
Developments in the use of Mathematica for Computable General Equilibrium analysis

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The use of Computable General Equilibrium modelling in evidence-based policy requires an advanced policy making frame of reference, advanced understanding of neoclassical economics and advanced operations research capabilities. This paper examines developments in the advanced operations research capability of a modern generalised mathematical software platform. Intertemporal general equilibrium modelling has become feasible over recent decades due to the development of powerful computer software and hardware. Software for this purpose has traditionally been highly specialised in its ability to define optimisation problems, presolve, and submit the modified mathematical specification to industrial strength optimisation algorithms. In the last two years, general purpose mathematical software has achieved industrial strength. For example, Mathematica now provides interior point optimisation, a technology that has taken three decades to evolve from mathematical research into a general application. It is now possible to take advantage of the many other attributes of general purpose modelling suites, for example graphics for data visualisation that greatly enhance the execution of research and communication of results to policy makers. This paper outlines techniques for the application of Mathematica to data mining of the GTAP database and in using interior point optimisation for Computable General Equilibrium modelling.

1. Introduction

This paper evaluates the use of general purpose computing environments in agile policy research. The amenity of general purpose computing environments is investigated in a typical policy problem applying benchmarking techniques to an extensive set of social and economic data. The availability and consistency of global economic data is evaluated for applying general purpose computing environments in Computable General Equilibrium (CGE) policy research.

2. Agile policy modelling

Once P. A. Samuelson (Dorfman et al. 1958) provided a solution to von Neumann's equations using linear programming, the key operational issue in using Computable General Equilibrium (CGE) models for policy research became the computationally intensive nature of models. The policy research community had little alternative but to focus on improving computation in order to achieve scale and scope in their models, particularly with nonlinear

production and utility functions. From this period through to the present day, a primary operational focus for equilibrium modellers has been computing throughput. As processing and memory capacity continued to double every two years, which has been called Moore's law, hardware capability increased exponentially.

In matching this with software capability, the discipline of operations research responded with better algorithms for algebraic processing, presolvers and industrial optimisers for linear and nonlinear problems. However, interactivity, data visualisation and graphical results communication were things of the future. SRI started to develop mouse driven cursors in the 1960s and the graphical user interface began to take shape at Xerox Park in the 1970s.

In the last two or three years, computing power and memory have become semi-infinite. At the same time, the proprietary techniques of industrial optimisation and algebraic processing have become commoditised in general purpose computing environments such as Mathematica and Matlab. Indeed, advanced formulations of these formerly proprietary techniques have become freely available in open source communities.

The arrival of such high powered operations research functionality in general purpose computing environments coincided with the evolution rich graphical user interfaces and data visualisation. This contrasts to output from single purpose and batch computing environments, which is often graphically processed in spreadsheets. The combination of graphical user interfaces with rich data visualisation tools has provided the ability for researchers to view results in multidimensional graphical forms. Even more importantly, it has given researchers the ability to quickly and effectively communicate results to policy makers.

These factors have led to general purpose computing environments providing a transition to high productivity and agile workspaces for policy researchers across a range of disciplines and in industry. When computing was expensive, considerable planning of models needed to be carried out before the models were processed. Development times were long due to the extra time spent in pre-processing and post-processing for what was essentially a one-off research implementation. Furthermore, the locked-down environment and expense of additional proprietary solvers would often lead to additional features being difficult or costly to implement.

Nowadays, computing is cheap and models can be iteratively developed in general purpose computing environments. While models still need to be as rigorous as ever, many more models, specification approaches and policy scenarios can be quickly tested. The increase in agility for CGE modellers and, in turn, for policy makers has materially increased the productivity of the evidence driven policy process.

Action research through learning by doing, rather than extended planning and specification, parallels the business and government preference for projects with small investments, short payback periods and near term exit strategies. This is sometimes expressed as the tracer bullet strategy of “ready, fire, aim”. Where tracer bullets are cheap, the best way of locating a target is just to start shooting and correcting through feedback.

