because similarity exhibits not only the same gross output multiplier, but also the same linkages between the two aspects of the net production (demand and income).

In the Leontief model, $\mathbf{x} = [\,\mathbf{I} - \mathbf{A}\,]^{-1}\,\mathbf{Yi}$, and in its alternative, $\mathbf{x} = \left[\,\mathbf{I} - \widetilde{\mathbf{A}}^T\,\right]^{-1}\mathbf{H}^T\mathbf{i}$, the causal relationship is indicated by two different aspects of the accounting formulation. Inverse matrices $\mathbf{Z}$ and $\widetilde{\mathbf{Z}}$ play the role of inter-connectors of the production and allocation processes. *Matrix* $\mathbf{Z}$ *captures only backward causal connection of the final demand, and matrix* $\widetilde{\mathbf{Z}}$ *indicates only forward causal linkages from the primary input. Matrix* $\mathbf{Z}$ *transmits forward outcome to value added, and matrix* $\widetilde{\mathbf{Z}}$ *allocates forward the required produced output to final demand.* A complete picture requires looking at **both sides of the same coin**.

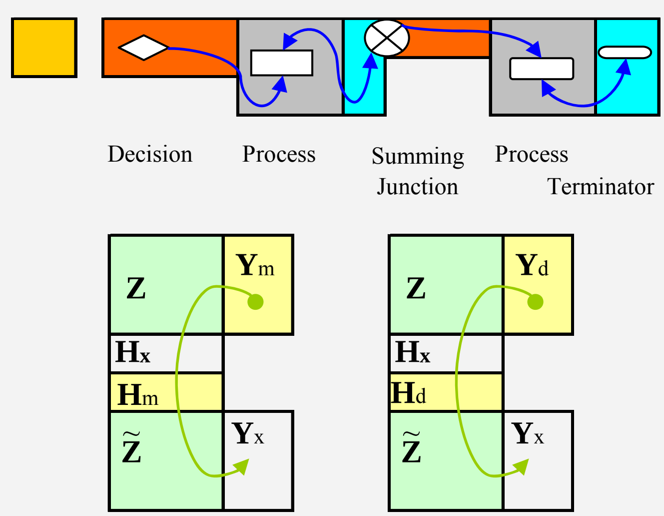
In light of the above presentation, linkage analysis is much more than the widely used but fallacious backward and forward linkages. Linkages should follow the positional relationship of the building blocks given by the data and the circular behaviour of the economy. Thus, linkages based on the interrelationships of *production* (Leontief inverse) have final demand as a causal block and value added is generated in the process of producing output. The causal element is linked to the rows, and the outcome of the process is associated with the columns of the Leontief inverse. The direction of the movement is counter-clock-wise. *Allocation* activity is presented with a clock-wise movement and is based on the similar Leontief inverse. Value added is the causal element linked to the columns of the Ghosh inverse. The relationship of final demand to allocated sectoral output is the outcome of the process, linked to the rows of the Ghosh inverse.

Production and allocation are inherently different faces of the same activity. Final demand and primary input ratios to sectoral gross output may be viewed as the outcome of the process. Thus final structure matrices relate the causal building block to the inter-relational structural building block to the end outcome. In the production presentation of the final structure, the causal linkage starts from the final demand and finishes at the primary inputs. The allocation presentation of the final structure starts with primary inputs and ends with final demand.

2 *Final demand & value added feedback & multiplier matrices*

Schema 3

Final Demand Feedback Structures



$\mathbf{Y}_m^T\mathbf{Z}^T\mathbf{H}_{\mathbf{x}}^T\mathbf{H}_m\tilde{\mathbf{Z}}\mathbf{Y}_{\mathbf{x}}$ $\quad$ $\mathbf{Y}_d^T\mathbf{Z}^T\mathbf{H}_{\mathbf{x}}^T\mathbf{H}_d\tilde{\mathbf{Z}}\mathbf{Y}_{\mathbf{x}}$

The production/final demand driven model $\mathbf{x} = \left[\mathbf{I} - \mathbf{A}\right]^{-1}\mathbf{Yi}$ and the allocation/value added driven (similar) model $\mathbf{x} = \left[\mathbf{I} - \tilde{\mathbf{A}}^T\right]^{-1}\mathbf{H}^T\mathbf{i}$ provide the basis that define the final structure matrices. Final structure matrices indicate complete linkages for the production and allocation processes, one seen separate from the other. However, production is not a separate process from allocation and vice versa, the two processes coexist. This coexistence of production and allocation is given by the multiplication of production based on the allocation based final structure matrix. The result is a feedback matrix. These square matrices are feedback because they indicate a route finishing at the starting point. They are final demand and a value added feedback matrices.

Final demand feedback structures (Schema 3) show the manner in which a final demand distribution affects final demand's relationship to gross output. In a parallel way, *value added feed-*

back structures (Schema 4) uncover value added distributional affect on value added portion to gross output.

Since the Leontief & Ghosh models provide the same gross output solution, they define the equation $\left[\mathbf{I}-\mathbf{A}\right]^{-1}\mathbf{Y}\,\mathbf{i}=\left[\mathbf{I}-\widetilde{\mathbf{A}}^{T}\right]^{-1}\mathbf{H}^{T}\,\mathbf{i}$ that links final demand and value added directly. This equation yields either a matrix relating the production matrix to a transposed allocation inverse, or the transposed allocation matrix to a Leontief inverse. The product of these two matrices is itself a different weighted multiplier that transmits effects from value added to final demand as well as from final demand to value added.

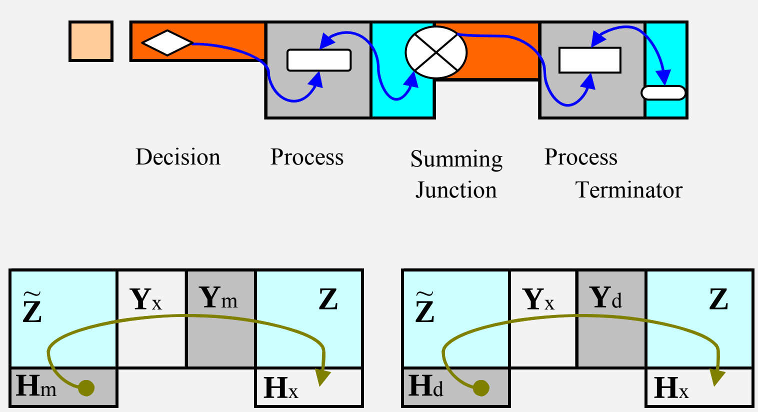
2.1 Final demand feedback Structure

The final demand feedback structure is a square matrix that takes into account explicitly both production and allocation processes. The starting point of the route that this matrix identifies is the distribution of final demand. The top portion in Scheme 4 identifies all matrix multiplications involved. Both distributions of final demand, $\mathbf{Y}_{\mathrm{m}}$ and $\mathbf{Y}_{\mathrm{d}}$, that may be used to identify policy and de facto decisions made, are the causal elements of the process. These decisions affect the process as the weighted multiplier matrices $\mathbf{ZY}_{\mathrm{m}}$ and $\mathbf{ZY}_{\mathrm{d}}$ depict. The result is transmitted by the Markov matrices to the finishing element of the process, the value added. Then, the share of value added to gross output is reflected in the value added distributions. The $\mathbf{H}_{\mathrm{x}}\mathbf{H}_{\mathrm{m}}$ and $\mathbf{H}_{\mathrm{x}}\mathbf{H}_{\mathrm{d}}$ are reflection matrices from the production to the allocation process. Matrices $\mathbf{ZH}_{x}\mathbf{H}_{m}\widetilde{\mathbf{Z}}$ or $\mathbf{ZH}_{x}\mathbf{H}_{d}\widetilde{\mathbf{Z}}$ are connector matrices. These matrices link the causal to the terminating aspects of interindustry accounting in final demand.

2.2 Value added feedback structure

In the same way that occurs in the final demand feedback matrix, the value added feedback structure is defined. In this case, since the process starts with decisions made in value added, the allocation final structure matrix is the first building block. The first process is the allocation and the summing junction that reflects the route that is on final demand. The reflection works through the production process and finishes at the terminal point of the value added share to the value of gross output.

Schema 4
Value Added Feedback Structures



$\mathbf{H}_{m}\widetilde{\mathbf{Z}}\mathbf{Y}_{\mathbf{x}}\mathbf{Y}_{m}^{T}\mathbf{Z}^{T}\mathbf{H}_{\mathbf{x}}^{T}$ $\quad$ $\mathbf{H}_{d}\widetilde{\mathbf{Z}}\mathbf{Y}_{\mathbf{x}}\mathbf{Y}_{d}^{T}\mathbf{Z}^{T}\mathbf{H}_{\mathbf{x}}^{T}$

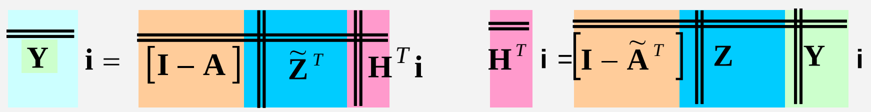
This value added feedback matrix is also a square matrix. The dimension of value added feedback matrix is smaller than the dimension of the final demand feedback matrix. Although there is a difference in their size, these two matrices provide the same quality of information, as having the same set of eigenvalues. Feedback matrices are not similar; $\mathbf{Y}_m^T\mathbf{Z}^T\mathbf{H}_\mathbf{x}^T\mathbf{H}_m\tilde{\mathbf{Z}}\mathbf{Y}_\mathbf{x}$ & $\mathbf{H}_m\tilde{\mathbf{Z}}\mathbf{Y}_\mathbf{x}\mathbf{Y}_m^T\mathbf{Z}^T\mathbf{H}_\mathbf{x}^T$ are Markov, while $\mathbf{Y}_d^T\mathbf{Z}^T\mathbf{H}_\mathbf{x}^T\mathbf{H}_d\tilde{\mathbf{Z}}\mathbf{Y}_\mathbf{x}$ and $\mathbf{H}_d\tilde{\mathbf{Z}}\mathbf{Y}_\mathbf{x}\mathbf{Y}_d^T\mathbf{Z}^T\mathbf{H}_\mathbf{x}^T$ are two dimensional symmetric matrices. The reason of their symmetry is that two dimensional final structure matrices of production & allocation activities are equal to each other.

2.3 Value Added effecting Final Demand & Final Demand effecting Value Added

The equality of the two models in terms of the output produced implies that final demand may be associated with value added and vice versa directly as $[\mathbf{I}-\mathbf{A}]^{-1}\mathbf{Y}\mathbf{i} = \left[\mathbf{I}-\tilde{\mathbf{A}}^T\right]^{-1}\mathbf{H}^T\mathbf{i}$. This equation formulates two different functional relations. The multiplier matrix where value added determines final demand and, the multiplier matrix where final demand influences value added (Schema 5).

Schema 5

Value Added determining Final Demand *Final Demand determining Value Added*



$$\mathbf{Y}\mathbf{i} = [\mathbf{I}-\mathbf{A}]\left[\mathbf{I}-\tilde{\mathbf{A}}^T\right]^{-1}\mathbf{H}^T\mathbf{i} \qquad \mathbf{H}^T\mathbf{i} = \left[\mathbf{I}-\tilde{\mathbf{A}}^T\right][\mathbf{I}-\mathbf{A}]^{-1}\mathbf{Y}\mathbf{i}$$

The multiplier matrix where value added (transposed) determines final demand is based on the interdependence of the allocation process given by (transposed) Ghosh inverse weighted (pre-multiplied) by the production matrix. The reverse defines the opposite route from final demand to the (transpose) value added. Here the Leontief inverse is weighted (pre-multiplied) by the transpose allocation matrix.

3 Final demand & Value Added relations via linkages and impact (1995 & 2000)

The Japanese 1995 & 2000 input-output tables in trillions of ¥ are given in Tables 1A & 1B. Intersectoral trade of manufacturing is the most significant element, followed by the sales of manufacturing to construction and services, manufacturing requirements from commerce and services, and intersectoral transaction of services. In the value added area, compensation of employees to manufacturing, commerce and service sectors preside over the other business. In final demand, private consumption from services, manufacturing, commerce, and real estate, capital formation in construction and manufacturing, as well as exports & imports in manufacturing are the most important figures of the table.

