WORLD INPUT-OUTPUT DATABASE

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TECHNICAL REPORT ON ADVANTAGES AND DISADVANTAGES OF TYPES OF INPUT-OUTPUT TABLES (PRODUCT-BY-PRODUCT OR INDUSTRY-BY-INDUSTRY)

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World Input-Output Database: Construction and Applications

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Table of contents

1 The Input-Output Framework

Following Rueda-Cantuche et al (2009), an input-output framework centres on the so-called supply and use tables. Roughly speaking, they can be seen as the output mix of industries and the industries' use of inputs, respectively. On the one hand, the supply table consists of an intermediate matrix of products produced by industries, plus additional column vectors comprising imports and several valuation adjustment items to convert total supply of products from basic prices into purchasers' prices, namely distribution margins (trade and transport) and net taxes on products. On the other hand, the use table may represent either domestically produced intermediate and final consumption or imported uses, both at basic and at purchasers' prices. Additional column vectors are shown regarding standard final demand components, i.e. final consumption, investment and exports; additional rows finally represent different components of gross value added, e.g. labour costs, capital use, other net taxes on production and net operating surplus (see Tables 1 and 2).

Table 1. Simplified overview of a supply table

	INDUSTRIES (NACE) / PRODUCTS (CPA)	OUTPUT OF INDUSTRIES (NACE)							IMPORTS			Total supply at basic prices	VALUATION		Total supply at purcha-sers' prices
		Agriculture	Industry	Construction	Trade	Private services	Government services	Total	Intra EU imports cif	Extra EU imports cif	Imports cif		Trade and transport margins	Taxes less subsidies on products	
No		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Products of agriculture	Production matrix (V^T)							Imports cif				Valuation items		
2	Products of industry														
3	Construction work														
4	Trade														
5	Private services														
6	Government services														
7	Total														
8	Cif/ fob adjustments on imports														
9	Direct purchases abroad by residents														
10	Output at basic prices	Total output of industries at basc prices							Total imports				Total		

Needless to say that total use of products at purchasers' prices (Table 2) should match total supply of products (Table 1) at the same valuation prices. This rectangular system (e.g. *m* industries and *n* products) turns out to be the most appropriate framework for balancing supply and demand and the best one to compile Gross Domestic Product (GDP). Indeed, it is not based on analytical assumptions but rather on direct statistical sources. Furthermore, symmetric (equal number of industries and of products) input-output tables (SIOTs) can be derived from the supply and use system. The dimension can be either product-by-product or industry-by-industry. This kind of symmetric system aims at grasping homogenous interrelationships either within products or within industries. The fact that SIOTs are square is highly relevant for input-output analysis. Productivity, energy and environmental analyses are well-known examples of impact studies for which SIOTs need to be constructed.

Notice that the valuation of the aforementioned supply and use tables is not coincident. On the one hand, the supply table is measured at basic prices, which means before products are conveyed to the markets, hence excluding trade and transport margins and net product taxes. On the other hand, the use table is measured at purchasers' prices, which means at the price either consumers or producers pay final or intermediate consumptions (including trade and transport margins and taxes less subsidies on products). For further purposes, i.e. the construction of SIOTs, both supply and use tables should be measured at basic prices. Accordingly, ten Raa and Rueda-Cantuche (2007) formalized an adjustment mechanism to convert the use table from purchasers to basic prices on the basis of given ratios of trade and transport margins and net product taxes. As mentioned by Eurostat (2008), basic prices is the preferable valuation concept in the supply and use framework in the sense that it provides a more homogeneous valuation as the different shares of product taxes less subsidies and of trade and transport margins are eliminated. Thus, for analytical purposes a valuation as homogeneous as possible is required as the input-output relations measured in monetary units are interpreted as technical relations.

Table 2. *Simplified overview of a use table*

	INDUSTRIES (NACE) / PRODUCTS (CPA)	OUTPUT OF INDUSTRIES (NACE)							FINAL USES									Total use at purchasers' prices
		Agriculture	Industry	Construction	Trade	Private services	Government services	Total	Final consumption expenditure by households	Final consumption expenditure by non-profit organisations	Final consumption expenditure by government	Gross fixed capital formation	Changes in valuables	Changes in inventories	Exports intra EU fob	Exports extra EU fob	Total	
No		1	2	3	4	5	6	7	7	8	9	10	11	12	13	14	15	16
1	Products of agriculture	Intermediate uses (U)							Final uses (Y)									
2	Products of industry																	
3	Construction work																	
4	Trade																	
5	Private services																	
6	Government services																	
7	Total	Total intermediate consumption							Total final uses of goods and services									
8	Cif/ fob adjustments on exports																	
9	Direct purchases abroad by residents																	
10	Domestic purchases. by non-residents																	
11	Total	Total intermediates							Total final uses									
12	Compensation of employees	Value added (W)																
13	Other net taxes on production																	
14	Consumption of fixed capital																	
15	Operating surplus, net																	
16	Total	Total value added at basic prices																
17	Output at basic prices	Total output of industries at basic prices																

The construction of SIOTs is a controversial issue in the literature. A product-by-product table describes the technological relations between products (Eurostat, 2008). The intermediate part depicts a sort of recipe how to produce each one of the products in terms of the amounts used of others, irrespective of the producing industry. Conversely, industry-by-industry tables describe inter-industry relations. The intermediate part describes for each industry the use of products of the (other) industries (Eurostat, 2008). However, product-by-product tables are more homogenous in their description of the transactions than industry-by-industry tables and they fit most types of input-output analysis. Nevertheless, product-by-product tables require labour intensive compilation tasks; they must be based on analytical assumptions that take final results away from actual market transactions and observations, and hence they make more difficult the integration of other statistical sources and the reporting on the transformation procedure. In addition, product-by-product tables must struggle with negatives depending on the assumed technology.

On the contrary, industry-by-industry tables are much closer to statistical sources than product-by-product tables; they allow an easier integration of other statistical databases, thus facilitating a more complete reporting on the compilation procedure. They are less labour intensive to compile, being based on pragmatic assumptions rather than on analytical hypotheses. But finally, the larger the secondary activities in the supply table are the more difficult it becomes to identify homogeneous cost structures in an industry-by-industry table. Industry-by-industry tables are compiled by several statistical offices including Denmark, the Netherlands, Norway, Canada and Finland, while most other countries compile product-by-product tables.

Basically, the choice of the type of SIOT is related to the treatment of secondary outputs. There are two basic approaches to eliminating secondary products. Both come from applying information from the use matrix to the supply matrix to reduce it to a purely diagonal one. Once this is done, the supply matrix contains no further useful information and is no longer presented. The transformed use matrix is what is referred to as an input-output matrix (UN, 2009, par. 28.47).

It follows that in deriving a product by product matrix in the simplest possible way, the final demand of the use matrix is unaltered. It already expresses demand by product and does not need changing. The intermediate consumption and value added parts of the matrix, though, need to be changed from an industry dimension to a product one. The row totals of the matrix already show the correct product totals so the exercise consists of reallocating entries from one column to another within the given row total (see Figure 1). This is called a technology approach. It assumes that the demand for intermediate consumption and labour and capital inputs are determined by the nature of the products made (UN, 2009, par. 28.48).

