

# A Simple Dynamic Waste Input-Output Analysis: Theory and Its Application to the Japanese Economy

YOKOYAMA Kazuyo <sup>\*</sup> , KAGAWA Shigemi <sup>†</sup>

## Abstract

The present paper proposes a simple dynamic waste input-output model definitely considering the waste generation and treatment structures and empirically examines the dependent relationship between capital accumulation and waste treatment and recycling focusing on the Japanese economy. From the empirical results, we find that not only the waste treatment strategies affect the balanced growth path but also the strategic shift from waste landfill to waste incineration brings about the increase in the balanced growth rate.

**Keyword:** Dynamic input-output model; balanced growth path; capital accumulation; waste treatment and recycling;

**JEL Classification Numbers:** O41, Q00, Q32

---

<sup>\*</sup>Graduate School of Environmental Studies, Tohoku University, Aoba-ku, Sendai, 980-8579, Japan [yokoyama@mail.kankyo.tohoku.ac.jp](mailto:yokoyama@mail.kankyo.tohoku.ac.jp)

<sup>†</sup>Faculty of Economics, Kyushu University, Higashi-ku, Fukuoka, 814-8581, Japan [kagawa@en.kyushu-u.ac.jp](mailto:kagawa@en.kyushu-u.ac.jp)

# 1 Introduction

Examining the relationship among economic growth, waste treatment and recycling, and environmental externalities is fundamental in evaluating and developing environmental and resource policies. The following dynamic input-output model proposed by Leontief (1953), Leontief (1970) is useful in analyzing the multi sector growth path based on the policy scenarios.

$$(x_1)_{i:t} = \sum_{j=1}^N (a_{11})_{ij} (x_1)_{j:t} + \sum_{j=1}^N (b_{11})_{ij} \{(x_1)_{j:t+1} - (x_1)_{j:t}\} + (f_1)_{i:t} \quad (i = 1, 2, \dots, N) \quad (1)$$

where  $(x_1)_{i:t}$  is the total output of goods  $i$  in time period  $t$ ,  $(a_{11})_{ij}$  is the technical coefficient showing the intermediate input requirement of goods  $i$  required to produce a unit of goods  $j$ ,  $(b_{11})_{ij}$  is the capital coefficient showing the capital goods  $i$  required to produce a unit of goods  $j$ ,  $(f_1)_{i:t}$  is the final demand of goods  $i$  in time period  $t$ . It should be noted that  $\sum_{j=1}^N b_{ij}(x_{j:t+1} - x_{j:t})$  represents the total intermediate input of capital goods  $i$  required for goods productions and the technical coefficients remain constant over time periods. In algebraic form, we can rewrite the following equation.

$$\mathbf{X}_{1:t} = \mathbf{A}_{11} \mathbf{X}_{1:t} + \mathbf{B}_{11} (\mathbf{X}_{1:t+1} - \mathbf{X}_{1:t}) + \mathbf{F}_{1:t} \quad (2)$$

where  $\mathbf{X}_{1:t}$  is the total output vector,  $\mathbf{A}_{11}$  technical coefficient matrix,  $\mathbf{B}_{11}$  capital coefficient matrix, and  $\mathbf{F}_{1:t}$  is the final demand vector.

In the past, there were many discussions of the dynamic Leontief system, for instance, on the relationship between time lag of output and dynamic stability (see Sargan (1958), Sargan (1961); Leontief (1961a), Leontief (1961b)), the relative stability and instability of the dynamic system (see Morishima (1958); Solow (1989); Jorgenson (1960), Jorgenson (1961); Tokoyama and Murakami (1972)), the singularity of the capital coefficient matrix (see Kendrick (1972); Luenberger and Arbel (1977); Meyer (1982)), generalization of the dynamic Leontief system (see Bródy (1974); Johansen (1978); Åberg and Persson (1981); ten Raa (1986b)), and numerical solution methods (see Almon (1963); Duchin and Szyld (1985); ten Raa (1986a)). More recently, Kurz and Salvadori (2000) developed a dynamic input-output model which outlines AK model, while Los (2001) proposed an analytical dynamic input-output model with some endogenous growth properties. However, the above-mentioned frameworks did not definitely deal with the joint-production structure such as waste generations and treatments.

Nakamura (1999) and Nakamura and Kondo (2002) proposed an analytical framework describing interdependence between goods productions and waste treatments and recycling, which is named Waste Input-Output model. The model enables us to evaluate life cycle scenarios considering resource and waste recycling strategies such as recycling of End of Life Electric

Home Appliances. For instance, Nakamura and Kondo (2006) compared the cheapest air conditioners (the low-end model) and the most expensive one (the high-end model) using the waste input-output model and found that in spite of largerst amount of input of production phase, the high-end model performs the best in terms of both global warming potential and landfill, while the low-end model performs the worst. Although the model is widely used, it is static and difficult to evaluate the relationship between economic growth and waste management strategies. Yokoyama et al. (2006) extended the static model into the dynamic one and evaluated the energy requirement and CO2 emission for the landfill mining activity, however the mathematical properties of the dynamic model and the effect of the waste management strategies on the balanced growth path were not explored. More recently, Dobos and Floriska (2007) generalized the dynamic input-output model to investigate the impact of recycling on the use of non-renewable resources. The important difference between Dobos and Floriska (2007) and the present paper lies in dealing with the waste from discarded capital goods. We definitely distinguished between the industrial and obsolete waste generations and recycling.

