The identification of key sectors by means of Data Envelopment Analysis (DEA): the case of EU-27

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Abstract
Standard input-output analysis generally takes the Leontief and the Ghosh inverses to provide, respectively, backward and forward multipliers for multiple dimensions (output, income…) as a starting point to identify key sectors in an economy. Then, it is difficult to say whether a sector is crucial for all dimensions (or most of all). Alternatively, hypothetical extraction methods have recently challenged the standard tools by providing the overall impact on the economy (without distinction between backward and forward effects) of the extraction of one single sector, only in terms of one dimension (*i.e.* production) but unfortunately not in terms of others (*i.e.* income, employment…). Anyhow, the identification of key sectors made in one way or another depends highly on a threshold defined most frequently (and arbitrarily) by the arithmetic mean of the values obtained, which, to a great extent, is sensitive to outliers. Therefore, rigid and robustless classifications come out from usual key sector analysis. In order to circumvent these controversial issues and thus find a more robust way to identify key sectors in an economy, this paper contributes to the literature by adding a more flexible approach (DEA) based on efficiency terms. We propose to use a single "key value" for the identification of key sectors that summarizes the (backward and forward) potential increases of multiple dimensions (production, income and employment, etc). Hence, our approach is independent from the quite often criticized methods for identifying key sectors. The empirical work is carried out for the 27 European Union economies.

Keywords: Input-output multipliers, key sectors, DEA, input-output linkages, composite indicators.

Note: The views expressed in this paper belong to the authors and should not be attributed to the European Commission or its services.

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1 Introduction

What is the relevance of an economic sector? In time of crisis, accurate answers are demanded by government agencies for policy-making and the planning of the assignation of subsidies in the most efficient way (in terms of repercussion). Answers can mainly be grouped in two main trends, *i.e.*: backward and forward linkages based on multiplier matrices; and differential output based on hypothetical extraction methods (HEM) in a broad sense. Anyway, there is no fully accepted consensus yet, on which the best evaluation procedure is. Each approach has its own pros and cons. Backward (BW) linkages, for instance, are constructed from the Leontief inverse whereas forward linkages (FW) use the inverse matrix from the Ghosh price model. While the Leontief quantity model has a clear technological interpretation, being well rooted in production theory, it was not until Dietzenbacher (1997) when the Ghosh price model was found to be as well interpreted as a Leontief price model. For a long time therefore, more conceptual credit has been given to BW linkages than to FW linkages since only the former were believed to trace the ripple effects implicit in the underlying technology. The notions of BW and FW linkages, started with Chenery and Watanabe (1958) and Rasmussen (1956), and were further developed by many authors. Schultz (1977), Sonis *et al.* (1995) and Dietzenbacher (2002) are good surveys of this literature and its development. Recently, the HEM has received ample attention again as an alternative to simple linkage measures. The basic idea behind HEM is that in order to elicit the economic role of a sector, we somehow need to simulate the impact of its elimination from the economy. If a given sector ceases to interact with the rest of sectors, what would the opportunity cost be measured in terms of lost gross output? A comprehensive recapitulation of HEM can be found in Miller and Lahr (2001) and recent applications include Sánchez-Chóliz and Duarte (2003).

Traditional linkage analysis is made comparing the sectorial values of the BW and FW linkages with respect to their arithmetic mean. The point is that, establishing a threshold that separates being a key sector or not accurately and robustly, is not only difficult (because of outliers) but also useless. For policy-making, the relative performance of sectors in terms of impacts might be more relevant than a rigid classification.

Anyway, both approaches, either multipliers or HEM, are incomplete since they are one-dimensional techniques trying to analyse a multi-dimensional phenomenon (production, income and employment), usually arriving to contradictory conclusions for each dimension. Those approaches do not tackle the issue comprehensively, but one dimension each time, then a new approach is needed in order to summarise the multi-dimensional information in a single indicator of relevance.

Following the approach initiated by Cherchye *et al.* (2006) about 'Benefit of the Doubt' composite indicators, this paper makes two interrelated contributions to the literature. Firstly, we introduce data envelopment analysis (DEA) in order to detect key sectors by means of a comprehensive indicator that summarizes the impact values of output, employment and income ("key value") solving controversial issues of multipliers analysis; and secondly, we decompose the "key value" in order to assess the relevance of a certain industry accurately, especially within the international context.

The paper is structured as follows: the next section discusses the main shortcomings of traditional linkages and HEM analysis. Section 3 presents a new approach on the basis of DEA to identify key activities. Then, the following section presents the data sources and the results of the empirical work carried out for the economies of the 27 Member States (MS) of the European Union (EU-27) for the year 2000. Finally, the last section concludes with a summary of the most prominent findings.

2 Controversial issues of the traditional key sector analysis

2.1 Multipliers Analysis

The BM and FM are given basically by pre-multiplying, respectively, the Leontief and the Ghosh inverse matrices by a coefficients vector, which will depend on the variable under consideration (income, employment, emissions, capital use…). For production multipliers, the coefficients vector turns out to be the unitary vector. For standard reference on input-output multipliers, see Miller and Blair (1985).

Following Dietzenbacher (1997), on the one hand backward multipliers can be interpreted as the change in the output value, when we increase final demand in one monetary unit (*ceteris paribus*), holding prices constant (Leontief quantity model). On the other hand, forward multipliers are interpreted as the potential change in the output

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value when we increase the value added in one monetary unit (*ceteris paribus*), holding the use of factor inputs constant (Ghosh price model).

Let us denote production backward and forward multipliers as PBM and PFM; income backward and forward multipliers as IBM and IFM; and employment backward and forward multipliers as EBM and EFM. Then, by normalizing their values using their respective arithmetic mean, the main advantage of this simple approach derives from their easy interpretation as being greater or smaller than one. In this sense, Table 1 shows the classification scheme.

Table 1

The traditional key sector analysis presents a chain of four interrelated problems, which we will analyze one by one: Firstly, it focuses on multipliers compared with their average (ratios), which is highly affected by outliers. Secondly, it "draws a line" to make a rigid classification in which some information about industries placed close to the inaccurate threshold is missing. Thirdly, and related to the previous one, it considers BM and FM equally important whatever the values of these multipliers are. And finally, it studies only one dimension (production, income or employment) each time, often obtaining contradictory conclusions depending on each dimension studied.

Table 2 and Table 3

Let us then analyze in detail the problems of one single country (the list of available countries with their relative codes is available at Table 2 and industries at Table 3) as an example, *i.e.*: Denmark (DK), whose multipliers are presented in Table 4. One of the main difficulties in traditional key sector analysis is the use of the average to classify sectors, which is due to the fact that most activities are concentrated around the average of linkages, which are incidentally highly affected by outliers (average reduces in more than a half if we compute average without outliers). And, as shown in Table 4, outliers (marked with an asterisk) usually exist among multipliers and are relevant (30 outliers out of 354 multipliers (6 multipliers types x 59 industries), this is 8.5% of cases for Denmark).

Then, we will focus our attention on a couple of alternatives to the arithmetic mean. In this sense, we propose to identify key sectors using: (a) the usual arithmetic

mean; (b) corrected averages (computed excluding outliers)[2]; and (c) the median, a rather simple descriptive statistic which is not affected by outliers. Table 4 shows the different classifications reported by each one for each multiplier dimension. Results under the (b) and (c) options are quite similar because outlier's effects have been neutralized. As it is shown in Table 4, the use of a different statistic to identify key sectors makes clearly a difference. There are significant differences in classification according to the different statistic taken into account.

Table 4

We have compared pair wise the identifications resulting from the three approaches: arithmetic mean, the corrected average and the median. That makes $\binom{3}{2} = 3$ possible comparisons *per* industry, and with 59 industries, this makes 3 x 59 = 177 different comparisons. Then, we calculated how many of them emerged with different identifications. In the case of production multipliers, this amounts to 20.34% (36 differences), 23.16% (41 differences) when studying income multipliers and 25.42% (45 differences) when checking employment multipliers (this may vary depending on the approach considered). In conclusion, we can say that between the 20% and 25% of our conclusions is dependant on the choice of the approach and not on data.

However, one way or another, this way of identifying key sectors relies on "drawing a line" (*e.g.* average, median, etc.) and putting industries into close boxes, which might not be quite satisfactory, since it loses information in terms of "distance to the line". *I.e.*: consider, at Table 4 and Figure 1, income multipliers for Recreational services (DK_{57}) and Hotel services (DK_{38}). They are both, quite near to the average line, which is not so accurate due to outliers. Bearing in mind that IBM_{57} (0.4728) is only 0.01 greater than IBM_{38} (0.4664) and that the mean yields 0.467, is it so informative to say that Hotel services (DK_{38}) is a weakly linked sector and Recreational services (DK_{57}) is a 'key sector' when both have similar IFM values? If we "move" the line to values determined by the corrected mean or the median (both are very close to each other), Hotel services (DK_{38}) becomes a 'key sector', too. Considering employment multipliers, this situation occurs for Agriculture (DK_{01}), Manufacture of Metal products

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[2] Consider an outlier to be a value outside the interval $\left(Q_1 - 1.5IQ, Q_3 + 1.5IQ, \right)$, where Q_1 is the first quartile,

(DK_{22}) and Membership org. services (DK_{56}). Then a kind of measure of the relative distance to the maximum, which let us state a sort of ranking that will be better than the traditional rigid classification.