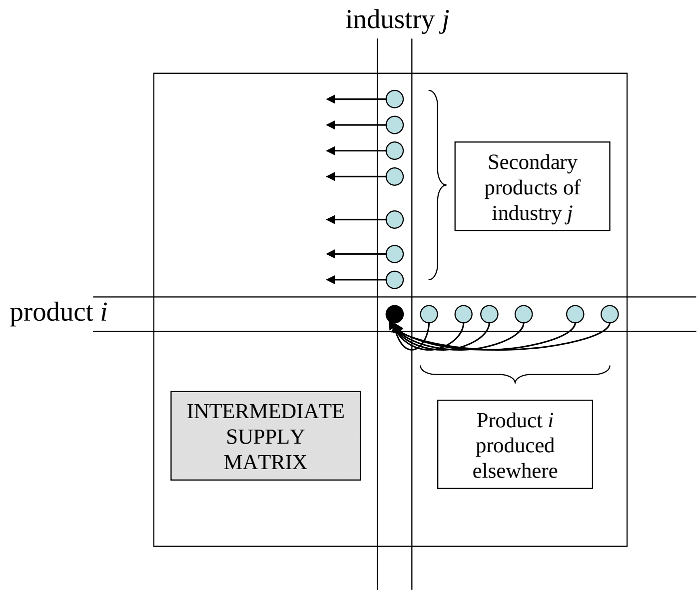

Figure 1: Transfers for a product table
(Rueda-Cantuche and ten Raa, 2009)

Regarding product-by-product tables, we may assume either products being produced with the same structure independently of the producing industry (product technology assumption) or being produced according to the sector that actually produces them (industry technology assumption). Nevertheless, there are other assumptions available in the literature, that were reviewed by Viet (1994) and more recently by ten Raa and Rueda-Cantuche (2003) who also provided their pros and cons from a theoretical perspective (see also Kop Jansen and ten Raa, 1990). For instance, Konijn (1994) and Konijn and Steenge (1995) proposed the activity technology model; the Japanese Office of Statistical Standards (1974), the lump-sum or aggregation method; and others proposed several hybrid technology assumptions, i.e. the mixed product and industry technology assumption (UN, 1968 and 1973; Gigantes (1970) and Armstrong (1975)); and the mixed product and by-product (Stone) technology assumption (ten Raa, Chakraborty and Small, 1984).

In deriving an industry by industry matrix in the simplest possible way, the value added part of the use matrix is unaltered and because the level of output will not alter, only the composition of intermediate consumption changes, not its total. Thus the exercise is one of reallocating items between rows but not between columns. In contrast to the product by product case, final demand will change and will show demand related to the industry supplying the products and not to the products themselves (see Figure 2). This is called a sales structure approach. It assumes that as the level of output of an industry changes, the pattern of sales will remain the same (UN, 2009, par. 28.49).

For the compilation of industry-by-industry tables, we identify only two major variants: the fixed industry sales structure assumption, where the industry deliveries are independent of the products delivered, and the fixed product sales structure, where they are instead independent of the producing industry.

Figure 3 finally provides a schematic summary of the input-output framework as explained so far.

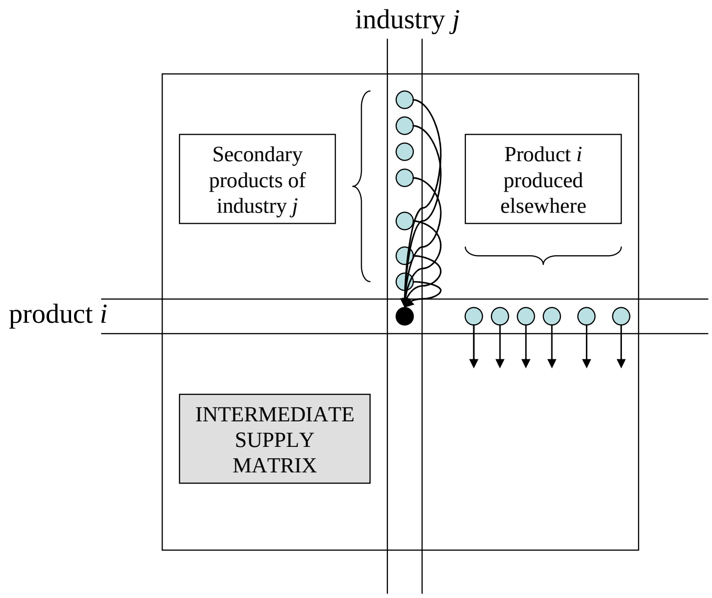

Figure 2: Transfers for an industry table (Rueda-Cantuche and ten Raa, 2009)

Next, let us define a use matrix, $\mathbf{U} = (u_{ij})_{\ i,j\ =\ 1,\ldots,\ n}$ of products *i* consumed by sector *j*, and a supply matrix $\mathbf{V}^{\mathbf{T}} = (v_{ij})_{\ i,j\ =\ 1,\ldots,\ n}$ where product *i* is produced by sector *j*, which is actually the transpose of the so-called make matrix **V**. According to Figure 1, models A, B, C and D can additionally be formalized on the basis of supply and use matrices as it is shown in Table 3. The main advantage of Table 3 is the simplicity of its notation, which is based on a reduced number of unknowns, i.e. the supply and use matrices and the final demand and gross value added matrices. Instead, relevant literature (including the European System of Accounts - ESA95 and the Revised System of National Accounts - SNA93) at this respect still inherits a different notation where the number of elements used to compile SIOTs is not so reduced, though sometimes rather more intuitive. Table 3 will serve us to provide in the Annex how to shift between the two types of tables using bridge matrices.

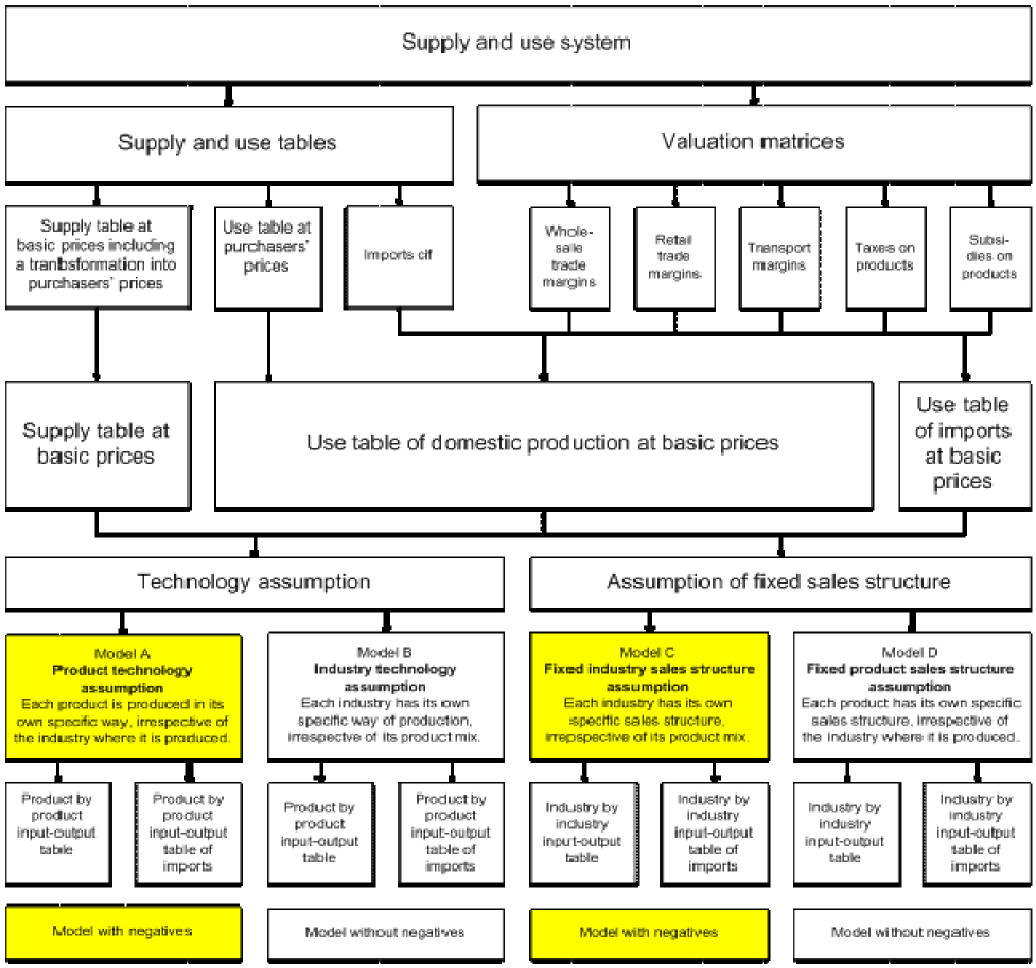

Figure 3. The Input-Output Framework

Many countries in the European Union compile product-by-product input-output tables with the product technology assumption (Model A). Sometimes large negative entries are removed in a manual balancing procedure. The use of the product technology model is preferred as the model is consistent with the use of product-by-product tables in input-output analysis. This cannot be said of the industry technology assumption that leads to a symmetric input-output table where the columns contain a mix of input structures. Some countries in the European Union compile industry-by-industry input-output tables. They apply Model D (fixed product sales structure) for the transformation of supply and use tables to input-output tables. Regarding the choice of methods for industry-by-industry tables the assumption of fixed product sales structures is clearly preferred, due to the unrealistic character of the alternative assumption of fixed industry sales structure.