The present paper derives the relationship between economic growth and waste management and empirically studies the effect of the incineration and landfill selection strategies on the balanced growth rate using the Japanese waste input-output table. Following the introduction, section 2 exposes the basic framework of the dynamic waste input-output model and formulates it, section 3 explains the data consturction, section 4 provides the results of the scenario analysis, and finally section 5 discusses and concludes.

## 2 Formulation of the dynamic waste input-output model

Figure 1 shows the basic framework of the dynamic waste input-output model. The Figure 1 shows that commodities are not only required as intermediate inputs but also purchased as final consumptions and capital investments (see the solid arrows A, B, and C). At the same time, wastes are generated from production processes and treated by waste treatment sectors (see the dotted arrow E) and the waste treatment sectors require commodities in order to treat the wastes (see the solid arrow D) and recover secondary resources (see the dotted arrow F). The wastes from the final consumptions and discarded capital goods are also flowing to the waste treatment sectors (see G and H).

In what follows, we formulate the dynamic waste input-output model. Although equations.(1) and (2) describe the dynamic interdependence of economic activities, the joint-products such as wastes are not dealt with considered. We can endogenously treat wastes within the dynamic model. Here let us assume that  $M$  industrial wastes are jointly generated by  $N$  ordinary

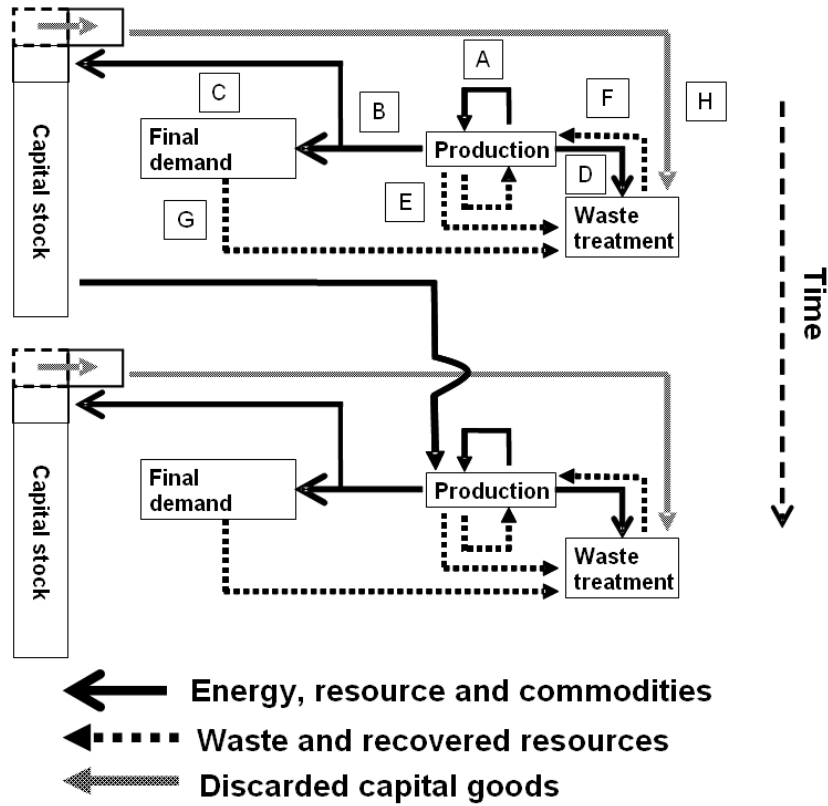


Fig. 1: Basic illustration of the dynamic waste input-output model

economic activities and treated by  $K$  waste treatment activities.

Since the waste treatment activities require both intermediate goods and capital goods in order to treat and recycling the waste, equations.(1) and (2) can be rewritten as

$$(x_1)_{i:t} = \sum_{j=1}^N (a_{11})_{ij} (x_1)_{j:t} + \sum_{k=1}^K (a_{12})_{ik} (x_2)_{k:t} + \sum_{j=1}^N (b_{11})_{ij} \{(x_1)_{j:t+1} - (x_1)_{j:t}\} \\ + \sum_{j=1}^K (b_{12})_{ik} \{(x_2)_{k:t+1} - (x_2)_{k:t}\} + (f_1)_{i:t} \quad (i = 1, 2, \dots, N) \quad (3)$$

In algebraic form,

$$X_{1:t} = A_{11}X_{1:t} + A_{12}X_{2:t} + B_{11}(X_{1:t+1} - X_{1:t}) + B_{12}(X_{2:t+1} - X_{2:t}) + F_{1:t} \quad (4)$$

where  $(x_2)_{k:t}$  is the activity level of waste treatment  $k$  in time period  $t$ ,  $(a_{12})_{ik}$  is the technical coefficient that represents the intermediate input requirement of goods  $i$  required to treat a unit of waste treatment  $k$ ,  $(b_{12})_{ik}$  is the capital coefficient that denotes the capital goods  $i$  accumulated to a unit activity of waste treatment activity  $k$ .

Equations.(3) and (4) expresses the dynamic material balance relating to goods productions and waste treatment. Subsequently, it is necessary to formulate the waste generations flowing to the waste treatment activities.

