

# Solow meets Leontief: Economic Growth and Energy Consumption

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## Abstract

This paper proposes a methodology that integrates a growth model with an input-output model to analyze the impacts of economic growth on the consumption of energy. The integration between the models is carry out by calibrating the growth module, which incorporates energetic inputs (renewable and nonrenewable) in the production function, and implementing shocks by the supply side (capital, labor, renewable and nonrenewable energy) in the input-output model. This allows us to verify the pattern of energy consumption for each sector in the input-output matrix. We apply this methodology to study the energy consumption of eleven economic sectors in Brazil, using data from the Brazilian National Accounts and Input-Output Matrix (IBGE) and the National Energy Report (BEN). We conduct experiments involving changes in technological progress growth rate, extraction and regeneration rates of both renewable and nonrenewable resources and population growth to analyze the impact of changes in the parameters of the model on the sectoral output growth rate and, consequently, on the consumption of energy in each economic sector.

Keywords: Growth, Input-Output Models, Model integration; Energy.

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# 1 Introduction

The relationship between energy use and output growth has received increasing attention in recent years. While energy is an essential input for growth and development in modern economies - economic production uses both renewable and nonrenewable resources as sources of energy - energy use is also expected to be a limiting factor to economic growth, as other factors of production such as labor and capital cannot do without energy. Limited natural resources imply a serious drag on growth that may eliminate most or all of the positive influence of technological progress on income per capita. However, the use of renewable resource may allow a sustained growth despite natural environment limitations. It can also be argued that the impact of energy use on growth will depend on the structure of the economy, energy intensity and the stage of economic growth of the country concerned. Moreover, if energy use and environment policies affect the rate of productivity and the growth of the population, they will also have effects on long-run growth.

The process of economic development has involved a strong growth of energy demand. Economic growth is a critical determinant of demands for energy and growth projections are essential for estimates of future demand and supply of energy. However, the expansion in energy consumption concomitant with economic development can potentially create serious problems. For instance, the growth in industrial production can place a severe strain on available domestic energy supplies, which in turn can lead to high energy prices domestically or an increase in imports of energy resources with consequences for the country's balance of payments. Economic growth can also create energy shortages since the rates depletion of exhaustible resources and regeneration of renewable resources may differ from the output growth rate.

Over the past years there has been an increase in the literature that deals with

the energetic topic. The use of different methodologies to study empirical questions is widely accepted in the literature. Although this paper does not seek to address and discuss advantages or disadvantages of different methods, we recognize the existence of a large range of approaches to model energy and natural resources. For instance, *(i)* econometric models (Adams and Shachmurove, 2008; Gan and Zhidong, 2008; Lee and Chang, 2008; Stern, 2007); *(ii)* input-output models [Anderson, Santos and Haines, 2007; Marriot, 2007, Morán and González, 2007; Kagawa and Inamurral, 2004; Alcántara and Padilla, 2003; Hawdon and Pearson, 1995; Hsu, 1989; Park, 1982); *(iii)* integrated models - econometric + input-output models (Rey, West and Janikas, 2004; Rey, 2000; Rey, 1998; Rey and Dev, 1997) and macro econometric models (Barker et al., 2007) and *(iv)* computable general equilibrium models (Sue Wing, 2008; Allan et al. 2007; Bjertnaes and Faehn, 2007; Wissema and Dellink, 2007; Vanden and Sue Wing, 2007; Otto and Reilly, 2007; Naqvi, 1998).

The main goal of this paper is to contribute in this discussion by proposing a novel approach to investigate the relationship between economic growth and energy consumption. This approach integrates an exogenous growth model and an input-output model to analyze energy use and economic growth at an economic sectoral level. Our point of departure for modeling economic growth is the neoclassical theory of growth originated by Solow (1956, 1976). This theory has been developed over the last years with important contributions to questions related to energy, the environment and economic growth (Cass, 1965; Koopmans, 1967), natural resources extraction and growth (Dasgupta and Heal, 1974; Stiglitz, 1974, Chakravorty et al., 1997, Martinet and Rotillon, 2007), environmental quality and income levels (Lopez, 1994; Brock and Taylor, 2004, 2005).

The input-output framework of analysis was developed by Wassily Leontief in the late 1920s and early 1930s. The input-output analysis is a method of systematically

quantifying the mutual interrelationships among the various sectors of a complex economic system. The input-output model is based on a fully determined general equilibrium model, where all the intermediate goods were expressed as a set of equations with sales and purchases of the intermediate industries forming the core of the system. Leontief (1941) designed a “closed model”, namely a model where all final demand and value added components were taken as endogenous. Later, Leontief (1951) reformulated the system to what is known as an “open model”, with the final demand and value added components treated exogenously.

It is important to highlight that the input-output literature covers issues across a wide range of topics (e.g. growth, welfare, interdependence); policy issues (e.g. income distribution, employment, investments, migration, energy consumption, and the environment); analytical frameworks (e.g. static, comparative static, dynamic, structural, spatial, and open versus closed); units and levels of analysis (e.g. enterprises, industries, metropolitan areas, regions, multiple regions, single nations, groups of countries, and the world)<sup>1</sup>. One of the most important features of input-output models is the idea of fixed technical coefficients. Economic policy, however, induce changes in these input-output coefficients. The literature presents a different range of approaches to address this issue, such as econometric models (Jorgenson and Wilcoxon, 1993), macroeconomic production functions (Raa, 2005) and applied general equilibrium models (Johansen, 1960).

Different input-output models are specified in order to study the interrelations between economic activities and their impacts on the environment, pollution and natural resources use. Park (1982) puts forward an input-output model to study direct, indirect and induced energy effects of a change in final demand and estimates the effects of technical change on energy consumption. Hawdon and Pearson (1995) constructed

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<sup>1</sup>See Leontief (1986), Ciaschini (1988) and Dietzenbacher and Lahr (2004) for more detailed discussions.

an input-output model to study interactions between energy and economic activities for the United Kingdom. Henry (1995) discusses the idea of capacity growth and the impacts on energy in an input-output framework and shows how a forward year by year uniform capacity growth across all sectors can be reached based on a specific annual growth rate. The specification of an energy sector in the model allows to deal explicitly with the importance of energy to any capacity expansion. Murthy et al. (1997) study carbon dioxide (CO<sub>2</sub>) emissions from energy consumption in India and argue that the input-output framework fits very well this kind of analysis due the possibility of verifying the direct and indirect emissions caused by variations in each category of final demand. Machado et al. (2001) measure the impacts of foreign trade on energy use and CO<sub>2</sub> emissions using a hybrid input-output calibrated to the Brazilian economy for 1985, 1990 and 1995<sup>2</sup>.

We present a growth model with renewable and nonrenewable resources and perfect mobility of commodities. Economic sectors differ with respect their capital intensity and energy use. Non-renewable resources are depleted as they are used. The stock of non-exhaustible resources is renewed each period at a given rate. Economic sectors differ with respect their capital intensity and energy use. To the best of our knowledge, this is the first study to propose an integrated exogenous growth model and input-output model to analyze energy consumption and economic growth at the economic sectoral level. The neoclassical growth model is a useful tool for gaining insights into the key factors that determine the ability of an economy to sustain itself in the long-run. An advantage of our analysis is that shocks to the input-output model are consistent with the solution of a well defined growth model, which minimizes the *ad hoc* structure of the shocks. The relationships between the economic agents are established in the input-output framework, which eliminate the necessity to model such

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<sup>2</sup>For other studies and references, see Lenzen and Dey (2002), Morán and González (2007), Ma and Stern (2008) and Bartz and Kelly (2008).

complex interactions. This integrated framework offers a simple methodology to study the relationship between growth and energy consumption, which relies mainly in the solution of the growth model and in the input-output structure of the economy. No econometric methods are used in our methodology.

Energy consumption is often used as a proxy for economic growth. On average, the consumption of energy grows at a rate of two percent per year and it is expected that it will double in 30 years. The growth rate is not uniform across countries. While the energy consumption grows at one percent in developed countries, this rate is four times higher in developing countries. In this paper, we contribute to the ongoing debate about the link between economic growth and energy consumption by study the Brazilian economy and its economic sectors.