We have assumed equally weighted criteria (for BM and FM), which might be the easiest weighing scheme, but not necessarily the best/fairest one. Although, we may have ignored it, considering both of the same importance, is by itself a weighing scheme. As stated by Cherchye *et al.* (2006), by keeping a weighing system fixed, eventual rankings still may depend on the particular (and so-called 'preliminary') normalization option.

Despite their common use, looking at single indicators separately seems controversial to us. Why should we consider that both BM and FM are equally important? For instance, why Recreational services (DK_{57}) should be better (as a key sector) than Food manufacture (DK_{09}), 'only' backward oriented (BW-O), just because the former has both PBM and PFM (1.57 and 2.30) greater that their respective averages (1.49 and 2.15)? Looking back our reasoning, and keeping in mind how sensible an average is to outliers, is the average so informative? If PBM_{09} (7.60) is roughly five times greater than PBM_{57}, then is it still so important to say that Recreational services (DK_{57}), is anyway, a 'key' sector and Food manufacture (DK_{09}) or is it not? The same applies for the Recreational services (DK_{57}), key sector, and Other business services (DK_{51}), 'only' FW-O, according to PFM. The former has both PBM and PFM (1.56 and 2.30) greater that their respective averages (1.49 and 2.15). If PFM_{51} (6.49) is roughly three times greater than PFM_{57}, when PBM_{51} (1.215) is close under the mean of PBM (1.57), then is it still so informative to say that Recreational services (DK_{57}) is, anyway, still 'key' sector and Other business services (DK_{51}) is not so relevant (see Figure 1 for production)?

Actually, using equal weights is not an option formulated from any reason but from the Ockham's razor: *"If competing hypotheses are equal in other respects, it is recommendable to select the hypothesis that introduces the fewest assumptions and postulates the fewest entities"*. But no hypotheses are less than a single one (equal weights): The point is that we can let data talk by themselves and select the best system,

Q_3, the third quartile and *IQ*, the inter-quartile range.

"giving the benefit of the doubt to indicators" as stated by Cherchye *et al.* (2006), so, why not to do it? Cherchye *et al.* (2006) present well-documented further discussion about the problems in the construction of composite indicators in relation to units of measurement, normalization processes and arguable fixed weighing schemes.

Finally, the problem becomes more important if we consider not only one dimension (production, income or employment) each time but more or even all at a single time. Table 4 shows the different classifications reported by each kind of multipliers.

How are we going to consider a sector, which is a key sector in terms of production but only BW-O in terms of income/employment? Look for example: Real State services (DK_{47}). It is key sector in terms of production but BW-O in terms of employment and income. Moreover, it is possible to find three different classifications, one for each dimension. Consider Hotel services (DK_{38}), it is Backward oriented in terms of production, weakly linked in terms of income and key sector in terms of employment! Again, why should we consider them (PM, IM and EM) equally important? And again, why don't we give the 'benefit of the doubt' to the indicators in order to obtain a more comprehensive and independent measure of the relevance of sectors? In key sector analysis, these are questions that need to be addressed.

2.2   Hypothetical Extraction Methods (HEM)

HEM, based on Schultz (1977), have been extensively generalized in diverse ways even including computable general equilibrium models. These techniques try to capture both backward and forward impacts not through multipliers but through the hypothetical loss in any dimension (production, income, employment, emissions …) if any specific industry would hypothetically disappear from the economy, *ceteris paribus*. An interested reader should see Miller & Lahr (2001) for a comprehensive review of HEM.

The advantages of this approach are that it combines both backward and forward impacts and avoids any issues regarding the sensitiveness of multipliers' analysis, described in the previous subsection. On the other hand, the disadvantages are related to the controversial plausibility of the hypothetical disappearance of an industry and the *ceteris paribus* assumption previously mentioned that would imply, for instance: fixed

final demands and fixed input coefficients between the actual and hypothetical-extraction scenarios.

The HEM key sector analysis solves the first three problems of multipliers analysis detailed in the previous subsection. Moreover, apart from the implausibility of HEM, the last controversy remains: it studies only one dimension (production, income or employment) each time, often obtaining contradictory conclusions depending on each dimension studied.

3   Key activities by means of DEA

DEA may offer a solution for the structural linkage assessment issue by means of calculating 'key-values' (DEA Score) instead of dealing with somewhat strict classifications affected by problems such as meaningful bounders and equality of weights in indicators. In this sense, Data Envelopment Analysis (DEA) may be instrumental in overcoming these limitations. It fills the informational gap in the 'right' set of weights by generating flexible 'benefit of the doubt'-weights for each evaluated activity. The dependence of the specific weighing scheme used to aggregate sub-indicators and the consequent disagreement among experts cannot be thus invoked to undermine the credibility of the resulting composite indicators (Cherchye *et al.*, 2006). In addition, DEA can deal with variables measured in different units (*e.g.* in monetary terms – output or income – and physical terms – employees or tons). Incidentally, DEA outcomes are rather easy to interpret. They may help to make a single ranking of sectors having a relative measure of their 'key-value' (in the range 0-1 or 0-100, easily understood as percentage).

Next paragraphs describe DEA and the related 'Benefit of the Doubt' method briefly. Please see Cherchye *et al.* (2006) for a detailed explanation. We will focus our attention on fundamental intuitions rather than on technical and computational aspects of DEA, which can be found in detail in specific textbooks such as Charnes *et al.* (1995), Cooper *et al.* (2000), Zhu (2003) or more recently under a new perspective in ten Raa (2008).

3.1   DEA fundamental intuitions

Following Farrell (1957), Data Envelopment Analysis (DEA) is a non-parametric linear-programming-based technique, which was further developed by Charnes *et al.*

(1978) and further extended by Banker *et al.* (1984). DEA generalizes the basic concept of efficiency, understood as productivity (ratio of outputs over inputs) and converts multiple input and output measures for a set of Decision-Making Units (DMU) into a single comprehensive measure of efficiency. DEA models identify a frontier of 'best-in-class' units that are used to measure the relative efficiency of remaining units in terms of their distance to the frontier.

Figure 2

Figure 2 depicts this approach graphically. Consider a production process[3] with two outputs (y_1 and y_2) and a single input (x). Each axis presents the values of output *per* input of each unit ($y_1/\mathbf{x}$, $y_2/\mathbf{x}$). Efficient units are the most distant from the origin (*O*) and draw the Possibilities Production Frontier (*PPF*). Efficiency scores (θ^*) are the relative distance to the frontier. Efficiency scores of efficient units (*A*, *B*, *C*, *D*) are:

$$\theta^* = \frac{\overline{OA}}{\overline{OA}} = \frac{\overline{OB}}{\overline{OB}} = \frac{\overline{OC}}{\overline{OC}} = \frac{\overline{OD}}{\overline{OD}} = 1 \Rightarrow Efficient$$

Analogously, efficiency score of inefficient units (*E*) are:

$$\theta^* = \frac{\overline{OE'}}{\overline{OE}} > 1 \Rightarrow Inefficient$$

To our purpose, we will relate 'key-values' of activities to their DEA efficiency using linkages as outputs of the DEA model.

The difficulty remains in determining the weighting system to aggregate inputs and output for each component of the efficiency ratio. The DEA linear program estimates fully feasible weights of inputs and outputs for each DMU, thereby obtaining the maximum value of the efficiency index for each unit. Thus, the resulting efficiency indexes are real maximum-efficiency upper limits (equal to or less than one for input-oriented models and equal to or greater than one for output-oriented models[4]). Then, DEA offers a solution for the choice of weights. The DEA weights assignation system

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[3] Please note the change in the usual IO notation about *x* and *y*. They are presented as it is usual in DEA literature, *x* for inputs and *y* for outputs, instead of *x* for production (output) and *y* for final demand as it is common in IO literature.

[4] In order to make an easier comprehension of 'key-value', DEA scores of our model with variable returns to scale and output orientation, $\theta^* \in [1, \infty)$, are presented as $1/\theta^* \in [0,1]$.

does not damage any particular activity, since the most profitable one will always be selected, among other DEA feasible options. The underlying system "awards" activities with a good performance (in terms of potential increase of production, income or employment) instead of "punishing" them due to a failure in the attainment of a certain variable. DEA calculates the *PPF* by non-parametric procedures with a fully flexible weighing scheme, overcoming the difficulties of the previous approaches (multipliers), fully detailed in section 2 with a fixed/arguable weighing scheme.

On the efficiency calculation process, DEA lets the introduction of inputs and outputs without market, and thus without price signals, like in production, income and employment multipliers. Besides, the capacity of DEA to manage variables of different natures is very useful when dealing with a combination of a set of variables with different units (*i.e.:* production, income and employment multipliers).

The variables to be considered in our empirical work will consist of six kinds of outputs (forward and backward multipliers of employment, income and production) and a dummy input (there will be no real inputs since outputs correspond to effects *per* unit of impulse). Then, our single input will be a fictitious variable with the same value for all activities (*i.e.*: 1), since all output variables are measures of impacts *per* unit of impulse.

Figure 3

Figure 3 is done by drawing *PPF* presented in figure 2 for each graph (one of each dimension) of figure 1. That is, it presents the facets of *PPF* for each dimension of Denmark. Obviously a single graph for all dimensions is impossible to draw since it would be a figure in R^6. However, an efficiency score can be computed for each industry-country as the relative distance to the *PPF* shown in figure 3.