If  $(\bar{a}_{21})_{ij}$  is defined as the waste generation coefficient representing waste  $i$  generated by unit production of goods  $j$  and  $(\bar{a}_{22})_{ik}$  is defined as the waste residue coefficient representing waste  $i$  generated by unit activity of waste treatment  $k$ ,  $(\bar{b}_{21})_{ij}$  the net waste generation coefficients representing waste  $i$  generated from scrapping unit of capital goods  $j$  and  $(\bar{b}_{22})_{ik}$  is defined as the net waste generation coefficient representing waste  $i$  generated by scrapping capital goods per unit activity of waste treatment  $k$ . Waste input output table is a hybrid type of input output table, that is combined both monetary based information and quantity based data about waste generation and recycling. Hence, note that  $(\bar{a}_{21})_{ij}$ ,  $(\bar{a}_{22})_{ik}$ ,  $(\bar{b}_{21})_{ij}$  and  $(\bar{b}_{22})_{ik}$  are derived on quantity based data.

We have the following dynamic waste generation equation.

$$(x_2)_{i:t} = \sum_{j=1}^N (\bar{a}_{21})_{ij} (x_1)_{j:t} + \sum_{k=1}^K (\bar{a}_{22})_{ik} (x_2)_{k:t} + \sum_{j=1}^N (\bar{b}_{21})_{ij} \{(x_2)_{j:t+1} - (x_2)_{j:t}\} \\ + \sum_{k=1}^K (\bar{b}_{22})_{ik} \{(x_2)_{k:t+1} - (x_2)_{k:t}\} + (\bar{f}_2)_{i:t} \quad (i = 1, 2, \dots, M) \quad (5)$$

or

$$\bar{X}_{2:t} = \bar{A}_{21}X_{1:t} + \bar{A}_{22}X_{2:t} + \bar{B}_{21}(X_{1:t+1} - X_{1:t}) + \bar{B}_{22}(X_{2:t+1} - X_{2:t}) + F_{2:t} \quad (6)$$

Here,  $F_{2:t} = (f_2)_{i:t}$  denotes the net waste generations mainly by household and government consumption.

Arranging equations.(5) and (6) yield

$$\begin{bmatrix} \mathbf{X}_{1:t} \\ \bar{\mathbf{X}}_{2:t} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11:t} & \mathbf{A}_{12:t} \\ \bar{\mathbf{A}}_{21:t} & \bar{\mathbf{A}}_{22:t} \end{bmatrix} \begin{bmatrix} \mathbf{X}_{1:t} \\ \mathbf{X}_{2:t} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{11:t} & \mathbf{B}_{12:t} \\ \bar{\mathbf{B}}_{21:t} & \bar{\mathbf{B}}_{22:t} \end{bmatrix} \left( \begin{bmatrix} \mathbf{X}_{1:t+1} \\ \mathbf{X}_{2:t+1} \end{bmatrix} - \begin{bmatrix} \mathbf{X}_{1:t} \\ \mathbf{X}_{2:t} \end{bmatrix} \right) + \begin{bmatrix} \mathbf{F}_{1:t} \\ \bar{\mathbf{F}}_{2:t} \end{bmatrix} \quad (7)$$

Such equation.(7) is the rectangular differential model, it is necessary to transform the square model as in Nakamura and Kondo (2002). Defining the waste allocation matrix  $S = (s_{ij})$  representing the share of waste  $j$  treated by waste treatment  $i$ . We can formulate the following the square dynamic system.

$$\begin{bmatrix} \mathbf{X}_{1:t} \\ \mathbf{X}_{2:t} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11:t} & \mathbf{A}_{12:t} \\ \mathbf{A}_{21:t} & \mathbf{A}_{22:t} \end{bmatrix} \begin{bmatrix} \mathbf{X}_{1:t} \\ \mathbf{X}_{2:t} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{11:t} & \mathbf{B}_{12:t} \\ \mathbf{B}_{21:t} & \mathbf{B}_{22:t} \end{bmatrix} \left( \begin{bmatrix} \mathbf{X}_{1:t+1} \\ \mathbf{X}_{2:t+1} \end{bmatrix} - \begin{bmatrix} \mathbf{X}_{1:t} \\ \mathbf{X}_{2:t} \end{bmatrix} \right) + \begin{bmatrix} \mathbf{F}_{1:t} \\ \mathbf{F}_{2:t} \end{bmatrix} \quad (8)$$

with  $\mathbf{A}_{21} = \mathbf{S}\bar{\mathbf{A}}_{21}$ ,  $\mathbf{A}_{22} = \mathbf{S}\bar{\mathbf{A}}_{22}$ ,  $\mathbf{B}_{21} = \mathbf{S}\bar{\mathbf{B}}_{21}$ ,  $\mathbf{B}_{22} = \mathbf{S}\bar{\mathbf{B}}_{22}$ ,  $\mathbf{F}_2 = \mathbf{S}\bar{\mathbf{F}}_2$ .

