Two new measures for assessing complexity as interrelatedness

João Dias*, J. Ferreira do Amaral and J. Carlos Lopes

ISEG - School of Economics and Management, Technical University of Lisbon, and UECE - Research Unit on Complexity and Economics

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ABSTRACT:

The study of complex systems has become more and more important and many attempts have been made to study such systems. In a previous paper, we developed a measure for assessing complexity as interrelatedness in Input-Output systems and applied it to a sample of relatively aggregated I-O tables of some OECD economies. However, the computation of this measure is very time demanding and becomes prohibitive for large I-O tables. In this paper we propose two new measures that are easy to apply and have not this computation burden, thus allowing us to measure complexity as interrelatedness of more disaggregated inter-industry relations. These measures are related to the trace and the degree of decomposability of a matrix. An application will be made to a group of OECD countries and the results will be compared with our previous measure.

* Corresponding author:

e-mail: jdias@iseg.utl.pt

Full address:
Prof. João Dias
ISEG - UECE
Rua do Quelhas, nº 6
1200-781 Lisboa
Portugal

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1. Introduction

The study of complexity has attracted many efforts in different fields in the last decades, including in biology, physics, computation science and social systems. Although this is true also in economics, we are not concerned here with complexity in general, and no attempt will be made here to present the main ideas, issues and results obtained up to now.

Our concern in this paper is with complexity as interrelatedness, applied to input-output systems. Some proposal have been made before, including those of Yan and Ames (1965) or Blin and Murphy (1974) who, in a certain way, refined the somewhat simplistic early attempt made by Peacock and Dosser (1957) and Wong (1954). The number of measures proposed has increased subsequently. A paper by Szymer in 1986 listed twenty-five alternatives to analyze connectedness or interrelatedness of economic systems as represented in an input-output framework. More recent proposals based on graph theory and structural path analysis have been suggested by Basu and Johnson (1996) and Sonis and Hewings (1998). Dietzembacher (1992) and Lantner and Carluer (2004) are other examples in this field.

In a previous paper, Amaral, Dias and Lopes (2007), we developed a new measure to assess complexity as interrelatedness in input-output systems. This index has some nice properties. However, an important drawback of this measure is its computation burden and, as such, only practical for IO tables of small dimension. In this paper, we further explore a feasible implementation based on an approximation to this measure and also explicitly consider the degree of decomposability of a matrix, both possibilities also presented in the ADL paper.

The paper will be structured as follows. In section 2, we briefly present our new measures and the link with the standard ADL index. Section 3 explores these new measures with an empirical application to a group of larger IO tables. Finally, some concluding remarks follow in section 4.

2. ADL index and two new measures

Let us consider the matrix A of technical coefficients, of order n, and a sub-matrix, a square block *A** of order m, with its diagonal elements taken from the diagonal of *A*. The degree of autonomy of *A** is defined as,

$$G_a(\mathbf{A*}) = \frac{\|\mathbf{A}*\|}{\|\mathbf{A}*\| + \|\mathbf{A}**\| + \|\mathbf{A}***\|},$$

where *A*** is the matrix of all the elements of the columns of *A* belonging to *A** but excluding the elements of *A** themselves. *A**** is similarly defined but for the rows. Here, the notation $\|M\|$ means the sum of all the elements of matrix *M*.

The complement of the degree of autonomy is the degree of dependency,

$$G_d(\mathbf{A*}) = 1 - G_a(\mathbf{A*}).$$

Based on this concept, for a given matrix *A* of technical coefficients, the first ADL index of complexity as interrelatedness is defined as,

$$(1)\quad G(\mathbf{A}) = \frac{(2^N - 2)G*(\mathbf{A})}{2^N - 2^{N-2} - 1} = \frac{\sum_k G_d\left(A_k^*\right)}{2^n - 2^{n-2} - 1},$$

with $G*(\mathrm{A}) = \dfrac{\sum_k G_d(\mathrm{A}_k*)}{2^N - 2}$.

The difficulty of this measure is due to the fact that the number of blocks *A** of A grows exponentially with the number of sectors, since we have, with n sectors, $2^n - 2$ such blocks.