Note that, the concept of economic efficiency is not the same as the degree of intersectorial linkage ('key-value') at all. In fact, they can develop in opposite ways (Karigiannis and Tzouvelekas, 2003), *i.e.* efficiency is related to low consumption of inputs whereas strong backward linkages are related to a great consumption of inputs. Our approach of 'key-value' as DEA score is built on the variables selection (IO multipliers). In this sense, the 'key-value' might be seen as a measure of the social efficiency for the economic development of an economy (*i.e.* how efficient an activity is

in terms of its potential impacts over the rest of the economy, how much a sector is a key sector).

For our application, we propose using the Variables Returns to Scale (VRS) model (Banker *et al.*, 1984) with Output orientation (VRS-O). Since we will analyze all economic activities, it is not consistent to assume constant returns to scale (CRS), which is a more restrictive assumption. Output orientation becomes fully justified since our model has no inputs. With regards to the analysis of IO multipliers, we will consider expanding outputs (that is what multipliers shows, the potential production, income and employment increase), rather than reducing inputs.

Then, the model specification (Banker *et al.*, 1984) will be:

$$\min \theta = \frac{v^t x^o - v_0}{u^t y^o} : \frac{v^t X - v_0 \bar{1}}{u^t Y} \geq 1, v \geq 0, u \geq 0, v_0 \text{ free} \qquad (1)$$

Where x^o and y^o are inputs and outputs of the assessed unit o, X and Y are inputs and outputs of all units, v^t and u^t are weights of each variable of unit o, v^0 and u^0 are independent terms of unit o for VRS assumption and, finally 1, is a unitary vector. The program minimizes the ratio of inputs over outputs. This has the same meaning that maximizing the ratio of output over inputs (productivity), in input oriented models. Restrictions ensure that optimal weights are feasible for any unit, this is: if applied to data of any other unit different from the assessed one, the minimized ratio is at least one, a possible value of the score.

Besides, it would be easy to include any other kind of multiplier, as the environmental ones, only taking into account its positive or negative sign to consider it as an input (more is worse/less is better) or output (more is better/less is worse) of the DEA model. In this sense, environmental multipliers should be considered as inputs and its inclusion would make unnecessary any fictitious input. The only warning about inclusion of variables is that as long as you include more variables in the model you lose discrimination power among the assessed units. Consider that if your model uses r outputs and s inputs, you could find up to $r \cdot s$ efficient units. This is because a unit with the highest ratio of one of the outputs to one of the inputs would be efficient (or close to be efficient). Then, it suggests that the number of units in the set should be substantially greater than $r \cdot s$, in order for there to be suitable discrimination between the assessed

units. Of course, it is an extreme situation, but you are warned! Another solution if this problem appears is to include some restrictions over weights that would also be useful for other reasons, as it will be discussed at the end of the next section.

Finally, it is obvious that the accuracy of the technique is highly dependant on the quality of data, as for any technique. In this sense, we must make clear that this DEA approach does not claim for any kind of multipliers as the best ones: multipliers *a la* Chenery and Watanabe (1958), multipliers *a la* Rassmussen (1956), Net multipliers *a la* Oosterhaven and Stelder (2002), multipliers of output shocks *a la* Cai and Leung (2004), etc. DEA analyzes data, it does not test them: our innovation remains on the way that previous works analyze multipliers, not in the way in which multipliers are obtained.

Moreover, DEA could be applied even to HEM results instead to multipliers considering a DEA output for each dimension (production, income, employment, emissions, etc). and summarizing all dimensions with DEA.

3.2 Isolation of country effects

For our ranking problem, it is easy to understand that society should not require the same from industries with different possibilities. This is due to the fact that, for example, the intensity of labour may be a non-discretional characteristic and the existence of some sectors may be due (or even should be due) to the existence of any natural resource (water, minerals, seas, …) in any country region. Obviously, its easy to see that this approach is applicable wherever you have a database of comparable Input-Output Tables with the same industry classification within nested spatial structure, say countries in an international organization (like the example described in the next section), regions in a country, etc. The only condition to apply this approach in a multilevel structure is that each unit of the level n belongs to a single group of the level $n+1$.

In this section a decomposition of industry key-values computed by DEA models is proposed. The idea of breaking efficiency down into different indexes is not new; it was introduced in Farrell (1957), and has been extended by several authors in many contexts. See, for instance, Färe and Primont (1984) and Sueyoshi *et al.* (1998).

In particular, the approach proposed by Amores and Contreras (2009) is followed. The authors used data of farms of different typologies to calculate farm and typology efficiency. The main idea behind their methodology was to isolate the effects on farm attainment that are due to different efforts of farms from those effects that are due to differences in the typology they belong. In order to achieve this decomposition for our problem, industries (59) of different countries (27) within the same country are compared first (59 industries *per* country). In this way, the differences in attainment between industry-country in the same country cannot be attributed to country factors. They are then compared with all industry-country of all countries (59·x 27 = 1593 industry-country), where differences between countries, if any, will be revealed. The approach is graphically depicted in Figure 4.

Figure 4

In this figure, industry-countries of two countries are considered: country 1, whose industries are represented by dots and country 2, whose industries are represented by squares. A thin line represents the DEA efficiency frontiers of the country of each industry-country. The thicker line shows the DEA efficiency frontier when both countries are considered.

Industry-country A belongs to country 2 and is inefficient in comparison with the industries of this country. The distance between A and B (the technically efficient point for this industry-country, given their inputs) represents the level of inefficiency of this industry-country. If all industry-countries of countries 1 and 2 are assessed together, then the efficient frontier is the envelope of the frontiers of country 1 and 2 (thick line). Industry-country A is more inefficient in this assessment, and its inefficiency is given by the distance between A and C.

This new distance represents two effects. Firstly, the lack of effort or ability of the industry-country that prevents it from being efficient in comparison with the industries of the same country. Secondly, the possibilities of the country to provide outputs since the maximum level of outputs for country 2 (point B) is below the value observed in the other country for the same level of inputs (point C).

To measure the first of these effects, which we call Intra-country Key Value (IKV), the distance OA/OB is used. To measure the second of these effects, which we

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call Country Key Value (CKV), the distance OB/OC is used. This measure of CKV can be considered as a measure of the benefits of the industries in this country to be efficient. A CKV of 100% is associated to the country whose industries generate the greatest impacts over the rest of its economy, whereas industry-countries showing CKV lower than 100% did not obtain results as high as those observed in other countries for industry-countries of similar multipliers. That is, the country cannot provide the possible maximum value, given the conditions of the country and the outputs of the industry-countries, and therefore their results are lower than 100% on Overall Key Value (OKV). DEA is used to calculate the IKV within each country and the efficiency of all industry-countries by considering all countries (OKV). The measure of CKV can then be found through decomposition since:

$$\underbrace{\frac{\overline{OA}}{\overline{OC}}}_{OKV} = \underbrace{\frac{\overline{OA}}{\overline{OB}}}_{IKV} \times \underbrace{\frac{\overline{OB}}{\overline{OC}}}_{CKV} \qquad (2)$$

Due to the fact that key-values are inverted scores of a DEA model VRS-O (1):

$$OKV, IKV, CKV \in [0,1] \qquad (3)$$

jointly (2) and (3), implies:

$$OKV \le IKV, CKV \qquad (4)$$

Perhaps, at this point, it is worth clarifying that decomposing industry-country efficiencies in this way and thereby ascribing different components of inefficiency to the country and industry-country, we are, in essence, making an initial diagnosis as to where potential inefficiencies/lack of interdependence lie. More precisely, when the multipliers of industry-countries are controlled, shortfalls in attainment appear to originate from the country factors or the industry-countries depending on the decomposition measures derived. It is a matter for further analysis, at both the industry-countries and the country level, to explain the apparent shortfalls in attainment and how they might be reduced.

For national economic planning, IKV is the main concern, although CKV may provide useful insights, about national factors that affect positively to industrial linking (transparent markets, free market policies, transportation infrastructure, etc) or on the

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other hand, that have a negative effect (burocracy, poor infrastructure, etc). To sum-up, it can let us answer the question: Why is not this industry a key-sector in the international framework? Or better, in our new language about key sectors identification: Why does not this sector have a low key value in comparison with what this sector makes in the rest of countries? Is it due to industrial issues or is it because of country factors?

For international (*e.g.*: EU, UN, OECD, etc) or interregional (USA, EU, any state) planning, the analysis of the mean and the standard deviation of CKV *per* country may provide information about national/regional effect on linking and its dispersion among the country whatever the industry is. Then, national/regional strengths or weaknesses may be detected for national/regional supporting plans.

The same analysis about IKV *per* industry informs us about sectors where interrelation level is high/low discounting country effects. Then, industrial strengths or weaknesses may be detected for industrial planning and outliers from this pattern may show industry-country particularities different from the general specificities of each sector.

At the end, OKV may detect best and worst generating effects in the whole group for benchmarking and policy making.

4   Data and results of the empirical work

The empirical work is carried out for the economies of 27 member states of the European Union (EU-27), available in Table 2, broken down into 59 industries/commodities, according to the A60 CPA (EC, 2002) available at Table 3. Dataset (input-output tables for 2000 at basic prices expressed in millions of current euros) was provided by Rueda-Cantuche *et al.* (2009). DEA scores were computed using a DEA Frontier, an add-in routine for MS-Excel available at Zhu (2003).