To study the impact of economy growth on energy consumption, we use the Brazilian National Accounts and Input-Output Matrix (IBGE) and the National Energy Report (BEN). We analyze eleven economic sectors: Agriculture, Mining, Nonmetallic minerals, Steel and nonferrous metals, Paper products and printing, Chemicals, Textiles, Food and beverages, Trade and services, Transportation and Public administration. More than eighty percent of the energy consumed by sectors such as Food and Beverages, Trade and Services, Public Administration and Paper Products and Printing comes from renewable natural resources. On the other hand, Mining, Chemicals, Transportation and Nonmetallic Minerals consume energy from nonrenewable resources heavily.

The calculated sectoral output growth rates indicate that the Trade and Services, Public Administration and Agriculture sectors have a much higher long-run growth rates than the other sectors as well as the economy. We observe that these three sectors have the lowest renewable and nonrenewable resources shares among all sectors analyzed. The remaining sectors have a very low output growth rate, lower than one

percent. For all sectors, except Public Administration, the consumption of energy in 2003 (our baseline year) is above the level of energy consumption associated with the sectoral long-run output growth rate. We then forecast the long-run energy consumption by sector in Brazil for the period 2004–2014, assuming that economic sectors grow at their long-run growth rates. We also conduct several experiments to analyze the impact of changes in the parameters of the model on the sectoral output growth rate and, consequently, on the consumption of energy in each economic sector. The experiments involve changes in technological progress growth rate, extraction and regeneration rates of both renewable and nonrenewable resources and population growth.

The paper is organized as follows: Section 2 presents the economy and the structure of the model. This section explains the key features of the integration between an exogenous growth model and the input-output model. The relationship between growth and energy use are also explored. In Section 3 we conduct a quantitative analysis of sectoral output growth and the consumption of renewable and nonrenewable resources. Some experiments are also discussed in this section. Finally, Section 4 offers concluding remarks.

## **2 An Integrated Model of Growth and Energy**

### **2.1 A Growth Model with Natural Resources**

In this section, we present a growth model with renewable and nonrenewable resources and perfect mobility of goods and services. The economic agents in this economy are households, firms, government and the rest of the world. Time runs in a discrete sequence of periods indexed by  $t$  (one year). All markets are assumed to be perfectly competitive, so economic agents take prices as given.

There are  $m$  economic sectors in this economy, producing  $m$  different goods and

services. In each sector, we assume that all production takes place in one representative profit-maximizing firm. The production function of a representative firm in sector  $i$  is given by:

$$Y_{it} = A_{it} K_{it}^{\alpha_i} L_{it}^{\beta_i} Z_{it}^{\kappa_i} E_{it}^{\varepsilon_i} \quad (1)$$

where  $Y_{it}$  represents sectoral output,  $K_{it}$  capital,  $L_{it}$  labor,  $Z_{it}$  renewable resources,  $E_{it}$  nonrenewable resources,  $\alpha_i, \beta_i, \kappa_i, \varepsilon_i > 0$  and  $\alpha_i + \beta_i + \kappa_i + \varepsilon_i = 1$ . The production function displays constant returns to scale. Technological progress is Hicks-neutral as it appears as an increasing variable,  $A_{it}$ , in  $A_{it}F(K_{it}, L_{it}, Z_{it}, E_{it})$ . The total factor productivity  $A_{it}$  for each economic sector changes over time at a constant rate, i.e.  $A_{it+1} = (1 + g_{A_i})A_{it}$ , for  $i = 1, 2, \dots, m$ . In this economy, economy sectors differ not only regarding the good produced but also with respect their capital intensity and energy use.

Equation (2) is the capital accumulation equation, assuming a given rate of depreciation  $\delta$ :

$$K_{t+1} = S_t + (1 - \delta)K_t \quad (2)$$

We assume that firms save an exogenous fraction,  $s$ , of total income in each period:

$$S_t = sY_t \quad (3)$$

where  $0 < s < 1$ .

A key feature of this model is the use of natural resources as sources of energy in the production of goods in this economy. Non-renewable resources are depleted as they are used. At the beginning of any period  $t$ ,  $N_t$  is the remaining stock of an exhaustible resource, for instance oil, gas, coal or metals. The part of this stock that is used as

energy input during period  $t$  will be denoted by  $E_t$ . The interaction between demand and supply involving current and future prices would determine how a given remaining stock of the exhaustible resource would be allocated over time. We assume that in each period a certain fraction  $s_i^E \in (0, 1)$  of the remaining stock is used in production in sector  $i$ , that is  $E_{it} = s_i^E N_t$ . Sectoral allocations of nonrenewable resources satisfy  $\sum_{i=1}^m s_i^E = s^E$ , where  $s^E > 0$  is the nonrenewable energy share in period  $t$ .

The stock of the nonrenewable resource will be reduced from one period to the next by the amount used in production in all  $m$  sectors in the first of the periods. The depletion of the natural resource is described by

$$N_{t+1} = N_t - \sum_{i=1}^m E_{it}$$

$$N_{t+1} = N_t - \sum_{i=1}^m s_i^E N_t \quad (4)$$

On the other hand, the accumulation equation for a renewable resource such as wood or (corn, sugar cane) ethanol can be describe as

$$R_{t+1} = z_t R_t - \sum_{i=1}^m s_i^Z R_t \quad (5)$$

where  $R_t$  is the stock of the renewable resource in period  $t$  and  $Z_{it}$  is the amount used as energy input for firms in sector  $i$  during period  $t$ . The regeneration rate  $z_t > 1$  implies how much of the stock of this resource is renewed each period. Similarly, in each period a fraction  $s_i^Z \in (0, 1)$  of the existing renewable resource is used in sector  $i$ , that is,  $Z_{it} = s_i^Z R_t$  and  $\sum_{i=1}^m s_i^Z = s^Z$ . Although we do not model the energy sector explicitly, equations (4) and (5) provide a simple and fair characterization of the dynamics of energy resources in an economy. An natural extension of this approach is to model the energy sector as a sector of the economy, where different energy commodities

would be treated differently. We leave this for future research.

The total amount of labor input used by firms in sector  $i$  is described as

$$L_{it+1} = (1 + g_{L_{it}})L_{it} \quad (6)$$

where  $g_{L_{it}}$  is the exogenous growth rate of the labor force in sector  $i$  and the total labor supply is given by  $L_t = \sum_{i=1}^m L_{it}$ .

In this economy there are  $m$  elements of each of the following aggregate final demand components: personal consumption  $C_t$ , investment  $I_t$ , government expenditures  $G_t$  and net exports  $NE_t = X_t - M_t$ . The goods market equilibrium in an small open economy requires

$$Y_t = C_t + I_t + G_t + X_t - M_t \quad (7)$$

where each of these aggregate components is obtained as the sum of the sector specific values, i.e.  $Y_t = \sum_{i=1}^m Y_{it}$  is total output,  $C_t = \sum_{i=1}^m C_{it}$  is total private consumption,  $I_t = \sum_{i=1}^m I_{it}$  is aggregate investment,  $G_t = \sum_{i=1}^m G_{it}$  is government expenditures,  $X_t = \sum_{i=1}^m X_{it}$  is aggregate exports and  $M_t = \sum_{i=1}^m M_{it}$  is aggregate imports.

