# Three way decomposition of the Efficiency of Andalusian Economy

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#### Abstract

Recent contributions to input-output analysis (ten Raa, 2006) propose a scheme for the efficiency evaluation of an industrial organization by the benchmarking of both, the firms and the industry as a whole as well. We extend this approach to trace the allocative efficiency of the whole economy by including the firms' supply and use micro-data for the computation of a measure of the efficiency of the whole economy. Additionally, we show how it can be decomposed in three components: firm efficiencies, industrial organizational efficiencies and industrial specialization efficiency of the economy. The empirical work is carried out for the Andalusian Economy.

Keywords: Input-Output, industrial organization, comparative advantage, allocative efficiency, efficiency decomposition.

## 1. Introduction

Specialization on comparative advantage provides efficiency gains to the whole economic system (David Ricardo). Allocative inefficiencies appear when that ricardian principle is not followed. Such inefficiencies can occur at firms level and at industry level. At the firms level, it if not all firms of an industry operate at minimum average cost conditions (competitive market equilibrium). Such inefficiencies in industrial organization are measureable with the model developed by ten Raa (2006). Such model relies on the benchmarking, not only of each firm of an industry, but also considering the reallocation of resources to firms which technology minimizes average cost. There is also scope for a wider analysis since allocative inefficiency may relay not only among firms in a specific industry but also, among different industries of an economy. The industrial specialization of an economy may be also inefficient in the sense that it is not focus on its comparative advantage. Such inefficiencies are not observed at industry levels, since it is an inter-industry problem. This paper tackles that issue, tracing allocative efficiency of the whole economy and its decomposition in different shares: : firm, industry, industrial organization, industrial specialization and economy efficiencies.

In the next section, we review a measure for the industrial organization efficiency. After that, we propose an inclusion of industrial specialization efficiency of the economy. In section 4, the economy-wide efficiency is analyzed and decomposed. An application is presented in next section. The paper ends with some conclusions. An appendix with details about database building has been added.

## 2. Review of Organization Efficiency

ten Raa (2006) proposes a scheme for the efficiency evaluation of an industrial organization by the benchmarking both of the firms and the industry as a whole as well. This approach is based on the efficiency gains available by the reallocation of resources to the firms which are more efficient in the production of the bundle of commodities.

Let us consider the input vector of a Decision Making Unit (DMU) *i* of the industry *k* by  $x_{ik}$  and its output vector by  $y_{ik}$ ,  $i = 1, ..., I_k$ ; k = 1, ..., K. For the determination of firm efficiency,  $\varepsilon_{ik}$ , a linear program is needed:

$$\max_{\lambda_{jk} \ge 0, \varepsilon_{ik}} 1/\varepsilon_{ik} : \sum_{j=1}^{I_k} \lambda_{jk} x_{jk} \le x_{ik}; \sum_{j=1}^{I_k} \lambda_{jk} y_{jk} \ge y_{ik}/\varepsilon_{ik}$$
(1)

where  $\varepsilon_{ik}$  are the efficiency scores ranging between 0 and 1.

These programs go on the line of Data Envelopment Analysis (DEA) models<sup>1</sup> with Constant Returns to Scale and Output orientation (DEA CRS-O). In this sense, this approach consists in the calculation of DEA CRS-O Scores for each firm, using as reference set for each firm its industry.

The equivalent dual program is:

$$\min_{p_{ik}, w_{ik} \ge 0} w_{ik} x_{ik} : p_{ik} y_{jk} \le w_{ik} x_{jk}, p_{ik} y_{ik} = 1, \ j = 1, \dots, I_k, k = 1, \dots, K$$
(2)

where  $w_{ik}$  and  $p_{ik}$  are the dual variables that solve each program and match the shadow prices of constraints of (1). Obviously, by the main theorem of linear programming, the primal and the dual programs have equal solution values:  $1/\varepsilon_{ik} = w_{ik}x_{ik}$ .

Following the approach of Färe and Grosskopf  $(2004)^2$ , ten Raa (2006) proposes the ensemble efficiency of an industry k,  $\varepsilon_k$ , as the solution of next program:

$$\max_{\lambda_{jk} \ge 0, \varepsilon_k} 1/\varepsilon_k : \sum_{j=1}^{I_k} \lambda_{jk} x_{jk} \le \sum_{i=1}^{I_k} x_{ik}, \sum_{j=1}^{I_k} \lambda_{jk} y_{jk} \ge \sum_{i=1}^{I_k} y_{ik} / \varepsilon_k$$
(3)

where  $\varepsilon_k$  is again a number between 0 and 1. The idea is to reallocate the combined inputs,  $\sum_{i=1}^{I_k} x_{ik}$  of all DMUs, as to get the maximum increase of the aggregate output

<sup>&</sup>lt;sup>1</sup> Details and complete DEA descriptions may be found in specific books such as Charnes et al. (1995) or Cooper et al (2000).