Focusing on the closed dynamic waste input-output model and solving in terms of  $(\mathbf{X}_{1:t}, \mathbf{X}_{2:t})$ , we can finally obtain:

$$\begin{bmatrix} \mathbf{X}_{1:t+1} \\ \mathbf{X}_{2:t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{11:t} & \mathbf{B}_{12:t} \\ \mathbf{B}_{21:t} & \mathbf{B}_{22:t} \end{bmatrix}^{-1} \left( \mathbf{I} - \begin{bmatrix} \mathbf{A}_{11:t} & \mathbf{A}_{12:t} \\ \mathbf{A}_{21:t} & \mathbf{A}_{22:t} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{11:t} & \mathbf{B}_{12:t} \\ \mathbf{B}_{21:t} & \mathbf{B}_{22:t} \end{bmatrix} \right) \begin{bmatrix} \mathbf{X}_{1:t} \\ \mathbf{X}_{2:t} \end{bmatrix} \quad (9)$$

$$= \left\{ \mathbf{I} + \begin{bmatrix} \mathbf{B}_{11:t} & \mathbf{B}_{12:t} \\ \mathbf{B}_{21:t} & \mathbf{B}_{22:t} \end{bmatrix}^{-1} \left( \mathbf{I} - \begin{bmatrix} \mathbf{A}_{11:t} & \mathbf{A}_{12:t} \\ \mathbf{A}_{21:t} & \mathbf{A}_{22:t} \end{bmatrix} \right) \right\} \begin{bmatrix} \mathbf{X}_{1:t} \\ \mathbf{X}_{2:t} \end{bmatrix} \quad (10)$$

If the Steenge and Thissen (2005) condition holds, we have the following condition

$$\left( \mathbf{I} + \begin{bmatrix} \mathbf{A}_{11:t} & \mathbf{A}_{12:t} \\ \mathbf{A}_{21:t} & \mathbf{A}_{22:t} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathbf{B}_{11:t} & \mathbf{B}_{12:t} \\ \mathbf{B}_{21:t} & \mathbf{B}_{22:t} \end{bmatrix} > \mathbf{0} \quad (11)$$

in the simple expression,

$$(\mathbf{I} + \mathbf{A}^*)^{-1} \mathbf{B}^* > \mathbf{0} \quad (12)$$

because the enlarged capital matrix is nonnegative. The closed dynamic system plays crucial role in determining the general solutions of equations.(10). Especially, the eigenvalues  $\lambda_i$ , ( $i = 1, \dots, N + K$ ) and eigenvectors  $\mathbf{u}_i$ , ( $i = 1, \dots, N + K$ ) of the enlarged matrix corresponding to equation.(10),  $\mathbf{E} = (\mathbf{I} - \mathbf{A}^*)^{-1} \mathbf{B}^*$  affect the particular solution of capital and waste accumulation.

The important point is that if the matrix  $\mathbf{E}$  is indecomposable and non-negative, there always exists positive dominant eigenvalue  $\lambda(\mathbf{E})$  corresponding to a positive eigenvector from Perron-Frobenius Theorem. In this case, the dynamic system shown in equation.(10) is dominated by the following eigenvalues and the corresponding eigenvectors.

$$\theta_i = 1 + \frac{1}{\lambda}, u_i \quad (i = 1, \dots, N + K) \quad (13)$$

The well-known relative stability condition can be written as

$$\theta_1 > |\theta_i| \Rightarrow 1 + \frac{1}{\lambda_1} > \left|1 + \frac{1}{\lambda_i}\right| \quad i = 2, \dots, N + K. \quad (14)$$

In this case, the general solution of the closed dynamic system can be obtained as

$$\mathbf{X}_t = \alpha_1 \left(1 + \frac{1}{\lambda_1}\right)^t \mathbf{U}_1 + \alpha_2 \left(1 + \frac{1}{\lambda_2}\right)^t \mathbf{U}_2 + \dots + \alpha_{K+N} \left(1 + \frac{1}{\lambda_{K+N}}\right)^t \mathbf{U}_{K+N} \quad (15)$$

$$= \sum_{i=1}^{K+N} \alpha_i \left(1 + \frac{1}{\lambda_i}\right)^t \mathbf{U}_i. \quad (16)$$

On the other hand, the general solution of standard dynamic Leontief model can be obtained as

$$\mathbf{X}_{1:t} = \beta_1 \left(1 + \frac{1}{\mu_1}\right)^t \mathbf{V}_1 + \beta_2 \left(1 + \frac{1}{\mu_2}\right)^t \mathbf{V}_2 + \dots + \beta_N \left(1 + \frac{1}{\mu_{K+N}}\right)^t \mathbf{V}_N \quad (17)$$

$$= \sum_{i=1}^N \beta_i \left(1 + \frac{1}{\mu_i}\right)^t \mathbf{V}_i \quad (18)$$

where  $\mu_i$  and  $\mathbf{V}_i$  are the eigenvalues and eigenvectors of the matrix  $(I - A_{11})^{-1}B_{11}$ .

Noting that enlarged of dynamic system is influenced by the waste allocation matrix showing the waste management strategies, it can be understood that the eigenvalues and eigenvectors are the function of  $\mathbf{S}$ , namely

$$\mathbf{X}_t = \sum_{i=1}^{K+N} \alpha_i \left(1 + \frac{1}{\lambda_i(\mathbf{S})}\right)^t \mathbf{U}_i(\mathbf{S}) \quad (19)$$

where  $1 + 1/\lambda_1(\mathbf{S})$  denotes Von Neumann growth rate.

For the modern sound material cycle society, it is very important to maximize the von Neumann growth rate considering the resource and waste management scenarios.