It is for this component that an approximation to *G(A)* will be considered for large IO systems. This is given by the upper limit to *G(A)* (with large n) given by,

$$(2)\ M_1 = 1 - \frac{1}{3}\frac{trace(\mathrm{A})}{\|\mathrm{A}\|} = 1 - \frac{T^*}{3}\ .$$

The second element that we may want to consider is the *network effect*, in order to take into account the connectedness of each sector with all the others in a given IO system. This second index is the degree of decomposability of a matrix *A*, and if defined as,

$$(3)\ M_2 = H(A) = 1 - \frac{Z(A)}{n^2 - n},$$

where $Z(A)$ is the number of zeros of the Leontieff inverse, that is, of $(I - A)^{-1}$. This is only relevant for reasonably disaggregated IO matrices since, for matrices of small size, M_2 will usually be equal to one, as in the case of the OECD matrices used by ADL.

Our third measure is the combination of M_1 and M_2, and is defined as,

$$(4)\ M_3 = M_1 \times M_2,$$

which integrates both the *dependency effect* and the *network effect*.

3. An empirical application

3.1 The approximation given by M_1=1-T*/3

We begin this section with an empirical application of M_1 to the same IO tables used by ADL, in order to compare the ADL measure and the M_1 approximation to it. Table 1 gives the ADL complexity index computed for a sample of OECD countries. The number of sector used in the IO tables was 17 and, for tables of this dimension, the computation is quite feasible.

In table 2 we used the approximation given by our upper limit 1-T*/3. In spite of the fact that M_1 gives higher values than the ADL measure (in this case, M_2=1), the hierarchy of countries for a given year or, for a given country, the evolution from the early seventies to the early nineties is the same in both tables. In fact, the correlation coefficient between these two measures is very high, equal to 0.998 for these 41 cases.

Table 1 ADL complexity index computed for 17 sector

	Early 1970s	Mid 1970s	Early 1980s	Mid 1980s	Early 1990s
Australia	0.747	0.731		0.756	0.767
Canada	0.779	0.778	0.765	0.766	0.765
Denmark	0.786	0.792	0.787	0.795	0.791
France	0.737	0.741	0.758	0.757	0.748
Germany		0.732		0.742	0.753
Italy				0.764	
Japan	0.740	0.740	0.742	0.723	0.719
Netherlands	0.754	0.769	0.777	0.782	
UK	0.729	0.770		0.751	0.737
US	0.712	0.720	0.731	0.741	0.732

Table 2. M_1 = 1-T*/3, for the same 17 sectors as in Table 1

	Early 1970s	Mid 1970s	Early 1980s	Mid 1980s	Early 1990s
Australia	0.914	0.907		0.918	0.923
Canada	0.928	0.928	0.922	0.922	0.921
Denmark	0.933	0.936	0.934	0.938	0.936
France	0.909	0.911	0.919	0.919	0.914
Germany		0.907		0.912	0.916
Italy				0.922	
Japan	0.910	0.910	0.911	0.903	0.901
Netherlands	0.918	0.926	0.929	0.932	
UK	0.905	0.925		0.916	0.909
US	0.898	0.902	0.907	0.911	0.907

3.2 The quality of the approximation as the number of sectors increases

As we have already stated before, the M_1 approximation is only necessary for large matrices. Therefore, it would be useful to compare the *G(A)* and M_1 as the number of sectors increases. Table 3 and Figure 1 in the Appendix show this comparison for the case of the United States in 1990. The number of sectors changes from 3 to 30. With 30 sectors, the computation of *G(A)* took about 2 days in a modern personal computer. For each additional sector, the time of computation is more or less multiplied by two. So, even with IO tables of moderate dimension, of around 40 sectors or so, it is not feasible to compute the ADL index, given the currently available computation power. But our empirical application for the United States shows that the gap between these two measures becomes smaller as the number of sectors increases. Similar results (not shown here) were obtained as well with data for other countries.

Table 3. *G(A)* and M_1 measures for the United States, 1990

	(a)	(b)		
Number of sectors	*G(A)*	M_1=1-T*/3	(b)-(a)	(b)/(a)
3	0.614	0.823	0.209	1.340
10	0.699	0.884	0.185	1.265
17	0.732	0.907	0.175	1.240
22	0.745	0.916	0.171	1.229
27	0.751	0.920	0.169	1.225
30	0.755	0.923	0.168	1.223

3.3 A first application of our new measures to EU countries

The first application of our M_1, M_2 and M_3 measures presented in this section is for a group of 19 EU countries, for which a set of IO tables in a comparable basis could be used. Table 4 lists our results for the case of M_1 and M_2. The values for M3 are presented in Table 5. Since our main purpose is simply to present an empirical application of our measures, we will not comment here on the hierarchy of values obtained for different countries or the recent evolution of these values in the last decade. Anyway, we note the strong increase of values of M_2 and M_3 in the case of Ireland, from the first to the last period considered here and we also note that, in general terms, M_1 and M_3 did not change substantially between 1995 and 2005, for the other EU economies.

