Supply-Use Tables: Simultaneously Balancing at Current and Constant Prices. A new Procedure.

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Abstract

According to the 1993 SNA and 1995 ESA recommendations, the annual Supply-Use Tables need to be compiled at both current and constant prices by the National Institutes of Statistics. The most appropriate way to obtain consistent Supply-Use Tables at both current and constant prices is to balance them simultaneously. The main complexity in balancing Supply-Use Tables and any other general accounting system simultaneously at both current and constant prices is the nonlinearity that inevitably occurs. Some attempts to solve the problem which are described in the literature are mainly based on the transformation of variables and definition of suitable matrices of variances and covariances of the system. In this paper, a suitable method is proposed to balance an extremely large set of National Accounts simultaneously at current and constant prices through the definition of a new procedure based on the predictor-corrector method. The distinctive features of the proposed balancing method are its flexibility, which is very high compared to the other methods in the literature, and its capability to allow the control of the consistency of the system of deflators. An application to the Italian 2006 Supply-Use tables has been worked out, and it has yielded good outcomes.

Keywords: I-O tables, National economic and social accounts, Simultaneous balancing method at current and constant prices, Statistical modelling, Very large national accounting balancing systems.

1 Introduction

Almost all National Institutes of Statistics publish a complete set of national accounting data at current prices and only a small number of aggregates at constant prices. The reason behind this practice is the considerable complexity of obtaining balanced accounting systems simultaneously at both constant and current prices. The most relevant problem to obtain a complete set of national accounts at constant prices is the impossibility of defining a complete and appropriate set of price indices. National accounts are, in fact, normally compiled and balanced first at current prices, and then converted to constant prices using a suitable system of deflators. It is known that, however, the accounting system at constant prices calculated in this way will be unbalanced and it is, therefore, necessary to balance it. This balancing procedure, moreover, determines modifications in the value of the deflators and the latter may not reflect the true economic dynamic of the country to which the accounts refer.

Some methods to solve the problem of balancing an accounting system simultaneously at current and constant prices are presented in the literature. They are essentially based on the balancing method originally proposed by Stone, Champernowne, and Meade (1942) and use an ad hoc transformation for variables, and the definition of a suitable matrix of variances and covariances to solve the nonlinearity of the accounting balancing system at both current and constant prices (Weale, 1988).

This work delineates a new method to balance an accounting system simultaneously at current and constant prices, which differs from the others in its approach to the problem. First, it is based on the method originally developed for balancing the Italian 1992 Symmetric Input-Output Table (SIOT, hereafter) (Nicolardi, 1998; 2000) that had its foundation in Byron (1978). The method proposed by Nicolardi is more flexible than Byron's technique and allows the balancing of extremely large accounting systems. Second, the new method defined in this work utilizes the predictor-corrector method to solve the nonlinearity of the balancing system. Finally, it allows the direct control of the magnitude of the variations of the deflators, which the balancing procedure normally causes, while in the literature the problem is managed in a roundabout way.

The paper is organized as follows. In Section 2, the methodology of balancing at current prices is presented. Section 3 introduces a method of simultaneous balancing at current and constant prices. In Section 4, some issues of the definition and calculation of variances are examined. Section 5 demonstrates an application of the method of simultaneous balancing. Section 6 presents some concluding remarks.

2 Methodological Overview

Stone approached the balancing of an accounting system that includes differently reliable estimates of the aggregates as a problem of constrained estimates obtained as a weighted linear combination of the initial estimates of the same aggregates. By considering a vector $\mathbf{x}$ of s accounting data, it is possible to express a system of k accounting equations in the form $G\mathbf{x}{=}\mathbf{h}$, where G is a $(k \times s)$ matrix of constraints and $\mathbf{h}$ is a vector of known values. In particular, when the accounting constraints are not satisfied, $G\mathbf{x}{\neq}\mathbf{h}$ and the system is unbalanced. In this latter case, starting from an initial vector of unbalanced estimates of $\mathbf{x}$, say $\hat{\mathbf{x}}$, Stone suggested obtaining a vector $\tilde{\mathbf{x}}$ that satisfies the accounting constraints in G through a generalized constrained estimator defined as follows:

$$\tilde{\mathbf{x}} = \hat{\mathbf{x}} - VG'\left(GVG'\right)^{-1}\left(G\tilde{\mathbf{x}} - \mathbf{h}\right) \tag{1}$$

where V is a prior estimate of the covariance matrix of $\hat{\mathbf{x}}$ and contains the *information* on the significance level of those estimates, and permits a reasoned distribution of the accounting residuals between them.[1] Subsequently, Byron

[1]The matrix V is normally used in a diagonal form, where the null covariances of the estimates are assumed. In practical terms, the inefficiency of the estimates, which this simplification yields, is negligible in most cases. Also, the methodologies utilized in compiling the national accounts do not usually allow the estimation of the correlations between the estimates