The problem can be written as

$$\mathbf{S} \in \arg \max 1 + \frac{1}{\lambda_1(\mathbf{S})} \quad (20)$$

$$s.t. \quad \sum_{j=1}^K s_{ij} = 1, \quad s_{ij} \geq 0. \quad (21)$$

Note that the allocation matrix  $\mathbf{S}$  represents the share of waste treated by waste treatment and determined by social regulation, waste treatment technology or political choice. If we can find an optimal waste allocation structure  $\mathbf{S}'$ , it is useful in evaluating the dynamically efficient allocation from the point of view of the relationship between economic growth and waste management.

### 3 Data construction

We set up the data mainly based on Waste Input Output Table for Japan(WIO table)(Nakamura (2004)) and the capital formation matrix from the supplementary table of Input Output Table for Japan(JIO table)(Ministry of Internal Affairs and Communications (2004)).

WIO table is developed on the basis of JIO table, and physical data about waste generation and recycling is collected from official data published by various kind of industrial organization and local governments and some interviews, while inventory data associated with waste treatment activity refers to the waste treatment simulation model(Matsuto (2005)). WIO table has 103 industrial sectors, 13 waste treatment sectors, and deals with 79 kind of wastes. Considering both capital accumulation and the amount of discarded capital goods, construction sector plays an important role. Thus we added more information about construction and civil engineering sectors by using the Input-output Tables of Subdivided Construction Sectors(Ministry of Land Infrastructure and Transport (2004)). In this table, construction and civil engineering sectors are divided into 42 sectors.

Making the extended capital coefficient matrix  $B^*$ , we assume that the waste from discarded capital goods only generated from construction and civil engineering sectors in this study, because it is much more work to divide all the discarded capital goods including cars, machinery and so on, from the process wastes. And life time is exogenously fixed in  $B^*$ , but it is not difficult to extend the model which determine the amount of obsolete waste generation through the life time distributions. However we focus on the dynamic property derived from the most simple DWIO model, these modifications and extensions should be discussed continuously as future issues.

### 4 Scenario Analysis

In this section, we apply the DWIO model to evaluate balanced growth rate considering the types of waste treatment facilities and its policy changes. In scenario 1, we consider the effects on the balanced growth rate, by selecting waste treatment options, incineration or landfill. In scenario 2, two different levels of capital accumulation in waste treatment sectors are considered, and its effect on balanced growth rate is evaluated.

#### 4.1 Scenario 1

In this scenario, as waste treatment options, incineration and landfill are considered about these 22 types of wastes. Originally the allocation between waste treatment sectors is shown in



Table 1. Here we consider how much balanced growth rate will change as the waste treatment policy changes.

Table. 1: Allocation matrix of the basic scenario

A part of Allocation matrix	Incineration (continuous type with generator)	Incineration (continuous type without generator)	Incineration (batch type)	Landfill
(H) Food waste	0.3087	0.1296	0.4617	0.1
(H) Old newspaper	0.3087	0.1296	0.4617	0.1
(H) Tired magazine	0.3087	0.1296	0.4617	0.1
(H) Waste cardboard	0.3087	0.1296	0.4617	0.1
(H) Paper drink box	0.3087	0.1296	0.4617	0.1
(H) Paper box, bag & package	0.3087	0.1296	0.4617	0.1
(H) Other waste paper	0.3087	0.1296	0.4617	0.1
(H) Waste textile	0.3087	0.1296	0.4617	0.1
(H) PET bottle	0.0343	0.0144	0.0513	0.9
(H) Other plastic bottle	0.0343	0.0144	0.0513	0.9
(H) Plastic container, cup & tray	0.2744	0.1152	0.4104	0.2
(H) Plastic bag, sheet & package	0.3087	0.1296	0.4617	0.1
(H) Other plastics article	0.0343	0.0144	0.0513	0.9
(B) Food waste	0.3087	0.1296	0.4617	0.1
(B) Old newspaper & magazine	0.3087	0.1296	0.4617	0.1
(B) Waste cardboard	0.3087	0.1296	0.4617	0.1
(B) Waste quality paper	0.3087	0.1296	0.4617	0.1
(B) Other waste paper	0.3087	0.1296	0.4617	0.1
(B) Waste textile	0.3087	0.1296	0.4617	0.1
(B) PET bottle	0.0343	0.0144	0.0513	0.9
(B) Waste styrofoam	0.3087	0.1296	0.4617	0.1
(B) Other waste plastics	0.0343	0.0144	0.0513	0.9

Fig.2 shows the relationship between balanced growth rate and the allocation of waste treatments. Here calculated balanced growth rate is 1.24 ~ 1.33%. From the figure, we can see that as the incineration ratio increases, the difference of balanced growth rate among scenarios is increasing.

Table. 2: Calculated balanced growth rate

Scenario parameters		Balanced growth rate(%)	Change rate(%)
Incineration	Landfill		
0	1	1.247	1.000
0.1	0.9	1.263	1.013
0.2	0.8	1.271	1.019
0.3	0.7	1.276	1.023
0.4	0.6	1.278	1.025
0.5	0.5	1.320	1.059
0.6	0.4	1.325	1.063
0.7	0.3	1.329	1.066
0.8	0.2	1.332	1.068
0.9	0.1	1.334	1.069

## 4.2 Scenario 2

In scenario 2, we consider two different levels of capital accumulation in waste treatment sectors, capital intensive type and labor intensive one respectively.