## 2.2 A Channel from Energy Use to Growth

A growth model with renewable and nonrenewable resources and perfect mobility of commodities consists of the following equations: (1), (2), (3), (4), (5), (6), (7) and  $A_{it+1} = (1 + g_i)A_{it}$ . In addition, the competitive factor markets imply that, in each period, each factor of production will earn its marginal product which depends on the predetermined level of  $K_t$ ,  $L_t$ ,  $Z_t$ ,  $E_t$  and  $A_t$ , in the economy. The following four equations complete the description of the model:

$$r_t = \alpha A_t \left( \frac{K_t}{L_t} \right)^{\alpha-1} \left( \frac{Z_t}{L_t} \right)^{\kappa} \left( \frac{E_t}{L_t} \right)^{\varepsilon} \quad (8)$$

$$w_t = \beta A_t \left( \frac{K_t}{L_t} \right)^\alpha \left( \frac{Z_t}{L_t} \right)^\kappa \left( \frac{E_t}{L_t} \right)^\varepsilon \quad (9)$$

$$v_t = \kappa A_t \left( \frac{K_t}{L_t} \right)^\alpha \left( \frac{Z_t}{L_t} \right)^{\kappa-1} \left( \frac{E_t}{L_t} \right)^\varepsilon \quad (10)$$

$$q_t = \varepsilon A_t \left( \frac{K_t}{L_t} \right)^\alpha \left( \frac{Z_t}{L_t} \right)^\kappa \left( \frac{E_t}{L_t} \right)^{\varepsilon-1} \quad (11)$$

One can easily verify that in the special case of  $\kappa = 0$  and  $\varepsilon = 0$ , where energy of any source is of no importance, the model boils down to a basic Solow model.

Inserting  $E_{it} = s_i^E N_t$  and  $Z_{it} = s_i^Z R_t$  into the production function (1) gives:

$$Y_{it} = A_{it} K_{it}^{\alpha_i} L_{it}^{\beta_i} (s_i^Z R_t)^{\kappa_i} (s_i^E N_t)^{\varepsilon_i}$$

Dividing both sides by  $L_{it}$

$$\begin{aligned} y_{it} &= A_{it} \left( \frac{K_{it}}{L_{it}} \right)^{\alpha_i} \left( \frac{s_i^Z R_t}{L_{it}} \right)^{\kappa_i} \left( \frac{s_i^E N_t}{L_{it}} \right)^{\varepsilon_i} \\ y_{it} &= A_{it} (s_i^Z)^{\kappa_i} (s_i^E)^{\varepsilon_i} k_{it}^{\alpha_i} R_t^{\kappa_i} N_t^{\varepsilon_i} L_{it}^{-\kappa_i - \varepsilon_i} \end{aligned} \quad (12)$$

where  $y_{it} = Y_{it}/L_{it}$  and  $k_{it} = K_{it}/L_{it}$ . Taking logs and time differences, we have:

$$g_{y_{it}} = \alpha_i g_{k_{it}} + g_{A_i} + \kappa_i g_{R_t} + \varepsilon_i g_{N_t} - (\kappa_i + \varepsilon_i) g_{L_{it}}$$

where  $g_{b_{it}} = \ln b_{it} - \ln b_{it-1}$ , for  $b = y, k, A, R, N$  and  $L$ . The growth rate of the stocks of the renewable ( $g_{R_t}$ ) and nonrenewable ( $g_{N_t}$ ) resources are approximately equal to  $(z_t - s^Z)$  and  $-s^E$ . Hence, the growth rate of output per worker in sector  $i$  is given by

$$g_{y_{it}} = \alpha_i g_{k_{it}} + g_{A_i} + \kappa_i (z_t - s^Z) - \varepsilon_i s^E - (\kappa_i + \varepsilon_i) g_{L_{it}} \quad (13)$$

If we assume that the dynamics of our model are such that the capital-output ratio ( $k_{it}/y_{it}$ ) converges toward a constant steady state level, the growth rates of output and

capital per worker must converge towards the same rates. The presence of technological change means there cannot be a steady state. Instead there is a balanced growth path along which capital, output and consumption grow at a common, constant rate. Setting  $g_{y_{it}} = g_{k_{it}}$  in (13) and solving it shows that both rates must converge towards a constant common level, approximately given by

$$g_{y_{it}} \approx \frac{1}{\beta_i + \kappa_i + \varepsilon_i} g_{A_{it}} + \frac{\kappa_i}{\beta_i + \kappa_i + \varepsilon_i} (z_t - s^Z) - \frac{\varepsilon_i}{\beta_i + \kappa_i + \varepsilon_i} s^E - \frac{\kappa_i + \varepsilon_i}{\beta_i + \kappa_i + \varepsilon_i} g_{L_{it}} \quad (14)$$

In our model, growth rates of the resources endowments are not responsive to prices and we do not consider any effect of prices on growth and energy consumption. We can highlight four aspects of this long-run sectoral output growth rate  $g_{y_{it}}$ . First, a given rate  $g_{A_{it}}$  of technological progress is more effective in creating economic growth than in the absence of natural resources, i.e.,  $1/(\beta_i + \kappa_i + \varepsilon_i)$  is greater than one. Second, population growth implies a drag on economic growth of the size  $-[(\kappa_i + \varepsilon_i)/(\beta_i + \kappa_i + \varepsilon_i)]g_{L_{it}}$ . Increasing amounts of labor in association with increasing amounts of capital will press on the limited amount of the nonrenewable natural resource and therefore, through diminishing returns, can imply a slower growth in income per worker than with no natural resources. In this economy, as the limited supply of nonrenewable resources disappears gradually through its use in production, the diminishing returns to capital and labor arising from the scarcity of this natural resource become more severe. This implies another drag on growth, explained by the term  $-[\varepsilon_i/(\beta_i + \kappa_i + \varepsilon_i)]s^E$  in (14). The larger is the extraction of nonrenewable resources,  $s^E$ , the faster the exhaustible resource will be depleted and the faster the negative influence from diminishing returns to the other factors will grow. Note however that the use of renewable resources as an input in the production process can help to overcome the drag on growth implied by the use of exhaustible resources. As the stock of renewable natural resources regenerates,

it has a positive effect on other factors and contributes to economics growth, given by the term  $[\kappa_i/\beta_i + \kappa_i + \varepsilon_i](z_t - s^Z)$ .

Differentiating (14) with respect to renewable and nonrenewable energy income shares,  $\kappa_i$  and  $\varepsilon_i$ , we obtain  $\partial g_{y_{it}}/\partial \kappa_i = (z_t - s^Z - g_{L_{it}})/(1 - \alpha_i)$  and  $\partial g_{y_{it}}/\partial \varepsilon_i = -(s^E + g_{L_{it}})/(1 - \alpha_i)$ , respectively. Note the while we cannot determine the relationship between sector output growth rate and the renewable resource share, since it will depend on the combination of net regeneration rate and population growth, the effect of a higher nonrenewable energy share is clearly negative, which is consistent with the drag on growth cause by the use of such exhaustible resources.

An interesting effect of a balance growth path is the impact of economic growth on the energy prices through the increasing use of natural resources as source of energy. In this economy, an increase in output  $Y_t$  leads to an increase in both renewable and nonrenewable resources,  $Z_t$  and  $E_t$ , respectively, since production uses these resources simultaneously and we do not consider energy saving technology in our model. The effect of this process on energy prices is not clear. For instance, the price of renewable resources  $v_t$  will increase if and only if  $\varepsilon/E_t > (1 - \kappa)/Z_t$ , and decrease otherwise. Similarly, if  $(1 - \varepsilon)/E_t < \kappa/Z_t$  the price of renewable resources  $q_t$  will increase along the balance growth path.

### 2.3 A Channel from Growth to Energy Use

In order to investigate the impact of economic growth on sectoral energy use, we integrate the macroeconomic model with a input-output model, where the final demand is the key link between these models. The main advantage of this approach is that shocks to the input-output model are consistent with the solution of a well defined growth model, which minimizes the *ad hoc* structure of the shocks.

The input-output table is a double-entry bookkeeping scheme. Table 1 shows the

input-output accounts for an economy with  $m$  sectors, in period  $t$ . For this economy composed by  $m$  sectors, each of the first  $m$  rows indicates the distribution of a sector's output and each of the first  $m$  columns shows the distribution of a sector's input.

This framework allows us to analyze the total sales of a specific sector and also the total purchase of the same sector. The sales of a product by the sector  $i$  (the sector that produces the goods) may be sold to a sector where it will be used as an input to the production process or as a good to be consumed as a final product by households, local or state or federal government or foreign consumers. These monetary flows can be represented as follows:

$$X_{it} = z_{i1t} + z_{i2t} + \dots + z_{imt} + F_{it}, \quad i = 1, \dots, m \quad (15)$$

where  $z_{ijt}$  is the monetary value of the sales from sector  $i$  to sector  $j$ ,  $F_{it} = C_{it} + I_{it} + G_{it} + NE_{it}$  is the value of sales of sector  $i$  goods to final consumers and  $X_{it}$  is the total value of goods produced by sector  $i$  in period  $t$ , for  $i, j = 1, \dots, m$ .