<sup>&</sup>lt;sup>2</sup> Lozano and Villa (2004) developed the same approach for input orientation and provide an interesting graphical interpretation.

vector,  $\sum_{i=1}^{I_k} y_{ik} / \varepsilon_k$ . This is obtained by inflating the aggregate output of industry k by the expansion factor  $1/\varepsilon_k$ . Economically, this model tries to maximize, not the individual performance of each firm, but the efficiency of the ensemble of firms of the same industry.

The equivalent dual program is:

$$\min_{p_k, w_k \ge 0} w_k \sum_{i=1}^{I_k} x_{ik} : p_k y_{jk} \le w_k x_{jk}, p_k \sum_{i=1}^{I_k} y_{ik} = 1, j = 1, \dots, I_K$$
(4)

where the dual variables  $w_k$  and  $p_k$  solve (4) and match the shadow prices of constrains of (3). Again, by the main theorem of linear programming, the primal and the dual program have equal solution values:  $1/\varepsilon_k = w_k \sum_{i=1}^{I_k} x_{ik}$ .

ten Raa (2006) defined industrial organization efficiency of the industry k,  $\varepsilon_k^o$  as follows:

$$\varepsilon_k^o = \varepsilon_k \cdot \sum_{i=1}^{I_k} \frac{s_{ik}}{\varepsilon_{ik}}$$
(5)

where  $\varepsilon_k$  is the ensemble efficiency determined by program (3),  $\varepsilon_{ik}$  are the efficiency scores of each firm determined by the set of programs (1) and  $s_{ik}$  are the revenue shares of each firm evaluated at the prices determined by dual program (4)<sup>3</sup>.

## 3. Industrial Specialization Efficiency

Reallocation of resources between industries may also be desirable, ten Raa and Mohnen (2002, 2006) tackle these reallocations of factors between sectors in the decomposition of Total Productivity Growth. ten Raa and Mohnen (2006) state the interest of making a further decomposition to consider firm's efficiency contribution, as we will present in the next section.

Industry efficiency must be calculated with model (3) instead using a model in line DEA-O CRS. For this step, we will work at the level of sectors (k = 1, ..., K), by pooling the vector of inputs and outputs within the firms of each industry k. The efficiency of the ensemble of industries (the whole economy),  $\varepsilon$ , calculated with industry data can be obtained by

$$\max_{\lambda_h \ge 0,\varepsilon} 1/\varepsilon : \sum_{h=1}^K \lambda_h \sum_{j=1}^{I_h} x_{jh} \le \sum_{k=1}^K \sum_{i=1}^{I_k} x_{ik}, \sum_{h=1}^K \lambda_h \sum_{j=1}^{I_h} y_{jh} \ge \sum_{k=1}^K \sum_{i=1}^{I_k} y_{ik} \bigg/ \varepsilon$$
(6)

The equivalent dual program is:

$$\min_{p,w\geq 0} w \sum_{k=1}^{K} \sum_{i=1}^{I_k} x_{ik} : p \sum_{h=1}^{K} \sum_{j=1}^{I_h} y_{jh} \le w \sum_{h=1}^{K} \sum_{j=1}^{I_h} x_{jh}, p \sum_{h=1}^{K} \sum_{j=1}^{I_h} y_{jh} = 1, h = 1, \dots K$$
(7)

where the dual variables *w* and *p* solve (7) and match the shadow prices of constrains of (6). Obviously, by the main theorem of linear programming, the primal and the dual program have, again, equal solution values:  $1/\varepsilon = w \sum_{k=1}^{K} \sum_{i=1}^{I_k} x_{ik}$ .

Following ten Raa's (2006) approach for industrial organization efficiency, industrial specialization efficiency,  $\varepsilon^s$ , is:

$$\varepsilon^{s} = \varepsilon \cdot \sum_{k=1}^{K} \frac{s_{k}}{\varepsilon_{k}}$$
(8)

where  $\varepsilon$  is the ensemble efficiency (whole economy efficiency) determined by program (6),  $\varepsilon_k$  are the efficiency scores of each industry determined by the set of programs (3) and  $s_k$  are the revenue shares of each industry evaluated at the prices determined by dual program (7).