The IKV column of Table 4 shows the 'intra-country key-values' (IKV) of each activity for Denmark (DK) (for space reasons, detailed results for all countries are not presented here, but they are available upon request to corresponding author) and this table also presents some descriptive statistics regarding comparative analysis between the different traditional classification approaches (a-c).

Some considerations should be highlighted, as far as they guarantee coherence with commonly used idea of key sectors, flexibility on values, comprehensive approach among dimensions, etc. They are as follows:

a) *Coherence with the commonly used idea of key sectors*: On average (see Table 5), IKV of key sectors are higher than those single oriented (either forward or backward) in any dimension, except employment, where average IKV is the same. The weakly linked sectors show the lowest average key values. This ranking keeps the same for minima. The opposite ranking applies to the coefficients of variation, this is, the dispersion gets reduced when IKV goes up, or key values of key sectors are more similar than key values of weakly linked sectors. Moreover, Spearman Correlation Coefficients between IKV and each of the six classifications considered in section 2 (Mean, Cor.Mean and Median of PM, IM and EM) varies between 0.7 and 0.9.

b) *Flexibility on values*: Among sectors presenting maxima IKV values (IKV>0.75) some sectors traditionally identified (see Table 1) appear as non-key sectors, but as FW-O or BW-O. Besides, some of the traditionally considered key sectors present IKV below the top values (IKV <0.75). A flexible approach like this one was one of the aims of the new approach that we were looking for in section 2. The necessity of this flexible approach was derived from the third problem of the traditional key sectors analysis (to consider BM and FM equally important whatever the values of these multipliers are). Then, our objective was to obtain an indicator that may be included in the top relevant industries, sectors which are not key sectors but present one of the very large multipliers (BM or FM). The results show that we have got it: in Table 4, we found[5] one BW-O sector (Food manufacture - DK_{09}) and one FW-O sector (Other business services - DK_{51}) in terms of production among the six sectors for which IKV is over 0.75. Please, remember now that Food manufacture (DK_{09}) and Other business services (DK_{51}), were used as examples in Section 2. There, they were used to exemplify the necessity to give flexibility to the fixed weighting scheme that kept BM and FM equally important and that made that Food manufacture (DK_{09}) and Other business

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[5] Classified according to mean. Table 6 presents frequency distributions of IKV according to every dimension/criteria.

services (DK_{51}) were not considered as key sectors although they had huge multipliers in one direction (back, for) and average multipliers in the other one. Now this issue is solved. Besides, in terms of income and employment, we found two BW-O industries (the new one is Real Estate services - DK_{47}) and the same FW-O sector. On the other hand, five traditionally considered key sectors in terms of production and income do not obtain the top IKV values (IKV<0.75): Retail trade (DK_{37}), Financial Intermediation services (DK_{44}), Public Administration (DK_{52}), Education services (DK_{53}) and Recreational services (DK_{57}). In terms of employment, we found an additional traditionally considered key sector (Hotel services - DK_{38}) which does not obtain a top IKV.

c) *Comprehensive approach among dimensions*: In section 2, we could not agree about a common classification across dimensions for Hotel services (DK_{38}) since it was BW-O, Key-Sector and Weakly Linked for production, income and employment respectively. The same happens with Real State services (DK47) since it was BW-O for production and Key-Sector for income and employment. Now we have a single measure for each of them: 0.317 and 1 respectively, which lets us have a summarised idea of the key-value of each of theses industries in Denmark.

The comparative analysis of OKV, IKV and CKV produce interesting results which are very suitable for policy making at a detailed level, as any key sector analysis. Anyway, some general ideas are:

d) *Major determinant factors of Key-Value*: Variability in IKV (0.7242) is considerably greater than in CKV (0.2548). This suggests that the major cause for key value is on sectorial causes instead of in national ones.

e) *The 'Keying'-Countries and the 'weak linking' ones*: Figure 5 shows that countries with the highest (over 0.7) mean CKV are: Greece, UK, Poland and Portugal. On the other hand, countries with the lowest mean CKV (below 0.5) are: Ireland, Malta and Italy.

f) *'Key Industries'*: Figure 6 shows Industry-countries with the highest (over 0.7) mean IKV, this is, the industry-countries level of interrelation skipping country effects, are: Food manufacture (09), Construction work (34), Wholesale trade (36),

Retail Trade (37), Real Estate services (47), other business services (51), Public administration (52), Education (53) and Health services (54). On the other hand, industries with the lowest mean IKV (below 0.15) are Fishing (03), Uranium ores (06), Metal ores (07), Tobacco manufacture (10), Leather industry (13), Office machinery industry (24) and Private Household with employees (59).

g) *Individual analysis is better than averaging*: The joint analysis of figures 5 and 6 comes to the conclusion that there is compensation among industries/countries with high/low key-values, then the specific study of each industry-country can provide more powerful insights that the mere analysis of averages, as it has done for the Danish case.

h) *'When a country is a burden...'*: Please, consider now Construction work (DK_{34}), Real Estate services (DK_{47}) and Other business services (DK_{51}), at Table 4. As can be observed, they have maxima IKVs (1), but lower OKVs (below 0.7) because their CKV is lower too (below 0.66). This implies that, although in the national framework these industries have big key-values, they do not have such key-values in the international framework because of national factors (their country is a "burden" for their level of interrelations considering what is possible for industry-countries in the rest of EU). There are 74 industry-countries out of 1593 (4.65%) along EU in that situation.

i) *'The Key goes to ...'*: The most interrelated industry-countries of the ensemble (OKV=1) are: Health services in Denmark (DK_{54}), Financial Intermediation in Greece (GR_{44}), Food (LT_{09}) and Education services in Lithuania (LT_{53}), Financial Intermediation in Luxembourg (LU_{44}), Retail trade in Latvia (LV_{37}), Construction work in Poland (PL_{34}) and Portugal (PT_{34}) and, finally, Agriculture in Romania (RO_{01}). These are the kind of industry-countries to study in depth in order to know why their key-values (level of interrelations across dimensions) are so high. Of course, we could extend this analysis to industry-countries with OKV over 0.75, but then specific analysis of IKV and CKV must be done, to determine where the 'under-attainment' lies. This is due to the fact that industry-countries presenting OKV=1, also report IKV=1 and CKV=1 necessarily (equations 2 and 3). This means that those most interrelated industry-countries previously mentioned are the

most interrelated within their own economy, but also considering the whole EU27. Then, their membership to their country is not a burden for their level of interrelations.

Besides, some limitations are found on this application:

j) *Comprehensive approach across dimensions, the other side of the coin*: There are sectors that are located (see Figure 2) close to the border lines where they would become key sectors from being single oriented sectors in any dimension. This does not mean that they must have similar IKV because it depends on the distance of all multipliers to their own frontiers-maxima of the studied unit (not to the borderline of being key sector-average), not only on that one where the value is close to the borderline. Effectively, Manufacture of Metal products (DK_{22}) and Furniture industry (DK_{30}) are close to borderlines to become key sectors, but IKV_{DK22} is 0.3963 while IKV_{DK30} is 0.1951. That could be attributed to the fact that they are close to different borderlines (FM *vs.* BM), Manufacture of Metal products (DK_{22}) to the border between FW-O and key sectors, while Furniture industry (DK_{30}) is close to the borderline between BW-O and Key-Sectors. But Agriculture (DK_{01}) and Membership org. services (DK_{56}) are also close to the same borderline (between FW-O and Key-Sectors) and IKV_{DK01} is 0.4754 while IKV_{DK56} is 0.2409. The point is that IKV also depends on the distance of the rest of the multiplier to its own frontier and on what is happening in that sense with the rest of dimensions (production, output and income), not only on that where the value is close to the border-line. It is not a real limitation, since it comes from the comprehensive approach used across dimensions (c), but it is advisable to be warned about it in order not to misunderstand the results.

k) *Too good in just one dimension is not so good*: The weighing system gives extremely high importance to those linkages that best perform. On the contrary, they give relatively low importance to those with inefficient performance. This would explain why the key value for Rubber (DK_{19}), a weakly linked sector, is greater than that of the Recreational services (DK_{57}), a key sector, albeit the former presents nearly to zero backward linkages in any dimension. Besides Services Auxiliary to Financial Intermediation (DK_{46}) only present a big

multiplier (PFM_{46}) while the rest of its multipliers are quite small, but even so, its IKV_{DK46} is 0.4712. Then, some limits to the weighing scheme should therefore be advisable to avoid overweighing the best dimension. Moreover, the restrictions on weights could perform the role of including policymaker preferences in the assessment tool, since they can limit the weight of any of the studied dimension in the evaluation process if such preferences exist (*e.g.*: special concern about employment instead of production, etc).

5 Conclusions

The standard identification of key sectors is based on the one-dimensional analysis of the mean of BM and FM or HEM. We have shown that no matter what kind of correction has been made to either of the multipliers threshold (with respect to the average, average without outliers or median), the problems of average dependency and multi-indicator (PBM, PFM, IBM, IFM, EBM and EFM) conclusions remain. Although HEM solves some of the problems, contradictory multi-dimensional conclusions remains. For multipliers analysis, between a fourth and a fifth of the conclusions about relevance of industries depend on the method instead of on data. Then, these approaches do not seem to be very robust. To solve this issue, a new approach for the identification of key activities is presented by using the so-called 'key-value' concept under a DEA approach in this paper.

Key value would, therefore, represent "how much" a sector is a key sector. No matter how close their backward and forward multipliers are to the mean (to become a key sector). Now, we talk a new language in terms of the comprehensive potential impacts of an activity over the rest of the economy, across dimensions with non-fixed weights among them, ranking all activities even key sectors.