These outstanding elements of final demand and value added have greater weight on the Leontief and Ghosh inverses that provide the total interdependencies in production and allocation activities. The aim of this paper has been to evaluate the way that the above elements of final demand and value added are linked to each other.

Table 1 A

1995 *In Trillions of Yen*	1 Agriculture	2 Mining	3 Manufacturing	4 Construction	5 Electric power	6 Commerce	7 Finance	8 Real estate	9 Transport	10 Communication	11 Public Admin.	12 Services	13 Act. not else classified	Intermediate Output
1 Agriculture, forestry and fishery	1.9	0.0	9.9	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.2	0.0	13.3
2 Mining	0.0	0.0	5.3	0.8	1.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7.4
3 Manufacturing	2.5	0.1	124.7	25.9	1.4	3.8	1.3	0.2	5.4	0.4	2.7	26.8	0.5	195.8
4 Construction	0.1	0.0	1.4	0.2	1.2	0.6	0.1	2.3	0.5	0.2	0.5	1.2	0.0	8.1
5 Electric power, Gas and water supply	0.1	0.0	5.9	0.6	2.5	1.2	0.2	0.2	0.9	0.2	0.9	4.6	0.1	17.3
6 Commerce	0.7	0.0	17.2	6.2	0.3	1.1	0.2	0.1	1.8	0.1	0.5	7.8	0.1	36.1
7 Finance and insurance	0.5	0.1	4.3	1.0	0.7	5.9	3.5	3.3	3.1	0.2	0.1	5.4	0.9	29.0
8 Real estate	0.0	0.0	1.1	0.3	0.3	3.8	0.7	0.5	0.8	0.2	0.0	2.8	0.1	10.6
9 Transport	0.7	0.4	9.3	4.7	0.7	5.3	0.7	0.2	5.3	0.4	0.8	3.9	0.1	32.6
10 Communication and broadcasting	0.0	0.0	0.9	0.5	0.1	1.9	0.7	0.0	0.3	0.9	0.4	3.7	0.0	9.5
11 Public administration	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.5
12 Services	0.2	0.1	20.7	7.0	2.4	5.3	3.8	1.0	6.6	2.0	1.9	14.3	0.3	65.6
13 Activities not elsewhere classified	0.2	0.0	2.3	0.2	0.2	0.6	0.1	0.5	0.2	0.1	0.4	1.2	0.0	6.0
Intermediate Input	6.8	0.8	203.2	47.5	11.1	29.6	11.4	8.3	24.9	4.7	8.1	72.8	2.6	431.9
1 Consumption expenditure outside households (row)	0.1	0.1	6.4	1.7	0.6	2.7	1.3	0.3	1.1	0.2	0.5	4.4	0.0	19.4
2 Compensation of employees	1.5	0.3	54.3	29.3	4.6	49.9	14.0	2.5	16.7	4.9	16.8	78.3	0.2	273.2
3 Operating surplus	5.2	0.2	20.1	3.1	3.6	11.3	6.0	28.4	2.8	1.5	0.0	15.2	2.3	99.7
4 Depreciation of fixed capital	1.8	0.2	16.8	4.5	5.4	5.0	3.7	20.8	3.3	2.8	0.8	15.5	0.3	80.8
5 Indirect taxes *	0.6	0.1	14.3	2.2	1.5	4.0	1.6	4.1	1.6	0.6	0.1	5.8	0.0	36.5
6 (less) Current subsidies	-0.2	0.0	-0.4	-0.2	-0.2	-0.2	-1.5	-0.2	-0.3	0.0	0.0	-1.1	0.0	-4.3
Value Added	9.0	0.9	111.4	40.6	15.3	72.7	24.9	55.9	25.2	10.1	18.1	118.2	2.9	505.2
Domestic production (gross inputs)	15.8	1.7	314.6	88.1	26.5	102.3	36.3	64.2	50.1	14.8	26.2	191.0	5.5	937.1

In Trillions of Yen	1 Consumption. Out Households	2 Consumption (Private)	3 Consumption Government	4 Capital Formation	5 Increase in Stocks	6 Exports	7 Imports	Total Final Demand	Total Demand - Gross Output
1 fishery	0.1	4.1	0.0	0.2	0.5	0.0	-2.4	2.5	15.8
2 Mining	0.0	0.0	0.0	0.0	0.0	0.0	-5.8	-5.8	1.7
3 Manufacturing	2.8	63.8	0.7	39.1	1.2	37.9	-26.7	118.8	314.6
4 Construction	0.0	0.0	0.0	80.0	0.0	0.0	0.0	80.0	88.1
5 supply	0.0	7.5	1.6	0.0	0.0	0.0	0.0	9.1	26.5
6 Commerce	2.2	50.5	0.0	10.4	0.2	3.1	-0.2	66.2	102.3
7 Finance and insurance	0.0	7.8	0.0	0.0	0.0	0.6	-1.0	7.4	36.3
8 Real estate	0.0	53.5	0.0	0.0	0.0	0.0	0.0	53.5	64.2
9 Transport	0.7	14.7	-0.1	0.8	0.2	3.7	-2.5	17.5	50.1
10 broadcasting	0.1	5.2	0.0	0.0	0.0	0.0	-0.1	5.3	14.8
11 Public administration	0.0	0.8	25.0	0.0	0.0	0.0	0.0	25.8	26.2
12 Services	13.5	64.0	41.9	9.2	0.0	1.3	-4.4	125.4	191.0
13 classified	0.0	0.0	0.0	0.0	0.0	0.0	-0.6	-0.5	5.5
Total	19.4	271.8	69.2	139.7	2.1	46.8	-43.7	505.2	937.1

Table 1 B

2000 In Trillions of Yen	1 Agriculture	2 Mining	3 Manufacturing	4 Construction	5 Electric power	6 Commerce	7 Finance	8 Real estate	9 Transport	10 Communication	11 Public Admin.	12 Services	13 Act. not else classified	Intermediate Output
1 Agriculture, forestry and fishery	1.6	0.0	8.4	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.3	0.0	11.5
2 Mining	0.0	0.0	7.4	0.7	2.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.1
3 Manufacturing	2.5	0.1	122.9	21.6	1.7	3.2	1.3	0.2	6.1	0.5	2.9	28.2	0.4	191.4
4 Construction	0.1	0.0	1.3	0.2	1.3	0.5	0.2	2.8	0.5	0.2	0.6	1.4	0.0	9.0
5 Electric power, Gas and water supply	0.1	0.0	6.3	0.5	1.6	1.2	0.2	0.2	0.9	0.3	1.0	5.5	0.1	18.1
6 Commerce	0.7	0.0	16.3	4.9	0.4	1.4	0.2	0.1	1.6	0.1	0.5	8.3	0.1	34.6
7 Finance and insurance	0.5	0.1	4.0	0.9	0.8	4.9	2.9	3.3	2.9	0.5	0.1	5.8	1.0	27.6
8 Real estate	0.0	0.0	0.9	0.3	0.2	2.9	0.6	0.4	0.7	0.4	0.0	2.7	0.0	9.1
9 Transport	0.6	0.4	8.2	4.0	0.7	4.6	0.7	0.1	5.0	0.5	1.1	4.2	0.2	30.5
10 Communication and broadcasting	0.0	0.0	1.1	0.9	0.1	2.5	0.8	0.1	0.4	2.7	0.5	4.9	0.1	14.2
11 Public administration	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.7
12 Services	0.2	0.1	23.1	6.4	2.8	6.3	5.0	1.7	6.7	3.6	2.8	19.3	0.3	78.2
13 Activities not elsewhere classified	0.1	0.0	1.7	0.3	0.1	0.6	0.3	0.3	0.2	0.1	0.0	0.7	0.0	4.4
Intermediate Input	6.3	0.7	201.5	40.9	11.7	28.3	12.1	9.2	25.0	8.8	9.5	82.3	2.9	439.4
1 Consumption expenditure outside households (row)	0.1	0.1	5.6	1.3	0.5	2.3	1.3	0.2	1.0	1.4	0.6	4.7	0.1	19.2
2 Compensation of employees	1.3	0.2	53.1	26.8	4.7	47.3	12.5	2.4	14.8	5.9	16.6	89.8	0.3	275.6
3 Operating surplus	4.7	0.2	16.9	1.4	3.5	10.0	9.0	29.6	2.6	1.5	0.0	16.7	0.4	96.5
4 Depreciation of fixed capital	1.5	0.1	16.7	4.1	5.0	4.8	3.4	20.7	3.0	3.8	9.5	20.3	0.4	93.4
5 Indirect taxes *	0.7	0.1	15.0	3.3	1.7	4.5	1.5	4.0	1.6	0.7	0.1	6.9	0.1	40.0
6 (less) Current subsidies	-0.2	0.0	-0.6	-0.3	-0.3	-0.2	-1.6	-0.2	-0.2	0.0	0.0	-1.5	0.0	-5.2
Total Value Added	8.1	0.7	106.6	36.5	15.3	68.6	26.0	56.6	22.9	13.3	26.7	136.9	1.3	519.5
Domestic production (gross inputs)	14.4	1.4	308.2	77.3	27.0	96.9	38.1	65.9	47.9	22.1	36.2	219.2	4.2	958.9

In Trillions of Yen	1 C. Out Households	2 Consumption (Private)	3 Consumption Government	4 Capital Formation	5 Increase in Stocks	6 Exports	7 Imports	Total Final Demand	Total Demand - Gross Output
1 Agriculture, forestry and fishery	0.1	3.9	0.0	0.2	0.8	0.1	-2.1	2.9	14.4
2 Mining	0.0	0.0	0.0	0.0	0.0	0.0	-8.7	-8.7	1.4
3 Manufacturing	3.3	61.6	0.5	39.7	-0.6	46.6	-34.3	116.8	308.2
4 Construction	0.0	0.0	0.0	68.3	0.0	0.0	0.0	68.3	77.3
5 Electric power, Gas and water supply	0.0	8.1	0.8	0.0	0.0	0.0	0.0	8.9	27.0
6 Commerce	1.9	45.9	0.0	10.7	0.1	4.5	-0.7	62.4	96.9
7 Finance and insurance	0.0	10.5	0.0	0.0	0.0	0.4	-0.4	10.5	38.1
8 Real estate	0.0	56.7	0.0	0.0	0.0	0.0	0.0	56.7	65.9
9 Transport	0.5	14.7	0.0	0.7	0.0	4.3	-2.9	17.4	47.9
10 Communication and broadcasting	0.2	7.8	0.0	0.0	0.0	0.1	-0.1	7.9	22.1
11 Public administration	0.0	0.7	34.8	0.0	0.0	0.0	0.0	35.5	36.2
12 Services	13.1	71.1	49.7	10.4	0.0	1.6	-4.8	141.0	219.2
13 Activities not elsewhere classified	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	-0.2	4.2
Total	19.2	281.0	85.7	130.0	0.3	57.5	-54.2	519.5	958.9

* (except custom duties and commodity taxes on imported goods)

Table 2 provides the distributions of total final demand, value added and gross output, as well as the output multiplier. This aggregate output multiplier of both the production/demand side, $\left(z_{11}y_1 + z_{12}y_2 + \cdots + z_{1n}y_n\right) \Big/ \sum_1^n y$, is the same to its equivalent $\left(h_1\tilde{z}_{11} + h_2\tilde{z}_{21} + \cdots + h_n\tilde{z}\right) \Big/ \sum_1^n h$ multiplier of the allocation/value added side. The logic behind this multiplier is that each element of the Leontief inverse row is weighed with the appropriate sectoral demand that exists for this sector. The same occurs with the Ghosh inverse. Each element of the appropriate column is multiplied by the associated percentage of the value added distribution. As is previously explained, the equality of the aggregate output multiplier is based on the same gross output that the two similar models yield, $\mathbf{x} = \mathbf{Zy} = \left(\mathbf{h}\tilde{\mathbf{Z}}\right)^T$. This multiplier is in agreement with available economic data. Large multiplier values exist in manufacturing and services followed by commerce and real estate. These are the sectors that provide the significant portion of output, comprise the largest amount of final demand and generate most of value added.