#### Table 1 - Input-Output Accounts

[ABOUT HERE]

Similarly, we can also analyze a sector purchases in order to engage in production. Firms in a particular sector purchase inputs from other producing or processing sectors. Also, in the production process firms make payments to the factors of production, i.e. labor (wages and salaries), land (rents), capital (interest, profits and depreciation charges), as well as direct and indirect taxes (considered to be payments for government services). Thus we can represent those monetary flows as follows:

$$X_{jt} = z_{1jt} + z_{2jt} + \dots + z_{mjt} + w_{jt} + ov_{jt}, \quad j = 1, \dots, m \quad (16)$$

where, in period  $t$ ,  $z_{jit}$  is the monetary value of the purchases of firms in sector  $j$  of goods produced in sector  $i$  and  $w_{jt}$  and  $ov_{jt}$  are wage and other payments made by firms in sector  $j$ , respectively.

The sum of the last column and last row of Table 1 imply the following relations, respectively

$$X_t = \sum_{i=1}^m z_{it} + W_t + OV_t \quad (17)$$

$$X_t = \sum_{j=1}^m z_{jt} + C_t + I_t + G_t + NE_t \quad (18)$$

where  $W_t$  and  $OV_t$  represent the total amount paid to the factors of production,  $C_t$  aggregate consumption,  $I_t$  aggregate investment,  $G_t$  total government expenditures and  $NE_t$  aggregate net exports. Equation (17) represents the total value of all economic activity in the economy found as the sum of all column sums in an input-output table and equation (18) is the total value of all economic activity found as the sum of all rows in an input-output table. Since  $\sum_{i=1}^m z_{it} = \sum_{j=1}^m z_{jt}$  (both are the total gross output summed over all  $m$  sectors), then

$$C_t + I_t + G_t + NE_t = W_t + OV_t \quad (19)$$

where the left-hand side is the gross domestic product and the right-hand side represents the national income.

Central to the input-output framework is the following identity:

$$X_t = A_t X_t + F_t \quad (20)$$

where  $X_t$  is an  $m$  by 1 vector of sectoral output,  $F_t$  is an  $m$  by 1 vector of final demands

and  $A_t$  is an  $m$  by  $m$  input-output coefficients matrix with a typical element

$$a_{ijt} = \frac{x_{ijt}}{X_{jt}} \quad (21)$$

In this framework, the sectoral consumption of renewable and nonrenewable energy can be written as follows:

$$Z_t = P_{rt}X_t \quad (22)$$

$$E_t = P_{nt}X_t \quad (23)$$

where  $Z_t$  and  $E_t$  are  $m$  by 1 vectors of sectoral energy consumption and  $P_{rt}$  and  $P_{nt}$  are  $m$  by  $m$  energy consumption coefficients matrices with typical elements  $z_{it} = Z_{it}/X_{it}$  and  $e_{it} = E_{it}/X_{it}$ , respectively. Combining equations (20), (22) and (23) we obtain expressions for the consumption of energy (renewable and nonrenewable) in each economic sector as a function of the sector's final demand, that is:

$$Z_t = P_{rt}(ID - A_t)^{-1}F_t \quad (24)$$

$$E_t = P_{nt}(ID - A_t)^{-1}F_t \quad (25)$$

where  $ID$  is an identity matrix and  $F_{it} = C_{it} + I_{it} + G_{it} + NE_{it}$  is a typical element of vector  $F_t$ .

It is important to emphasize the role of aggregation in the integration of our growth model and the input-output model. The aggregate final demand and total production of this economy are given by, respectively

$$F_t = \sum_{i=1}^m F_{it} = \sum_{i=1}^m (C_{it} + I_{it} + G_{it} + NE_{it}) \quad (26)$$

$$Y_t = \sum_{i=1}^m Y_{it} \quad (27)$$

where  $Y_{it}$  is the output produced in sector  $i$  in period  $t$ , according to the production function (1). Moreover, in equilibrium,  $Y_{it} = F_{it}$ , i.e. sector  $i$  output is equal to the value of sales of sector  $i$  goods to final consumers. Hence, we can substitute each element of vector  $F_t$  by  $Y_{it}$  and rewrite expressions (24) and (25) as follows

$$Z_t = P_{rt}(ID - A_t)^{-1}Y_t \quad (28)$$

$$E_t = P_{nt}(ID - A_t)^{-1}Y_t \quad (29)$$

$$TE_t = P_t(ID - A_t)^{-1}Y_t \quad (30)$$

where  $TE_t$  denotes a vector of total sectoral energy consumption, renewable plus non-renewable, in period  $t$  and  $P_t$  is a  $m$  by  $m$  energy consumption coefficients matrix.

Having constructed these expressions, we can investigate the impact of economic growth on the sectoral energy consumption of renewable and nonrenewable resources. The consumption of energy in period  $t + 1$ , associated with a given growth rate of sectoral output,  $g_{yit}$ , calculated in (14), is determined by

$$Z_{t+1} = P_{rt}(ID - A_t)^{-1}Y_{t+1} \quad (31)$$

$$E_{t+1} = P_{nt}(ID - A_t)^{-1}Y_{t+1} \quad (32)$$

where elements of vector  $Y_{t+1}$  is given by  $Y_{it+1} = g_{yit}Y_{it}$ , for  $i = 1, \dots, m$ .<sup>3</sup>

Note that since the consumption of energy is determined along the balance growth path, according to the depletion of exhaustible resources and the regeneration of re-

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<sup>3</sup>Our approach, in particular equations (28)–(30), can be related to the concept of generalized Leontief production function (GLp). See Saunders (2008).

newable resources expressions (4) and (5), respectively, this economy will eventually converge from a stage where both resources are used in production to a stage where the only source of energy comes from renewable resources. In the long-run, the equilibrium depletion rate is zero ( $s^E = 0$ ) since exhaustible resources are no longer available (which also implies that  $\varepsilon_i = 0$ ) and the rate of regeneration of renewable resources  $z_t$  must be equal to  $1 + s^Z$ . After the economy reaches this point, the output growth rate in sector  $i$  is approximately given by

$$\tilde{g}_{y_{it}} \approx \frac{1}{\beta_i + \kappa_i} g_{A_{it}} + \frac{\kappa_i}{\beta_i + \kappa_i} (1 - g_{L_{it}}) \quad (33)$$

If we assume that the labor and renewable shares ( $\beta_i$  and  $\kappa_i$ ) remain the same, i.e., the depletion of exhaustible resources is reflected in a higher capital share  $\alpha_i$ , with firms substituting energy from nonrenewable resources for physical capital, the output growth rate  $\tilde{g}_{y_{it}}$  is higher than  $g_{y_{it}}$ , given by expression (14). That is, the economy grows faster when only renewable resources are used in production. This feature of our analysis also contributes to the literature that study the use of energy and transition between nonrenewable and renewable energy in a wider perspective (see for instance, Tahvonen and Salo (2001)).

## 3 Quantitative Analysis

### 3.1 Data

This section describes the database used in this paper. Brazil is a growing developing economy and an energy power. Figure 1 shows that the consumption of energy and the GDP in Brazil have grown over the past three decades at similar rates, keeping the ratio energy consumption to the GDP at a steady level. According to Proops

(1988) there is a consensus about the relationship between countries' energy use (E) and their national income, or product (Y). If the energy use/income ratio (E/Y) is plotted against time, many developed countries exhibit a "humped" shape. On the other hand, developing countries, as Brazil, display a rising trend as showed in Figure 1.