Table 3. Transformation of supply and use tables into symmetric input-output tables

	MODEL A Product by product Product technology based	MODEL B Product by product Industry technology based	MODEL C Industry by industry Fixed industry sales structure	MODEL D Industry by industry Fixed product sales structure
Input coefficients	$\mathbf{A}_A(\mathbf{U},\mathbf{V}) = \mathbf{U}\mathbf{V}^{\text{-}\mathbf{T}}$	$\mathbf{A}_B(\mathbf{U},\mathbf{V}) =$ $\mathbf{U}\left(diag(\mathbf{Ve})\right)^{-1}\mathbf{V}\left(diag(\mathbf{V}^{\mathbf{T}}\mathbf{e})\right)^{-1}$	$\mathbf{A}_C(\mathbf{U},\mathbf{V}) =$ $diag(\mathbf{Ve})\mathbf{V}^{\text{-}\mathbf{T}}\mathbf{U}\left(diag(\mathbf{Ve})\right)^{-1}$	$\mathbf{A}_D(\mathbf{U},\mathbf{V}) =$ $\mathbf{V}\left(diag(\mathbf{V}^{\mathbf{T}}\mathbf{e})\right)^{-1}\mathbf{U}\left(diag(\mathbf{Ve})\right)^{-1}$
Intermediates	$\mathbf{Z}_A = \mathbf{A}_A\left(\mathbf{U},\mathbf{V}\right)diag(\mathbf{V}^{\mathbf{T}}\mathbf{e})$	$\mathbf{Z}_B = \mathbf{A}_B(\mathbf{U},\mathbf{V})diag(\mathbf{V}^{\mathbf{T}}\mathbf{e})$	$\mathbf{Z}_C = \mathbf{A}_C(\mathbf{U},\mathbf{V})diag(\mathbf{Ve})$	$\mathbf{Z}_D = \mathbf{A}_D(\mathbf{U},\mathbf{V})diag(\mathbf{Ve})$
Final demand	$\mathbf{F}_A = \mathbf{Y}$	$\mathbf{F}_B = \mathbf{Y}$	$\mathbf{F}_C = diag(\mathbf{Ve})\mathbf{V}^{\text{-}\mathbf{T}}\mathbf{Y}$	$\mathbf{F}_D = \mathbf{V}\left(diag(\mathbf{V}^{\mathbf{T}}\mathbf{e})\right)^{-1}\mathbf{Y}$
Value Added (VA)	$\mathbf{VA}_A = \mathbf{W}\,\mathbf{V}^{\text{-}\mathbf{T}}diag(\mathbf{V}^{\mathbf{T}}\mathbf{e})$	$\mathbf{VA}_B = \mathbf{W}\left(diag(\mathbf{Ve})\right)^{-1}\mathbf{V}$	$\mathbf{VA}_C = \mathbf{W}$	$\mathbf{VA}_D = \mathbf{W}$
Output	$\mathbf{q}_A = \left(\mathbf{I}-\mathbf{A}_A(\mathbf{U},\mathbf{V})\right)^{-1}\mathbf{F}_A\,\mathbf{e}$	$\mathbf{q}_B = \left(\mathbf{I}-\mathbf{A}_B(\mathbf{U},\mathbf{V})\right)^{-1}\mathbf{F}_B\,\mathbf{e}$	$\mathbf{g}_C = \left(\mathbf{I}-\mathbf{A}_C(\mathbf{U},\mathbf{V})\right)^{-1}\mathbf{F}_C\,\mathbf{e}$	$\mathbf{g}_D = \left(\mathbf{I}-\mathbf{A}_D(\mathbf{U},\mathbf{V})\right)^{-1}\mathbf{F}_D\,\mathbf{e}$
Negatives	YES	NO	YES	NO

Legend for the transformation of supply and use tables into input-output tables at basic prices

A = Technical coefficients matrix
$\mathbf{V}^{\mathbf{T}}$ = Supply matrix
U = Use matrix
Y = Matrix for final demand by product and category
W = Matrix of gross value added by component and by industry
e = Column vector of ones

NOTE: **T** will denote transposition and –1 inversion of a matrix. Since the two operations commute, their composition may be denoted **–T**. Also, *diag* will denote diagonalization whether by suppression of the off-diagonal elements of a square matrix or by placement of the elements of a vector.

While product-by-product input-output tables are believed to be more homogeneous, industry-by-industry input-output tables are closer to statistical sources and actual observations. In empirical research it depends on the objectives of analysis which type of input-output table is best suited for economic analysis, which will be addressed in more detail in next sections. Annual supply and use tables at basic prices provide the user with flexible access to input-output data for various applications.

2 The Choice of type of Input-Output Table in the UN and European Systems of National Accounts

The choice of technology assumption in the construction of product by product SIOTs has played a relevant role in the various systems of national accounts and handbooks/manuals published by the United Nations (UN) and Eurostat. To the contrary, the choice of type of SIOTs (product by product or industry by industry) has been almost fully neglected. In this section, we will explore the treatment of this issue by the two most recent systems of national accounts published by the UN and Eurostat together with their respective handbooks or manuals.

2.1 SNA93, UN Handbook of IO Compilation (1999) and SNA08

Essentially, the SNA93 (UN, 1993) states that only product by product tables will be described in detailed since they are often proved as most useful (par. 15.150) but however the SNA93 does not provide any justification for this assortment and simply ignores industry by industry tables.

It was not until the publication of the UN Handbook of Input-Output Compilation and Analysis (UN, 1999) when industry by industry tables received a more detailed treatment, although still not too far reaching. After providing the definitions of product and industry SIOTs (par. 4.41), the UN Handbook asserts that industry by industry SIOTs are much less useful than product by product SIOTs because an industry might represent a group of establishments, part of which may be artificially created by mathematical methods (e.g. extrapolation) and therefore, does not reflect any "realistic" picture of the economy. It follows that market shares might be less stable and consequently, the input-output (IO) analysis made on the basis of the Leontief inverse of an industry by industry SIOT should not have a significant time lag involved. Concerning IO modeling, the UN Handbook (par. 4.60) also states that "…*this model (the Leontief quantity model based on industry by industry SIOTs) is however of almost no interest to analysts since final demand is, rarely, in terms of industry outputs*…"

With an increasing interest for industry by industry SIOTs, the new System of National Accounts - SNA08 (UN, 2009) includes now one section specifically for these kinds of tables (pars. 28.57 to 28.63). As to the choice of type of SIOTs, the SNA08 states that both product by product and industry by industry SIOTs serve different analytical functions. (…) "*For example, to ensure that price indices are strictly consistent, a product by product matrix is to be preferred. For a link to labour market questions, an industry by industry table may be more useful. Although traditionally a lot of interest focused on the product by product tables, this was accompanied in large part by an attention to the underlying technology. Increasingly the economic interaction of different industries has brought more interest in the industry by industry tables*." It is also interesting to remark that in one of the annexes (par. A4.21), the SNA08 recognizes a change of emphasis from the physical view of input-output economics (the rationale was to have a unit or establishment that related as far as possible to only one activity in only one location so that the link to the physical processes of production was as clear as possible) to an economic point of view, and from product by product matrices to industry by industry ones. Maybe at this point, the WIOD project could serve as a proof of this reasoning provided that the input-output tables to be generated under the project will be of the industry by industry type.

To sum up, the choice of type of SIOT is increasingly playing a bigger role in the most recent systems of national accounts but however, it remains unclear which types of tables are to be used for what type of analysis. There are few instances here and there but without a clear structure or even clear recommendations. Consequently, this deliverable will particularly provide in next sections some useful guidelines at this respect for input-output practitioners.

2.2 ESA95, the Eurostat Manual of Supply, Use and IO Tables (2008) and the ESA08 (draft version)

Unfortunately, to the knowledge of the authors, neither the ESA95 nor the draft version of the European System of Accounts - ESA08 (Eurostat, 2009) mentions explicitly the issue of the choice of type of SIOTs. The ESA95 just offers a flexible approach to compile industry by industry SIOTs or product by product SIOTs according to the objective of economic analysis. As in the SNA93, it is recommended to compile the latter tables although industry by industry tables are also accepted if the industries are close to homogenous units of production (Eurostat, 2008; p.31). Nevertheless, the Eurostat Manual (2008) considerably deals with this issue in its chapter 11.