This basis for action research is provided by general purpose computing environments, particularly those with “all in” functionality so researchers' creativity is enhanced rather than restricted. While there are many general purpose computing environments available for use, this investigation mainly focuses on Mathematica. However, the findings of this research are equally applicable to other general purpose computing environments.

3. Mathematica

Mathematica is one of the leading “all in” general purpose computing environments in which all of Wolfram's functionality is present and no separate algorithms or optimisers need to be licensed. Similar commercial products include Maple and Sage. Open source programs taking a similar approach to Mathematica include Ascend, Axiom (FriCAS and OpenAxiom), Maxima (Macsyma), Ocaml and yacas (“yet another computer algebra system”). Matlab and the open source programs Octave and Scilab are general purpose procedural processors, rather than functional processors, and used primarily for matrix manipulation.

Mathematica's algebraic and symbolic environment provides considerable amenity as a development environment for complex models. Its fundamental strength comes from functional processing using LISP-like list management, Prolog-like pattern management and graph processing. In addition, Mathematica has database management capabilities and includes Country Databases with extensive economic data sets as part of the “all in” philosophy (Wolfram Research 2010). In addition, Wolfram's freely available Alpha platform provides coverage of almost a reference library of data. The confluence of Mathematica computational power and Alpha's reference data brings the prospect of widely deployable, multidisciplinary knowledge based computing systems.

Mathematica's symbolic and functional processing is differentiated from general purpose numeric procedural processors, such as Matlab, C++, Fortran and Basic. In contrast to procedural programming, model development in Mathematica is at a very high level of abstraction. Each function has considerable sophistication including alternative automatically selected algorithms and acyclic processing to sort computation order. The ability to hold constraints in symbolic form is a very important advantage in complex optimisation models.

Mathematica's operations research global and local optimisers, include an implementation of perhaps the most advanced nonlinear interior point optimisation algorithm available, the COIN Project's IPOPT solver. A unique advantage provided by symbolic and functional processing is the ability to use symbolic constraints in optimisations. One area that Mathematica could further develop is the provision of Karush Kuhn Tucker multipliers from nonlinear optimisation, similar to the Lagrange multipliers exposed by Mathematica's DualLinearProgramming function.

Graphical output and data visualisation is an important feature of Mathematica, with major advantages for research productivity and communication with policy makers, as shown in the next section.

4. Policy Benchmarking in a General Purpose Computing Environment

An example of the use of Mathematica in optimisation is drawn from Australia's recent debate on its future level of population, dependence on raw material exports and critical mass of industry. In recent decades, Australian raw material exports of energy and mineral commodities such as coal and iron ore have supplanted Australia's more traditional national endowments of wool and wheat. Indeed, over the last two years, energy and mineral exports to China, Japan and other Asian countries largely insulated Australia from the 2009 Global Financial Crisis and cemented its reputation as the world's Goldilocks economy.

Mining has always been central to the Australian economy. During the 1859 Gold Rush, mining comprised an estimated 15% to 20% of Australia's GDP (Battellino 2010). This proportion declined to 5% by 1909 and since that time has provided about 4.5% of Australia's GDP. However, this understates the effect on Australia's economy. Buoyancy in Australia's terms of trade as a result of commodity exports together with a low critical mass of only 21 million people has led to the widespread loss of industry, which has also resulted in an iniquitous cycle of productivity loss. In recent years, in an example of the "Dutch disease" where large scale job loss occurred as a result of The Netherlands discovering North Sea oil, Australia's soaring terms of trade has led to significant manufacturing closures including Bonds textiles and the Mitsubishi automobile manufacturing plant.‡1

Since the Global Financial Crisis, in major part due to the mismatch between current wages and the consumption vector, leaders around the world have begun to recognise the primacy of employment and importance of manufacturing as a fundamental generator of wealth. For example, in his Announcement on the American Auto Industry, President Barack Obama (2009) said "We cannot, and must not, and we will not let our auto industry simply vanish. This industry is like no other -- it's an emblem of the American spirit; a once and future symbol of America's success. It's what helped build the middle class and sustained it throughout the 20th century. It's a source of deep pride for the generations of American workers It's a pillar of our economy that has held up the dreams of millions of our people".