[ABOUT HERE]

Figure 1 - Energy Consumption and GDP - Brazil

It is important to highlight that the rising trend of (E/Y) in Brazil was interrupted because of a shortage occurred in the end of 90's. At the beginning of the 70's the energy consumption was around 60,595 millions of tep (tone equivalent petroleum) and during all decade the average growth was 5.4% by year. At 80's there was a decrease in the growth of energy demand (average growth was 2%). This could be due to the decrease in the economic growth. During the 90's there was not any significant difference at the behavior of energy consumption. In the last ten years, the energy consumption has increased by an average rate of 3.3% per year. In 2005, the consumption of energy was 182,612 millions of tep or, equivalently, one tep per capita. However, the consumption of energy per capita in Brazil is only a quarter of the consumption in OECD countries - 1,39 tone equivalent petroleum (tep) versus 5,5 tep, respectively.

The Brazilian energy consumption is highly concentrated on renewable sources of energy. Since 1970, the consumption of diesel oil and electricity have presented an increasing pattern, and more recently (after 1999) the consumption of natural gas has also increased (Figure 2)<sup>4</sup>.

[ABOUT HERE]

Figure 2 - Sources of Energy - Brazil

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<sup>4</sup>The classification of the sources of energy is presented in the Appendix A.1.

To study the impact of economy growth on energy consumption, we use the Brazilian National Accounts and Input-Output Matrix (IBGE) and the National Energy Report (BEN). We analyze eleven economic sectors: Agriculture, Mining, Nonmetallic Minerals, Steel and Nonferrous Metals, Paper Products and Printing, Chemicals, Textiles, Food and Beverages, Trade and Services, Transportation and Public Administration.

Taking data from 2003 (the last year used to parameterize the model) we can see the distribution of renewable and nonrenewable use of energy among sectors. These figures are showed on Table 2. We can highlight that Food and Beverages, Trade and services and Nonmetallic minerals are the sectors that use more intensively renewable energy. On the other side, Transportation and Chemicals are the ones consume energy from nonrenewable resources heavily.

To parameterize the macroeconomic module, we use data collected for the period 1990 to 2003 for household consumption, investment, government consumption, exports, imports and number of workers. It is important to highlight that for the period 1990 to 1996 the data were collected directly from the final demand vector of the Brazilian input-output table. For the remaining period, we also use the vector of final demand of the Brazilian input-output tables, estimated by Guilhoto and Sesso Filho (2004). Data from input-output matrices enable us to construct the technical coefficients matrix  $A$ , the vectors,  $K$  and  $L$ , respectively, capital and labor by sector and the vector of sectoral output by sector,  $X$ . In this exercise, we assume that the input-output coefficients and energy consumption coefficients (matrices  $A_t$ ,  $P_{rt}$  and  $P_{nt}$ ) remain constant, at their period  $t$  levels.

To specify the energy module of our model, we use data from the National Energy Report (EPE). This publication enables us to collect data for energy use by sector and by type (i.e renewable,  $Z$ , or non renewable,  $E$ , sources). The data are collected

for 15 sectors (Energy sector, Residential, Agriculture, Mining, Nonmetallic Minerals, Steel, Nonferrous steel, Paper products and printings, Chemicals, Textiles, Food and Beverages, Trade and Services, Transportation, Public Administration and Other Sectors. Thus, it was necessary to implement a compatibilization between the data from input-output system and the energetic balance data because the input-output tables are disaggregated for 42 sectors and the data from National Energy Report are disaggregated for 15 sectors. Thus, we implement an aggregation of the input-output sectors in order to make the two data source compatible<sup>5</sup>.

Table 2 - Energy Use by Economic Sectors (2003)

[ABOUT HERE]

### 3.2 Output Growth Rates

In this section, we proceed to parameterize the model in order to calculate the output growth rate for each sector in this economy. There is a balanced growth path along which capital, output and consumption grow at a common, constant rate. The parameters of the model are  $\alpha_i$ ,  $\beta_i$ ,  $\kappa_i$ ,  $\varepsilon_i$ ,  $g_{A_i}$ ,  $g_{L_{it}}$ ,  $z_t$ ,  $s^Z$  and  $s^E$ . The growth rate of output per worker in sector  $i$  is given by equation (14), and it is repeated here for convenience:

$$g_{y_{it}} \approx \frac{1}{\beta_i + \kappa_i + \varepsilon_i} g_{A_i} + \frac{\kappa_i}{\beta_i + \kappa_i + \varepsilon_i} (z_t - s^Z) - \frac{\varepsilon_i}{\beta_i + \kappa_i + \varepsilon_i} s^E - \frac{\kappa_i + \varepsilon_i}{\beta_i + \kappa_i + \varepsilon_i} g_{L_{it}}$$

where  $\beta_i$ ,  $\kappa_i$  and  $\varepsilon_i$  are labor, renewable energy and nonrenewable energy shares,  $g_{A_i}$  is the total factor productivity growth rate,  $g_{L_{it}}$  is the growth rate of the labor force,  $z_t$  and  $s^Z$  are the regeneration rate of renewable resources and fraction of the existing renewable resources used in all sectors, respectively, and  $s^E$  is a fraction of the remaining stock of nonrenewable resources used in total production.

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<sup>5</sup>See Appendix A.2 for details.

Table 3 summarizes these values for each sector, which are obtained as the historical average for the period between 1990 to 2003. We follow Gollin (2002) to calculate the labor share for each economic sector. Labor shares vary considerably across sectors, ranging from 0.23 in the Mining sector to 0.65 in the Public Administration sector. The mean is 0.35, with a standard deviation of 0.11. Our values for capital share are consistent with those provided by Bugarin et al. (2005) for the Brazilian economy. As expected, renewable and nonrenewable resources shares,  $\kappa_i$  and  $\varepsilon_i$  respectively, also assume quite different values across economics sectors. These shares are higher for sectors where production uses energy intensively, for instance, Steel, Nonferrous Metals, Nonmetallic Mineral and Transportation. Differences regarding the kind of energy used in the production process can also be observed from the data in Table 2. Note that sectors such as Transportation and Paper Products and Printing use more renewable energy resources than Chemicals, for example.

Table 3 - Parameter Values and Sectoral Output Growth Rates

[ABOUT HERE]

In order to assign values to the parameters  $z$ ,  $s^Z$  and  $s^E$ , we use the concept of National Energy Supply from the Brazilian National Energy Report (EPE). Given the limitations to obtain information about the total stock of renewable and nonrenewable resources for the Brazilian economy, we treat the national supply of energy as the total stock of energy available in a particular year. The regeneration rate of renewable resources net of the fraction used in the production process,  $(z - s^Z)$ , is equal to 0.0218. This implies that the Brazilian total stock of renewable resources increases, on average, 2.18 percent. With respect to nonrenewable resources, we find an average value for the extraction rate  $s^E$  of 0.0371, corresponding to 3.71 percent of the remaining stock of exhaustible resources being used each year. Note that, at any rate, an assumption

of  $s^E < \delta$  is plausible as annual depreciation rates for aggregate capital are usually estimated to at least 0.05. To overcome problems related to this strategy, we will conduct experiments assuming different values of these parameters.

The calculated sectoral output growth rates suggest that the Trade and Services, Public Administration and Agriculture sectors have a much higher long-run growth rates than the other sectors as well as the economy ( $g_{yt} = 0.0166$ ). Interestingly, these three sectors have the lowest renewable and nonrenewable resources shares ( $\kappa_i + \varepsilon_i$ ) among the sectors analyzed. The remaining sectors have a very low output growth rate, lower than one percent. For instance, the long-run output growth rate for sector Paper Products and Printing is very close to zero.

### 3.3 Energy Consumption

Once we calculate the output growth rate for each economic sector, we use the input-output framework to investigate the impact of economic growth on the consumption of energy. We choose the year of 2003 to construct the input-output and energy consumption coefficient matrices, since it is the last available data for the Brazilian input-output accounts.

The consumption of energy  $En_{2003}$  for each economic sector measured in tep (tone equivalent petroleum), for the year 2003, is presented in Table 4. Note, for instance, that although the Food and Beverages sector consumes much more energy than sectors like Nonmetallic Mineral and Paper Products and Printing, its intensity of energy use is lower than these sectors' coefficients.