Following the Eurostat Manual (2008, p. 301), (…) "*product-by-product input-output tables are theoretically more homogeneous in their description of the transactions than industry-by-industry tables, since a single element of the latter can refer to products that are characteristic in other industries. This supports the assumption that in practice product-by-product tables generally are better suited for many types of input-output analysis. This is the main reason for ESA 1995 to favour product-by-product tables for economic analysis. The transmission programme of ESA 1995 requires Member States of the European Union to transmit product-by-product tables. However, the transmission of industry-by-industry input-output tables is also accepted provided that industry-by-industry tables are a good approximation of product-by-product input-output tables. While product-by-product input-output tables are believed to be more homogeneous, industry-by-industry input-output tables are closer to statistical sources and actual observations. In empirical research it depends on the objectives of analysis which type of input-output table is better suited for economic analysis. For example, it seems more feasible to use product-by-product input-output tables for productivity analysis or the analysis of new technologies in the economy. On the other hand, industry-by-industry input-output tables are possibly the better option if the economic impact of a major tax reform is studied on the basis of input-output data* (…)". Similarly to the UN Systems of National Accounts (SNA93 and SNA08) , there is also here only a general remark on the suitability of the type of SIOT, which cannot be considered as a clear guidance on which types of tables are to be used for what type of analysis.

Broadly speaking, very little secondary output reported in the supply table would lead to fade away the distinction between products and industries. So, a relatively low level of secondary activities reported in the European Union supply tables may well suggest, as one can read in the Eurostat Manual (2008, p. 309), that the difference between product by product SIOTs and industry by industry SIOTs is relatively small and consequently, both transformations can be regarded as valid options for impact analysis. However, (…) “*it must be noticed that secondary activities vary considerably across sectors even the general level is low* (…)” (Eurostat, 2008; p. 309).

The Eurostat Manual (2008, p. 340) argues that "*the type of tables that best fulfils the standard quality criteria is the industry by industry table based on the assumption of fixed product sales structures and the product by product SIOT based on the product technology assumption. These types of tables reflect the accumulated experience and current practice of those countries most permanently involved in the compilation of SIOTs*." As for analytical purposes, it follows that "*in practice all analytical uses of SIOTs must implicitly assume an industry technology, no matter how the tables have been originally compiled*." In other words, any type of input-output analysis must necessarily assume an industry technology since widely different baskets of products are assumed to be produced with identical input structures. Furthermore, product by product tables based on the product technology assumption already in the process of its construction relies on the assumption of an industry technology approach as soon as it is realised that the number of products exceed the number of industries. The Eurostat Manual (2008) continues with the argument even stating that "any product by product table is in practice a manipulated industry by industry table (…)" (p. 340). Nevertheless, (…) the Eurostat Manual recognizes that "*there is no ideal target type of table against which to measure the quality of the outcome.*"

Focusing on the two aforementioned models (Models A and D) to construct product by product tables and industry by industry tables, respectively, the Eurostat Manual defines a set of quality features of both types of SIOTs (p. 340-341):

Transparency

Industry by industry SIOTs provide more transparency than product by product SIOTs because the fixed product sales structure assumption can be derived from the supply and use tables without too much effort and in such a way that negatives do not appear. Conversely, the product technology assumption is usually applied in a complex context requiring a balancing procedure to treat the negative elements that may arise and thus, causing less transparency.

Comparability

Industry by industry SIOTs guarantee more comparability with national accounts data since they are closer to statistical sources, survey results and actual observations. To the contrary, product by product tables have been compiled in an analytical step which creates less comparability with the sources but at the same time guarantees more comparability across nations.

Inputs

Product by product SIOTs have a clear input structure in terms of products for intermediate use and value added for the compensation of labour and capital for homogenous branches. However, in industry by industry SIOTs, mixed bundles of goods and services rather than homogeneous products are reported for intermediate and final uses.

Resources and timeliness

The compilation of product by product tables based on the product technology assumption requires more resources and balancing efforts due to the treatment of the negatives that may appear. Consequently, publication may be delayed. However, industry by industry tables can be directly derived from supply and use tables with less resource intensive efforts.

Analytical potential

The Eurostat Manual (2008, p. 341) states that "*industry by industry tables are well suited for specific analytical purposes which are related to industries (tax reform, impact analysis, fiscal policy, monetary policy, etc.)*" while product by product tables "*are well suited for many other specific analytical purposes which are related to homogeneous production units (productivity, comparison of cost structures, employment effects, energy policy, environmental policy, etc.)*" Although useful, this distinction just enumerates possible applications without a clear guidance on which types of tables are to be used for what type of analysis, which will hopefully be provided by this deliverable.

To cut a long story short, the choice of type of SIOT is not a relevant issue in the two most recent ESAs (1995 and 2008) although the Eurostat Manual (2008) gives much more insight into the matter than any of the UN documents. However, we still think that a deeper and clearer connection between standard input-output applications and the use of product by product and/or industry by industry tables is needed.

3 The relevance of the applications: the quantity and the price models in input-output analysis

3.1 The quantity and price models in input-output analysis

The main purpose of this section is to present briefly the theoretical background of the two most commonly and broadly used models in input-output analysis, i.e. the quantity and the price models. It will follow a discussion on the choice of type of SIOT for each type of model together with some guidelines.

Dietzenbacher (1997) considered the following SIOT in money terms (say, euros in the case of WIOD) for period 0:

$\mathbf{X_0}$	$\mathbf{f_0}$	$\mathbf{x_0}$
$\mathbf{v'_0}$	-	$\mathbf{v'_0 e}$
$\mathbf{x'_0}$	$\mathbf{e'f_0}$	

$\mathbf{X_0}$ is the *n* x *n* matrix of intermediate uses; its typical element x_{ij}^0 denotes the value (in euros) of the deliveries from industry (product) *i* to industry (product) *j*, which will depend on the type of SIOT used. Dietzenbacher (1997) did not however distinguish in his paper between the two types of SIOTs referring implicitly all the time to industry by industry tables. The column vector $\mathbf{f_0}$ can be interpreted as sectoral (product) final demands including private and government consumption, investments and net exports[1]. The row vector $\mathbf{v'_0}$ gives the value added in each industry (product or homogenous branch), containing, for instance, payments for the labour and capital primary factors. The value of each industry (product) output is given

[1] Dietzenbacher (1997) made this assumption without loss of generality and for the sake of notational convenience.

by the elements of the vector $\mathbf{x_0}$ while $\mathbf{e}$ denotes the *n*-dimension column vector of ones. Column-wise, a SIOT depicts input structures and row-wise, output structures. Since the total value of outputs equals the total value of inputs, for each industry (product), the following sets of accounting equations are obtained:

$$\mathbf{x}_0 = \mathbf{X}_0\mathbf{e} + \mathbf{f}_0 \tag{1}$$
$$\mathbf{x}'_0 = \mathbf{e}'\mathbf{X}_0 + \mathbf{v}'_0 \tag{2}$$

It follows that the input coefficients are defined as the industry (product) *i*'s input into industry (product) *j* as a fraction of the purchaser's output (x_j^0). They are obtained as $a_{ij}^0 = x_{ij}^0 / x_j^0$, or in matrix terms, as $\mathbf{A}_0 = \mathbf{X}_0\hat{\mathbf{x}}_0^{-1}$ where $\hat{\mathbf{x}}_0$ denotes a diagonal matrix. Then, equation (1) may be written as:

$$\mathbf{x}_0 = \mathbf{A}_0\mathbf{x}_0 + \mathbf{f}_0 \tag{3}$$

In a similar way, the output coefficients denote the industry (product) *i*'s delivery to industry (product) *j* as a fraction of the seller's output (x_i^0). They are obtained as $b_{ij}^0 = x_{ij}^0 / x_i^0$ or, in matrix terms, as $\mathbf{B}_0 = \hat{\mathbf{x}}_0^{-1}\mathbf{X}_0$. Subsequently, equation (2) may be rewritten as

$$\mathbf{x}'_0 = \mathbf{x}'_0\mathbf{B}_0 + \mathbf{v}'_0 \tag{4}$$

From the accounting equations (3) and (4), it is usual to obtain the so called **Leontief quantity model** and the **Ghosh price model**, respectively. However, we must include also two other types of models that are not so often treated in the input-output literature but that deserve however to be mentioned for the sake of comprehensiveness.