Even more poignant for Australia are the recent comments by Russian President Dimitri Medvedev (2009) who criticised the "shamefully low" productivity of Russia's current generation resulting in the situation that Russia's budget was financed by raw material exports. He vowed to end Russia's "humiliating dependence on raw material exports" and to enhance productivity for "worthwhile positions in the world division of labour".

Australia's Finance Minister has recently echoed these sentiments, seeking a return to the industrial vitality shown by Australia in the 1990s (Janda & Hyam 2010). Unfortunately, Australian competitiveness is intricately linked to its exchange rate. Australian manufacturing struggled in the 1960s when the mining boom caused Australia's exchange rate to increase by 30%. Following large inflation in the 1970s, strict wage controls linking wage rises to productivity led to Australia's exchange rate declining 40% through the 1980s and 1990s. This is what greatly enhanced Australian industrial competitiveness and materially contributed to the new Australian export industries referred to by the Minister. Unfortunately,

the current energy boom that saved Australia from the worst effects of the Global Financial Crisis has also propelled Australia's exchange rate upwards by 30%.

The future of industry in any country is closely tied to the issue of population. In Australia, as in many Western countries, births are lower than the natural replacement rate and future population is exogenously determined by migration policy. For this reason migration is a highly controversial domestic issue.

The Australian Treasury has considered the issues of population and migration from the perspective of the ratio of working people per person aged 65 and over (Swan 2010). In many Western countries this ratio is precipitously declining, for example in Japan the ratio is expected to decline to only 1.4 by 2050. The Australian Treasury forecasts that the Australian ratio will fall to 2.7 by 2050, compared to 5.0 today and 7.5 forty years ago. Australia's Treasurer has proposed that Australia's population be increased from the current 21 million to 36 million people in 2050, through continuing migration.

Economic commentators have noted that China needs to develop its own internal market and not depend on the world for export sales. The same is true for Australia and this raises the issue of Australia's critical mass and whether it is adequately managing its endowments, in addition to mining resources, on a world scale.

Australia's domestic population policy is also not insensitive to regional and global demographics and other Western role models such as America and Canada. With the world's population expected to increase from the current 6.8 billion to 9.5 billion by 2050, the population of Australia assumes greater importance as a geopolitical issue. Australia is surrounded by populous nations throughout North and South East Asia, such as China and India, which are experiencing high economic growth and the emergence of a wealthy middle class. America, which provides a major role model for Australia, expects its population to grow from 310 to 440 million people by 2050.

Mathematica's "all in" general purpose computing environment and databases may be used to investigate Australia's management performance compared to world's best practice. Figure 1 is drawn from the Country Database within Mathematica. It shows Australia on a notionally efficient frontier of GDP and population.

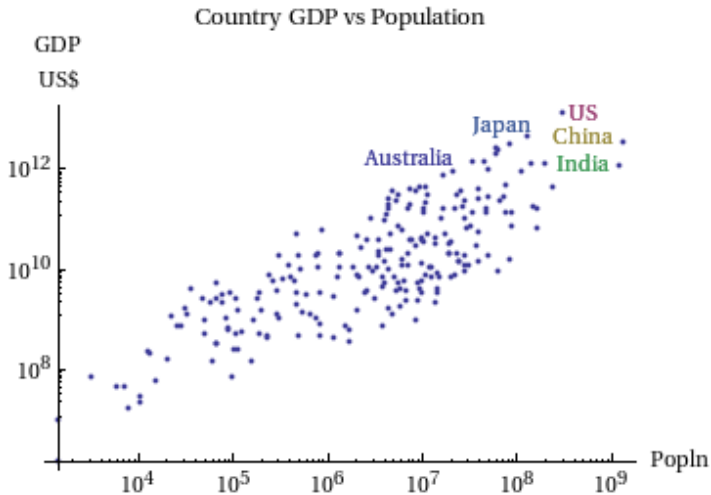


Fig. 1 Country GDP versus Population (Source: Mathematica Country Database)

Figure 2 shows how Australia compares to other countries in terms of GDP at purchasing power parity and Employment. Australia (AU) is nearest but one (NL) to the plot origin. An envelope of best practice is drawn from The Netherlands (NL) to the USA (US) and from the USA (US) to China (C).