Table 2

Distributions of total final demand, value added, gross output & its multiplier	**Total Final Demand**		**Total Value Added**		**Gross Output**		**Output Multiplier**	
	1995	**2000**	**1995**	**2000**	**1995**	**2000**	**1995**	**2000**
Agriculture, forestry and fishery	0.5%	0.6%	1.8%	1.6%	1.7%	1.5%	0.028	0.031
Mining	-1.1%	-1.7%	0.2%	0.1%	0.2%	0.1%	0.003	0.003
Manufacturing	23.5%	22.5%	22.0%	20.5%	33.6%	32.1%	0.593	0.623
Construction	15.8%	13.2%	8.0%	7.0%	9.4%	8.1%	0.149	0.174
Electric power, Gas and water supply	1.8%	1.7%	3.0%	2.9%	2.8%	2.8%	0.052	0.052
Commerce	13.1%	12.0%	14.4%	13.2%	10.9%	10.1%	0.187	0.203
Finance and insurance	1.5%	2.0%	4.9%	5.0%	3.9%	4.0%	0.073	0.072
Real estate	10.6%	10.9%	11.1%	10.9%	6.8%	6.9%	0.127	0.127
Transport	3.5%	3.3%	5.0%	4.4%	5.3%	5.0%	0.092	0.099
Communication and broadcasting	1.0%	1.5%	2.0%	2.6%	1.6%	2.3%	0.043	0.029
Public administration	5.1%	6.8%	3.6%	5.1%	2.8%	3.8%	0.070	0.052
Services	24.8%	27.1%	23.4%	26.4%	20.4%	22.9%	0.422	0.378
Activities not elsewhere classified	-0.1%	0.0%	0.6%	0.2%	0.6%	0.4%	0.008	0.011
Total	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**		

Figure 3, presents the traditional row and column summations of the Leontief and Ghosh inverses and shows that there is a sizeable variability. The row summation of Ghosh inverse provides very large multiplier values of mining for both input-output tables, and the column summation of the Ghosh inverse for manufacturing. These numerical values, ranging from 12.6 to 17.3, are not acceptable as multipliers; therefore the Ghosh's inverse was declared implausible.

The reason for this implausibility, as it was mentioned in the methodological part of the paper is the fact that we apply Procrustian logic. Not all sectors are equivalent, as Table 2 shows. While construction is significant in its final demand proportion, it follows behind while one observes its contribution to value added or gross output. There is NO base for the assumption that we should evaluate the impact of a unit of final demand or value added in one sector, assuming all others have zero value demand or value added. Neither, we have any evidence that all sectors are equivalent. Thus, the reality shown in Table 2 should lead us towards our impact (multiplier) evaluations. The last two columns of Table 2 indicate not only that Leontief and Ghosh models

Figure 3

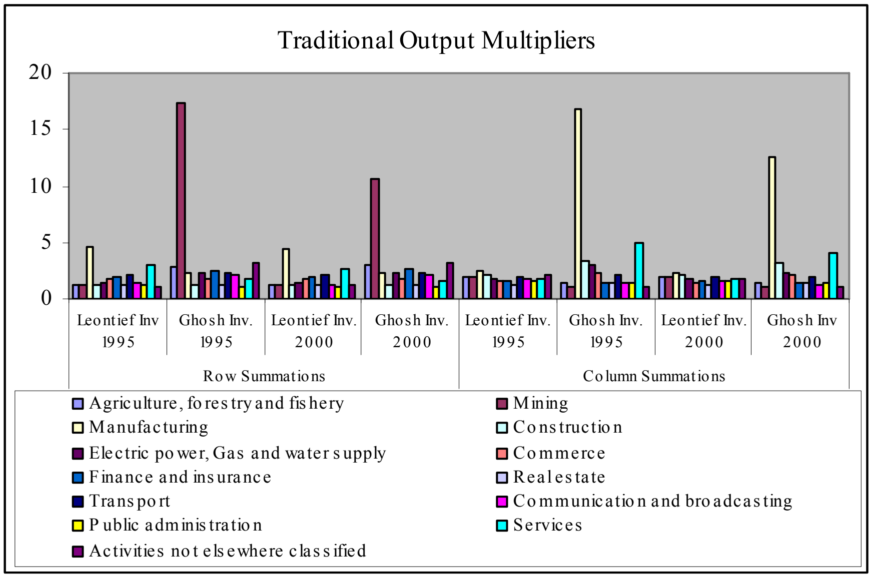


Table 4

$\mathbf{Y}_m$ (distribution of a unit of each type of final demand) & $\mathbf{Y}_d$ (distribution of a unit of final demand)

	Consum. Out HH.		Consumption (Private)		Consumption Government		Capital Formation		Increase in Stocks		Exports		Imports		Total Final Demand	
	1995	**2000**	**1995**	**2000**	**1995**	**2000**	**1995**	**2000**	**1995**	**2000**	**1995**	**2000**	**1995**	**2000**	**1995**	**2000**
Distribution of each type of final demand																
Agriculture	0.01	0.00	0.02	0.01	0.00	0.00	0.00	0.00	0.23	2.80	0.00	0.00	0.05	0.04	0.01	0.01
Mining	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	-0.04	0.00	0.00	0.13	0.16	-0.01	-0.02
Manufacturing	0.15	0.17	0.23	0.22	0.01	0.01	0.28	0.31	0.58	-2.30	0.81	0.81	0.61	0.63	0.24	0.22
Construction	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.53	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.13
Electric power	0.00	0.00	0.03	0.03	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.02
Commerce	0.11	0.10	0.19	0.16	0.00	0.00	0.07	0.08	0.09	0.42	0.07	0.08	0.00	0.01	0.13	0.12
Finance	0.00	0.00	0.03	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.01	0.01	0.02
Real estate	0.00	0.00	0.20	0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.11
Transport	0.04	0.03	0.05	0.05	0.00	0.00	0.01	0.01	0.08	0.12	0.08	0.07	0.06	0.05	0.03	0.03
Communication	0.01	0.01	0.02	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02
Public administ.	0.00	0.00	0.00	0.00	0.36	0.41	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.07
Services	0.69	0.68	0.24	0.25	0.61	0.58	0.07	0.08	0.00	0.00	0.03	0.03	0.10	0.09	0.25	0.27
Other Activities	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
Total	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Distribution of a total final demand unit																
Agriculture	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
Mining	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.02	-0.01	-0.02
Manufacturing	0.01	0.01	0.13	0.12	0.00	0.00	0.08	0.08	0.00	0.00	0.07	0.09	-0.05	-0.07	0.24	0.22
Construction	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.13
Electric power	0.00	0.00	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.02
Commerce	0.00	0.00	0.10	0.09	0.00	0.00	0.02	0.02	0.00	0.00	0.01	0.01	0.00	0.00	0.13	0.12
Finance	0.00	0.00	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02
Real estate	0.00	0.00	0.11	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.11
Transport	0.00	0.00	0.03	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	-0.01	0.03	0.03
Communication	0.00	0.00	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02
Public administ.	0.00	0.00	0.00	0.00	0.05	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.07
Services	0.03	0.03	0.13	0.14	0.08	0.10	0.02	0.02	0.00	0.00	0.00	0.00	-0.01	-0.01	0.25	0.27
Other Activities	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Total	0.038	0.037	0.538	0.541	0.137	0.165	0.277	0.250	0.004	0.001	0.093	0.111	-0.087	-0.104	**1**	**1**

yield the same gross output multiplier, but this multiplier is **not** implausible and it is realistic.

Thus, the important driving forces are the distributions of final demand (Table 3) and value added (Table 4). Not all aspects of final demand are equivalent as the matrix $\mathbf{Y}_m$ indicates. Matrix $\mathbf{Y}_m$ is useful whenever one is interested in examining the impact of a ¥ allocated to any type of final demand equivalently.

The sectoral breakdown of these figures is more important because it indicates to which sectors these specific components of demand are channeled. Most (83%) of the Private consumption expenditure is channeled to four sectors. The service sector absorbs 25.3%, Manufacturing 21.9% Real estate 20.2% and Commerce 16.3% of a unit of private consumption expenditure for 2000. Construction absorbs more than half of the gross domestic fixed capital formation, that is 13.2% of final demand, and almost the half of this value is channeled to manufacturing. Manufacturing is the dominant player for exporting (81%) as well as importing (63%) activity, while mining sector (16%) is noticeable in the import column.

Private consumption expenditure concentrates 54% of final demand, exports are more or less equal to imports with 11% and 10% respectively, with gross domestic fixed capital formation absorbing one forth of the final demand and the consumption expenditure of the general government being the 16.5% of final demand. These specific characteristics given in the $\mathbf{Y}_{\mathrm{d}}$ matrix are those that provide the tone and purpose for the analysis, and these characteristics are captured when the weighted inverse is evaluated.

It is noticeable that this is not a positive column, since imports in the mining sector are greater than domestic production. The service sector dominates the economy with 27.1% of final demand, mainly allocated to private (13.7%) and general government (9.6%) of the consumption expenditures. The manufacturing sector follows with 22.5% of final demand, indicating that there is a strong industrial base in the economy, and the demand of the manufacturing sector is driven by private consumption expenditures (11.9%), exports (9%), fixed capital formation (7.6%) and imports (6.6%). Construction (13.2%) is allocated only to fixed capital formation; Commerce (12%) and real estate are mainly affected by private consumption (8.8% & 10.9%) respectively.

Table 4 provides comparable information for the value added distributions $\mathbf{H}_m$ & $\mathbf{H}_d$. The compensation of employees is just a percentage point lower than the share of private consumption to the net output and it is distributed mainly to services, manufacturing and commerce. Operating surplus and depreciation of fixed capital have almost the same share. While operating surplus is mainly concentrated into real estate, depreciation of fixed capital has similar allocation to real estate and service.

Looking at the sectoral distribution of the value added, services (26.4%) has less than a percentage point difference from final demand distribution; manufacturing and real estate have the same weight in value added as in final demand (20.5% and 10.9%) respectively, and commerce with 13.2% has a little more weight in value added than its final demand (12%).

Seventy one percent of the value added allocation to construction comes from employee compensation, while for services the equivalent assessment is 65% and for manufacturing 49%.

Most of the real estate value added is allocated to operating surplus and to depreciation of fixed capital.