(1978) proposed an alternative approach to estimating $\tilde{\mathbf{x}}$ that can be used to solve large econometric systems. He considered a constrained quadratic loss function defined as follows:

$$Z = \frac{1}{2}(\tilde{\mathbf{x}} - \hat{\mathbf{x}})' \, V^{-1} \, (\tilde{\mathbf{x}} - \hat{\mathbf{x}}) + \lambda \, (G\tilde{\mathbf{x}} - \mathbf{h}) \tag{2}$$

in which λ is the vector of Lagrange multipliers. In (2), $\tilde{\mathbf{x}}$ has to be as close as possible, in a quadratic loss sense, to $\hat{\mathbf{x}}$ and satisfy, at the same time, the accounting restrictions. The first order conditions on (2) are:

$$\begin{aligned} \tilde{\lambda} &= (GVG')^{-1} \, (G\hat{\mathbf{x}} - \mathbf{h}) \quad (a) \\ \tilde{\mathbf{x}} &= \hat{\mathbf{x}} - \, VG'\tilde{\lambda} \qquad\qquad\quad (b) \end{aligned} \tag{3}$$

Although (3) is equivalent to (1), it allows the estimation problem to be solved under less restrictive conditions than (1), and, at the same time, the complexity of the estimation process to be reduced so that $\tilde{\mathbf{x}}$ can be worked out through a smaller linear equation system. Furthermore, Byron used the conjugate gradient method to estimate large accounting systems by applying (3). Recalling that (3a) is a system of linear equations and the matrix (GVG') is symmetric positive definite when the accounting system is exactly defined, the same relation (3a) can be rewritten as follows:[2]

$$(GVG')\,\lambda = A\,\lambda = \mathbf{a} \tag{4}$$

The solution to (4) can be obtained by using the following iterative procedure, i.e. the conjugate gradient algorithm:

$$\begin{aligned} \pi_0 &= \rho_0 = \theta - A\,\lambda_0 \\ \alpha_i &= \rho'_i\,\rho_i/\pi'_i\,A\,\pi_i \\ \lambda_{i+1} &= \lambda_i + \alpha_i\,\pi_i \\ \rho_{i+1} &= \rho_i - \alpha_i\,A\,\pi_i \\ \beta_i &= \rho'_{i+1}\,\rho_{i+1}/\rho'_i\,\rho_i \\ \pi_{i+1} &= \rho_{i+1} + \beta_i\,\pi_i \end{aligned} \tag{5}$$

where π and ρ are the gradient-based direction vectors, λ_0 is a vector of initial values for λ, and i and i+1 refer to the iteration count. The iterative process is stopped when the condition $(\theta\text{-}A\lambda_i)<\epsilon$ is verified, with ϵ sufficiently small. Substituting λ_i for $\hat{\lambda}$ in (3b) $\tilde{\mathbf{x}}$ is estimated. The convergence process can be accelerated by a suitable normalization of (GVG') (Byron, 1978).

Also applying the balancing method proposed by Byron, the problem of calculating and managing (GVG') remains significant. Nicolardi (1998, 2000) has proposed a method of decomposing the accounting matrix into blocks to allow both a significant reduction of the quantity of data to memorize and the calculation of the product (GVG') without defining G element by element.

of different flows.

[2]In fact, the matrix (GVG') cannot be positive definite mainly in two cases: when G has not full row rank, and this means that two or more constraint conditions are linearly dependent, and when the variances and covariances of the elements in a generic equation are all equal to 0. If the latter is not verified, it can be demonstrated that the matrix (GVG') is nonsingular when $rank(V)\geq k$. In all cases, however, an appropriate reformulation of the system of equations can solve the problem.

The development of this methodology started from the observation that in (5) (GVG') appears in the second and fourth relation within the following product:

$$A\pi = W(GVG')\,\pi \tag{6}$$

where W is the matrix of normalization proposed by Byron (1978). In (6) it can be seen that determining the product starting with the last pair of elements and proceeding to the first, the subsequent results are always formed by vectors, that is:

$$\begin{array}{c} G'_{(s\times k)}\,\pi_{(k)} = \gamma_{(s)} \\ V_{(s\times s)}\,\gamma_{(s)} = \gamma'_{(s)} \\ G_{(s\times k)}\,\gamma'_{(s)} = \gamma''_{(s)} \\ W_{(k\times k)}\,\gamma''_{(s)} = \gamma'''_{(s)} \end{array} \tag{7}$$

where the subscripts are the dimensions of the different matrices and vectors. As V and W are diagonal matrices and can be stored in vectorial form, it can easily be seen how the quantity of data to be memorized to obtain the product (6) is significantly reduced.