Fig. 2 GDP at Parity versus Employed Persons with best practice envelope and constant scale trajectory (Source: Mathematica Country Database)

While Australia (AU) is not on the segment of the envelope joining The Netherlands and USA, a parallel trajectory through Australia (AU), Canada (CA), Italy (IT), France (FR), United Kingdom (GB), Germany (DE) and Japan (JP) shows that Australia's economic and employment performance is nevertheless consistent with other economies.

The research question of whether Australia can better manage its resources may now be investigated through benchmarking Australia to the best performing countries as proxies for world's best efficiency. This involves moving from the two dimensional analysis in Figure 2 to a constrained optimisation that establishes whether Australia can expand its performance given constraints on land area and other geophysical factors such as fresh water, soil fragility and social factors such as egalitarianism.

For the purpose of this research, the broad assumption will be made that in the future technology will address the water and soil fragility issues. For example, an energy technology such as fast breeder nuclear fission or laser inertial fusion may provide unlimited desalination capacity and thereby remove Australia's water constraint (Williams 2010).

The GINI index may be used as a proxy for Australian egalitarianism. This Index measures inequality of income distribution and ranges from 0 (or no inequality) to 1. A small index number implies that the distribution of income in a society is fair. Some GINI indexes in ascending order of inequality are Denmark 0.24, France and Germany 0.28, Australia 0.30, Canada 0.32, United Kingdom 0.34, Russia 0.41, USA 0.45 and China 0.47.

This benchmarking research into whether Australia is achieving world's best efficiency in its management of resources was undertaken using Data Envelopment Analysis (DEA) in Mathematica. The results suggest that Australia has increasing returns to scale and needs to increase the size of its economy to achieve optimum scale. Indeed, expansion in the order of 11.6 fold is theoretically feasible, implying a population of about 260 million. In order to achieve this, Australia would need to replicate American productivity practices. Indeed, if the GINI index constraint is relaxed, the benchmarking proposal suggests that a Taiwan-style of social structure and business efficiency would allow the population to increase 100-fold.

A population size of 260 million would be an order of magnitude increase rather than the incremental increase suggested by Australia's Treasurer. Such a large increase in population implies a series of megacities across Australia's north, the size of Beijing and Shanghai but designed with New York's sustainability and vivacity. New cities have not yet become a topic for discussion in Australia, although it has been suggested that a new major city in the Armidale-Tamworth area might be the sort of construction and enterprise driver that the state of New South Wales needs, to recover from its lacklustre economic and employment growth.

5. Computable General Equilibrium Policy Tool

5.1 Extension of Benchmarking to global policy tools

The benchmarking approach shown in the previous section may be extended to more complex policy analysis (ten Raa 2005; 2008). Nettleton (2010a; 2010c) shows how Computable

General Equilibrium (CGE) under constrained resource usage may be processed using Mathematica's general purpose computing environment, symbolic processing, database mining and implementation of the IPOPT interior point optimiser. Nettleton's CGE policy tool is called the Spatial Climate Economic Policy Tool for Regional Equilibria (Sceptre).

5.2 Data sources

Nettleton (2010b) has highlighted the importance of consistency in policy driven evidence research. This applies as much to the underlying data set as to the framework of analysis (Nettleton 2010c). There are four integrated sources of data for CGE policy studies. These are National Accounting Matrices including Environmental Accounts (NAMEAs), the OECD Input Output tables and bilateral trade data, Global Trade Analysis Data Project (GTAP) and EXIOPOL. The advantages and disadvantages of each have been evaluated.

5.2.1 NAMEA

National Account Matrices including Environmental Accounts (NAMEAs) are national accounts of environmental emissions of 10 to 15 gases. De Haan & Keuning (1996) describe how NAMEAs provide the direct contributions of individual industries to environmental pressures, in both absolute and relative terms. For example, ores, biomass, CO₂, CO, N₂O, NH₃, NO_x, SO₂, CH₄, NMVOC, Pb, PM10, nutrient pollutants, value added and full-time-equivalent jobs produced per tonne of mineral consumed. Input Output analysis of NAMEA data reconstructs the production chain, notwithstanding it may not be homogeneous.