Quantity models
Equation (3) rests on the assumption of fixed technical coefficients being the new industry (product) output vector ($\mathbf{x}_1$) required for an exogenously specified new final demand vector ($\mathbf{f}_1$) such that,

$$\mathbf{x}_1 = (\mathbf{I} - \mathbf{A}_0)^{-1}\mathbf{f}_1 \tag{5}$$

Given a shock in the physical amounts consumed by final users of a product (or of the bundle of products produced by a certain industry, both primarily and secondarily produced), then the effect on the total output value of the industry (product) output is given by $\mathbf{x}_1$. Notice that in this so called **Leontief quantity model** there is no change in prices.

Furthermore, equation (5) can also be expressed as a ratio per unit of output value of the period 0 as[2],

$$\hat{\mathbf{x}}_0^{-1}\mathbf{x}_1 = \hat{\mathbf{x}}_0^{-1}(\mathbf{I} - \mathbf{A}_0)^{-1}\hat{\mathbf{x}}_0\hat{\mathbf{x}}_0^{-1}\mathbf{f}_1 = (\mathbf{I} - \mathbf{B}_0)^{-1}\hat{\mathbf{x}}_0^{-1}\mathbf{f}_1 \tag{6}$$

[2] The relationship between the Leontief and the Ghosh inverses can be found in Miller and Blair (2009, p. 548).

which gives the variation rate of the quantities produced to meet the new final demand. That is, the new output total value ($\mathbf{x}_1$) results from the multiplication of old prices ($\mathbf{p}_0$) by the new quantities demanded ($\mathbf{q}_1$) such as,

$$\mathbf{x}_1 = \hat{\mathbf{p}}_0 \mathbf{q}_1 \tag{7}$$

whilst the old output values result from the amounts consumed valued at prices of period 0, as

$$\hat{\mathbf{x}}_0^{-1} = \left(\hat{\mathbf{p}}_0 \hat{\mathbf{q}}_0\right)^{-1} = \hat{\mathbf{q}}_0^{-1} \hat{\mathbf{p}}_0^{-1} \tag{8}$$

Then, by replacing the right-hand side (RHS) of equation (6) by equations (7) and (8), it is straightforward that,

$$\hat{\mathbf{q}}_0^{-1} \hat{\mathbf{p}}_0^{-1} \hat{\mathbf{p}}_0 \mathbf{q}_1 = \hat{\mathbf{q}}_0^{-1} \mathbf{q}_1 = (\mathbf{I} - \mathbf{B}_0)^{-1} \hat{\mathbf{x}}_0^{-1} \mathbf{f}_1 \tag{9}$$

which is the so called **Ghosh quantity model** (Dietzenbacher, 1997). A change in the final demand shares over the total output value of period 0 caused by variations in the quantities demanded will lead to changes in the quantities produced.

Price models

Equation (4) is based on the assumption of fixed output coefficients. For a new value added vector ($\mathbf{v'}_2$), the new total output values are calculated by,

$$\mathbf{x'}_2 = \mathbf{v'}_2 \ (\mathbf{I} - \mathbf{B}_0)^{-1} \tag{10}$$

Given a price change in any of the primary factors used (generally speaking, capital and labour), then the effect on the output value of the industry (product) output is given by $\mathbf{x}_2$. Notice that in this so called **Ghosh price model** there is no change in quantities consumed of primary inputs and of goods and services.

Moreover, equation (10) can also be expressed as a ratio per unit of output value of the period 0 as,

$$\mathbf{x'}_2 \hat{\mathbf{x}}_0^{-1} = \mathbf{v'}_2 \hat{\mathbf{x}}_0^{-1} \hat{\mathbf{x}}_0 (\mathbf{I} - \mathbf{B}_0)^{-1} \hat{\mathbf{x}}_0^{-1} = \mathbf{v'}_2 \hat{\mathbf{x}}_0^{-1} \ (\mathbf{I} - \mathbf{A}_0)^{-1} \tag{11}$$

which gives the price variation of products generated by the variation in the prices of primary factors. That is, the new output total value ($\mathbf{x}_2$) results from the multiplication of old quantities produced ($\mathbf{q}_0$) by the new prices ($\mathbf{p}_2$) such as,

$$\mathbf{x'}_2 = \mathbf{p'}_2 \hat{\mathbf{q}}_0 \tag{12}$$

while the old output values result from the amounts consumed valued at prices of period 0, as

$$\hat{\mathbf{x}}_0^{-1} = \left(\hat{\mathbf{p}}_0 \hat{\mathbf{q}}_0\right)^{-1} = \hat{\mathbf{q}}_0^{-1} \hat{\mathbf{p}}_0^{-1} \tag{13}$$

Threfore, by replacing the RHS of equation (11) by equations (12) and (13), it is easy to obtain that,

$$\mathbf{p}'_2\hat{\mathbf{q}}_0\hat{\mathbf{q}}_0^{-1}\hat{\mathbf{p}}_0^{-1} = \mathbf{p}'_2\hat{\mathbf{p}}_0^{-1} = \mathbf{v}'_2\hat{\mathbf{x}}_0^{-1}(\mathbf{I}-\mathbf{A}_0)^{-1} \quad (14)$$

which is the so called **Leontief price model** or **supply-driven model** (Dietzenbacher, 1997). A change in value added shares over the total output value of period 0 caused by variations in the prices of primary inputs will lead to changes in product prices.

3.2 The relationship between the models and the choice of type of input-output table

Quantity models

The Ghosh and Leontief quantity models are demand driven models. They both measure the effects on the output (in physical and monetary values, respectively) of a change in final demand. To that purpose, the use of product by product tables would imply to assume a shock in the final demand of a specific product irrespectively of the industry that actually produced it. For instance, for an increase in the households' purchase of electric cars against fuel based vehicles one would need a product by product table in order to quantify the effects on the quantities of energy inputs supplied to meet such new demand. Furthermore, if greenhouse gas direct emissions are available on a product basis, the total effects on the environment can be easily calculated with a product by product table by multiplying the new output value $\mathbf{x}_1$ (from equation 5) by the emission levels per product output. Nevertheless, emission coefficients are mostly available on an industry basis, which then makes product by product tables unsuitable. Furthermore, if one eventually uses an industry by industry table the calculated effects would be caused instead by a change in the final demand of the bundle of goods and services produced by a specific industry, which is not necessarily that of a specific commodity. All in all, in the case of environmental analysis, the kind of data available and the objective of the analysis definitely play a major role in the choice of type of SIOT to be used.

Input-output analysis is also applied to labour market analyses through the calculation of employment multipliers under the Leontief quantity model. Due to the fact that employment data are usually recorded by firms and therefore grouped by industries, industry by industry tables may be more appropriate than product by product tables. It is not very likely to find employment data related to products. Moreover, one must bear in mind that the effects on employment thus calculated using industry by industry tables will be caused by a change in the final demand of a mixed bundle of goods and services produced by a certain sector, which does not necessarily be a single specific commodity.

The input-output quantity models are used to evaluate the effects of introducing a new product technology as well. Provided that the new technology refers to a single product and that it can be easily subtracted from its mother branch, the Leontief and Ghosh quantity models would allow for evaluating the effects on the output value (and physical amounts produced) of the other competing products. At this respect, product by product tables seems to be more suitable than industry by industry tables, where each industry produces more than one single product. Clearly, the new demand for a new product (e.g. electric cars) will drive a set of direct and indirect effects on the other product outputs.

The calculation of value added and income (wages and salaries) multipliers are also a matter of interest in the input-output literature. It is quite intuitive that the compensation of employees and the value added are clearly linked to industries rather than to products or homogenous branches. Industry by industry tables keep a direct link to the original statistical sources. Bearing this in mind, industry by industry tables are in this case also preferable to product by product tables although the IO literature admit several impact analyses on the basis of value added/income related to homogenous branches of activities.