The total energy consumption coefficients matrix  $P$  is key to this analysis. A typical element of this matrix is calculated as the ratio between total energy consumption and production, in a particular sector, and it is interpreted as the intensity of energy use. Table 4 presents this coefficients for the year 2003. Among the sectors we study,

Transportation and Steel, Nonferrous Metals are the sectors with the highest intensity of energy use. On the other hand, the intensity of energy use in Trade and Services and Public Administration is very small.

Next, we investigate the impact of economic growth on the consumption of energy in each sector. In particular, this analysis is based on equation (30), which expresses the energy consumption in each sector associated with a given growth rate of sectoral output,  $g_{yit}$ , presented in the previous section. We denote the levels of energy consumption in each sector in the year 2003 as the energy constraint, meaning the total amount of energy available to that sector in a particular period of time. We then verify if this consumption of energy is consistent with a long-run output growth rate for the economic sector. That is, if in 2003 the output of sector  $i$  were to grow at  $g_{yit}$ , what would be the consumption of energy associated with this sectoral growth. We then compare this result with energy constraint given by the energy consumption level of 2003 (Table 4).

For all sectors, except Public Administration, the consumption of energy in 2003 is above the level of energy consumption associated with the sectoral long-run output growth rate. The calculated consumption of energy in the sectors Agriculture, Food and Beverages and Textiles is very close to the levels observed in 2003. It implies that, if these sectors grow at their long run output growth rate they are closer to the sectoral energy constraint than other sectors, for instance, Nonmetallic Mineral and Steel, Nonferrous Metals. Similar patterns are observed if we assume that all sectors grow at the same rate, i.e. the economy long-run output growth rate  $g_{y_t} = 0.0166$ .

Given the observation that energy consumption levels associated with long-run growth rates are below the energy consumption of 2003, a natural question to ask is what would be the energy consumption of in Brazil if economic sectors continue to grow at their long-run growth rates. We answer this question by forecasting the long-run

energy consumption by sector in Brazil for the next ten year, that is, for the period 2004 – 2014. To make our analysis more informative, we study the consumption of renewable and nonrenewable energy separately.

Table 4 - Intensity of Energy Use and Energy Consumption

[ABOUT HERE]

Figure 3 and 4 show that the energy consumption of renewable resources will increase over time, as expected. The consumption of energy grows at a rate consistent with the balanced growth path of output. The dotted line refers to the consumption of non-renewable energy and the solid line represents the consumption of renewable resources.

[ABOUT HERE]

Figure 3 - Energy Consumption Forecast - Renewable and Non-Renewable Resources

[ABOUT HERE]

Figure 4 - Energy Consumption Forecast - Renewable and Non-Renewable Resources

Finally, we present the total consumption of energy (renewable and non-renewable) for the economy as a whole and compare to the total stock of energy. A non-renewable resources depletion rate of 0.0371 and a regeneration rate of renewable resources net of the fraction used in the production process equal to 0.0218 imply a decreasing stock of natural resources for the Brazilian economy as shown in Figure 5.

[ABOUT HERE]

Figure 5 - Energy Consumption Forecast - Economy

Total Energy - Renewable and Non-Renewable

### 3.4 Some Experiments

In this section, we conduct several experiments to analyze the impact of changes in the parameters of the model on the sectoral output growth rate and, consequently, on the consumption of energy in each economic sector. The experiments will involve changes in key aspects of the Brazilian economy, in particular, technological progress, extraction and regeneration rates of both renewable and nonrenewable resources and population growth. The experiments are as follows: *(i)* increase the technological progress ( $\uparrow g_{A_i}$ ) by 50 percent; *(ii)* increase the regeneration rate of renewable resources ( $\uparrow (z_t - s^Z)$ ) by 100 percent; *(iii)* increase the degeneration of nonrenewable resources ( $\uparrow s^E$ ) by 100 percent; *(iv)* increase the population growth ( $\uparrow g_{L_{it}}$ ) by 50 percent.

In Table 5, we report the output and total energy consumption growth rates for each sector and experiment, as well as the baseline growth rates for comparison. As expected, according to equation (14), the first two experiments lead to a higher growth rate of sectoral output, i.e., a higher technological progress and regeneration rate leads the economy to grow faster. On the other hand, an increase in the degeneration rate of nonrenewable resources or a population growth implies lower output growth rates. The total consumption of energy (renewable and nonrenewable) follows the same pattern of the sectoral output.

Table 5 - Output and Energy Consumption Growth Rates

[ABOUT HERE]

## 4 Conclusion

This paper studies the impacts of economic growth on the consumption of energy. We propose a methodology that integrates a growth model with an input-output model. This model integration contributes to the literature by potentially minimizing these

separated model weaknesses. Our approach presents clear advantages when compared to the each approach alone, i.e. integrated model versus a growth model and an input-output model. In particular, regarding the Leontief approach, our model incorporates technological progress through a Cobb-Douglas production function, uses a calibration process to make forecasts and allows for substitutability between inputs in the production process. These features are not standard or regular in the input-output approach. For the macroeconomic approach, the relationships between the economic agents are established in the input-output framework, which eliminate the necessity to model such complex interactions. Several studies suggest that in order to better understand the development over time of energy use, we have to use a disaggregated microeconomic approach. This is the approach proposed in this paper. A possible extension of this study is to close the model for consumption and consider the residential energy use. An natural extension of this approach is to model the energy sector as a sector of the economy, where different energy commodities would be treated differently. We leave this for future research.

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# Appendix

## A.1 - Sources of Energy

<ol style="list-style-type: none"> <li><b>1. Natural Gas</b></li> <li><b>2. Renewable Resources</b> <ul style="list-style-type: none"> <li>Coal</li> <li>Wood</li> <li>Sugar Cane Bagasse</li> <li>Other sources</li> </ul> </li> <li><b>3. Electricity</b></li> <li><b>4. Diesel</b></li> </ol>	<ol style="list-style-type: none"> <li><b>5. Other Sources</b> <ul style="list-style-type: none"> <li>Charcoal</li> <li>Kerosene</li> <li>Liquefied Petroleum Gas</li> <li>Coke Gas</li> <li>Naphtha</li> <li>Natural Gas</li> <li>Steam Coal</li> </ul> </li> </ol>
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## A.2 - Compatibilization Between National Energy Report and Input-Output Table Sectors - Brazil

<ol style="list-style-type: none"> <li><b>1. Agriculture</b> <ul style="list-style-type: none"> <li><i>1. Agriculture</i></li> </ul> </li> <li><b>2. Mining</b> <ul style="list-style-type: none"> <li><i>2. Mining</i></li> <li><i>3. Petroleum</i></li> </ul> </li> <li><b>3. Nonmetallic Minerals</b> <ul style="list-style-type: none"> <li><i>4. Nonmetallic minerals</i></li> </ul> </li> <li><b>4. Steel and Nonferrous Metals</b> <ul style="list-style-type: none"> <li><i>5. Steel</i></li> </ul> </li> <li><b>5. Nonferrous Metals</b> <ul style="list-style-type: none"> <li><i>6. Nonferrous Metals</i></li> <li><i>7. Other Metals products</i></li> </ul> </li> <li><b>6. Paper and Printing Products</b> <ul style="list-style-type: none"> <li><i>8. Paper and Printing Products</i></li> <li><i>9. Rubber</i></li> </ul> </li> <li><b>7. Chemicals</b> <ul style="list-style-type: none"> <li><i>10. Chemical Elements</i></li> <li><i>11. Petroleum Refining</i></li> <li><i>12. Other Chemicals</i></li> </ul> </li> <li><b>8. Textile</b> <ul style="list-style-type: none"> <li><i>20. Textile Industry</i></li> <li><i>21. Clothing</i></li> <li><i>22. Footwear</i></li> </ul> </li> <li><b>9. Food and Beverages</b> <ul style="list-style-type: none"> <li><i>13. Coffee</i></li> <li><i>14. Processed Vegetables</i></li> <li><i>15. Meat packing plants</i></li> <li><i>16. Dairy Products</i></li> </ul> </li> </ol>	<ol style="list-style-type: none"> <li><i>17. Sugar</i></li> <li><i>18. Vegetable oil mills</i></li> <li><i>19. Other food products</i></li> <li><b>10. Trade and Services</b> <ul style="list-style-type: none"> <li><i>35. Trade</i></li> <li><i>36. Financial Institutions</i></li> <li><i>37. Personal Services</i></li> <li><i>38. Business Services</i></li> <li><i>39. Real Estate</i></li> <li><i>40. Community Services</i></li> </ul> </li> <li><b>11. Transportation</b> <ul style="list-style-type: none"> <li><i>41. Transportation</i></li> </ul> </li> <li><b>12. Public Administration</b> <ul style="list-style-type: none"> <li><i>42. Public Administration</i></li> </ul> </li> <li><b>13. Other Sectors</b> <ul style="list-style-type: none"> <li><i>23. Machinery</i></li> <li><i>24. Electrical Equipment</i></li> <li><i>25. Electronic Equipment</i></li> <li><i>26. Automobile, Trucks and Buses</i></li> <li><i>27. Transportation Equipment</i></li> <li><i>28. Wood Products and Furniture</i></li> <li><i>29. Pharmaceuticals and Veterinary</i></li> <li><i>30. Plastics</i></li> <li><i>31. Other Manufacturing</i></li> <li><i>32. Electric, Gas, Sanitary Services</i></li> <li><i>33. Construction</i></li> <li><i>34. Communication</i></li> </ul> </li> </ol>
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Table 1 - Input-Output Accounts