As basement scenario, present end of life vehicles(ELVs) treatment is considered, actively machinery treatment with shredder machine and nibbler in scenario A (Capital intensive type), and ELV dismantling by hand in scenario B (Labor intensive type). In this scenario, we assumed that capital intensive type requires 1.3 times as much capital accumulation of machinery in waste treatment sector as the base scenario. On the other hand, labor intensive type requires 0.7 times as much as the base scenario. This assumption is based on the interview with the auto dismantlers, and the amount of capital accumulation about machinery, such as shredder machine and nibbler is increased or decreased respectively.

Table. 3: Calculated balanced growth rate

	Labor intensive	Present situation	Capital intensive
BGR	1.32369	1.32383	1.32390
Differences between scenarios about BGR	99.989	100	100.005

(BGR:Balanced growth rate)

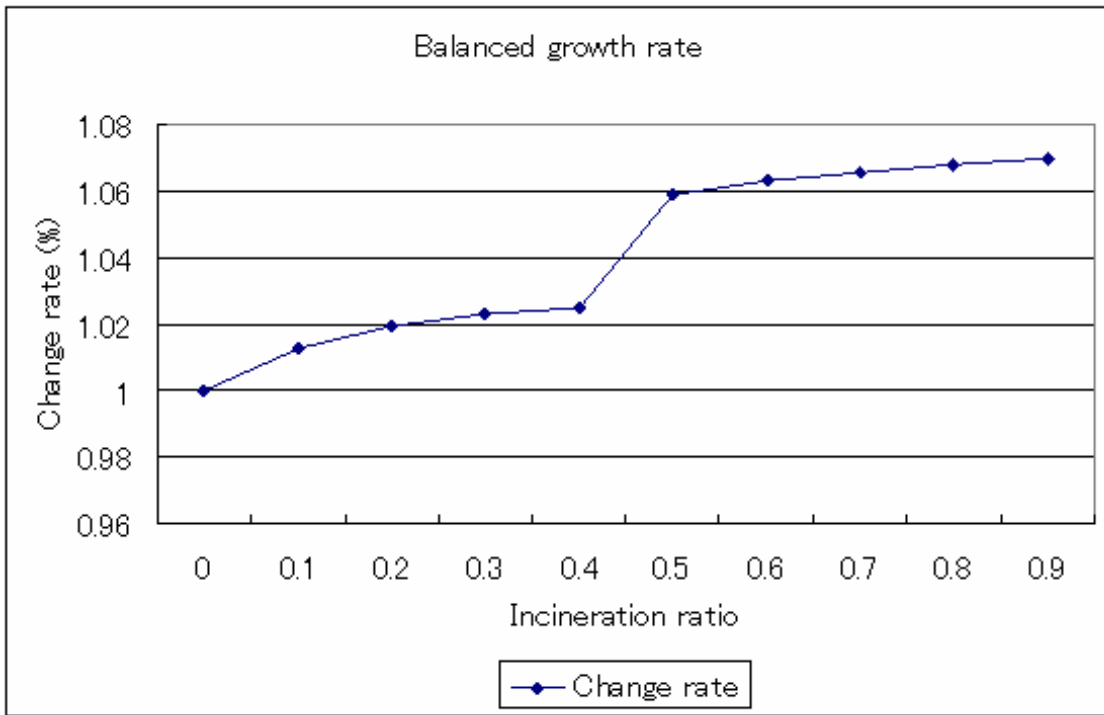


Fig. 2: Balanced growth rate associated with each scenarios

The calculated balanced growth rate about three scenarios are presented in Table3. From the result, the calculated balanced growth rates are not so big different among the scenarios. Figure 3,4 and 5 shows the difference in factors of eigen vectors between scenarios. Factors of industrial sectors are represented in Figure 3,4, Figure 5 shows them of waste treatment sectors.

### 4.3 Discussions

Through the scenario analysis, we introduced the Dynamic Waste Input Output model and examined its property. in focusing on the balanced growth path concerning capital accumulation and waste treatment, we explored the dependent relationship between economic and waste treatment activity and derived some fundamental implications. The functional relationship discusses above is useful for analytically discussing the long-run dependent relationship between waste generations and economic growth. Numerical examination based on the implications will make it more clear this functional relationship between capital and waste accumulation process.