As a summarizing remark, the IO quantity models are driven by changes in the amounts of goods and services consumed or demanded. The use of product by product tables is preferable since the shock can be easily assigned to a single product and the output effects can also be related to homogenous branches of activities. To the contrary, the use of industry by industry tables in this context would lead to measure the effects of a variation in the demanded quantity of a mixed bundle of goods and services produced by a certain industry on the industry output values and amounts of (mixed) goods and services produced. Insofar, the choice favours clearly product by product tables almost in all cases. However, the Leontief quantity model is extensively used to account for many different kinds of multiplier effects, e.g. environmental, employment, income… that needs data that are almost solely available on an industry basis. To some extent, this justifies the use of industry by industry tables in some situations. Therefore, it seems to be a clear trade-off. Either one assumes that the additional data (environmental, employment, income…) is on a product basis and uses product by product tables to measure the effects on the output value (also in physical terms) of changes in final demand of single products, or one assumes that the additional data is on an industry basis and uses industry by industry tables, although being aware that the derived effects on total output values are referred not to single products but to a mixed bundle of goods and services produced by a certain industry.

Price models

The Ghosh and Leontief price models measure the effects of variations in the prices of primary inputs on the output value and on the prices of goods and services, respectively. The amount of factor inputs used remains unchanged and so the amounts of goods and services produced. These models are seen as supply-side driven models preferably to be used in cases of shortage of supply or excess of demand. Variations in salaries and wages per hour, in profit rates, in fixed capital use rates or in net tax rates[3] on production will generate changes in prices of goods and services and output value that could be quantified through the price IO models. As a result, industry by industry tables seems to be more suitable for these kind of analyses since initial changes are referred to the different components of the value added, which are directly linked to the surveyed firms data and/or groups of firms (industries) data. Indeed, statistical data on labour costs are referred to workers employed in industries and not in homogenous branches of activity. Fiscal policies (excluding taxes on products) on taxes and subsidies on production (e.g. environmental tax) are commonly referred to polluter industries rather than to homogenous branches. Moreover, profit rates are also related to firms and industries rather than to products.

Nevertheless, the price changes obtained through the IO price models using an industry by industry SIOT are not reflecting single product price variations but variations in the prices of a mixed bundle of goods and services produced by an industry. So, there is a clear trade-off again at this respect. Either one assumes that changes in primary inputs occur in homogenous branches and uses product by product tables to calculate single product price changes or one assumes that the price variations of primary factors occur in industries and

[3] Generally speaking, the taxes less subsidies on production included in the value added at basic prices are those that are not payable per unit of some good or service produced or transacted (ESA95).

uses industry by industry tables to obtain mixed product price changes. The choice is eventually up to the user.

Supply-use tables

Two major trade-offs have been identified concerning the choice of type of SIOT to be used in impact analysis. The main difficulty underlying the two trade-offs is referred to the symmetry of the SIOTs. They are defined as product by product or industry by industry type. Hence, if one is interested in estimating, for instance, the effects of an increase in the labour costs of the electricity sector (industry) on the prices of fuels (product), then the choice of type of SIOT would lead to provide two different answers but however none of them the correct one. On the one hand, if we use product by product tables we will be assigning the increase of labour costs to a homogenous branch of activity and not to the electricity sector and on the other hand, if we use industry by industry tables, the price effects will correspond to a mixed basket of goods and services of the fuel producing industry rather than to fuel.

To solve this issue, supply-use tables are clearly the best choice since they are defined on a product by industry basis rather than on a product or industry basis. However, there has been very little research on the application of supply and use tables to impact analysis. To the knowledge of the authors, the single contributions at this respect can be found in ten Raa and Rueda-Cantuche (2007) and in Rueda-Cantuche and Amores (2009). The former authors proved that employment and output multipliers (from the Leontief quantity model) can be derived from supply and use data by regressing employment (output) by industries on the net output[4] by products. Therefore, a change in the net output of products (implicitly a change in the final demand) will cause a variation in the employment (output) of industries. The interested reader may find more details in the cited paper. The latter contribution relates to environmental input-output impact analysis and applied the same concept to carbon dioxide emissions in Denmark. This line of research can be further extended methodologically under the WIOD project to include time series of multiregional supply-use systems. Insofar, it has been applied only to a single-country for one year only.

4 Conclusions and recommendations for WIOD

This section summarizes the main conclusions and recommendations that can be drawn for the WIOD project.

The construction of SIOTs is a controversial issue in the input-output literature as regard the choice of model to construct both product by product and industry by industry SIOTs, especially the former ones. However, there has been so far little attention paid on the choice of type of SIOT to carry out impact analyses let alone other input-output applications. The UN and Eurostat systems of national accounts just simply refer to this issue vaguely and basically recommend nothing except that the purpose of the analysis will determine the choice of type to be used. Moreover, there are no explicit guidelines for the user to make the correct choice accordingly with its own purpose.

Independently of the purpose of the analysis, both types of SIOTs have its own advantages and disadvantages. On the one hand, the product by product tables are more homogeneous in their description of the transactions being one of the most commonly used

[4] Ten Raa and Rueda-Cantuche (2007) defined net output as the difference between the intermediate parts of the supply and use matrices, which incidentally makes the final demand vector if one sums the elements of the net output matrix over columns.

tables in input-output analysis (productivity, comparison of costs structures, employment effects, energy policy…) and have a clear input structure in terms of products for intermediate uses and value added for the compensation of labour and capital for homogenous branches. However, product-by-product tables require labour intensive compilation tasks; they must be based on analytical assumptions that take final results away from actual market transactions and observations, and hence, they make more difficult the integration of other statistical sources and the reporting on the transformation procedure.

On the other hand, industry by industry tables are much closer to statistical sources; they allow for an easier comparability with other statistical databases; they are less labour intensive to compile, being based on pragmatic assumptions rather than on analytical hypotheses. Nevertheless, the larger the secondary activities in the supply table are the more difficult it becomes to identify homogeneous cost structures in an industry-by-industry table.

In practice, most of the worldwide countries compile product by product tables although there are some hardly negligible countries like Denmark, the Netherlands, Norway, Canada and Finland that compile industry by industry SIOTs. Nevertheless, one can always shift from one type to another as it is shown in the Annex.

The choice of type of SIOT is playing increasingly a relevant role in the most recent systems of national accounts but still they provide unclear guidelines on the type of table to be used for what type of analysis. There is no clear structure or even clear recommendations.

In empirical research, it depends on the objectives of the analysis which type of table is best suited for economic analysis. Particularly in impact analyses, questions like, for example, what fuel price effects would generate an increase in the labour costs of the electricity industry cannot really be answered by input-output price models as it is generally thought. Moreover, this is even independently of the type of SIOT used. Either one assumes that changes in primary costs (labour) occur in homogeneous branches rather than in industries and therefore uses product by product tables or one assumes that the price changes of primary factors effectively occur in industries and thus, uses industry by industry tables. Nonetheless, the corresponding reported price effects will be those of the fuel industry rather than those of the fuel product itself.

As regard input-output quantity models there is also a trade-off in the case of impact analyses related to environment, employment… or any economic dimension for which data is mainly available on an industry basis. Either one assumes that the additional data external to the input-output system (employment, emissions…) is on a product basis and uses a product by product table to evaluate the total effects of a change in the amount of the final demand consumed of a single product (like e.g. bio-fuels) or one assumes that the additional data is on an industry basis and uses industry by industry tables. Nevertheless, the derived total effects on employment, emissions… will correspond to a change in the output of a mixed bundle of goods and services produced by a certain industry rather than to changes in single product outputs.

Two major trade-offs have been identified concerning the choice of type of SIOT to be used in input-output impact analyses. The main shortcoming underlying this issue is related to the symmetry of SIOTs. They are defined as either product by product or industry by industry type. To solve this matter efficiently, supply and use tables are clearly the best choice since they are defined on a product by industry basis rather than solely on a product or industry basis. It is therefore advisable that the WIOD project could follow the lines of the pioneering works of ten Raa and Rueda-Cantuche (2007) and Rueda-Cantuche and Amores (2009) and continue exploring the use of supply and use tables in the calculation of input-output impact multipliers of any kind. Of course, one can always come back to standard input-output analysis bearing in mind the methodological trade-offs addressed in this paper.