	Inter	Sectoral	Purchases		Final	Demand			Total
									Output
Inter	$z_{11t}$	$z_{12t}$	...	$z_{1mt}$	$C_{1t}$	$I_{1t}$	$G_{1t}$	$NE_{1t}$	$X_{1t}$
Sectoral	$z_{21t}$	$z_{22t}$	...	$z_{2mt}$	$C_{2t}$	$I_{2t}$	$G_{2t}$	$NE_{2t}$	$X_{2t}$
Sales	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.
	$z_{m1t}$	$z_{m2t}$	...	$z_{mmt}$	$C_{mt}$	$I_{mt}$	$G_{mt}$	$NE_{mt}$	$X_{mt}$
Wages	$w_{1t}$	$w_{2t}$	...	$w_m$					$W_t$
Others	$ov_{1t}$	$ov_{2t}$	...	$ov_m$					$OV_t$
Total									
outlays	$X_{1t}$	$X_{2t}$	...	$X_{mt}$	$C_t$	$I_t$	$G_t$	$NE_t$	$X_t$

Table 2 - Energy Use by Economic Sectors (2003)

Sectors	Renewable	Nonrenewable
	Energy	Energy
Agriculture	39.55	60.45
Mining	31.47	68.53
Nonmetallic minerals	84.31	15.69
Steel and nonferrous metals	32.56	67.44
Paper products and printing	81.23	18.77
Chemicals	28.04	71.96
Textiles	63.88	36.12
Food and Beverages	91.94	8.06
Trade and Services	86.12	13.88
Transportation	23.80	76.20
Public Administration	79.44	20.56

Source: National Energy Report (2005)

Table 3 - Parameter Values and Sectoral Output Growth Rates

	$\alpha_i$	$\beta_i$	$\kappa_i$	$\varepsilon_i$	$g_{A_i}$	$g_{L_i}$	$g_{y_{it}}$
Agriculture	0.6684	0.2731	0.0119	0.0466	0.0092	0.016	0.0205
Mining	0.5423	0.2308	0.0882	0.1387	0.0012	-0.013	0.0021
Nonmetallic Mineral	0.3381	0.3537	0.1305	0.1777	0.0054	0.001	0.0020
Steel, Nonferrous Metals	0.1834	0.2922	0.2572	0.2672	0.0008	-0.009	0.0017
Paper Products and Printing	0.4037	0.3735	0.0680	0.1548	0.0079	0.016	0.0003
Chemicals	0.4375	0.2667	0.1636	0.1322	0.0114	0.016	0.0094
Food and Beverages	0.5515	0.3320	0.0638	0.0527	0.0058	0.029	0.0043
Textiles	0.4488	0.3497	0.0532	0.1483	0.0123	0.022	0.0062
Trade and Services	0.6289	0.2993	0.0505	0.0213	0.0384	0.038	0.0969
Transportation	0.3107	0.3787	0.0126	0.2980	0.0229	0.036	0.0013
Public Administration	0.3045	0.6481	0.0297	0.0177	0.0278	0.032	0.0378

Table 4 - Intensity of Energy Use and Energy Consumption

	$p_{i2003}$	$En_{2003}$	$\hat{En}_{g_{yit}}$	$En_{2003} - \hat{En}_{g_{yit}}$ (%)
Agriculture	0.0314	8,149,978	7,766,623	-4.7
Mining	0.0333	2,495,390	1,907,059	-23.6
Nonmetallic Mineral	0.0792	2,808,450	717,215	-74.5
Steel, Nonferrous Metals	0.1462	23,185,217	9,340,771	-59.7
Paper Products and Printing	0.0833	6,529,913	4,965,165	-24.0
Chemicals	0.0241	6,600,595	4,745,847	-28.1
Food and Beverages	0.0669	16,614,956	16,017,275	-3.6
Textiles	0.0171	1,080,282	1,026,853	-4.9
Trade and Services	0.0060	4,993,986	3,848,921	-22.9
Transportation	0.5027	53,140,147	44,095,442	-17.0
Public Administration	0.0097	3,179,753	3,256,740	2.4

Table 5 - Output and Energy Consumption Growth Rates

Sectors	Sectoral Output Growth Rate ( $g_{yit}$ )					Total Energy Consumption Growth Rate				
	Baseline	Experiments				Baseline	Experiments			
		1	2	3	4		1	2	3	4
Agriculture	2.05	3.44	2.13	1.53	1.91	1.78	3.09	1.97	1.30	1.53
Mining	0.21	0.34	0.63	-0.92	0.53	2.16	3.67	2.52	1.36	2.05
Nonmetallic Mineral	0.20	0.61	0.63	-0.79	0.18	1.53	2.67	1.90	0.78	1.41
Steel, Nonferrous Metals	0.17	0.22	0.86	-1.04	0.47	0.78	1.29	1.38	-0.27	0.94
Paper Products	0.03	0.70	0.28	-0.93	-0.26	3.04	5.15	3.28	2.43	2.73
Chemicals	0.94	1.95	1.58	0.07	0.53	3.32	5.61	3.63	2.70	2.96
Food and Beverages	0.43	1.08	0.74	-0.01	0.06	1.10	2.11	1.39	0.67	0.74
Textiles	0.62	1.73	0.83	-0.38	0.21	1.14	2.48	1.35	0.22	0.74
Trade and Services	9.69	14.86	9.98	9.48	9.32	8.31	12.85	8.58	8.06	7.96
Transportation	0.13	1.80	0.17	-1.47	-0.68	1.63	3.63	1.76	0.49	1.02
Public Administration	3.78	5.78	3.87	3.69	3.67	3.81	5.83	3.90	3.71	3.69
Economy	1.66	2.96	1.97	0.84	1.45	1.97	3.72	2.20	1.14	1.57