Empirical results have shown how this technique keeps coherence with the traditional idea of what a key sector is, but it solves controversial issues of traditional analysis presented in section 2 (outliers, equal relevance of indicators, multidimensional analysis, etc).

A decomposition of those potential impacts has been performed, differencing country related impacts from industrial ones. In this sense, empirical results suggest that industrial factors are more relevant on key value than country factors. In some cases, the

country factors are a burden for the linking level of specific industries. Anyway, it is also shown that the individual analysis of results is more informative for policy making than the analysis of averaged results since a compensation process may be produced.

Finally, it is shown that DEA procedures need careful specifications in order to avoid activities with almost null linkages, but obtaining good scores.

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Tables and Figures:

Figure 1: Multipliers and key sector analysis *per* dimension for Denmark

Note: In order to obtain clearer graphs in a suitable size, income and employment multipliers for Health services (DK_{54}) were not plotted. (Mean: Solid line. Median: Broken line)
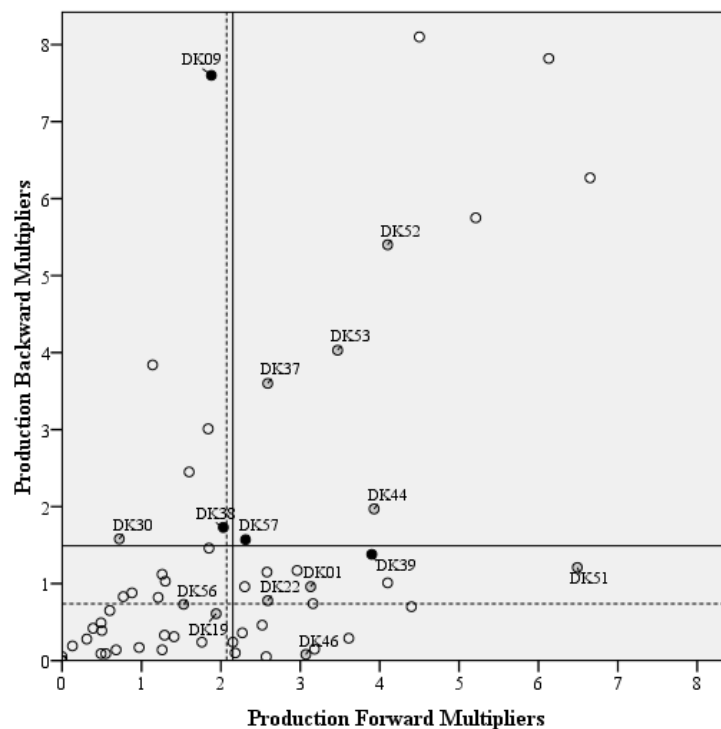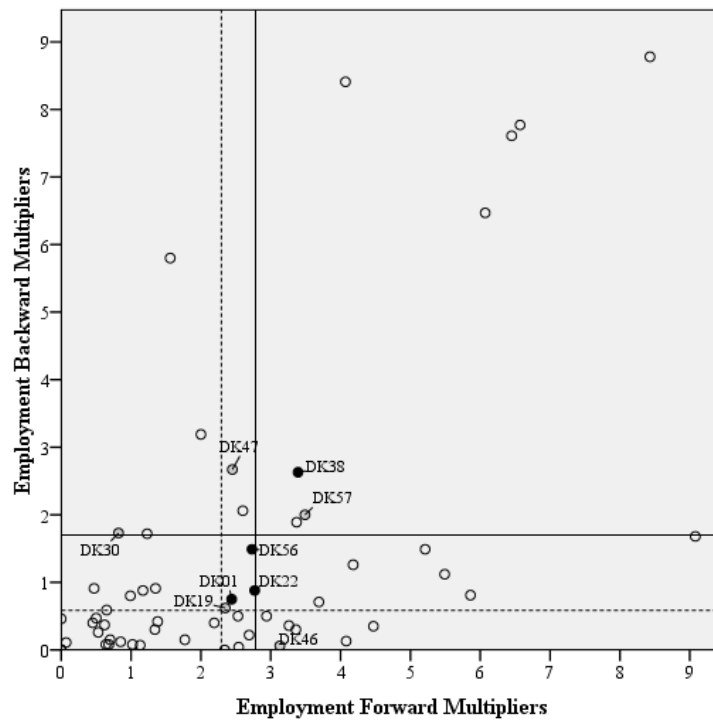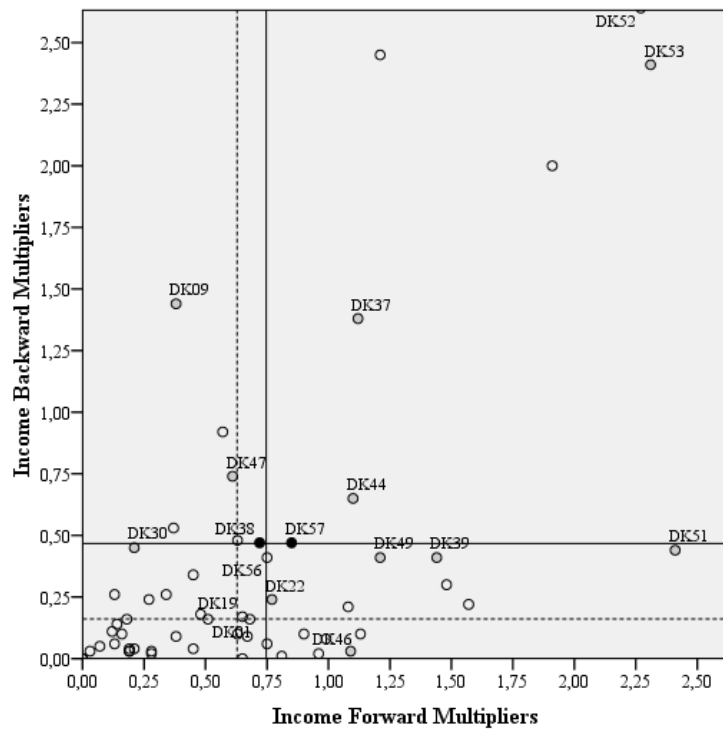
Figure 2: Graphical approach of efficiency scores calculation (PPF: Red line)
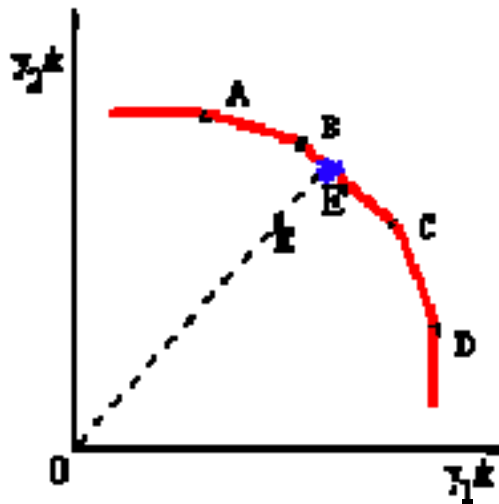
Figure 3: Possibilities Production Frontier facets for the Danish Case (Wide line: PPF facets)
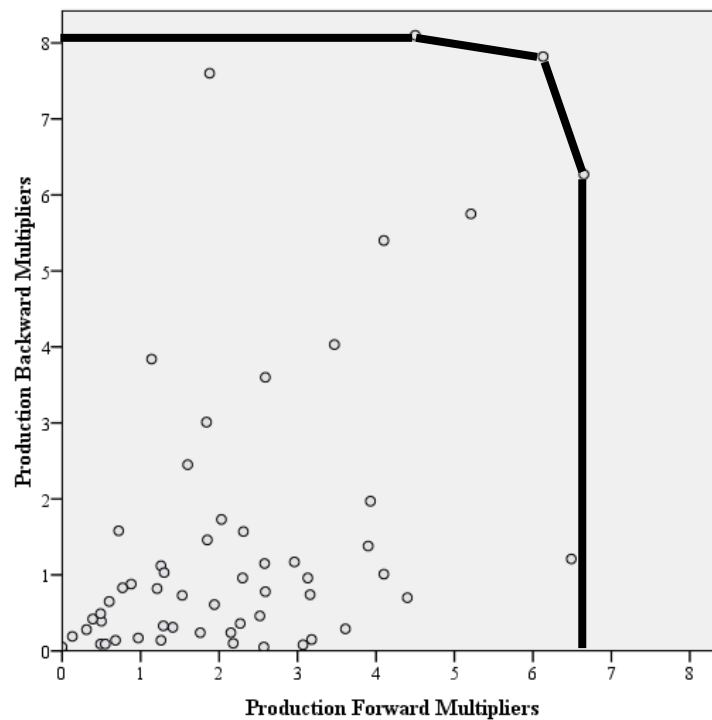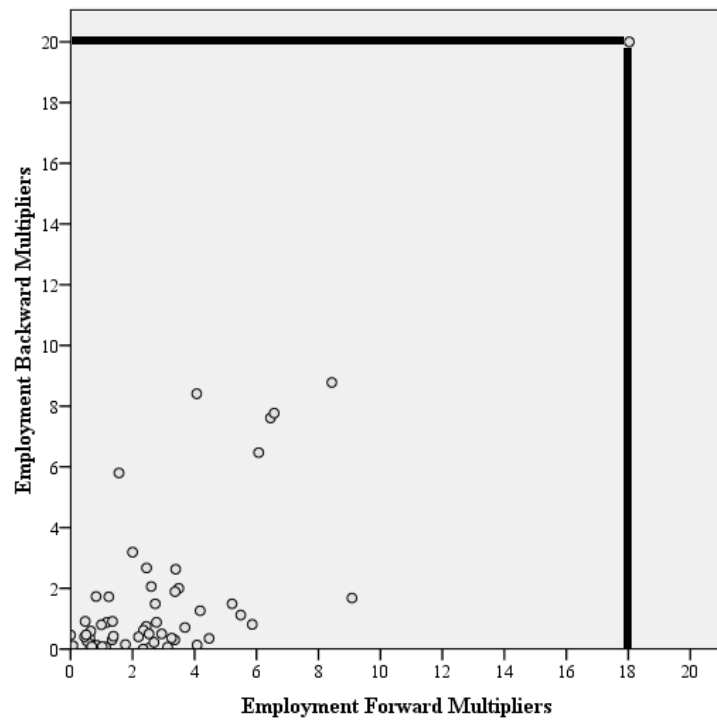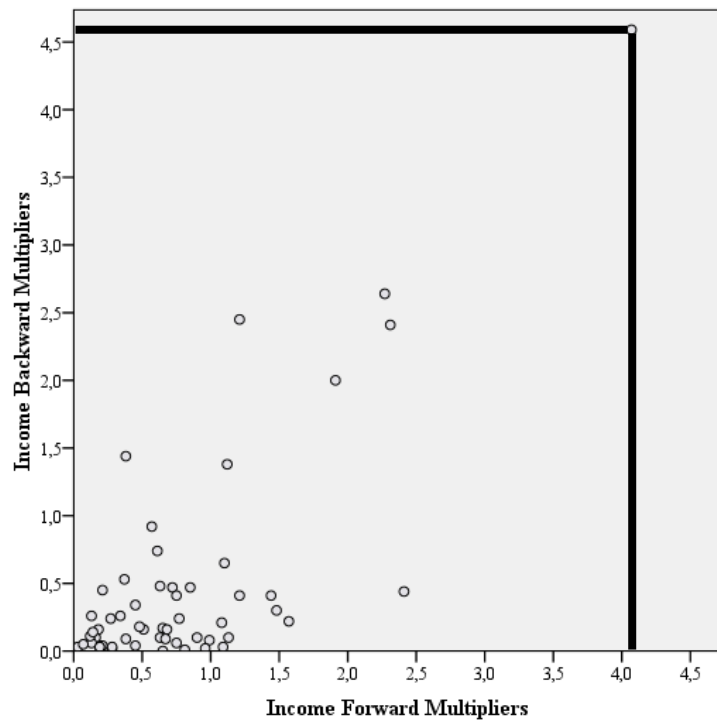
Figure 4: Isolation of Country Effects
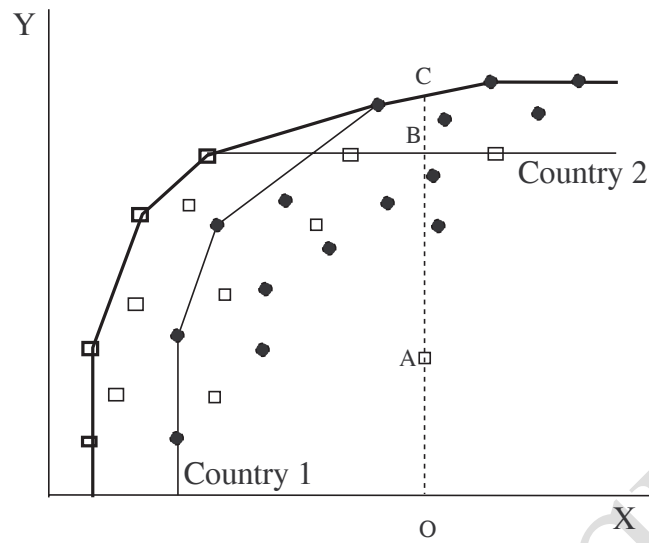
Figure 5:

Key Value components per country
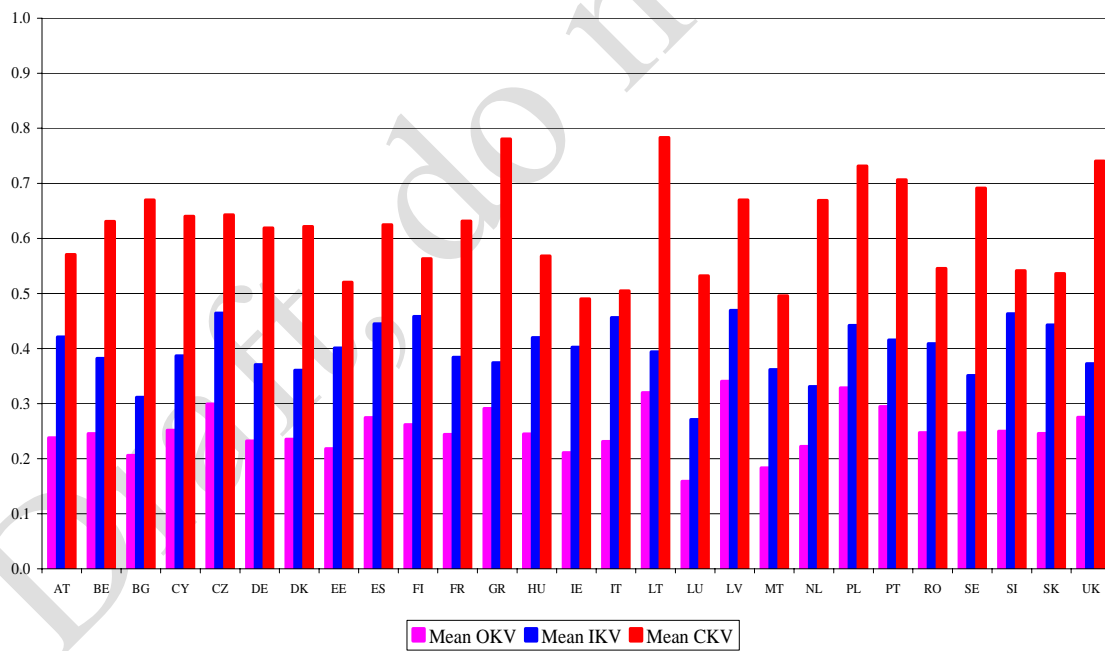
Figure 6:

Key Value components per industry
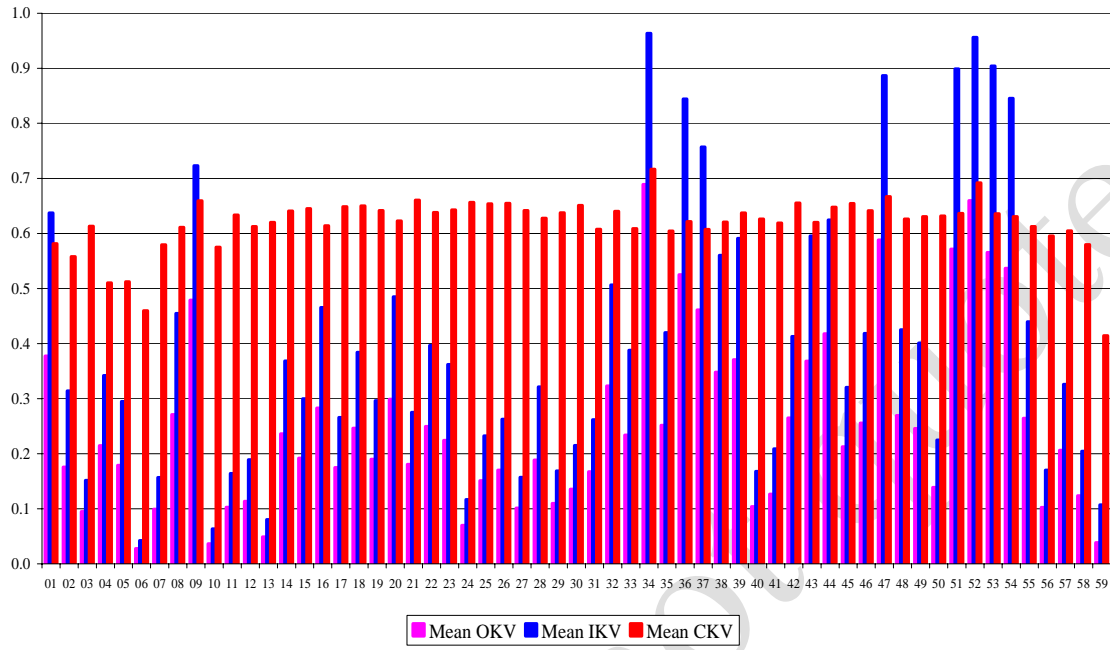
Table 1: Traditional Key Sector Analysis

	Forward Multiplier<1	Forward Multiplier>1
Backward Multiplier>1	Backward-Oriented	Key-Sector
Backward Multiplier<1	Weakly Linked	Forward-Oriented