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Annex

Bridge matrices to switch between different types of symmetric input-output tables

Following the notation provided by the Eurostat Manual (2008, pp. 348-352) and denoting as ^ the diagonalization whether by suppression of the off-diagonal elements of a square matrix or by placement of the elements of a vector, the bridge matrices of technical coefficients that make possible to switch between the different types of SIOTs are presented in Table A.1. As regard intermediates, final demand, value added and output calculations (see Table 3), the reader should not find any difficulty in deriving the corresponding bridges matrices from those depicted in Table A.1 for technical coefficients. Hence, we will focus our attention exclusively on the bridge matrices for technical coefficients matrices. In this Annex, we will demonstrate mathematically all the results provided in Table A.1. ´

Following the Eurostat Manual's (2008, p.349) notation, let us denote $\mathbf{g}$ as the column vector of industry output; $\mathbf{q}$ as the column vector of product output; $\mathbf{C} = \mathbf{V}^{\mathrm{T}}\hat{\mathbf{g}}^{-1}$ as the product-mix matrix with share of each product in industry outputs (supply table); $\mathbf{D} = \mathbf{V}\hat{\mathbf{q}}^{-1}$ as the market shares matrix with contribution of each industry to the product output (supply table); and $\mathbf{Z} = \mathbf{U}\hat{\mathbf{g}}^{-1}$ as the inputs requirements for products per unit of output of an industry (use table). In addition, Table A.1 shows in its main diagonal the different models for product by product tables and industry by industry tables using the same notation.

Theorem 1 (From Model A to B and vice versa)

Let $\mathbf{A_A}(\mathbf{U},\mathbf{V})$ be a technical coefficients matrix of a product by product SIOT constructed under the product technology model, then the product by product technical coefficients matrix $\mathbf{A_B}(\mathbf{U},\mathbf{V})$ on the basis of the industry technology model is given by,

$$\mathbf{A_B}(\mathbf{U},\mathbf{V}) = \mathbf{A_A}(\mathbf{U},\mathbf{V})\,\mathbf{CD}$$

and conversely,

$$\mathbf{A_A}(\mathbf{U},\mathbf{V}) = \mathbf{A_B}(\mathbf{U},\mathbf{V})\mathbf{D}^{-1}\mathbf{C}^{-1}$$

Proof.- The necessity proof is as follows. By definition, a product by product technical coefficient matrix using the industry technology model is:

$$\mathbf{A_B}(\mathbf{U},\mathbf{V}) = \mathbf{ZD} \qquad \text{(A.1)}$$

then, by using $\mathbf{C}^{-1}\mathbf{C} = \mathbf{I}$ and operating in (A.1), we obtain

$$\mathbf{A_B}(\mathbf{U},\mathbf{V}) = \mathbf{ZC}^{-1}\mathbf{CD} = \mathbf{A_A}(\mathbf{U},\mathbf{V})\mathbf{CD}$$

The sufficient condition departures from the product technology assumption:

$$\mathbf{A_A}(\mathbf{U},\mathbf{V}) = \mathbf{ZC}^{-1}. \qquad \text{(A.2)}$$

Then, by post-multiplying the RHS of the equation (A.2) by $(\mathbf{CD})(\mathbf{CD})^{-1} = \mathbf{I}$ we obtain,

$$\mathbf{A_A}(\mathbf{U},\mathbf{V}) = \mathbf{ZC}^{-1}\mathbf{CD}(\mathbf{CD})^{-1} = \mathbf{ZD}(\mathbf{CD})^{-1} = \mathbf{A_B}(\mathbf{U},\mathbf{V})\mathbf{D}^{-1}\mathbf{C}^{-1}$$

Theorem 2 (From Model A to C and vice versa)

Let $\mathbf{A_A}(\mathbf{U},\mathbf{V})$ be a technical coefficients matrix of a product by product SIOT constructed under the product technology model, then the industry by industry technical coefficients matrix $\mathbf{A_C}(\mathbf{U},\mathbf{V})$ on the basis of the fixed industry sales structure model is given by,

$$\mathbf{A_C}(\mathbf{U},\mathbf{V}) = \mathbf{C}^{-1}\mathbf{A_A}(\mathbf{U},\mathbf{V})\mathbf{C}$$

and conversely,

$$\mathbf{A_A}(\mathbf{U},\mathbf{V}) = \mathbf{CA_C}(\mathbf{U},\mathbf{V})\mathbf{C}^{-1}$$

Proof.- By definition, an industry by industry technical coefficient matrix using the fixed industry sales structure is:

$$\mathbf{A_C}(\mathbf{U},\mathbf{V}) = \mathbf{C}^{-1}\mathbf{Z} \qquad \text{(A.3)}$$

then, by using $\mathbf{C}^{-1}\mathbf{C} = \mathbf{I}$ and operating in (A.3), we obtain

$$\mathbf{A_C}(\mathbf{U},\mathbf{V}) = \mathbf{C}^{-1}\mathbf{ZC}^{-1}\mathbf{C} = \mathbf{C}^{-1}\mathbf{ZC}^{-1}\mathbf{C} = \mathbf{C}^{-1}\mathbf{A_A}(\mathbf{U},\mathbf{V})\,\mathbf{C}$$

The sufficient condition begins with the product technology assumption:

$$\mathbf{A_A}(\mathbf{U},\mathbf{V}) = \mathbf{ZC}^{-1}. \qquad \text{(A.4)}$$

Then, by pre-multiplying the RHS of equation (A.4) by $\mathbf{CC}^{-1} = \mathbf{I}$ we obtain,

$$\mathbf{A_A}(\mathbf{U},\mathbf{V}) = \mathbf{CC}^{-1}\mathbf{ZC}^{-1} = \mathbf{CA_C}(\mathbf{U},\mathbf{V})\mathbf{C}^{-1}$$

Theorem 3 (From Model A to D and vice versa)

Let $\mathbf{A_A}(\mathbf{U},\mathbf{V})$ be a technical coefficients matrix of a product by product SIOT constructed under the product technology model, then the industry by industry technical coefficients matrix $\mathbf{A}_\mathrm{D}(\mathbf{U},\mathbf{V})$ on the basis of the fixed product sales structure model is given by,

$$\mathbf{A}_\mathrm{D}(\mathbf{U},\mathbf{V}) = \mathbf{DA_A}(\mathbf{U},\mathbf{V})\mathbf{C}$$

and conversely,

$$\mathbf{A_A}(\mathbf{U},\mathbf{V}) = \mathbf{D}^{-1}\mathbf{A}_\mathrm{D}(\mathbf{U},\mathbf{V})\mathbf{C}^{-1}$$

Proof.- By definition, an industry by industry technical coefficient matrix using the fixed product sales structure is:

$$\mathbf{A}_{\mathrm{D}}(\mathbf{U},\mathbf{V}) = \mathbf{DZ} \qquad \text{(A.5)}$$

then, by using $\mathbf{C}^{-1}\mathbf{C} = \mathbf{I}$ and operating in (A.5), we obtain

$$\mathbf{A}_{\mathrm{D}}(\mathbf{U},\mathbf{V}) = \mathbf{DZC}^{-1}\mathbf{C} = \mathbf{DA}_{\mathbf{A}}(\mathbf{U},\mathbf{V})\,\mathbf{C}$$

The sufficient condition departs from the product technology assumption:

$$\mathbf{A}_{\mathbf{A}}(\mathbf{U},\mathbf{V}) = \mathbf{ZC}^{-1}. \qquad \text{(A.6)}$$

Then, by pre-multiplying the RHS of equation (A.6) by $\mathbf{D}^{-1}\mathbf{D} = \mathbf{I}$ we obtain,

$$\mathbf{A}_{\mathrm{A}}(\mathbf{U},\mathbf{V}) = \mathbf{D}^{-1}\mathbf{DZC}^{-1} = \mathbf{D}^{-1}\mathbf{A}_{\mathrm{D}}(\mathbf{U},\mathbf{V})\mathbf{C}^{-1}$$

Theorem 4 (From Model B to C and vice versa)