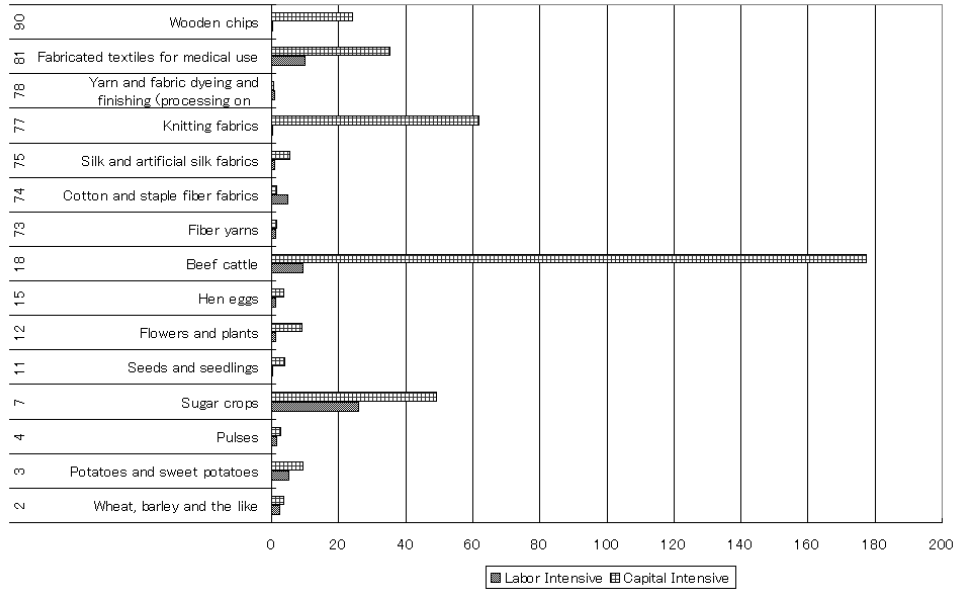


Fig. 3: Difference in factors of eigen vectors between scenarios:1

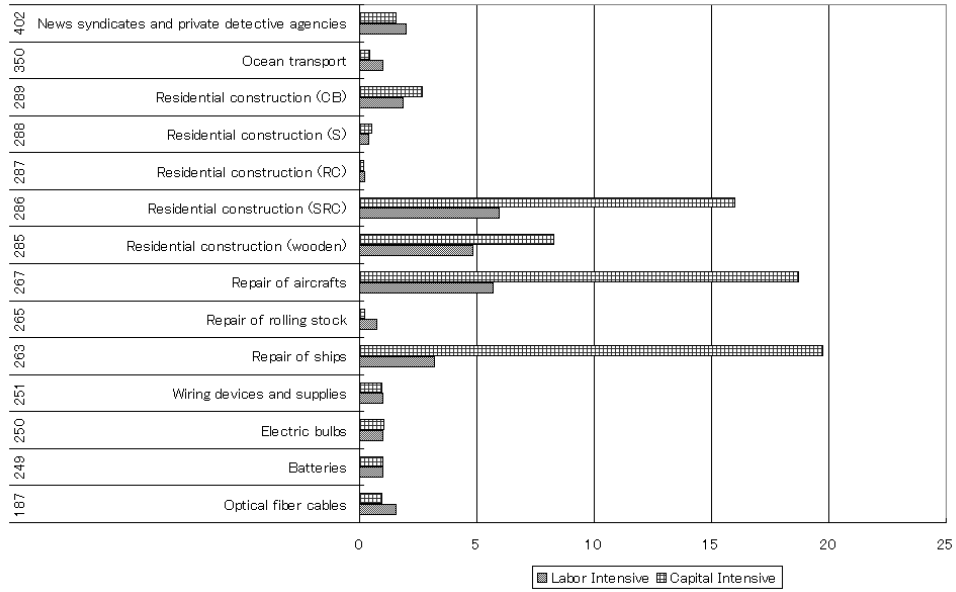


Fig. 4: Difference in factors of eigen vectors between scenarios:1

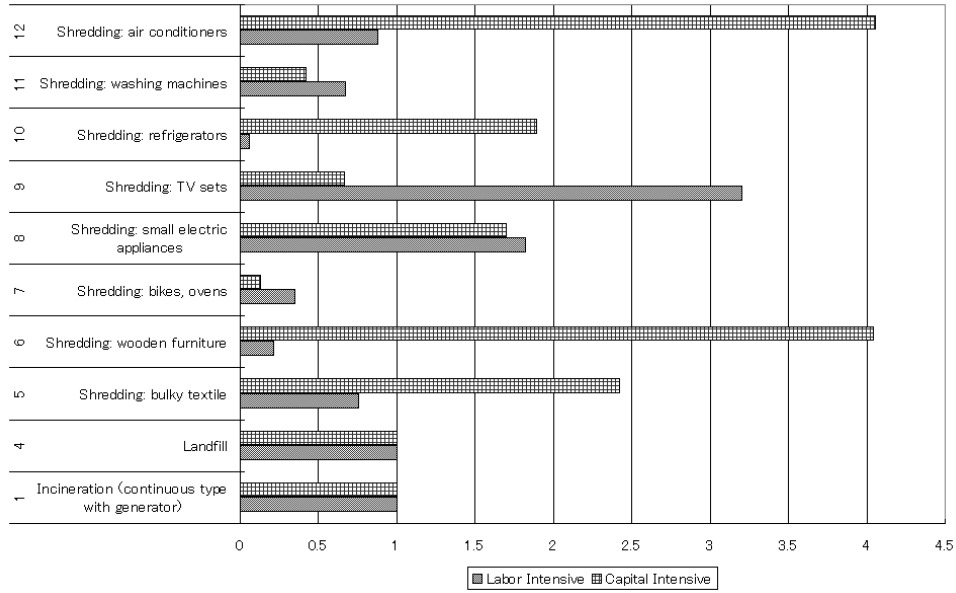


Fig. 5: Difference in factors of eigen vectors between scenarios:1

## 5 Conclusion

In this paper, we introduced the Dynamic Waste Input Output model and examined its property. Capital stock is a driving force of economic growth, as well as, potential wastes that will be generated in the future. In this framework, we treated with such capital accumulation and waste from discarded capital goods as B matrix. For sustainable management of wastes and exhaustible resources accumulated in capital stock,  $B_{21}$  and  $B_{22}$  should be managed or controlled about some kind of rare metals or exhaustible resources.