The submission of NAMEAs by European Union member countries is voluntary, which contrasts to the requirements of the United Nations revised system of National Accounts (SNA93) to submit an input output table every five years and annual Source and Use tables.

NAMEA matrices are used in Input Output analysis for evaluating efficiencies and targeting environmental policies. However, according to Tukker (2008), the information within NAMEAs is merely sufficient to analyse global warming impact and perhaps acidification but not the range of analysis required for external costs, total material requirements and ecological footprints.

5.2.2 OECD

In November 2007, the OECD released its 2006 edition of harmonised Input Output tables, Industry Database (STAN) and Bilateral Trade Database (BTD) (OECD 2007a; 2007b). The OECD estimates that between 85% and 95% of world trade is covered in its Bilateral Trade Database. The Input Output tables cover 28 OECD countries (all members except Iceland and Mexico) and 10 non-member countries (Argentina, Brazil, China, Chinese Taipei, India, Indonesia, Israel, Russia and South Africa). This has increased from 18 OECD countries and 2 non-OECD countries (Brazil and China) in the previous edition. The OECD's data provides an important component of comprehensive global data models and has become a foundation of all world economic databases, such as the Global Trade Analysis Project (GTAP).

5.2.3 EXIOPOL

Tukker (2008) describes "A New Environmental Accounting Framework Using Externality Data and Input-Output Tools for Policy Analysis" (EXIOPOL). This is a Euro 5 million

collaborative project of 37 institutes funded by the European Union with the 2010 objective of building a world multiregional Input Output model (MRIO) from officially reported data as well as OECD and GTAP data.

Environmental themes will be linked to the MRIO model, including the interactions and spill overs between countries of global warming, acidification, eutrophication and photochemical oxidants. The results will be used to estimate the external costs of environmental impacts and applying these results to major policy questions.

The EXIOPOL project expects to unify current work in Input-Output analysis, material flow analysis and life cycle assessment of products at the company (or micro) level. It also hopes to contribute new insights on cost-effectiveness and cost-benefit analysis to many EU Policy fields including *inter alia* a policy for integrated products, strategy for natural resources, action plans for environmental technologies, sustainable consumption and production. This will involve scenario-analysis at regional (or meso-) level and national or world (macro) level using input output analysis (given exogenous technology, emission and demand scenarios), CGE models and macro econometric models.

5.2.4 GTAP

The Global Trade Analysis Project (GTAP) Version 7 database of national input output models, trade data and energy data has 2004 data for 57 sectors and 113 regions. It relies heavily on OECD's harmonised input output and STAN bilateral trade data and World Bank data. GTAP's focus on the factors of production and a world economy MRIO table are exceedingly useful in analysis. In addition to the standard GTAP7 database files, GTAP community members have also provided consistent data for greenhouse gas emissions from fossil fuel combustion in Gg CO₂ (Giga Grams of CO₂) corresponding to International Energy Agency energy volumes data (Lee 2008).

In contrast to the lack of integration and limited scope of NAMEAs and OECD data, GTAP's compelling advantage is its availability, consistency and geographic coverage. While commodity classification has some inconsistencies with the OECD, GTAP is progressively resolving these issues. In the future EXIOPOL may provide valuable enhancements to the GTAP database.

5.3 Mining the GTAP 7 database

The GTAP 7 database (Hertel 1999; Hertel & Walmsley 2008) may be aggregated using GTAP utility functions in the GTAPAgg package. §4. Nettleton (Nettleton 2010a, Appendix 7) shows the preliminary aggregation of GTAP data into three global regions (NAFTA, EU25 and the Rest of the World), three commodities (food, manufacturing and services) and three factors of production (land, labour and capital). Aggregation produces a proprietary GEMPACK “har” format, which may be viewed and transformed for use in other database systems, such as Mathematica's HSQLDB database engine.