Let $\mathbf{A}_{\mathrm{B}}(\mathbf{U},\mathbf{V})$ be a technical coefficients matrix of a product by product SIOT constructed under the industry technology model, then the industry by industry technical coefficients matrix $\mathbf{A}_{\mathbf{C}}(\mathbf{U},\mathbf{V})$ on the basis of the fixed industry sales structure model is given by,

$$\mathbf{A}_{\mathrm{C}}(\mathbf{U},\mathbf{V}) = \mathbf{C}^{-1}\mathbf{A}_{\mathrm{B}}(\mathbf{U},\mathbf{V})\mathbf{D}^{-1}$$

and conversely,

$$\mathbf{A}_{\mathrm{B}}(\mathbf{U},\mathbf{V}) = \mathbf{CA}_{\mathrm{C}}(\mathbf{U},\mathbf{V})\mathbf{D}$$

Proof.- By definition, an industry by industry technical coefficient matrix using the fixed industry sales structure is:

$$\mathbf{A}_{\mathrm{C}}(\mathbf{U},\mathbf{V}) = \mathbf{C}^{-1}\mathbf{Z} \qquad \text{(A.7)}$$

then, by using $\mathbf{DD}^{-1} = \mathbf{I}$ and operating in (A.7), we obtain

$$\mathbf{A}_{\mathrm{C}}(\mathbf{U},\mathbf{V}) = \mathbf{C}^{-1}\mathbf{ZDD}^{-1} = \mathbf{C}^{-1}\mathbf{A}_{\mathrm{B}}(\mathbf{U},\mathbf{V})\,\mathbf{D}^{-1}$$

The sufficient condition starts from the industry technology assumption:

$$\mathbf{A}_{\mathrm{B}}(\mathbf{U},\mathbf{V}) = \mathbf{ZD}. \qquad \text{(A.8)}$$

Then, by pre-multiplying the RHS of equation (A.8) by $\mathbf{CC}^{-1} = \mathbf{I}$ we obtain,

$$\mathbf{A}_{\mathrm{B}}(\mathbf{U},\mathbf{V}) = \mathbf{CC}^{-1}\mathbf{ZD} = \mathbf{CA}_{\mathbf{C}}(\mathbf{U},\mathbf{V})\mathbf{D}$$

Theorem 5 (From Model B to D and vice versa)

Let $\mathbf{A}_B(\mathbf{U},\mathbf{V})$ be a technical coefficients matrix of a product by product SIOT constructed under the industry technology model, then the industry by industry technical coefficients matrix $\mathbf{A}_D(\mathbf{U},\mathbf{V})$ on the basis of the fixed product sales structure model is given by,

$$\mathbf{A}_D(\mathbf{U},\mathbf{V}) = \mathbf{D}\mathbf{A}_B(\mathbf{U},\mathbf{V})\mathbf{D}^{-1}$$

and conversely,

$$\mathbf{A}_B(\mathbf{U},\mathbf{V}) = \mathbf{D}^{-1}\mathbf{A}_D(\mathbf{U},\mathbf{V})\mathbf{D}$$

Proof.- By definition, an industry by industry technical coefficient matrix using the fixed product sales structure is:

$$\mathbf{A}_D(\mathbf{U},\mathbf{V}) = \mathbf{D}\mathbf{Z} \qquad \text{(A.9)}$$

then, by using $\mathbf{D}\mathbf{D}^{-1} = \mathbf{I}$ and operating in (A.9), we obtain

$$\mathbf{A}_D(\mathbf{U},\mathbf{V}) = \mathbf{D}\mathbf{Z}\mathbf{D}\mathbf{D}^{-1} = \mathbf{D}\mathbf{A}_B(\mathbf{U},\mathbf{V})\,\mathbf{D}^{-1}$$

The sufficient condition begins with the industry technology assumption:

$$\mathbf{A}_B(\mathbf{U},\mathbf{V}) = \mathbf{Z}\mathbf{D}\,. \qquad \text{(A.10)}$$

Then, by pre-multiplying the RHS of equation (A.10) by $\mathbf{D}^{-1}\mathbf{D} = \mathbf{I}$ we obtain,

$$\mathbf{A}_B(\mathbf{U},\mathbf{V}) = \mathbf{D}^{-1}\mathbf{D}\mathbf{Z}\mathbf{D} = \mathbf{D}^{-1}\mathbf{A}_D(\mathbf{U},\mathbf{V})\mathbf{D}$$

Theorem 6 (From Model C to D and vice versa)

Let $\mathbf{A}_C(\mathbf{U},\mathbf{V})$ be a technical coefficients matrix of an industry by industry SIOT constructed under the fixed industry sales structure model, then the industry by industry technical coefficients matrix $\mathbf{A}_D(\mathbf{U},\mathbf{V})$ on the basis of the fixed product sales structure model is given by,

$$\mathbf{A}_D(\mathbf{U},\mathbf{V}) = \mathbf{D}\mathbf{C}\mathbf{A}_C(\mathbf{U},\mathbf{V})$$

and conversely,

$$\mathbf{A}_C(\mathbf{U},\mathbf{V}) = \mathbf{C}^{-1}\mathbf{D}^{-1}\mathbf{A}_D(\mathbf{U},\mathbf{V})$$

Proof.- By definition, an industry by industry technical coefficient matrix using the fixed product sales structure is:

$$\mathbf{A}_D(\mathbf{U},\mathbf{V}) = \mathbf{D}\mathbf{Z} \qquad \text{(A.11)}$$

then, by using $\mathbf{C}\mathbf{C}^{-1} = \mathbf{I}$ and operating in (A.11), we obtain

$$\mathbf{A}_{D}(\mathbf{U},\mathbf{V}) = \mathbf{D}\mathbf{C}\mathbf{C}^{-1}\mathbf{Z} = \mathbf{D}\mathbf{C}\mathbf{A}_{C}(\mathbf{U},\mathbf{V})$$

The sufficient condition begins with the fixed industry sales structure:

$$\mathbf{A}_{C}(\mathbf{U},\mathbf{V}) = \mathbf{C}^{-1}\mathbf{Z}. \qquad (A.12)$$

Then, by using $\mathbf{D}^{-1}\mathbf{D} = \mathbf{I}$ we obtain that equation (12) becomes,

$$\mathbf{A}_{C}(\mathbf{U},\mathbf{V}) = \mathbf{C}^{-1}\mathbf{D}^{-1}\mathbf{D}\mathbf{Z} = \mathbf{C}^{-1}\mathbf{D}^{-1}\mathbf{A}_{D}(\mathbf{U},\mathbf{V})$$

Table A.1. Bridge matrices for technical coefficients to switch between different types of SIOTs

To: / From:	MODEL A Product by product Product technology based	MODEL B Product by product Industry technology based	MODEL C Industry by industry Fixed industry sales structure	MODEL D Industry by industry Fixed product sales structure
Model A	$\mathbf{A_A(U,V) = Z\,C^{-1}}$	$\mathbf{A_B(U,V) = A_A CD}$	$\mathbf{A_C(U,V) = C^{-1}A_A C}$	$\mathbf{A_D(U,V) = DA_A C}$
Model B	$\mathbf{A_A(U,V) = A_B D^{-1}C^{-1}}$	$\mathbf{A_B(U,V) = Z\,D}$	$\mathbf{A_C(U,V) = C^{-1}A_B D^{-1}}$	$\mathbf{A_D(U,V) = DA_B D^{-1}}$
Model C	$\mathbf{A_A(U,V) = CA_C C^{-1}}$	$\mathbf{A_B(U,V) = CA_C D}$	$\mathbf{A_C(U,V) = C^{-1}Z}$	$\mathbf{A_D(U,V) = DCA_C}$
Model D	$\mathbf{A_A(U,V) = D^{-1}A_D C^{-1}}$	$\mathbf{A_B(U,V) = D^{-1}A_D D}$	$\mathbf{A_C(U,V) = C^{-1}D^{-1}A_D}$	$\mathbf{A_D(U,V) = D\,Z}$

Legend
A = Technical coefficients matrix
$\mathbf{V}^T$ = Supply matrix
U = Use matrix
e = Column vector of ones
Z = Inputs requirements for products per unit of output of an industry (use table)
C = Product-mix matrix with share of each product in output of an industry (supply table)
D = Market shares matrix with contribution of each industry to the output of a product (supply table)