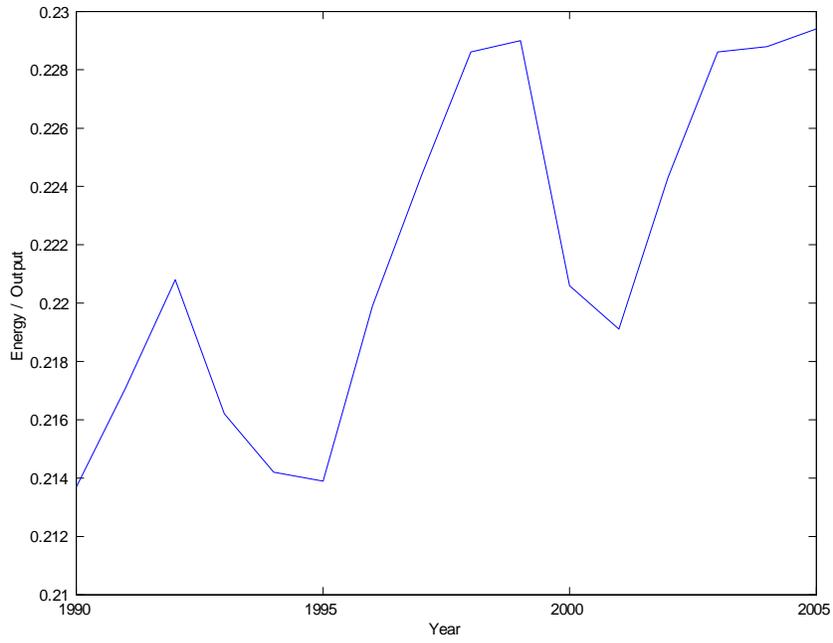


Figure 1 - Energy Consumption and GDP - Brazil

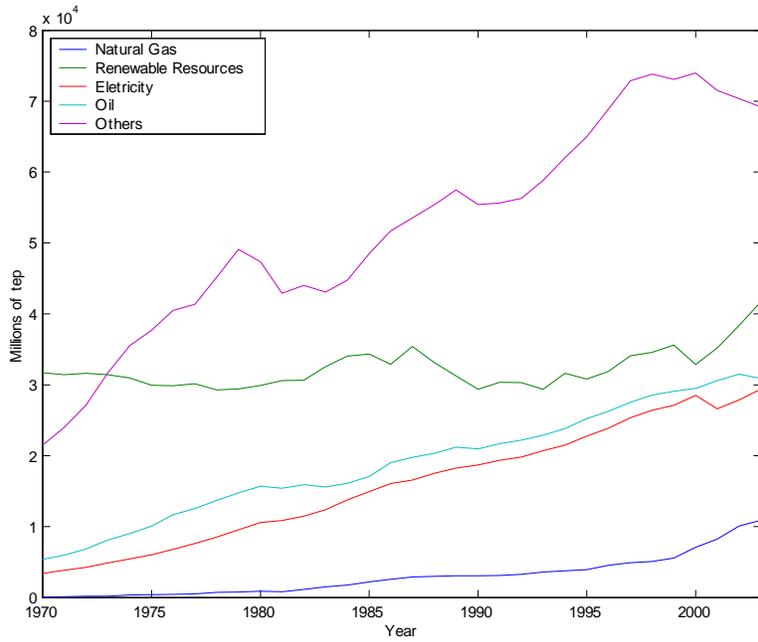


Figure 2 - Sources of Energy - Brazil

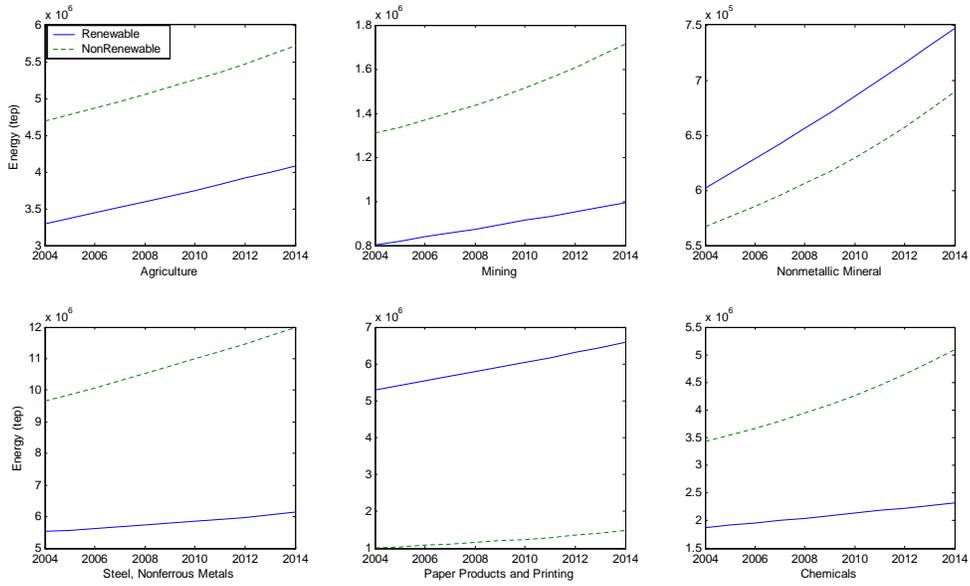


Figure 3 - Energy Consumption Forecast  
Renewable and Non-Renewable Resources

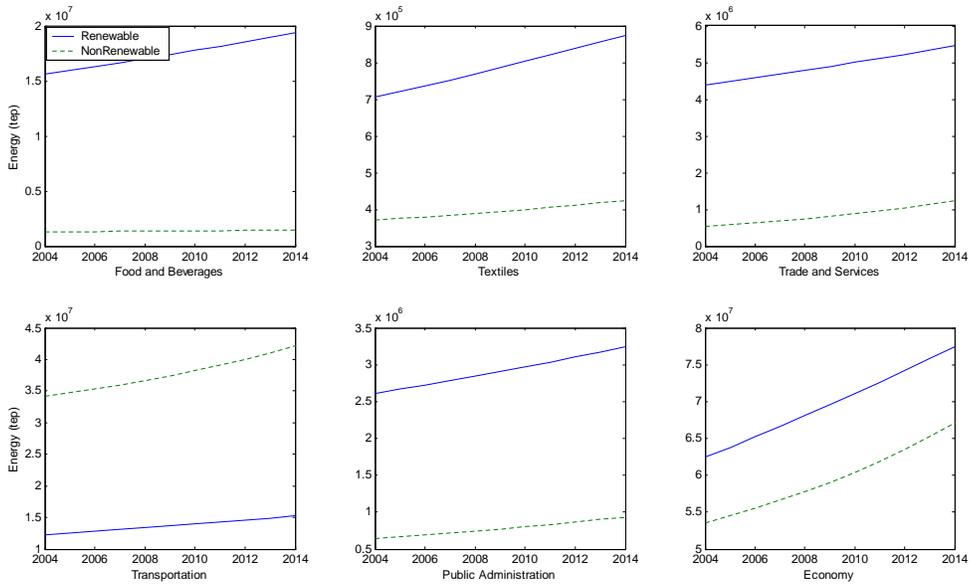


Figure 4 - Energy Consumption Forecast  
Renewable and Non-Renewable Resources

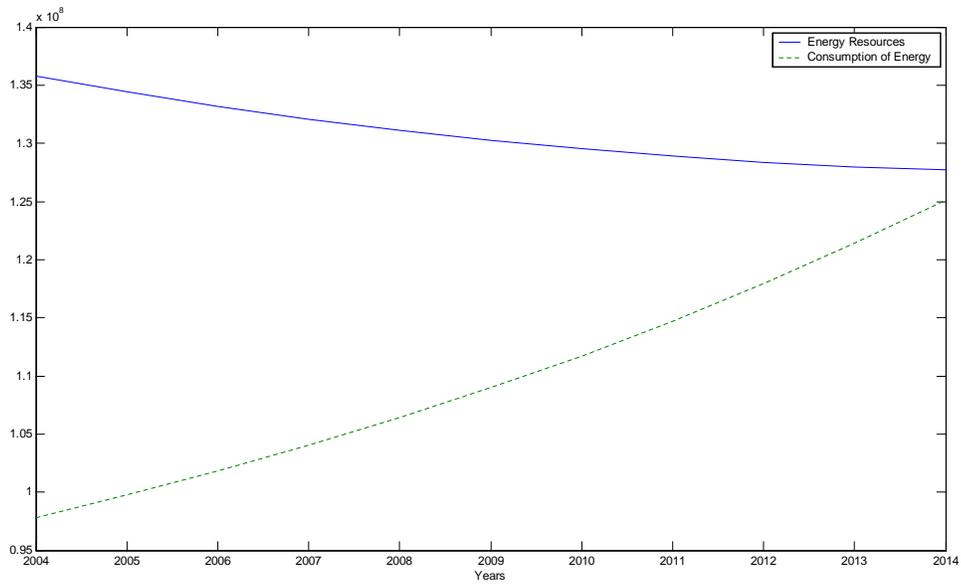


Figure 5 - Energy Consumption Forecast - Economy  
Total Energy - Renewable and Non-Renewable