As part of its “all in” computing environment, Mathematica's database management facilities provide the ability to mine data sources. In order to utilise GTAP's data within Mathematica, it is necessary to further mine the data by converting the table structure designed for GTAP's

own CGE model by reconciliation to GTAP's generic Social Accounting Matrix (SAM) (McDonald & Patterson 2004, p.6).

Lastly, the ability to expand the policy research using Mathematica's own social, economic, financial and natural science databases is very valuable, for example population and growth, employment and GINI index as shown in the above policy benchmarking example. Nettleton (2010a, Appendix 7) describes the parallel mining of GTAP data and Mathematica Country Databases for Computable General Equilibrium.

6. Conclusion

This paper introduced the topic of agile policy modelling. It found that the confluence of computing processor and memory power, algorithm design, graphical user interfaces and data visualisation in general purpose computing environments has greatly increased productivity and creativity in modelling policy. General purpose computing environments providing “all in” functionality including database mining capabilities were found to enhance modellers' capabilities.

The compelling advantages of inbuilt economic databases, operations research functionality, data visualisation and graphical communication with an “all in” general purpose computing environment were demonstrated through a policy modelling example. This policy research benchmarked Australia's efficiency in using its continental land endowment while maintaining the Australian trait of social egalitarianism.

The extension of benchmarking techniques into Computable General Equilibrium (CGE) policy tool were investigated. It was found that the Global Trade Analysis Project (GTAP) provided a comprehensive and consistent set of Input Output data within the framework of a Social Accounting Matrix. The importance of database management and query functions within an “all in” general purpose computing environment was highlighted.

Notes and references

‡1 Of course, this issue is not restricted to countries as small as Australia. For example, in 2010, BMW ceased its own engine development because the cost could not be amortised over BMW's volume of production. Previously BMW's own engine development had been a core competence for Series 5 & 7. BMW has now joint ventured with Peugeot. In contrast, Volkswagen is able to sustain its own engine development given production of 8 million vehicles.

‡2 Mathematica commands for Figure 1 GDP vs Country (see <http://reference.wolfram.com/mathematica/ref/CountryData.html>)

(*** Data Mining and Plot of GDP vs. Population ***)

Show[

```
ListLogLogPlot[Tooltip[{CountryData[#, "Population"], CountryData[#, "GDP"]}, CountryData[#, "Name"]]  
& /@ CountryData["Countries"], AxesLabel -> {"Popln", Labeled["GDP", "US$"]}, PlotLabel -> "Country  
GDP vs Population"],
```

```
ListLogLogPlot[{{7 10^6, 1.5 10^12}}, {{5 10^8, 1.2 10^13}}, {{5 10^8, 4 10^12}}, {{5 10^8, 1.2  
10^12}}, {{7 10^7, 7 10^12}}],
```

```
PlotMarkers -> {"Australia", "US", "China", "India", "Japan"}]]
```

‡3 The Mathematica commands for Benchmarking Australia's Population to World Best Practices and Figure 2