Table 2: List of Countries & Codes

Code	Country
AT	Austria
BE	Belgium
BG	Bulgaria
CY	Cyprus
CZ	Czech
DK	Denmark
EE	Estonia
FI	Finland
FR	France
DE	Germany
GR	Greece
HU	Hungary
IE	Ireland
IT	Italy
LV	Latvia
LT	Lithuania
LU	Luxembourg
MT	Malta
NL	Netherlands
PL	Poland
PT	Portugal
RO	Romania
SK	Slovakia
SI	Slovenia
ES	Spain
SE	Sweden
UK	UK

Source: EC (2003)

Table 3: A60 CPA (1/2)

Code	Industry
01	Products of agriculture, hunting and related services
02	Products of forestry, logging and related services
03	Fish and other fishing products; services incidental of fishing
04	Coal and lignite; peat
05	Crude petroleum and natural gas; services incidental to oil and gas extraction excluding surveying
06	Uranium and thorium ores
07	Metal ores
08	Other mining and quarrying products
09	Food products and beverages
10	Tobacco products
11	Textiles
12	Wearing apparel; furs
13	Leather and leather products
14	Wood and products of wood and cork (except furniture); articles of straw and plaiting materials
15	Pulp, paper and paper products
16	Printed matter and recorded media
17	Coke, refined petroleum products and nuclear fuels
18	Chemicals, chemical products and man-made fibres
19	Rubber and plastic products
20	Other non-metallic mineral products
21	Basic metals
22	Fabricated metal products, except machinery and equipment
23	Machinery and equipment n.e.c.
24	Office machinery and computers
25	Electrical machinery and apparatus n.e.c.
26	Radio, television and communication equipment and apparatus
27	Medical, precision and optical instruments, watches and clocks
28	Motor vehicles, trailers and semi-trailers
29	Other transport equipment
30	Furniture; other manufactured goods n.e.c.

Table 3: A60 CPA (2/2)

Code	Industry
31	Secondary raw materials
32	Electrical energy, gas, steam and hot water
33	Collected and purified water, distribution services of water
34	Construction work
35	Trade, maintenance and repair services of motor vehicles and motorcycles; retail sale of automotive fuel
36	Wholesale trade and commission trade services, except of motor vehicles and motorcycles
37	Retail trade services, except motor vehicles & motorcycles; repair services personal & household goods
38	Hotel and restaurant services
39	Land transport; transport via pipeline services
40	Water transport services
41	Air transport services
42	Supporting and auxiliary transport services; travel agency services
43	Post and telecommunication services
44	Financial intermediation services, except insurance and pension funding services
45	Insurance and pension funding services, except compulsory social security services
46	Services auxiliary to financial intermediation
47	Real estate services
48	Renting services of machinery and equipment without operator and of personal and household goods
49	Computer and related services
50	Research and development services
51	Other business services
52	Public administration and defence services; compulsory social security services
53	Education services
54	Health and social work services
55	Sewage and refuse disposal services, sanitation and similar services
56	Membership organisation services n.e.c.
57	Recreational, cultural and sporting services
58	Other services
59	Private households with employed persons

Source: A60 CPA, EC (2000)

Table 4: Linkages, Classifications and Key values for Denmark, 2000 (1/3)

Sector	Production BM	Production FM	Income BM	Income FM	Employment BM	Employment FM	OKV	IKV	CKV	Mean PM	Mean IM	Mean EM	CorMean PM	CorMean IM	CorMean EM	Median PM	Median IM	Median EM
DK01	0.962	3.130	0.161	0.509	0.753	2.442	0.259	0.475	0.545	F	W	W	K	W	K	K	W	K
DK02	0.144	0.684	0.037	0.185	0.153	0.700	0.058	0.104	0.555	W	W	W	F	W	W	W	W	W
DK03	0.167	0.972	0.037	0.211	0.124	0.855	0.081	0.148	0.547	W	W	W	F	W	W	W	W	W
DK04	0.000	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.00	W	W	W	W	W	W	W	W	W
DK05	0.957	2.297	0.030	0.186	0.081	0.638	0.186	0.345	0.538	F	W	W	K	W	W	K	W	W
DK06	0.000	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.00	W	W	W	W	W	W	W	W	W
DK07	0.000	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.00	W	W	W	W	W	W	W	W	W
DK08	0.051	2.569	0.012	0.807	0.040	2.543	0.220	0.394	0.558	F	F	W	F	F	F	F	F	F
DK09	7.599 *	1.881	1.439 *	0.384	5.797 *	1.557	0.454	0.939	0.484	B	B	B	K	B	B	B	B	B
DK10	0.188	0.134	0.032	0.028	0.109	0.072	0.016	0.024	0.674	W	W	W	W	W	W	W	W	W
DK11	0.390	0.496	0.096	0.160	0.371	0.620	0.051	0.077	0.664	W	W	W	F	W	W	W	W	W
DK12	0.282	0.311	0.062	0.133	0.258	0.531	0.038	0.049	0.777	W	W	W	F	W	W	W	W	W
DK13	0.086	0.490	0.016	0.281	0.068	1.132	0.067	0.079	0.852	W	W	W	F	W	W	W	W	W
DK14	0.362	2.266	0.104	0.628	0.396	2.189	0.193	0.346	0.557	F	W	W	F	W	W	F	W	W
DK15	0.240	2.152	0.064	0.750	0.216	2.687	0.186	0.331	0.562	F	F	W	F	F	F	F	F	F
DK16	0.695	4.399	0.217	1.569	0.810	5.864	0.381	0.677	0.563	F	F	F	F	K	K	F	K	K
DK17	0.883	0.883	0.029	0.276	0.085	1.020	0.098	0.137	0.716	W	W	W	K	W	W	B	W	W
DK18	2.448	1.598	0.528	0.367	1.716	1.232	0.196	0.307	0.638	B	B	W	K	B	B	B	B	B
DK19	0.610	1.940	0.171	0.646	0.619	2.350	0.170	0.298	0.570	W	W	W	F	F	F	F	K	B
DK20	0.288	3.609	0.085	0.991	0.296	3.358	0.306	0.552	0.555	F	F	F	F	F	F	F	F	F
DK21	0.329	1.292	0.086	0.381	0.301	1.342	0.110	0.198	0.556	W	W	W	F	W	W	W	W	W
DK22	0.785	2.586	0.243	0.771	0.884	2.771	0.226	0.396	0.570	F	F	W	K	K	K	K	K	K
DK23	3.009	1.842	0.920	0.575	3.193	2.002	0.259	0.375	0.691	B	B	B	K	B	B	B	B	B
DK24	0.094	0.553	0.026	0.187	0.085	0.683	0.048	0.085	0.565	W	W	W	F	W	W	W	W	W
DK25	1.125	1.257	0.256	0.335	0.882	1.173	0.130	0.193	0.672	W	W	W	K	B	B	B	B	B
DK26	0.649	0.599	0.161	0.177	0.586	0.646	0.068	0.093	0.729	W	W	W	F	W	W	W	W	W

Table 4: Linkages, Classifications and Key values for Denmark, 2000 (2/3)

Sector	Production BM	Production FM	Income BM	Income FM	Employment BM	Employment FM	OKV	IKV	CKV	Mean PM	Mean IM	Mean EM	CorMean PM	CorMean IM	CorMean EM	Median PM	Median IM	Median EM
DK27	0.835	0.774	0.235	0.269	0.801	0.994	0.091	0.121	0.753	W	W	W	K	B	B	B	B	B
DK28	0.420	0.387	0.110	0.120	0.403	0.451	0.044	0.060	0.730	W	W	W	F	W	W	W	W	W
DK29	0.493	0.495	0.140	0.144	0.474	0.497	0.055	0.077	0.719	W	W	W	F	W	W	W	W	W
DK30	1.577	0.723	0.447	0.208	1.732	0.822	0.116	0.195	0.595	B	W	W	K	B	B	B	B	B
DK31	0.100	2.179	0.000	0.653	0.001	2.336	0.186	0.334	0.557	F	W	W	F	F	F	F	F	W
DK32	1.149	2.581	0.160	0.682	0.501	2.529	0.224	0.394	0.568	F	W	W	K	F	F	K	F	F
DK33	0.141	1.261	0.039	0.445	0.147	1.769	0.109	0.194	0.561	W	W	W	F	W	W	W	W	W
DK34	8.096 *	4.498	2.448 *	1.210	8.409 *	4.069	0.662	1	0.662	K	K	K	K	K	K	K	K	K
DK35	1.459	1.848	0.478	0.634	2.061	2.603	0.200	0.287	0.698	W	B	B	K	K	K	B	K	K
DK36	5.745 *	5.210	2.000 *	1.911	6.467 *	6.067	0.643	0.815	0.789	K	K	K	K	K	K	K	K	K
DK37	3.604 *	2.591	1.377 *	1.118	7.608 *	6.454	0.366	0.457	0.802	K	K	K	K	K	K	K	K	K
DK38	1.734	2.029	0.466	0.716	2.625	3.386	0.223	0.317	0.703	B	W	K	K	K	K	K	K	K
DK39	1.381	3.898	0.414	1.437	1.485	5.206	0.361	0.602	0.599	F	F	F	K	K	K	K	K	K
DK40	3.842 *	1.137	0.256	0.131	0.914	0.469	0.235	0.475	0.495	B	W	W	K	B	B	B	B	B
DK41	0.823	1.209	0.183	0.484	0.419	1.384	0.132	0.189	0.700	W	W	W	K	W	W	B	B	W
DK42	0.739	3.164	0.207	1.083	0.711	3.692	0.273	0.486	0.561	F	F	F	K	K	F	F	K	K
DK43	1.014	4.100	0.301	1.478	1.116	5.490	0.359	0.632	0.568	F	F	F	K	K	K	K	K	K
DK44	1.972	3.931	0.653	1.096	1.887	3.367	0.375	0.603	0.622	K	K	K	K	K	K	K	K	K
DK45	1.033	1.297	0.343	0.447	0.914	1.349	0.141	0.201	0.700	W	W	W	K	B	B	B	B	B
DK46	0.077	3.073	0.020	0.957	0.056	3.135	0.263	0.471	0.558	F	F	F	F	F	F	F	F	F
DK47	6.273 *	6.649 *	0.736	0.606	2.671	2.453	0.604	1	0.604	K	B	B	K	B	K	K	B	K
DK48	0.149	3.177	0.032	1.091	0.130	4.077	0.274	0.489	0.561	F	F	F	F	F	F	F	F	F
DK49	1.170	2.959	0.406	1.213	1.262	4.182	0.293	0.460	0.637	F	F	F	K	K	K	K	K	K
DK50	0.236	1.755	0.101	0.897	0.358	3.263	0.214	0.278	0.770	W	F	F	F	F	F	W	F	F
DK51	1.215	6.488	0.443	2.411 *	1.678	9.076 *	0.576	1	0.576	F	F	F	K	K	K	K	K	K
DK52	5.395 *	4.099	2.640 *	2.266	7.770 *	6.572	0.625	0.688	0.909	K	K	K	K	K	K	K	K	K