Table 4

$\mathbf{H}_m$ (distribution of a unit of each type of value added) & $\mathbf{H}_d$ (distribution of a unit of value added)

	Consumption out. HH		Compensation of employees		Operating surplus		Depreciation of fixed capital		Indirect taxes *		(less) Current subsidies		Total Value Added	
	1995	**2000**	**1995**	**2000**	**1995**	**2000**	**1995**	**2000**	**1995**	**2000**	**1995**	**2000**	**1995**	**2000**
Distribution of each type of value added														
Agriculture	0.01	0.01	0.01	0.00	0.05	0.05	0.02	0.02	0.02	0.02	0.05	0.03	0.02	0.02
Mining	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Manufacturing	0.33	0.29	0.20	0.19	0.20	0.17	0.21	0.18	0.39	0.37	0.10	0.12	0.22	0.21
Construction	0.09	0.07	0.11	0.10	0.03	0.01	0.06	0.04	0.06	0.08	0.04	0.07	0.08	0.07
Electric P.	0.03	0.03	0.02	0.02	0.04	0.04	0.07	0.05	0.04	0.04	0.05	0.05	0.03	0.03
Commerce	0.14	0.12	0.18	0.17	0.11	0.10	0.06	0.05	0.11	0.11	0.04	0.04	0.14	0.13
Finance	0.07	0.07	0.05	0.05	0.06	0.09	0.05	0.04	0.04	0.04	0.35	0.32	0.05	0.05
Real Estate	0.01	0.01	0.01	0.01	0.28	0.31	0.26	0.22	0.11	0.10	0.04	0.04	0.11	0.11
Transport	0.06	0.05	0.06	0.05	0.03	0.03	0.04	0.03	0.04	0.04	0.08	0.04	0.05	0.04
Communic.	0.01	0.07	0.02	0.02	0.02	0.02	0.04	0.04	0.02	0.02	0.00	0.00	0.02	0.03
Public administ.	0.03	0.03	0.06	0.06	0.00	0.00	0.01	0.10	0.00	0.00	0.00	0.00	0.04	0.05
Services	0.23	0.24	0.29	0.33	0.15	0.17	0.19	0.22	0.16	0.17	0.25	0.29	0.23	0.26
Other Activities	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
Total	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Distribution of a totalvalue added unit														
Agriculture	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.02
Mining	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Manufacturing	0.01	0.01	0.11	0.10	0.04	0.03	0.03	0.03	0.03	0.03	0.00	0.00	0.22	0.21
Construction	0.00	0.00	0.06	0.05	0.01	0.00	0.01	0.01	0.00	0.01	0.00	0.00	0.08	0.07
Electric P.	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.03	0.03
Commerce	0.01	0.00	0.10	0.09	0.02	0.02	0.01	0.01	0.01	0.01	0.00	0.00	0.14	0.13
Finance	0.00	0.00	0.03	0.02	0.01	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.05	0.05
Real Estate	0.00	0.00	0.00	0.00	0.06	0.06	0.04	0.04	0.01	0.01	0.00	0.00	0.11	0.11
Transport	0.00	0.00	0.03	0.03	0.01	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.05	0.04
Communic.	0.00	0.00	0.01	0.01	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.02	0.03
Public administ.	0.00	0.00	0.03	0.03	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.04	0.05
Services	0.01	0.01	0.15	0.17	0.03	0.03	0.03	0.04	0.01	0.01	0.00	0.00	0.23	0.26
Other Activities	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
Total	0.038	0.037	0.541	0.531	0.197	0.186	0.160	0.180	0.072	0.077	-0.009	-0.010	1	1

Given the total interdependencies provided by the appropriate inverses, and their appropriate backward and forward linkages, production and allocation based final structure matrices are calculated. Augustinovics' final structure matrices are Markov matrices and the production based final structure matrix is different from the allocation based final structure matrix, as one may observe in Table 5. In the contrary, production and allocation based final structure matrices driven by a two-dimension distribution are exactly the same. This is because as we already mentioned production and allocation are the two sides of the same activity, and one ¥ of value added has the same overall impact as one ¥ of final demand. Private consumption dominates the field, with a slide decline from 54.1% to 53.8% from 1995 to 2000. During the same time period, capital formation increased 2.7%, and government consumption declined from 16.5% to 13.7%.

Table 5

Final Structure Matrices (*see Schema 2*)

Augustinovics Production Final Structure — Hx95.Z95.Ym95 / Hx00.Z00.Ym00

	Hx95.Z95.Ym95: Consumption. Out Households	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total	Hx00.Z00.Ym00: Consumption. Out Households	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total
Consumption out HH	0.040	0.036	0.034	0.042	-0.024	0.046	0.050		0.040	0.036	0.037	0.045	0.043	0.048	0.049	
Compensation of employees	0.590	0.472	0.602	0.594	-0.021	0.530	0.511		0.593	0.476	0.672	0.589	0.474	0.524	0.503	
Operating surplus	0.156	0.236	0.106	0.140	0.881	0.176	0.190		0.168	0.240	0.125	0.159	0.252	0.194	0.215	
Depreciation of fixed capital	0.152	0.190	0.218	0.141	0.173	0.153	0.160		0.141	0.184	0.124	0.136	0.154	0.148	0.154	
Indirect taxes *	0.073	0.078	0.048	0.093	0.027	0.104	0.100		0.067	0.074	0.049	0.078	0.089	0.095	0.090	
(less) Current subsidies	-0.011	-0.011	-0.008	-0.010	-0.036	-0.010	-0.011		-0.010	-0.010	-0.008	-0.008	-0.011	-0.009	-0.011	
Total	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**		**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	

Augustinovics Allocation Final Structure — Hm95.Zs95.Yx95 / Hm00.Zs00.Yx00

	Hm95.Zs95.Yx95: Consumption. Out Households	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total	Hm00.Zs00.Yx00: Consumption. Out Households	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total
Consumption out HH	0.040	0.524	0.152	0.286	0.000	0.139	-0.140	**1.000**	0.040	0.497	0.133	0.321	0.005	0.116	-0.111	**1.000**
Compensation of employees	0.041	0.481	0.187	0.280	0.000	0.111	-0.100	**1.000**	0.042	0.473	0.170	0.301	0.004	0.090	-0.081	**1.000**
Operating surplus	0.031	0.686	0.094	0.188	0.003	0.105	-0.106	**1.000**	0.033	0.655	0.087	0.223	0.005	0.091	-0.094	**1.000**
Depreciation of fixed capital	0.031	0.571	0.200	0.196	0.001	0.094	-0.093	**1.000**	0.034	0.619	0.106	0.235	0.004	0.086	-0.083	**1.000**
Indirect taxes *	0.035	0.547	0.102	0.302	0.000	0.150	-0.136	**1.000**	0.036	0.551	0.093	0.300	0.005	0.122	-0.108	**1.000**
(less) Current subsidies	0.040	0.588	0.135	0.244	0.002	0.107	-0.117	**1.000**	0.043	0.601	0.123	0.246	0.005	0.097	-0.115	**1.000**

Adamou Production Final Structure — Hx95.Z95.Yd95 / Hx00.Z00.Yd00

	Hx95.Z95.Yd95: Consumption. Out Households	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total	Hx00.Z00.Yd00: Consumption. Out Households	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total
Consumption out HH	0.001	0.019	0.006	0.011	0.000	0.005	-0.005	0.037	0.002	0.019	0.005	0.012	0.000	0.004	-0.004	0.038
Compensation of employees	0.022	0.255	0.099	0.149	0.000	0.059	-0.053	0.531	0.023	0.256	0.092	0.163	0.002	0.049	-0.044	0.541
Operating surplus	0.006	0.127	0.018	0.035	0.000	0.019	-0.020	0.186	0.006	0.129	0.017	0.044	0.001	0.018	-0.019	0.197
Depreciation of fixed capital	0.006	0.103	0.036	0.035	0.000	0.017	-0.017	0.180	0.005	0.099	0.017	0.038	0.001	0.014	-0.013	0.160
Indirect taxes *	0.003	0.042	0.008	0.023	0.000	0.012	-0.010	0.077	0.003	0.040	0.007	0.022	0.000	0.009	-0.008	0.072
(less) Current subsidies	0.000	-0.006	-0.001	-0.002	0.000	-0.001	0.001	-0.010	0.000	-0.005	-0.001	-0.002	0.000	-0.001	0.001	-0.009
Total	0.037	0.541	0.165	0.250	0.001	0.111	-0.104	**1.000**	0.038	0.538	0.137	0.277	0.004	0.093	-0.087	**1.000**

Adamou Allocation Final Structure — Hd95.Zs95.Yx95 / Hd00.Zs00.Yx00

	Hd95.Zs95.Yx95: Consumption. Out Households	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total	Hd00.Zs00.Yx00: Consumption. Out Households	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total
Consumption out HH	0.001	0.019	0.006	0.011	0.000	0.005	-0.005	0.037	0.002	0.019	0.005	0.012	0.000	0.004	-0.004	0.038
Compensation of employees	0.022	0.255	0.099	0.149	0.000	0.059	-0.053	0.531	0.023	0.256	0.092	0.163	0.002	0.049	-0.044	0.541
Operating surplus	0.006	0.127	0.018	0.035	0.000	0.019	-0.020	0.186	0.006	0.129	0.017	0.044	0.001	0.018	-0.019	0.197
Depreciation of fixed capital	0.006	0.103	0.036	0.035	0.000	0.017	-0.017	0.180	0.005	0.099	0.017	0.038	0.001	0.014	-0.013	0.160
Indirect taxes *	0.003	0.042	0.008	0.023	0.000	0.012	-0.010	0.077	0.003	0.040	0.007	0.022	0.000	0.009	-0.008	0.072
(less) Current subsidies	0.000	-0.006	-0.001	-0.002	0.000	-0.001	0.001	-0.010	0.000	-0.005	-0.001	-0.002	0.000	-0.001	0.001	-0.009
Total	0.037	0.541	0.165	0.250	0.001	0.111	-0.104	**1.000**	0.038	0.538	0.137	0.277	0.004	0.093	-0.087	**1.000**

From the value added side of the picture, employee compensation increased from 53.1% to 54.1% within five years. Operating surplus went to 19.7% from 18.6%, but there is a decline from 18% to 16% of the depreciation of capital.

The product of the two final structures defines the feedback matrices. Table 6 provides the value added feedback and Table 7 the final demand feedback.

Markov matrices show that more than half of the value added ¥ returns to employee compensation, followed by the operating surplus that receives about 20% and capital depreciation varies from 15.7% to 18.4%.

In the two dimension case, a multiplication of a matrix to itself provides a symmetric[23] matrix in a linear algebra sense. This matrix is not a distribution matrix, since the total sum of its elements

[23] The i^{th} row is the same as the i^{th} column. This is obvious when one observes the right side matrices of the Table 6.

is not a unit. This is a percentage indicating that from one ¥ of value added only a fraction comes back to value added. This feedback effect declines for value added from 40.7% to 40.2%. The two dimension feedback matrix shows how this is distributed to value added. Almost half of this (20.4% & 20.5%) is feedback of overall employee compensation. Half of the overall employee compensation feedback is the direct employee compensation feedback (10.4% & 10.5%).

Table 6

Value Added Feedback Matrices

	FSMpm95.Transpose[FSMam95]						FSMpd95.Transpose[FSMad95]						
	Consumption out HH	Compensation	Operating surplus	Depreciation of fixed capital	Indirect taxes *	(less) Current subsidies	Consumption out HH	Compensation	Operating surplus	Depreciation of fixed capital	Indirect taxes *	(less) Current subsidies	Total
Consumption out HH	0.037	0.037	0.036	0.036	0.037	0.037	0.001	0.008	0.003	0.003	0.001	0.000	0.015
Compensation of employees	0.534	0.538	0.512	0.527	0.530	0.525	0.008	0.104	0.042	0.037	0.016	-0.002	0.205
Operating surplus	0.183	0.179	0.203	0.187	0.188	0.192	0.003	0.042	0.019	0.016	0.007	-0.001	0.085
Depreciation of fixed capital	0.178	0.179	0.181	0.184	0.175	0.180	0.003	0.037	0.016	0.014	0.006	-0.001	0.074
Indirect taxes *	0.078	0.077	0.078	0.075	0.080	0.077	0.001	0.016	0.007	0.006	0.003	0.000	0.032
(less) Current subsidies	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	0.000	-0.002	-0.001	-0.001	0.000	0.000	-0.004
Total	1	1	1	1	1	1	0.015	0.205	0.085	0.074	0.032	-0.004	0.407

	FSMpm00.Transpose[FSMam00]						FSMpd00.Transpose[FSMad00]						
Consumption out HH	0.039	0.039	0.038	0.038	0.039	0.038	0.001	0.008	0.003	0.003	0.001	0.000	0.015
Compensation of employees	0.546	0.550	0.524	0.529	0.535	0.534	0.008	0.105	0.044	0.034	0.015	-0.002	0.204
Operating surplus	0.194	0.191	0.208	0.205	0.200	0.202	0.003	0.044	0.020	0.015	0.007	-0.001	0.088
Depreciation of fixed capital	0.158	0.157	0.166	0.164	0.161	0.163	0.003	0.034	0.015	0.012	0.005	-0.001	0.069
Indirect taxes *	0.073	0.071	0.073	0.073	0.074	0.072	0.001	0.015	0.007	0.005	0.002	0.000	0.030
(less) Current subsidies	-0.008	-0.008	-0.009	-0.009	-0.009	-0.009	0.000	-0.002	-0.001	-0.001	0.000	0.000	-0.004
Total	1	1	1	1	1	1	0.015	0.204	0.088	0.069	0.030	-0.004	0.402

Table 7 shows the final demand feedback on value added. Private consumption dominates the field of the Markov-type feedback with more than half of every ¥ of each type of final demand to return back to private consumption. Capital formation and government consumption follow for both years of the analysis.