Finally, note that if we repeat in this section the same approach than the previous section, but working with industry data, we will find two problems:

<sup>&</sup>lt;sup>3</sup> In the averaging procedure described in (5), weighed harmonic mean is used because it is the most suitable procedure for averaging productivities or performances, Casas Sánchez and Santos Peña (1996, pp. 78-81).

- a) Two measures of industry efficiency, one calculated only within industry data,  $\varepsilon_k$ , and other calculated with data of all sectors (DEA)
- b) This last measure has a problem. We are treating industries as comparable DMUs, when they produce different products.

#### 4. Economy Efficiency: A three way Efficiency Decomposition

In this section, we present a single measurement of the whole Economy Efficiency,  $\varepsilon$ , instead of the measurements of each sector/firm. For this purpose, usual DEA techniques can not be applied, since a reference set is needed, or do not even be considered due it will imply comparing different economies with a technique that assumes the comparability of the DMUs.

On the other hand, our measurement,  $\varepsilon$ , will come from the own economy data. We build the efficiency measurement from the lowest level (firm) to the highest one (the whole economy) by a nesting decomposition of different efficiency measurements to isolate the effects at each level. Substituting (5) in (8) and reordering:

$$\varepsilon = \frac{\varepsilon^{s}}{\sum_{k=1}^{K} \frac{s_{k} \cdot \sum_{i=1}^{I_{k}} s_{ik} / \varepsilon_{ik}}{\varepsilon_{k}^{o}}}$$
(11)

where  $\varepsilon^s$  is the industrial specialization efficiency calculated by (8),  $\varepsilon_k^o$  is the Organizational Efficiency of industry *k*, determined by (5),  $\varepsilon_{ik}$  are the efficiency scores of each firm determined by the set of programs (1), and  $s_{ik}$  and  $s_k$  are the revenue shares of each firm and each industry respectively, evaluated at the prices determined by dual programs (4) and (7).

#### 5. Application to the Andalusian Economy

# 6. Conclusion

# **Data Appendix**

The IEA (Instituto de Estadística de Andalucía - Regional Statistical Office of Andalusia) provided the cross-section inputs and outputs establishment data. These data were used for the elaboration of the Input-Output Andalusian Framework 2000 -MIOAN00 (IEA, 2006), which is the input-output table for Andalusia, based on the European System of Accounts (ESA-95) published by EUROSTAT (1996). IEA publishes two use tables, which differ by valuation. One is valued at purchasers' prices and the other at basic prices, which is the same as the former but excluding trade and transport margins and net commodity taxes (see Viet, 1994, p. 28). Trade and transport margins needs simply be reallocated from the commodities where they are included, at purchasers' values, to the use matrix rows of trade and transport services. The make table is published exclusively at basic prices. The United Nations System of National Accounts (SNA) recommends basic values; production costs of good and services are measured before they are conveyed to the market for consumption so that the effects of tax and subsidy policies as well as of differences in types of economic transactions are isolated. Valuations must be in basic prices. ten Raa and Rueda-Cantuche (2007a) detail the procedure, including the assumed equality of margins and net commodity taxes between establishments in a given industry, consuming a given commodity.

There is a single capital type and a single labour type. Data for each establishment is obtained from capital consumption and total equivalent employees figures in the IO dataset. The capital endowment and the total labour force are the sum across establishments of their capital consumption and total equivalent employees figures.

Sales and purchases were classified into 86 commodities. 36,108 observations were considered: 36,086 obtained by IEA from an specific survey done to build MIOAN00 while the other 22 observations represent data of 22 sectors which data are

obtained by IEA from different statistical sources when building MIOAN00, not by specifically surveying establishments: The list of this latter group of sectors is:

k	Industry
01	Growing of vegetables and horticultural specialties
02	Growing of Vineyard and Olive
03	Other agricultural products and services
04	Livestock and Hunting
05	Forestry and related service activities
06	Fishing
46	Manufacture of electricity
48	Collection, purification and distribution of water
61	Financial intermediation
62	Insurance and pension funding
64	Real estate activities
67	Research and Development Services
73	Other service to firms n.e.c.
74	Public administration and defence; compulsory social security
75	Non-market education
76	Market education
77	Non-market Health and veterinary activities
78	Market Health and veterinary activities
79	Non-market Social services
82	Activities of organizations
84	Other entertainment, cultural and sport activities n.e.c.
85	Other personal services
86	Activities of households as employers of domestic staff

We do not claim that the data were measured without error. Particularly, basic prices building follow some usual assumptions. For a sensitivity analysis we refer to ten Raa (2005).

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