Table 4. M_1 and M_2 for 19 EU countries, 49 sectors, 1995 and 2005

	M_1=1-T*/3		M_2=H(A)	
	1995	2005	1995	2005
Belgium	0.934	0.953	0.980	0.980
Czech Rep.	-	0.893	-	0.980
Denmark	0.948	0.947	1.000	1.000
Estonia	0.909	0.942	0.959	1.000
Finland	0.942	0.939	1.000	1.000
France	0.908	0.910	0.980	0.980
Germany	0.924	0.923	1.000	1.000
Greece	0.933	0.934	0.980	0.980
Hungary	0.964	0.960	1.000	1.000
Ireland	0.737	0.922	0.796	1.000
Italy	0.946	0.948	1.000	1.000
Latvia	0.909	0.898	0.959	0.959
Lithuania	-	0.921	-	0.959
Netherlands	0.921	0.933	1.000	1.000
Portugal	0.907	0.906	0.980	0.980
Slovakia	0.925	0.920	1.000	1.000
Slovenia	0.914	0.913	0.959	1.000
Spain	0.931	0.925	0.980	0.980
UK	0.932	-	1.000	-

Data from Eurostat. In some cases, the years are around 1995 or 2005

Table 5. M3 for 19 EU countries, 49 sectors, 1995 and 2005

	1995	2005
Belgium	0.915	0.933
Czech Rep.	-	0.893
Denmark	0.948	0.947
Estonia	0.909	0.942
Finland	0.942	0.939
France	0.908	0.910
Germany	0.924	0.923
Greece	0.933	0.934
Hungary	0.964	0.960
Ireland	0.737	0.922
Italy	0.946	0.948
Latvia	0.909	0.898
Lithuania	-	0.921
Netherlands	0.921	0.933
Portugal	0.907	0.906
Slovakia	0.925	0.920
Slovenia	0.914	0.913
Spain	0.931	0.925
UK	0.932	-

3.4 An illustration with larger matrices.

An empirical application of our M_1, M_2 and M_3 measures is presented here for Taiwan in 2004 and Japan in 2000. For Taiwan, two domestic IO matrices of different size where used in Table 6, one with 49 sectors and the other much more disaggregated, with 161 sectors. Somewhat contrarily to our expectation, M_2 did not change significantly from 49 to 161 sectors. By contrast, M_1 and M_3 increased with the level of disaggregation, as we also noted in Table 3 for the United States. In the case of Japan, the network effect is much more significant, thus putting M_3 at a lower level.

Table 6. M_1-M_3 for 49 and 161 sectors, Taiwan, 2004

	Number of sectors	M_1	M_2	M_3
Taiwan, 2004	49	0.938	0.980	0.919
	161	0.968	0.981	0.950
Japan, 2000	104	0.955	0.884	0.844

4. Concluding remarks

In a previous paper we proposed a new measure of economic complexity as interrelatedness in input-output systems which has some nice properties but has a practical limitation, in that its computation is very time demanding and becomes prohibitive for large IO tables. In this paper we used an approximation to that measure. We showed that this is highly correlated with the ADL index but has not any computation burden. In fact, it can be applied to very large matrices with hundreds of sectors and the time of computation is of only a few seconds. An empirical application is made for matrices of moderate size in the case of 19 EU countries. Another application is also made for larger matrices available for Japan and Taiwan.

In the paper we also used another measure, de degree of decomposability of a matrix, in order to take into account the network effect of complexity as interrelatedness. This is usually not relevant in IO matrices of very small size, but may be very significant in large IO systems. In our case, it shows an interesting increase for Ireland between 1995 and 2005. It is also relevant in the case of Japan but, interestingly enough, does not seem significant for Taiwan, even for a relatively disaggregated IO matrix of this country.

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Appendix

Figure 1. *G(A)* and 1-T*/3, from 3 to 30 sector, US 1990.