The symmetric matrices indicate that the overall final demand feedback is slightly smaller that the analogous value added, but improves from 36.1 to 36.4 between these five years. About 18% is the private consumption over all feedback on itself, while the direct private consumption feedback by itself is the same for both years of the tables analyzed (9.4%). The feedback from exports to exports and import to imports indicate a slight decline. The feedback from capital formation to capital formation shows an increase from 9.4% to 10.5%. Half of this feedback (4.7% & 5.2%) is absorbed by private consumption. A careful look shows that private consumption absorbs half of exports and imports feedback as well as half of government consumptions feedback as well as half of its own overall feedback.

Table 7

Final Demand Feedback on Value Added Matrices

Transpose[FSMam95].FSMpm95

	Consumption. Out HH	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total
Consumption. Out HH	0.037	0.036	0.038	0.038	0.030	0.037	0.037	
Consumption (Private)	0.532	0.552	0.526	0.529	0.674	0.539	0.542	
Consumption Government	0.168	0.161	0.175	0.167	0.108	0.163	0.162	
Capital Formation	0.255	0.245	0.254	0.258	0.186	0.254	0.252	
Increase in Stocks	0.000	0.001	0.000	0.000	0.002	0.000	0.001	
Exports	0.111	0.110	0.109	0.112	0.103	0.112	0.112	
Imports	-0.104	-0.104	-0.102	-0.105	-0.104	-0.106	-0.106	
Total	1.000	1.000	1.000	1.000	1.000	1.000	1.000	

Transpose[FSMam00].FSMpm00

	Consumption. Out HH	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total
Consumption. Out HH	0.039	0.038	0.040	0.039	0.038	0.038	0.038	
Consumption (Private)	0.529	0.549	0.518	0.528	0.548	0.538	0.542	
Consumption Government	0.141	0.132	0.147	0.141	0.131	0.136	0.134	
Capital Formation	0.280	0.272	0.285	0.281	0.273	0.278	0.276	
Increase in Stocks	0.004	0.004	0.004	0.004	0.004	0.004	0.004	
Exports	0.093	0.093	0.092	0.093	0.093	0.094	0.094	
Imports	-0.086	-0.087	-0.085	-0.086	-0.088	-0.087	-0.087	
Total	1.000	1.000	1.000	1.000	1.000	1.000	1.000	

Transpose[FSMad95].FSMpd95

	Consumption. Out HH	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total
Consumption. Out HH	0.001	0.007	0.002	0.004	0.001	0.002	-0.001	0.015
Consumption (Private)	0.007	0.094	0.032	0.047	0.000	0.020	-0.018	0.182
Consumption Government	0.002	0.032	0.012	0.017	0.000	0.007	-0.006	0.063
Capital Formation	0.004	0.047	0.017	0.025	0.000	0.010	-0.010	0.094
Increase in Stocks	0.001	0.000	0.000	0.000	0.003	0.000	0.000	0.004
Exports	0.002	0.020	0.007	0.010	0.000	0.004	-0.004	0.039
Imports	-0.001	-0.018	-0.006	-0.010	0.000	-0.004	0.004	-0.036
Total	0.015	0.182	0.063	0.094	0.004	0.039	-0.036	0.361

Transpose[FSMad00].FSMpd00

	Consumption. Out HH	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total
Consumption. Out HH	0.001	0.007	0.002	0.004	0.000	0.001	-0.001	0.015
Consumption (Private)	0.007	0.094	0.028	0.052	0.001	0.017	-0.015	0.183
Consumption Government	0.002	0.028	0.009	0.017	0.000	0.005	-0.005	0.057
Capital Formation	0.004	0.052	0.017	0.031	0.000	0.009	-0.009	0.105
Increase in Stocks	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.001
Exports	0.001	0.017	0.005	0.009	0.000	0.003	-0.003	0.033
Imports	-0.001	-0.015	-0.005	-0.009	0.000	-0.003	0.002	-0.030
Total	0.015	0.183	0.057	0.105	0.001	0.033	-0.030	0.364

Final structure and feedback matrices indicate linkages between final demand and value added. The *final demand value added multipliers* are based on the identity $\left[\mathbf{I}-\mathbf{A}\right]^{-1}\mathbf{Y}\,\mathbf{i} = \left[\mathbf{I}-\widetilde{\mathbf{A}}^{T}\right]^{-1}\mathbf{H}^{T}\,\mathbf{i}$ and explained in *Schema 5*. The final demand/value added multipliers allow one to pinpoint a particular sector and its role in this process. The formation in *Schema 5* shows how via the allocation matrix and the Leontief inverse, or the production matrix and the Ghosh inverse final demand is translated into value added and vice versa. Table 8 presents *multipliers* that show the *impact of value added into final demand* as Markov and two dimensional distributions.

One ¥ of total value added feedback given in Table 8 , is not that different either, although it is not exactly the same as the analogous of that indicated in feedback matrices. For example a value added ¥ according to Table 8, is distributed 51.8% to compensation, 19.9% to operating surplus and 18% to depreciation of fixed capital, while according to Table 5 the same distribution indicates 53.1%, 19.9% & 18%. For all practical purposes, these differences are normal while vast numerical calculations are preformed, and they do not alter the validity of the analysis.

The production matrix post-multiplied to the transposed Ghosh inverse play the role of a multiplier matrix that absorbs row wise causal effects from the transposed value added and generates the matrix that the summation of row elements yields total final demand. Thus, one may examine the impact of either the Markov or the two dimensional distribution of value added on final demand. This is what Table 8 indicates. In the top part the impact of a Markov distribution is given, and in the bottom part the effect of the two dimensional value added distribution to final demand is provided. Both presentations allow the role of processing sectors to be identified. Although there is not a one to one correlation between these sectoral multipliers, the correlation coefficient among them is significant, indicating that they provide the same information without noticeable variability.

Table 8

Multipliers indicating the sectoral impact of value added to final demand

	Pr95.Transpose[Zs95].Transpose[Hm95]							Pr00.Transpose[Zs00].Transpose[Hm00]						
	Consumption out HH	Compensation	Operating surplus	Depreciation of fixed capital	Indirect taxes *	(less) Current subsidies	Total	Consumption out HH	Compensation	Operating surplus	Depreciation of fixed capital	Indirect taxes *	(less) Current subsidies	Total
Agriculture	-0.008	-0.005	0.040	0.007	0.003	0.029	0.065	-0.011	-0.006	0.040	0.009	-0.002	0.044	0.074
Mining	-0.019	-0.016	-0.016	-0.017	-0.024	-0.016	-0.108	-0.012	-0.011	-0.011	-0.013	-0.016	-0.009	-0.071
Manufacturing	0.323	0.197	0.237	0.199	0.394	0.175	1.525	0.338	0.193	0.263	0.236	0.409	0.184	1.623
Construction	0.149	0.165	0.064	0.089	0.158	0.133	0.757	0.185	0.189	0.100	0.121	0.155	0.129	0.879
Electric power	0.017	0.004	0.028	0.042	0.030	0.044	0.165	0.017	0.003	0.026	0.056	0.027	0.042	0.170
Commerce	0.108	0.153	0.108	0.049	0.085	0.068	0.571	0.115	0.163	0.115	0.064	0.084	0.082	0.623
Finance	0.037	0.018	0.056	0.008	0.006	0.287	0.412	0.033	0.019	0.018	0.010	0.008	0.316	0.404
Real estate	0.013	0.008	0.310	0.222	0.099	0.061	0.713	0.011	0.003	0.283	0.255	0.108	0.058	0.717
Transport	0.041	0.041	0.023	0.025	0.028	0.050	0.207	0.039	0.044	0.017	0.031	0.027	0.089	0.247
Communication	0.060	0.009	0.008	0.033	0.009	-0.010	0.110	0.001	0.007	0.008	0.028	0.008	-0.012	0.040
Public administ.	0.052	0.078	0.015	0.117	0.021	0.018	0.300	0.045	0.077	0.014	0.025	0.018	0.017	0.196
Services	0.256	0.327	0.196	0.227	0.186	0.309	1.502	0.240	0.291	0.184	0.217	0.180	0.281	1.394
Other Activities	0.002	-0.002	0.001	0.003	-0.004	0.002	0.003	-0.005	-0.005	0.016	-0.003	-0.007	0.001	-0.004
	Pr95.Transpose[Zs95].Transpose[Hd95]							Pr00.Transpose[Zs00].Transpose[Hd00]						
Agriculture	0.000	-0.003	0.007	0.001	0.000	0.000	0.006	0.000	-0.003	0.008	0.001	0.000	0.000	0.005
Mining	-0.001	-0.008	-0.003	-0.003	-0.002	0.000	-0.017	0.000	-0.006	-0.002	-0.002	-0.001	0.000	-0.011
Manufacturing	0.012	0.105	0.044	0.036	0.030	-0.002	0.225	0.013	0.105	0.052	0.038	0.030	-0.002	0.235
Construction	0.005	0.087	0.012	0.016	0.012	-0.001	0.132	0.007	0.102	0.020	0.019	0.011	-0.001	0.158
Electric power	0.001	0.002	0.005	0.008	0.002	0.000	0.017	0.001	0.002	0.005	0.009	0.002	0.000	0.018
Commerce	0.004	0.081	0.020	0.009	0.007	-0.001	0.120	0.004	0.088	0.023	0.010	0.006	-0.001	0.131
Finance	0.001	0.009	0.010	0.001	0.000	-0.003	0.020	0.001	0.010	0.003	0.002	0.001	-0.003	0.015
Real estate	0.000	0.004	0.058	0.040	0.008	-0.001	0.109	0.000	0.002	0.056	0.041	0.008	0.000	0.106
Transport	0.002	0.022	0.004	0.004	0.002	-0.001	0.033	0.001	0.024	0.003	0.005	0.002	-0.001	0.035
Communication	0.002	0.005	0.002	0.006	0.001	0.000	0.015	0.000	0.004	0.002	0.005	0.001	0.000	0.010
Public administ.	0.002	0.041	0.003	0.021	0.002	0.000	0.068	0.002	0.041	0.003	0.004	0.001	0.000	0.051
Services	0.009	0.173	0.036	0.041	0.014	-0.003	0.271	0.009	0.157	0.036	0.035	0.013	-0.002	0.248
Other Activities	0.000	-0.001	0.000	0.001	0.000	0.000	0.000	0.000	-0.003	0.003	-0.001	0.000	0.000	-0.001
Total	0.038	0.518	0.199	0.180	0.076	-0.011	1.000	0.038	0.522	0.212	0.166	0.072	-0.010	1.000

Table 9 is comparable to the previous one. In this table, the multiplier matrix is the product of transposed allocation to the Leontief inverse. The causal effect from final demand is captured either as Markov (top) or two dimensional distributions (bottom). Again, there is no considerable variation between this table and the previously presented final demand feedback structures.