GDP vs Employment figure are:

```
(*** Data Mining and Plot of GDP vs. Employment ***)
initvariables = {"CountryCode", "GDPAAtParity", "LaborForce", "UnemploymentFraction", "LandArea",
"GiniIndex"};
displayvariables = initvariables[{{1, 2, 3, 5, 6}}];
data1 = Outer[CountryData, CountryData["Countries"], initvariables, 1];
(*Remove missing values*) data2 = Pick[data1, Map[NumericQ[#] &, Total[data1[All, 2 ;;], {2}]]];
(*Sort the variables*) data3 = Transpose[Sort[data2, #1[[2]] > #2[[2]] &]];
(*Calculate Employed persons*) data4 = Transpose[{data3[[1]], data3[[2]]*10^-12, data3[[3]]*(1 -
data3[[4]])*10^-6, data3[[5]]*10^-6, data3[[6]]};
(*Determine the position of Australia in the list*) austposn = Position[data4, "AU"][[1, 1]];
(*Choose how many more countries after Australia will be included*) afteraust = 3;
cutoff = austposn + afteraust;
data = Take[data4, cutoff];
aust = data[[austposn]];
points = ListLogLogPlot[Map[Tooltip[{data[[#, 3]], data[[#, 2]]}, data[[#, 1]]] &, Range[Length[data]]],
AxesLabel -> {Labeled["Empl.", "mill"], Labeled["GDPAAtParity", "US$tr"]};
expansionline = ListLogLogPlot[Map[{aust[[3]], aust[[2]]} # &, Range[.5, 19.5, 1]], Joined -> True, PlotStyle
-> Purple];
frontierline = ListLogLogPlot[Map[{data[[#, 3]], data[[#, 2]]} &, {19, 1, 2}], Joined -> True];
pointnames = Table[Graphics[Text[Style[" " <> StringTake[data[[i, 1]], 2], Blue, Italic, Tiny], {Log[data[[i,
3]], Log[data[[i, 2]]]}], {i, cutoff}];
(*Show[points,expansionline,frontierline]*)
Show[points, pointnames, expansionline, frontierline, PlotLabel -> "Best Practice GDP vs Employment"]
*** Benchmarking using Dual Linear Programming ***
all = Transpose[data];
names = all[[1]];
(*Constant returns to scale*)
a = Join[Map[Prepend[all[[#]], -aust[[#]]] &, Range[2, 3]], Map[Prepend[all[[#]], 0] &, Range[4,
Length[all]]];
(*GDP>current, Population>current, Area<current*)
b = Join[ Map[{0, 1} &, Range[2, 3]], Map[{aust[[#]], -1} &, Range[4, Length[all]]];
c = Prepend[Table[0, {cutoff}], -1];
{x, shadowprice, slacklower, slackupper} = DualLinearProgramming[c, a, b];
Print["expansion factor e & proportions of countries practices that Australia should investigate"];
TableForm[x, TableHeadings -> {Prepend[names, "e"]}];
Print["Contributions to Australia's practices in terms of resources"];
TableForm[Map[#*x &, a], TableHeadings -> {Rest[displayvariables], Prepend[names, "e"]}];
Print["Shadow Prices of an extra unit of resources in Australia"];
TableForm[shadowprice, TableHeadings -> {Rest[displayvariables]}];
(** From a Cost Perspective **)
(*Constant returns to scale*)
ca = Join[Map[Prepend[all[[#]], 0] &, Range[2, 3]], Map[Prepend[all[[#]], -aust[[#]]] &,
Range[4, Length[all]]];
(*GDP>current, Population>current, Area<current*)
cb = Join[Map[{aust[[#]], 1} &, Range[2, 3]], Map[{0, -1} &, Range[4, Length[all]]];
cc = Prepend[Table[0, {cutoff}], 1];
{cx, cshadowprice, cslacklower, cslackupper} = DualLinearProgramming[cc, ca, cb]
(*Variable returns to scale*)
sa = Append[ca, 1 - cc];
sb = Append[cb, {1, 0}];
sc = cc;
{sx, sshadowprice, sslacklower, sslackupper} = DualLinearProgramming[sc, sa, sb]
(*Non-increasing returns to scale*)
ra = Append[ca, 1 - cc];
rb = Append[cb, {1, -1}];
```

```

rc = cc;
{rx, rshadowprice, rslacklower, rslackupper} = DualLinearProgramming[rc, ra, rb]
(*Scale efficiency, if the ratio is 1 then the production unit is operating at maximum scale and if less than 1 it is
not operating at optimum scale*)
se = cx[[1]]/sx[[1]];
(*If Scale efficiency is less than 1, it is necessary to calculate cx/rx. If the ratio is equal to 1 then the production
unit has increasing returns to scale and needs to increase its size to achieve optimum scale. If less than 1 then
the production unit is subject to decreasing returns to scale and needs to decrease its size.*)
sse = cx[[1]]/rx[[1]];
Print["Scale efficiency: ", se];
Print["Second scale efficiency: ", sse];

```

‡4 GTAPAgg contributed to GTAP by Mark Horridge, Centre of Policy Studies, Monash University, Melbourne, Australia

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