Table 4: Linkages, Classifications and Key values for Denmark, 2000 (3/3)

Sector	Multipliers Production BM	FM	Income BM	FM	Employment BM	FM	Key Value OKV	IKV	CKV	Classification Mean PM	IM	EM	CorMean PM	IM	EM	Median PM	IM	EM
DK53	4.025 *	3.465	2.406 *	2.307 *	8.780 *	8.430 *	0.565	0.567	0.997	K	K	K	K	K	K	K	K	K
DK54	7.815 *	6.133	4.593 *	4.070 *	20.003 *	18.033 *	1	1	1	K	K	K	K	K	K	K	K	K
DK55	0.461	2.519	0.099	1.127	0.350	4.469	0.269	0.394	0.682	F	F	F	F	F	F	F	F	F
DK56	0.732	1.527	0.412	0.746	1.489	2.733	0.180	0.241	0.747	W	W	W	K	K	K	W	K	K
DK57	1.565	2.308	0.473	0.854	1.997	3.492	0.243	0.359	0.678	K	K	K	K	K	K	K	K	K
DK58	0.306	1.414	0.095	0.669	0.499	2.937	0.160	0.224	0.716	W	W	F	F	F	F	W	F	F
DK59	0.050	0.001	0.050	0.066	0.458	0.001	0.016	0.023	0.699	W	W	W	W	W	W	W	W	W

	Production BM	FM	Income BM	FM	Employment BM	FM	OKV	IKV	CKV									
Q_1	0.238	0.828	0.045	0.210	0.184	0.838	0.095	0.142	0.560	Count of sectors classification								
Q_2	0.739	1.881	0.161	0.628	0.586	2.350	0.196	0.331	0.622									
Q_3	1.512	3.101	0.445	1.037	1.697	3.439	0.284	0.481	0.709	K	9	8	9	31	18	19	18	19	20
Mean	1.491	2.150	0.467	0.747	1.763	2.783	0.235	0.360	0.621	B	6	5	4	0	9	8	11	10	9
C.Mean	0.711	2.072	0.205	0.630	0.753	2.297	n.a.	n.a.	n.a.	F	18	14	12	23	11	12	11	10	9
Max	8.096	6.649	4.593	4.070	20.003	18.033	1	1	1	W	26	32	34	5	21	20	19	20	21
CV	1.368	0.756	1.748	0.967	1.837	1.036	0.838	0.756	0.292									

Key: Multipliers

- **BM:** Backward Multipliers
- **FM:** Forward Multipliers
- **PM:** Production Multipliers
- **IM:** Income Multipliers
- **EM:** Employment Multipliers

Descriptives

- Q_1: 1st quartile
- Q_2: 2nd quartile (Median)
- Q_3: 3rd quartile
- **Mean:** Arithmetic Mean
- **C.Mean:** Corrected Arithmetic Mean (computed without outliers, *)
- **Max:** Maximun

Key Value

- **OKV:** Overall Key Value (Eq. 1 EU dataset)
- **IKV:** Inner Key Value (Eq. 1, DK dataset)
- **CKV:** Country Key Value (Eq. 2)
- **0.000:** Value lower than 10^{-4}, rounded but not null.
- ***:** Outlier (value outside 1.5 times the inter-quartile range).

Classifications

- **K:** Key-activity (BM>1, FM>1)
- **B:** Backward Oriented (BM>1, FM<1)
- **F:** Forward Oriented (BM<1, FM>1)
- **W:** Weakly Linked (BM<1, FM<1)

Source:

- **Multipliers:** Rueda-Cantuche (2009)
- **Descriptives:** own elaboration
- **Classifications:** own elaboration
- **Key Values:** own elaboration

Table 5: IKV per kind of sector for Denmark

Average IKV	Mean			CorMean			Median		
	PM	IM	EM	PM	IM	EM	PM	IM	EM
K	0.72	0.69	0.65	0.63	0.59	0.61	0.62	0.57	0.60
B	0.43	0.58	0.65	0.31	0.42	0.35	0.31	0.40	0.34
F	0.49	0.51	0.52	0.44	0.38	0.39	0.43	0.39	0.39
W	0.13	0.18	0.19	0.11	0.13	0.11	0.10	0.13	0.12

Coefficient of Variation of IKV	Mean			CorMean			Median		
	PM	IM	EM	PM	IM	EM	PM	IM	EM
K	0.32	0.32	0.37	0.32	0.32	0.37	0.32	0.32	0.37
B	0.55	0.55	0.49	0.55	0.55	0.49	0.55	0.55	0.49
F	0.33	0.34	0.37	0.33	0.34	0.37	0.33	0.34	0.37
W	0.70	0.75	0.73	0.70	0.75	0.73	0.70	0.75	0.73

Maximun IKV	Mean			CorMean			Median		
	PM	IM	EM	PM	IM	EM	PM	IM	EM
K	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
B	0.94	1.00	1.00	0.94	1.00	0.94	0.94	1.00	0.94
F	1.00	1.00	1.00	0.68	0.55	0.55	0.68	0.55	0.55
W	0.30	0.48	0.48	0.30	0.48	0.35	0.28	0.48	0.35

Minimun IKV	Mean			CorMean			Median		
	PM	IM	EM	PM	IM	EM	PM	IM	EM
K	0.36	0.36	0.32	0.35	0.24	0.24	0.32	0.24	0.24
B	0.20	0.29	0.29	0.12	0.12	0.12	0.12	0.12	0.12
F	0.33	0.28	0.22	0.33	0.22	0.22	0.30	0.22	0.22
W	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Key:

PM: Production Multipliers	**K:** Key-activity (BM>1, FM>1)
IM: Income Multipliers	**B:** Backward Oriented (BM>1, FM<1)
EM: Employment Multipliers	**F:** Forward Oriented (BM<1, FM>1)
BM: Backward Multipliers	**W:** Weakly Linked (BM<1, FM<1)
FM: Forward Multipliers	**0.00:** Value lower than 10^{-3}, rounded but not null.

Source:

IKV: own elaboration based on Rueda-Cantuche (2009)

Table 6: Frequency distributions of IKV for Denmark according to key-sector clasifications

Mean

Production Multipliers

	0-0.25	0.25-0.5	0.5-0.75	0.75-1
K		2	3	4
B	1	4		1
F		13	4	1
W	23	3		

Income Multipliers

	0-0.25	0.25-0.5	0.5-0.75	0.75-1
K		2	3	3
B		3		2
F		9	4	1
W	24	8		

Employment Multipliers

	0-0.25	0.25-0.5	0.5-0.75	0.75-1
K		3	3	3
B		2		2
F	1	6	4	1
W	23	11		

Corrected Mean

Production Multipliers

	0-0.25	0.25-0.5	0.5-0.75	0.75-1
K		8	5	5
B	7	5		1
F		7	2	
W	17	2		

Income Multipliers

	0-0.25	0.25-0.5	0.5-0.75	0.75-1
K	1	7	6	4
B	4	3		2
F	1	9	1	
W	18	3		

Employment Multipliers

	0-0.25	0.25-0.5	0.5-0.75	0.75-1
K	1	7	6	5
B	4	3		1
F	1	10	1	
W	18	2		

Median

Production Multipliers

	0-0.25	0.25-0.5	0.5-0.75	0.75-1
K		8	5	5
B	6	4		1
F		9	2	
W	18	1		

Income Multipliers

	0-0.25	0.25-0.5	0.5-0.75	0.75-1
K	1	8	6	4
B	5	3		2
F	1	8	1	
W	17	3		

Employment Multipliers

	0-0.25	0.25-0.5	0.5-0.75	0.75-1
K	1	8	6	5
B	4	4		1
F	1	7	1	
W	18	3		

Key:

B: Backward Oriented (BM>1, FM<1)   **K:** Key-activity (BM>1, FM>1)

W: Weakly Linked (BM<1, FM<1)   **F:** Forward Oriented (BM<1, FM>1)

Source:

IKV: own elaboration based on Rueda-Cantuche (2009)