Table 9
Multipliers indicating the sectoral impact of final demand to value added

Transpose[Al95].Z95.Ym95

	Consumption. Out Households	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total
Agriculture	0.016	0.021	0.007	0.017	2.733	0.026	0.060	2.880
Mining	0.014	0.016	0.009	0.027	-0.114	0.037	0.191	0.179
Manufacturing	0.063	0.071	-0.021	0.250	-3.032	0.545	-0.451	-2.576
Construction	-0.076	-0.064	-0.038	0.434	0.200	-0.142	-0.199	0.116
Electric power	-0.002	0.018	0.020	-0.028	0.139	-0.029	-0.254	-0.137
Commerce	0.112	0.155	0.010	0.135	0.414	0.122	0.048	0.996
Finance	0.024	0.072	0.014	0.032	0.103	0.044	0.048	0.337
Real estate	0.007	0.205	0.004	-0.012	0.000	0.006	0.005	0.214
Transport	0.031	0.058	0.009	0.045	0.204	0.097	0.115	0.559
Communication	0.022	0.036	0.012	0.013	0.006	0.010	0.010	0.109
Public administ.	-0.020	-0.014	0.391	-0.019	0.024	-0.022	-0.019	0.323
Services	0.664	0.230	0.600	0.093	-0.190	-0.003	0.068	1.462
Other Activities	0.004	0.004	-0.005	0.007	0.004	0.008	0.013	0.033

Transpose[Al00].Z00.Ym00

	Consumption. Out Households	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total
Agriculture	0.018	0.025	0.009	0.020	0.257	0.031	0.078	0.437
Mining	0.009	0.011	0.007	0.020	0.040	0.025	0.153	0.264
Manufacturing	0.082	0.128	0.009	0.320	0.258	0.654	0.035	1.486
Construction	-0.087	-0.077	-0.050	0.471	-0.140	-0.157	-0.197	-0.237
Electric power	0.010	0.031	0.041	-0.004	-0.012	0.004	-0.100	-0.031
Commerce	0.123	0.174	0.013	0.132	0.124	0.105	0.032	0.702
Finance	0.030	0.068	0.020	0.034	0.045	0.052	0.066	0.315
Real estate	0.012	0.205	0.007	-0.005	0.008	0.008	0.005	0.240
Transport	0.036	0.061	0.006	0.050	0.117	0.105	0.108	0.484
Communication	0.019	0.027	0.013	0.009	0.007	0.008	0.008	0.092
Public administ.	-0.017	-0.011	0.348	-0.016	-0.016	-0.020	-0.018	0.249
Services	0.672	0.203	0.615	0.078	-0.046	-0.008	0.072	1.587
Other Activities	0.007	0.007	0.005	0.008	0.011	0.011	0.023	0.071

Transpose[Al95].Z95.Yd95

	Consumption. Out Households	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total
Agriculture	0.001	0.012	0.001	0.004	0.000	0.003	-0.006	0.014
Mining	0.001	0.008	0.001	0.007	0.000	0.004	-0.020	0.001
Manufacturing	0.002	0.038	-0.004	0.063	0.000	0.060	0.047	0.207
Construction	-0.003	-0.034	-0.006	0.108	0.000	-0.016	0.021	0.070
Electric power	0.000	0.010	0.003	-0.007	0.000	-0.003	0.027	0.029
Commerce	0.004	0.084	0.002	0.034	0.000	0.013	-0.005	0.132
Finance	0.001	0.039	0.002	0.008	0.000	0.005	-0.005	0.050
Real estate	0.000	0.111	0.001	-0.003	-0.001	0.001	-0.001	0.108
Transport	0.001	0.031	0.001	0.011	0.000	0.011	-0.012	0.044
Communication	0.001	0.020	0.002	0.003	0.001	0.001	-0.001	0.027
Public administ.	-0.001	-0.007	0.065	-0.005	0.000	-0.002	0.002	0.051
Services	0.024	0.124	0.099	0.023	0.000	0.000	-0.007	0.264
Other Activities	0.000	0.002	-0.001	0.002	0.000	0.001	-0.001	0.003
Total	0.032	0.437	0.167	0.249	0.000	0.077	0.038	1.000

Transpose[Al00].Z00.Yd00

	Consumption. Out Households	Consumption (Private)	Consumption Government	Capital Formation	Increase in Stocks	Exports	Imports	Total
Agriculture	0.001	0.013	0.001	0.005	0.001	0.003	-0.007	0.018
Mining	0.000	0.006	0.001	0.005	0.000	0.002	-0.013	0.002
Manufacturing	0.003	0.069	0.001	0.089	0.001	0.061	-0.003	0.220
Construction	-0.003	-0.042	-0.007	0.130	-0.001	-0.015	0.017	0.080
Electric power	0.000	0.017	0.006	-0.001	0.000	0.000	0.009	0.030
Commerce	0.005	0.093	0.002	0.037	0.001	0.010	-0.003	0.144
Finance	0.001	0.037	0.003	0.010	0.000	0.005	-0.006	0.049
Real estate	0.000	0.110	0.001	-0.001	0.000	0.001	0.000	0.111
Transport	0.001	0.033	0.001	0.014	0.000	0.010	-0.009	0.050
Communication	0.001	0.015	0.002	0.002	0.000	0.001	-0.001	0.020
Public administ.	-0.001	-0.006	0.048	-0.005	0.000	-0.002	0.002	0.036
Services	0.026	0.109	0.084	0.022	0.000	-0.001	-0.006	0.234
Other Activities	0.000	0.004	0.001	0.002	0.000	0.001	-0.002	0.006
Total	0.035	0.458	0.143	0.309	0.003	0.076	-0.023	1.000

Table 10
Final Demand – Value Added Multiplier

	Hd		Yd		Hd	Yd
	1995	2000	1995	2000	00-95	00-95
Agriculture	0.006	0.005	0.014	0.018	-0.001	0.004
Mining	-0.017	-0.011	0.001	0.002	0.005	0.000
Manufacturing	0.225	0.235	0.207	0.220	0.010	0.014
Construction	0.132	0.158	0.070	0.080	0.027	0.010
Electric power	0.017	0.018	0.029	0.030	0.001	0.001
Commerce	0.120	0.131	0.132	0.144	0.011	0.012
Finance	0.020	0.015	0.050	0.049	-0.006	-0.001
Real estate	0.109	0.106	0.108	0.111	-0.003	0.003
Transport	0.033	0.035	0.044	0.050	0.001	0.006
Communication	0.015	0.010	0.027	0.020	-0.005	-0.007
Public administ.	0.068	0.051	0.051	0.036	-0.017	-0.016
Services	0.271	0.248	0.264	0.234	-0.023	-0.030
Other Activities	0.000	-0.001	0.003	0.006	-0.001	0.003

Table 10 summarises the previous two tables providing the sectoral overall multipliers that come from the two dimensional distributions in value added, $\mathbf{H}_d$, and final demand, $\mathbf{Y}_d$. It is shown that services, manufacturing and contraction value added affect final demand. Although this happens since final demand influence value added mainly through services, manufacturing and commerce.

Figure 4

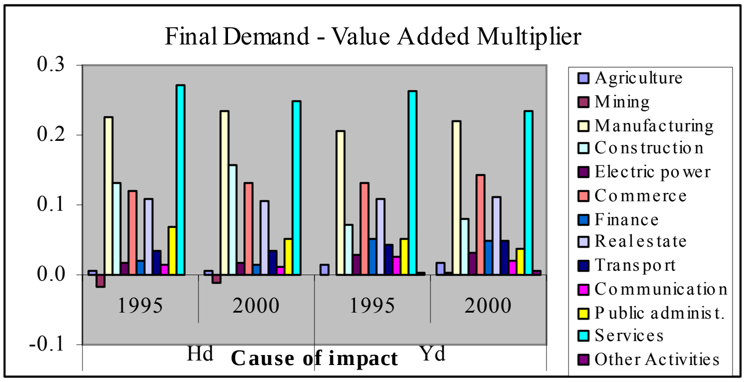


Figure 4 shows the outcome of Table 10. As one may observe, the role of services, although dominant, illustrates some decline, while on the contrary the role of manufacturing displays some strength. Likewise, the role of public administration as well as finance and insurance sectors diminishes in the final demand value added linkages.

4 *Conclusion*

This paper showed both theoretically and empirically, that linkage is an essential and important aspect of interindustry analysis. Technically, linkages are defined by a matrix multiplication. The columns and rows in the interindustry system are defined by the processing sectors as well as various types or final demand and value added. Each element of a matrix identifies how a column is related to a row. The interindustry transaction matrix depicts the relationship of purchases and sales among the processing sectors, the final demand matrix presents the sales of industrial sectors to the different types of final demand, and the value added matrix shows the non processing sectoral payments.

Having a complete set of input output data; the summation of the row elements gives the sectoral gross output and total value added, while the summation of the column elements provides the same sectoral output and the various aspects of final demand. From this we derive the *Production – Value Added Direct Markov Distribution* $\begin{bmatrix} \mathbf{A} \\ \mathbf{H}_x \end{bmatrix}$ (column wise) and its equivalent *Allocation – Final Demand Direct Markov Distribution* $\begin{bmatrix} \tilde{\mathbf{A}} & \mathbf{Y}_x \end{bmatrix}$ (row wise). These matrices show how a unit or sectoral product is distributed, either as purchases or sales. This is the basis for linear similarity that is utilized in this paper.

Linear similarity implies that two matrices, $\mathbf{A}$ and $\tilde{\mathbf{A}}$, may look different, but they possess the same structural properties if they $\mathbf{A}=diag(\mathbf{x})\,\tilde{\mathbf{A}}\,diag(\mathbf{x})^{-1}$ exists. Precisely, this is the case here, since $\mathbf{A}=\mathbf{X}\,diag(\mathbf{x})^{-1}$ and $\tilde{\mathbf{A}} = diag(\mathbf{x})^{-1}\mathbf{X}$.

Hence ***output multipliers*** are defined when we take into account total sectoral interdependences, as they are given by the Leontief inverse for production activity, and its similar (Ghosh) inverse for allocation activity. Multiplier $\mathbf{ZY}_m \leftarrow$ provides the impact of a yen in each final demand category, $\mathbf{ZY}_d \leftarrow$ the impact of a yen in final demand. The first multiplier assumes, for example, that private and public consumption is treated equivalently, while the second takes into account their difference in magnitude. These two multipliers relate final demand distributions to the rows of Leontief inverse. In the same manner, multipliers $\mapsto \mathbf{H}_m\tilde{\mathbf{Z}}$ and $\mapsto \mathbf{H}_d\tilde{\mathbf{Z}}$ relate value added distributions to the columns of the Ghosh inverse. These multipliers provide the impact of value added to gross sectoral output through the total interrelation of allocation. These multipliers may be decomposed based on the Taylor decomposition as $(\mathbf{I}+\mathbf{A}+\mathbf{A}^2+\mathbf{A}^3+\ldots)\mathbf{Y}_\mathrm{m}$ & $(\mathbf{I}+\mathbf{A}+\mathbf{A}^2+\mathbf{A}^3+\ldots)\mathbf{Y}_\mathrm{d}$ as well as $\mathbf{H}_\mathrm{m}(\mathrm{I}+\tilde{\mathbf{A}}+\tilde{\mathbf{A}}^2+\tilde{\mathbf{A}}^3+\ldots)$ & $\mathbf{H}_\mathrm{d}(\mathrm{I}+\tilde{\mathbf{A}}+\tilde{\mathbf{A}}^2+\tilde{\mathbf{A}}^3+\ldots)$.

The columns of the Leontief inverse and the rows of the Ghosh inverse provide meaningful linkage interactions with value added and final demand ratios to sectoral output, $\mathbf{H}_\mathrm{x}$ and $\mathbf{Y}_\mathrm{x}$ respectfully. These are the *Production – Value Added* Total Markov Distribution $\leftarrow \mathbf{H}_x\mathbf{Z}$ and *Allocation– Final Demand* Total Markov Distribution $\rightarrow \tilde{\mathbf{Z}}\mathbf{Y}_x$ of a sectoral output unit. These relationships provide the total aspect, in an analogous way that $\begin{bmatrix} \mathbf{A} \\ \mathbf{H}_x \end{bmatrix}$ and $\begin{bmatrix} \tilde{\mathbf{A}} & \mathbf{Y}_x \end{bmatrix}$ indicate the direct aspect of the distribution of a unit of sectoral output. As a result, they can be decomposed as $\mathbf{H}_\mathrm{x}(\mathbf{I}+\mathbf{A}+\mathbf{A}^2+\mathbf{A}^3+\mathbf{A}^4\ \ldots)$ and as $(\mathrm{I}+\tilde{\mathbf{A}}+\tilde{\mathbf{A}}^2+\tilde{\mathbf{A}}^3+\tilde{\mathbf{A}}^4+\ldots)\mathbf{Y}_\mathrm{x}$.