Although an emphasis is placed on the simple dynamic input-output model, it is natural that more complex dynamic phenomena, such as time gestation of capital stock and temporally distributed activities should be mathematically considered. At the same time, it would be important to examine the impact of price formation on waste accumulation. In order to attain it, we must connect this consideration with previous studies in terms of duality. This is an important and interesting study for environmental and ecological economics.

## Acknowledgments

This research was supported by a Waste Management Research Grant (2006) from the Ministry of the Environment, Japan.

## References

- ALMON, C. (1963): “Numerical solution of a modified Leontief dynamic system for consistent forecasting or indicative planning,” *Econometrica*, 31, 665–678.
- BRÓDY, A. (1974): *Proportions, Prices, and Planning*. Amsterdam, North-Holland.
- DUCHIN, F., AND D. B. SZYLD (1985): “A dynamic input-output model with assured positive output,” *Metroeconomica*, 37, 269–282.
- JOHANSEN, L. (1978): “On the theory of dynamic input-output models with different time profiles of capital construction and finite life-time of capital equipment,” *Journal of Economic Theory*, 19, 513–533.
- JORGENSON, D. W. (1960): “A dual stability theorem,” *Econometrica*, 28, 892–899.
- (1961): “Stability of a dynamic input-output system,” *Review of Economic Studies*, 28, 105–116.
- KENDRICK, D. (1972): “On the Leontief dynamic inverse,” *Quarterly Journal of Economics*, 86, 693–696.
- KURZ, H. D., AND N. SALVADORI (2000): “The Dynamic Leontief Model and the Theory of Endogenous Growth,” *Economic Systems Research*, 12(2), 255–265.
- LEONTIEF, W. (1970): “Environmental Repercussions and the Economic Structure: An Input-Output Approach,” *The Review of Economics and Statistics*, 52(3), 262–271.
- LEONTIEF, W. W. (1953): “Dynamic analysis,” in *Studies in the Structure of American Economy*. Oxford University Press.
- (1961a): “Lags and the stability of dynamic systems,” *Econometrica*, 29, 659–669.
- (1961b): “Lags and the stability of dynamic systems: a rejoinder,” *Econometrica*, 29, 674–675.

- LOS, B. (2001): “Endogenous Growth and Structural Change in a Dynamic Input-Output Model,” *Economic Systems Research*, 13(1), 3–34.
- LUENBERGER, D. G., AND A. ARBEL (1977): “Singular dynamic Leontief systems,” *Econometrica*, 45, 991–995.
- MATSUTO, T. (2005): *Toshi-Gomi Syori system no Bunseki, Keikaku, Hyouka (Analysis, planning and assessment of municipal solid waste treatment)*. Gihodou Shuppan Co.,Ltd, in Japanese.
- MEYER, U. (1982): “Why singularity of dynamic Leontief systems does not matter,” in *Input-Output Analysis Volume .* Northampton, Edward Elger Publishing.
- MINISTRY OF INTERNAL AFFAIRS AND COMMUNICATIONS (2004): “2000 Input-Output Table for Japan,” .
- MINISTRY OF LAND INFRASTRUCTURE AND TRANSPORT (ed.) (2004): *Input-output Tables of Subdivided Construction Sectors*. Ministry of Land, Infrastructure and Transport.
- MORISHIMA, M. (1958): “Prices, interest and profits in a dynamic Leontief system,” *Econometrica*, 26, 358–380.
- NAKAMURA, S. (1999): “An Interindustry Approach to Analyzing Economic and Environmental Effects of the Recycling of Waste,” *Ecological Economics*, 28(1), 133–145.
- NAKAMURA, S. (2004): *Waste Input Output Table 2000(ver.0.04b)*. <http://www.f.waseda.jp/nakashin/WIO.html>.
- NAKAMURA, S., AND Y. KONDO (2002): “Input-Output Analysis of Waste Management,” *Journal of Industrial Ecology*, 6(1), 39–64.
- ÅBERG, M., AND H. PERSSON (1981): “A note on a closed input-output model with finite life-times and gestation lags,” *Journal of Economic Theory*, 24, 446–452.
- SARGAN, J. D. (1958): “The instability of the Leontief dynamic model,” *Econometrica*, 26, 381–392.
- (1961): “Lags and the stability of dynamic systems: a reply,” *Econometrica*, 29, 670–673.
- SOLOW, R. M. (1989): “Competitive valuation in a dynamic input-output system,” *Econometrica*, 27, 30–53.

- STEENGE, A. E., AND M. J. P. M. THISSEN (2005): "A New Matrix Theorem: Intepretation in Terms of Internal Trade Structure and Implications for Dynamic Systems," *Journal of Economics*, 84(1), 71–94.
- TEN RAA, T. (1986a): "Applied dynamic input-output with distributed activities," *European Economic Review*, 30, 805–831.
- (1986b): "Dynamic input-output analysis with distributed activities," *Review of Economics and Statistics*, 68, 300–310.
- TOKOYAMA, K., AND Y. MURAKAMI (1972): "Relative stability in two types of dynamic Leontief models," *International Economic Review*, 13, 408–415.