The above paragraphs indicate clearly that columns of rows of the Leontief and its similar inverses cannot be treated in the same way. The processes of capturing and transmitting in linear algebra are based on the distinction of a multiplication as pro- and post- multiplication from and to the appropriate matrices. Thus, Leontief inverse captures impact from final demand, and this affects its rows, $\mathbf{ZY}_d \leftarrow$, while it transmits results to the relationship of value added to gross output, and this affects its columns, $\leftarrow \mathbf{H}_x\mathbf{Z}$. The Ghosh inverse captures impact from the value added, and this affects its columns, $\mapsto \mathbf{H}_d\tilde{\mathbf{Z}}$, and transmits results to the relationship of final demand to gross output, and this affects its rows, $\rightarrow \tilde{\mathbf{Z}}\mathbf{Y}_x$.

Final Structure Matrices are distributions depicting a complete movement, from the origin (final demand or value added) to the destination (relationship of value added or final demand to gross output). As a result there is a ***Production*** *Final Structure Matrix* defined by Augusztinovics, $\mathbf{H}_x\mathbf{ZY}_m \leftarrow$, which is a Markov distribution, and the two dimensional distribution defined in this paper, $\mathbf{H}_x\mathbf{ZY}_d \leftarrow$. The Augusztinovics ***Allocation*** *Final Structure Matrix* is $\rightarrow \mathbf{H}_m\tilde{\mathbf{Z}}\mathbf{Y}_x$ and its two dimensional alternative, $\rightarrow \mathbf{H}_d\tilde{\mathbf{Z}}\mathbf{Y}_x$.

Feedback structures identify the linkage between the causality of an origin point (distribution) of the process to the result (its relationship to the value of output). Since we have two different points of origin, final demand and value added, we have two feedback structures. The final demand feedback structures are $\mathbf{Y}_m^T\mathbf{Z}^T\mathbf{H}_\mathbf{x}^T\mathbf{H}_m\tilde{\mathbf{Z}}\mathbf{Y}_\mathbf{x}$, & $\mathbf{Y}_d^T\mathbf{Z}^T\mathbf{H}_\mathbf{x}^T\mathbf{H}_d\tilde{\mathbf{Z}}\mathbf{Y}_\mathbf{x}$ and the value added feedback structures are $\mathbf{H}_m\widetilde{\mathbf{Z}}\mathbf{Y}_\mathbf{x}\mathbf{Y}_m^T\mathbf{Z}^T\mathbf{H}_\mathbf{x}^T$ & $\mathbf{H}_d\widetilde{\mathbf{Z}}\mathbf{Y}_\mathbf{x}\mathbf{Y}_d^T\mathbf{Z}^T\mathbf{H}_\mathbf{x}^T$. These are square matrices. Final demand feedback structures have the dimensions of final demand, while the value added feedback structures have the dimensions of value added. A noticeable fact is that $\mathbf{Y}_d^T\mathbf{Z}^T\mathbf{H}_\mathbf{x}^T\mathbf{H}_d\widetilde{\mathbf{Z}}\mathbf{Y}_\mathbf{x}$ has the same eigenvalues as matrix $\mathbf{H}_d\widetilde{\mathbf{Z}}\mathbf{Y}_\mathbf{x}\mathbf{Y}_d^T\mathbf{Z}^T\mathbf{H}_\mathbf{x}^T$. Matrices $\mathbf{Y}_\mathbf{x}\mathbf{Y}_d^T$ & $\mathbf{H}_\mathbf{x}^T\mathbf{H}_d$ are summing junctions, connectors for the production and allocation processes. The decision is when the distribution of final demand or value added is specified, and the termination when final demand or value added is contrasted to the realized value of gross sectoral output.

The relationship between final demand and value added in the sectoral aspect is given with the multipliers that final demand determines value added $\mathbf{H}^T\mathbf{i} = f(\mathbf{Yi})$, and value added determines final demand $\mathbf{Yi} = f(\mathbf{H}^T\mathbf{i})$. These matrices are: the allocation weighted Leontief inverse $[\mathbf{I}-\widetilde{\mathbf{A}}^T][\mathbf{I}-\mathbf{A}]^{-1}$, and the production weighted Ghosh inverse $[\mathbf{I}-\mathbf{A}][\mathbf{I}-\widetilde{\mathbf{A}}^T]^{-1}$.

Table 11
Linkages as Multipliers

Sectoral Multipliers	Capturing impact	Determining outcome
$\mathbf{ZY}_m \leftarrow$	A yen in all types of final demand distributed appropriately to the industrial sectors	Value of sectoral gross output
$\mathbf{ZY}_d \leftarrow$	A yen of final demand distributed appropriately to the industrial sectors & types of final demand	Value of sectoral gross output
$\mapsto \mathbf{H}_m\widetilde{\mathbf{Z}}$	A yen in all types of value added distributed appropriately to the industrial sectors	Value of sectoral gross output
$\mapsto \mathbf{H}_d\widetilde{\mathbf{Z}}$	A yen of value added distributed appropriately to the industrial sectors and types of value added	Value of sectoral gross output
$\mathbf{H}^T\mathbf{i} = f(\mathbf{Yi})$ $f = [\mathbf{I}-\widetilde{\mathbf{A}}^T][\mathbf{I}-\mathbf{A}]^{-1}$	Final demand, or a distribution of final demand	Value added, or a distribution of value added
$\mathbf{Yi} = f(\mathbf{H}^T\mathbf{i})$ $f = [\mathbf{I}-\mathbf{A}][\mathbf{I}-\widetilde{\mathbf{A}}^T]^{-1}$	Value added, or a distribution of value added	Final demand, or a distribution of final demand

What is being revealed is that the aggregate output multiplier is the same in both approaches. The proposed weighted multiplier is a correction to the traditional output multiplier. The Rasmussen indices are rejected as linkage measurement since they are simply linear transformations of the row and column summations of the appropriate Leontief and Ghosh inverses. Linkage is something that links. In genetics it is in the DNA, in mechanics it is in cams and levers. Linkages are multiplier matrices in interindustry analysis. As matrices, linkages link an action to its outcome. The Leontief and its similar inverse are the two basic linkage matrices. Since there are similar matrices, they have the same structural characteristics. These linkage measurements are appropriately weighted as Tables 11 &12 summarize.

Table 12

Linkages as Distributions

Column	Row	Two dimensional
$\leftarrow \mathbf{H}_x\mathbf{Z}$ Total distribution of a unit of output to value added categories	$\rightarrow \widetilde{\mathbf{Z}}\mathbf{Y}_x$ Total distribution of a unit of output to final demand categories	
$\mathbf{H}_x\mathbf{Z}\mathbf{Y}_m \leftarrow$ Distribution of a unit in each final demand category to value added categories	$\rightarrow \mathbf{H}_m\widetilde{\mathbf{Z}}\mathbf{Y}_x$ Distribution of a unit in each value added category to final demand categories	$\rightarrow \mathbf{H}_d\widetilde{\mathbf{Z}}\mathbf{Y}_x = \mathbf{H}_x\mathbf{Z}\mathbf{Y}_d \leftarrow$ Distribution of a final demand (value added) unit to the categories of value added (final demand)
$\mathbf{H}_m\widetilde{\mathbf{Z}}\mathbf{Y}_\mathbf{x}\mathbf{Y}_m^T\mathbf{Z}^T\mathbf{H}_\mathbf{x}^T$ Feedback of a unit in each value added category to the value added relationship to output	$\mathbf{Y}_m^T\mathbf{Z}^T\mathbf{H}_\mathbf{x}^T\mathbf{H}_m\widetilde{\mathbf{Z}}\mathbf{Y}_\mathbf{x}$ Feedback of a unit in each final demand category to the final demand relationship to output	$\mathbf{Y}_d^T\mathbf{Z}^T\mathbf{H}_\mathbf{x}^T\mathbf{H}_d\widetilde{\mathbf{Z}}\mathbf{Y}_\mathbf{x}$ $\mathbf{H}_d\widetilde{\mathbf{Z}}\mathbf{Y}_\mathbf{x}\mathbf{Y}_d^T\mathbf{Z}^T\mathbf{H}_\mathbf{x}^T$ Feedback of a final demand (value added) unit to final demand (value added) relationship to output ***(same eigenvalues)***

Distributions do not transmit an action providing outcome. They provide decompositions of a unit of final demand or value added or their categorical types. In that sense, they do indicate another aspect of a linkage.

Significant work has been recently published dealing with the linkage issue within the input-output framework. This paper did not deal with this discussion due to the enormity of associated information. Definitely, for a complete presentation of such a literature review, an analysis is required so that the topic can be comprehensively examined.

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Mathematical Appendix

Weighed Multipliers

Production – weighted by the Distribution Final Demand __multipliers__

$\mathbf{ZY}_m \leftarrow$ Augusztinovics type (1A)

$$\begin{bmatrix} \dfrac{z_{11}c_1 + z_{12}c_2 + \cdots + z_{1n}c_n}{\sum c} & \dfrac{z_{11}g_1 + z_{12}g_2 + \cdots + z_{1n}g_n}{\sum g} & \cdots & \dfrac{z_{11}m_1 + z_{12}m_2 + \cdots + z_{1n}m_n}{\sum m} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{z_{n1}c_1 + z_{n2}c_2 + \cdots + z_{nn}c_n}{\sum c} & \dfrac{z_{n1}g_1 + z_{n2}g_2 + \cdots + z_{nn}g_n}{\sum g} & \cdots & \dfrac{z_{n1}m_1 + z_{n2}m_2 + \cdots + z_{nn}m_n}{\sum m} \end{bmatrix}$$

$\mathbf{ZY}_d \leftarrow$ Adamou type (1B)

$$\begin{bmatrix} \dfrac{z_{11}c_1 + z_{12}c_2 + \cdots + z_{1n}c_n}{\sum y} & \dfrac{z_{11}g_1 + z_{12}g_2 + \cdots + z_{1n}g_n}{\sum y} & \cdots & \dfrac{z_{11}m_1 + z_{12}m_2 + \cdots + z_{1n}m_n}{\sum y} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{z_{n1}c_1 + z_{n2}c_2 + \cdots + z_{nn}c_n}{\sum y} & \dfrac{z_{n1}g_1 + z_{n2}g_2 + \cdots + z_{nn}g_n}{\sum y} & \cdots & \dfrac{z_{n1}m_1 + z_{n2}m_2 + \cdots + z_{nn}m_n}{\sum y} \end{bmatrix}$$

Allocation– weighted by the Distribution Value Added __multipliers__

$\mapsto \mathbf{H}_m\widetilde{\mathbf{Z}}$ Augusztinovics type (1C)

$$\begin{bmatrix} \left(w_1\widetilde{z}_{11} + w_2\widetilde{z}_{21} + \cdots + w_n\widetilde{z}_{n1}\right)\Big/\sum w & \cdots & \left(w_1\widetilde{z}_{1n} + w_2\widetilde{z}_{2n} + \cdots + w_n\widetilde{z}_{nn}\right)\Big/\sum w \\ \vdots & \ddots & \vdots \\ \left(v_1\widetilde{z}_{11} + v_2\widetilde{z}_{21} + \cdots + v_n\widetilde{z}_{n1}\right)\Big/\sum v & \cdots & \left(v_1\widetilde{z}_{1n} + v_2\widetilde{z}_{2n} + \cdots + v_n\widetilde{z}_{nn}\right)\Big/\sum v \end{bmatrix}$$

$\mapsto \mathbf{H}_d\widetilde{\mathbf{Z}}$ Adamou type (1D)

$$\begin{bmatrix} \left(w_1\widetilde{z}_{11} + w_2\widetilde{z}_{21} + \cdots + w_n\widetilde{z}_{n1}\right)\Big/\sum h & \cdots & \left(w_1\widetilde{z}_{1n} + w_2\widetilde{z}_{2n} + \cdots + w_n\widetilde{z}_{nn}\right)\Big/\sum h \\ \vdots & \ddots & \vdots \\ \left(v_1\widetilde{z}_{11} + v_2\widetilde{z}_{21} + \cdots + v_n\widetilde{z}_{n1}\right)\Big/\sum h & \cdots & \left(v_1\widetilde{z}_{1n} + v_2\widetilde{z}_{2n} + \cdots + v_n\widetilde{z}_{nn}\right)\Big/\sum h \end{bmatrix}$$

Production – Value Added <u>Total</u> Markov <u>Distribution</u> $\leftarrow \mathbf{H}_x\mathbf{Z}$ (2A)

$$\begin{bmatrix} \left(\frac{w_1}{x_1}\right)z_{11}+\left(\frac{w_2}{x_2}\right)z_{21}+\cdots+\left(\frac{w_n}{x_n}\right)z_{n1} & \cdots & \left(\frac{w_1}{x_1}\right)z_{1n}+\left(\frac{w_2}{x_2}\right)z_{2n}+\cdots+\left(\frac{w_n}{x_n}\right)z_{nn} \\ \vdots & \ddots & \vdots \\ \left(\frac{v_1}{x_1}\right)z_{11}+\left(\frac{v_2}{x_2}\right)z_{21}+\cdots+\left(\frac{v_n}{x_n}\right)z_{n1} & \cdots & \left(\frac{v_1}{x_1}\right)z_{1n}+\left(\frac{v_2}{x_2}\right)z_{2n}+\cdots+\left(\frac{v_n}{x_n}\right)z_{nn} \end{bmatrix}$$

Analogous to *Production – Value Added <u>Direct</u> Markov <u>Distribution</u>* $\begin{bmatrix} \mathbf{A} \\ \mathbf{H}_x \end{bmatrix}$ (2A1)

Allocation– Final Demand <u>Total</u> Markov <u>Distribution</u> $\rightarrow \tilde{\mathbf{Z}}\mathbf{Y}_x$ (2B)

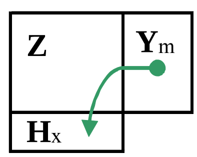
$$\begin{bmatrix} \tilde{z}_{11}\left(\frac{y_{C1}}{x_1}\right)+\tilde{z}_{12}\left(\frac{y_{C2}}{x_1}\right)+\cdots+\tilde{z}_{1n}\left(\frac{y_{Cn}}{x_n}\right) & \cdots & \tilde{z}_{11}\left(\frac{y_{M1}}{x_1}\right)+\tilde{z}_{12}\left(\frac{y_{M2}}{x_1}\right)+\cdots+\tilde{z}_{1n}\left(\frac{y_{Mn}}{x_n}\right) \\ \vdots & \ddots & \vdots \\ \tilde{z}_{n1}\left(\frac{y_{C1}}{x_1}\right)+\tilde{z}_{n2}\left(\frac{y_{C2}}{x_1}\right)+\cdots+\tilde{z}_{nn}\left(\frac{y_{Cn}}{x_n}\right) & \cdots & \tilde{z}_{n1}\left(\frac{y_{M1}}{x_1}\right)+\tilde{z}_{n2}\left(\frac{y_{M2}}{x_1}\right)+\cdots+\tilde{z}_{nn}\left(\frac{y_{Mn}}{x_n}\right) \end{bmatrix}$$

Analogous to *Allocation – Final Demand <u>Direct</u> Markov <u>Distribution</u>* $\begin{bmatrix} \tilde{\mathbf{A}} & \mathbf{Y}_x \end{bmatrix}$ (2B1)

Final Structure Matrices (3)

Production Final Structure Matrix *Augusztinovics type* $\mathbf{H}_x\mathbf{Z}\mathbf{Y}_m \leftarrow$ (3A)

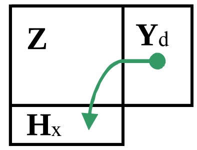
<u>Markov Distribution</u>



$$\begin{bmatrix} \frac{w_1}{x_1}\frac{z_{11}y_{C1}+\cdots+z_{1n}y_{Cn}}{\sum y_C}+\cdots+\frac{w_n}{x_n}\frac{z_{n1}y_{C1}+\cdots+z_{nn}y_{Cn}}{\sum y_C} & \cdots & \frac{w_1}{x_1}\frac{z_{11}y_{M1}+\cdots+z_{1n}y_{Mn}}{\sum y_M}+\cdots+\frac{w_n}{x_n}\frac{z_{n1}y_{M1}+\cdots+z_{nn}y_{Mn}}{\sum y_M} \\ \frac{t_1}{x_1}\frac{z_{11}y_{C1}+\cdots+z_{1n}y_{Cn}}{\sum y_C}+\cdots+\frac{t_n}{x_n}\frac{z_{n1}y_{C1}+\cdots+z_{nn}y_{Cn}}{\sum y_C} & \cdots & \frac{t_1}{x_1}\frac{z_{11}y_{M1}+\cdots+z_{1n}y_{Mn}}{\sum y_M}+\cdots+\frac{t_n}{x_n}\frac{z_{n1}y_{M1}+\cdots+z_{nn}y_{Mn}}{\sum y_M} \\ \vdots & \ddots & \vdots \\ \frac{v_1}{x_1}\frac{z_{11}y_{C1}+\cdots+z_{1n}y_{Cn}}{\sum y_C}+\cdots+\frac{v_n}{x_n}\frac{z_{n1}y_{C1}+\cdots+z_{nn}y_{Cn}}{\sum y_C} & \cdots & \frac{v_1}{x_1}\frac{z_{11}y_{M1}+\cdots+z_{1n}y_{Mn}}{\sum y_M}+\cdots+\frac{v_n}{x_n}\frac{z_{n1}y_{M1}+\cdots+z_{nn}y_{Mn}}{\sum y_M} \end{bmatrix}$$

Production Final Structure Matrix, **Adamou type** $\mathbf{H}_x\mathbf{Z}\mathbf{Y}_d \leftarrow$ (3B)

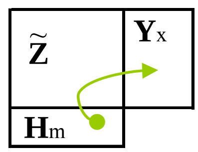
Two – Dimensional Distribution



$$\begin{bmatrix} \frac{w_1}{x_1}\frac{z_{11}y_{C1}+\cdots+z_{1n}y_{Cn}}{\sum y}+\cdots+\frac{w_n}{x_n}\frac{z_{n1}y_{C1}+\cdots+z_{nn}y_{Cn}}{\sum y} & \cdots & \frac{w_1}{x_1}\frac{z_{11}y_{M1}+\cdots+z_{1n}y_{Mn}}{\sum y}+\cdots+\frac{w_n}{x_n}\frac{z_{n1}y_{M1}+\cdots+z_{nn}y_{Mn}}{\sum y} \\ \frac{t_1}{x_1}\frac{z_{11}y_{C1}+\cdots+z_{1n}y_{Cn}}{\sum y}+\cdots+\frac{t_n}{x_n}\frac{z_{n1}y_{C1}+\cdots+z_{nn}y_{Cn}}{\sum y} & \cdots & \frac{t_1}{x_1}\frac{z_{11}y_{M1}+\cdots+z_{1n}y_{Mn}}{\sum y}+\cdots+\frac{t_n}{x_n}\frac{z_{n1}y_{M1}+\cdots+z_{nn}y_{Mn}}{\sum y} \\ \vdots & \ddots & \vdots \\ \frac{v_1}{x_1}\frac{z_{11}y_{C1}+\cdots+z_{1n}y_{Cn}}{\sum y}+\cdots+\frac{v_n}{x_n}\frac{z_{n1}y_{C1}+\cdots+z_{nn}y_{Cn}}{\sum y} & \cdots & \frac{v_1}{x_1}\frac{z_{11}y_{M1}+\cdots+z_{1n}y_{Mn}}{\sum y}+\cdots+\frac{v_n}{x_n}\frac{z_{n1}y_{M1}+\cdots+z_{nn}y_{Mn}}{\sum y} \end{bmatrix}$$

Allocation Final Structure Matrix Augusztinovics type $\rightarrow \mathbf{H}_m\tilde{\mathbf{Z}}\mathbf{Y}_x$ (3C)

Markov Distribution



$$\begin{matrix} \frac{y_{C1}}{x_1}\frac{w_1\tilde{z}_{11}+\cdots+w_n\tilde{z}_{n1}}{\sum w}+\cdots+\frac{y_{Cn}}{x_n}\frac{w_1\tilde{z}_{1n}+\cdots+w_n\tilde{z}_{nn}}{\sum w} & \cdots & \frac{y_{M1}}{x_1}\frac{w_1\tilde{z}_{11}+\cdots+w_n\tilde{z}_{n1}}{\sum w}+\cdots+\frac{y_{Mn}}{x_n}\frac{w_1\tilde{z}_{1n}+\cdots+w_n\tilde{z}_{nn}}{\sum w} \\ \frac{y_{C1}}{x_1}\frac{t_1\tilde{z}_{11}+\cdots+t_n\tilde{z}_{n1}}{\sum t}+\cdots+\frac{y_{Cn}}{x_n}\frac{t_1\tilde{z}_{1n}+\cdots+t_n\tilde{z}_{nn}}{\sum t} & \cdots & \frac{y_{M1}}{x_1}\frac{t_1\tilde{z}_{11}+\cdots+t_n\tilde{z}_{n1}}{\sum t}+\cdots+\frac{y_{Mn}}{x_n}\frac{t_1\tilde{z}_{1n}+\cdots+t_n\tilde{z}_{nn}}{\sum t} \\ \vdots & \ddots & \vdots \\ \frac{y_{C1}}{x_1}\frac{v_1\tilde{z}_{11}+\cdots+v_n\tilde{z}_{n1}}{\sum v}+\cdots+\frac{y_{Cn}}{x_n}\frac{v_1\tilde{z}_{1n}+\cdots+v_n\tilde{z}_{nn}}{\sum v} & \cdots & \frac{y_{M1}}{x_1}\frac{v_1\tilde{z}_{11}+\cdots+v_n\tilde{z}_{n1}}{\sum v}+\cdots+\frac{y_{Mn}}{x_n}\frac{v_1\tilde{z}_{1n}+\cdots+v_n\tilde{z}_{nn}}{\sum v} \end{matrix}$$

Allocation Final Structure Matrix Adamou type $\rightarrow \mathbf{H}_d\tilde{\mathbf{Z}}\mathbf{Y}_x$ (3D)

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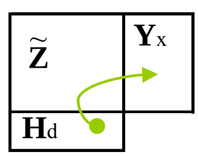

$$\begin{matrix} \frac{y_{C1}}{x_1}\frac{w_1\tilde{z}_{11}+\cdots+w_n\tilde{z}_{n1}}{\sum h}+\cdots+\frac{y_{Cn}}{x_n}\frac{w_1\tilde{z}_{1n}+\cdots+w_n\tilde{z}_{nn}}{\sum h} & \cdots & \frac{y_{M1}}{x_1}\frac{w_1\tilde{z}_{11}+\cdots+w_n\tilde{z}_{n1}}{\sum h}+\cdots+\frac{y_{Mn}}{x_n}\frac{w_1\tilde{z}_{1n}+\cdots+w_n\tilde{z}_{nn}}{\sum h} \\ \frac{y_{C1}}{x_1}\frac{t_1\tilde{z}_{11}+\cdots+t_n\tilde{z}_{n1}}{\sum h}+\cdots+\frac{y_{Cn}}{x_n}\frac{t_1\tilde{z}_{1n}+\cdots+t_n\tilde{z}_{nn}}{\sum h} & \cdots & \frac{y_{M1}}{x_1}\frac{t_1\tilde{z}_{11}+\cdots+t_n\tilde{z}_{n1}}{\sum h}+\cdots+\frac{y_{Mn}}{x_n}\frac{t_1\tilde{z}_{1n}+\cdots+t_n\tilde{z}_{nn}}{\sum h} \\ \vdots & \ddots & \vdots \\ \frac{y_{C1}}{x_1}\frac{v_1\tilde{z}_{11}+\cdots+v_n\tilde{z}_{n1}}{\sum h}+\cdots+\frac{y_{Cn}}{x_n}\frac{v_1\tilde{z}_{1n}+\cdots+v_n\tilde{z}_{nn}}{\sum h} & \cdots & \frac{y_{M1}}{x_1}\frac{v_1\tilde{z}_{11}+\cdots+v_n\tilde{z}_{n1}}{\sum h}+\cdots+\frac{y_{Mn}}{x_n}\frac{v_1\tilde{z}_{1n}+\cdots+v_n\tilde{z}_{nn}}{\sum h} \end{matrix}$$