In (7), however, the calculation of the product (G'π) remains a problem because of the very large dimensions that G can reach. To resolve this problem it was observed that the accounting matrices often have a block configuration, so that it was possible to reformulate the product (G'π) through a linear combination of suitable decomposition of π in blocks without having to determine G (Nicolardi, 1998; 2000). A simple balancing scheme as defined below in (8a), where a_{ij}, b_{ij} and c_{ij} are respectively the ijth elements of the three generic ($n\times m$) matrices of accounting data A, B and C, can easily be rewritten in a compact form (8b) as follows:

$$\begin{array}{l} \left\{ \begin{array}{l} a_{ij} = b_{ij} + c_{ij}\ \forall i,j \\ \sum_j a_{ij} = \sum_j b_{ij} + \sum_j c_{ij}\ \forall i \\ \sum_i a_{ij} = \sum_i b_{ij} + \sum_i c_{ij}\ \forall j \\ \sum_{i,j} a_{ij} = \sum_{i,j} b_{ij} + \sum_{i,j} c_{ij} \end{array} \right. \qquad (a) \\ \\ \left\{ \begin{array}{l} A - B - C = 0 \\ A\,\mathbf{i}_m - B\,\mathbf{i}_m - C\,\mathbf{i}_m = \mathbf{0} \\ \mathbf{i}'_n\,A - \mathbf{i}'_n\,B - \mathbf{i}'_n\,C = \mathbf{0} \\ \mathbf{i}'_n A\,\mathbf{i}_m - \mathbf{i}'_n\,B\mathbf{i}_m - \mathbf{i}'_n\,C\mathbf{i}\mathbf{B}_m = \mathbf{0} \end{array} \right. \qquad (b) \end{array} \tag{8}$$

where $\mathbf{i}_n$ and $\mathbf{i}_m$ are vectors of ones of dimension n and m respectively.[3] In (8b) only q matrices, vectors and scalars are formed by initial data, while the remainder come from their transformations, e.g. $A\mathbf{i}_m$. By means of the formulation (8b) it is now possible to calculate the vectors γ and $\gamma^{''}$ in (7) without explicitly using G.

In particular, in order to compute the product $G^{'}\pi$ in the first of (7), π has to be decomposed into k blocks $\Im_i$ (matrices, vectors and scalars) equal to the number of macro-equations in the scheme (8b). Each of the k blocks has the same dimensions as the data matrices, or their transformations, which enter each macro-equation. Through the application of the scheme (8b), $k = 4$

[3] In (8a) and (8b) the balancing equations 2-4 are already implicit in the first equation, but they are normally used explicitly so as to be able to express the constraint conditions on the marginal distributions of the accounting matrices.

blocks with dimensions, respectively, ($n \times m$), ($n \times 1$), ($1 \times m$) and (1×1) are obtained. The solution of the above product $G^{'}\pi$ is given by the q blocks $\Re_i$ that are calculated as a linear combination of the blocks $\Im_j$, where j corresponds to the macro-equations in which each of the initial q matrices of the scheme (8b), ordered according to the sequential position in the balancing scheme and without repetition, is present.

For instance, when the scheme (8b) is used, $q = 3$ since the only matrices of initial data are A, B and C, and the ordered vector of initial matrices is $\varsigma = (A, B, C)$. If a ($q \times k$) matrix Λ is specified, where each row i contains the identification numbers of the macro-equations where the ith matrix in ς, or its transformations, is present, the ith block $\Re_i$ can be worked out by using the first of (9) below. In that formula, Λ_i denotes the ith row of the matrix Λ and the superscript el indicates that all the sums are to be computed element by element.[4] Each block $\Im_j$ has the same algebraic sign of the ith matrix in ς in each kth equation. The second and third of (9) show the matrix Λ and the formulae to calculate each block $\Re_i$ as follows:

$$
\Re_i = \left(\sum_{j \in \Lambda_i} \Im_j \right)^{el} \quad i = 1, \ldots, q
$$

$$
\Lambda = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix} \tag{9}
$$

$$
\begin{cases} \Re_1 = +\Im_1 + \Im_2 \mathbf{i'}_m + \mathbf{i}_n \Im_3 + \Im_4 \mathbf{i}_n \mathbf{i'}_m \\ \Re_2 = -\Im_1 - \Im_2 \mathbf{i'}_m - \mathbf{i}_n \Im_3 - \Im_4 \mathbf{i}_n \mathbf{i'}_m \\ \Re_3 = -\Im_1 - \Im_2 \mathbf{i'}_m - \mathbf{i}_n \Im_3 - \Im_4 \mathbf{i}_n \mathbf{i'}_m \end{cases}
$$

The sequential vectorization of the q blocks yields γ.

To determine $\gamma^{''}$ in the third of (7), $\gamma^{'}$ have to be decomposed into q blocks $\Im_i$ with the same dimensions as the ith data matrix in ς. The solution of the product $G\gamma^{'}$ is given by the k blocks $\Re_i$ as resulting from the linear combination of the blocks $\Im_j$ with the same position as the matrices of ς_j in each macro-equation of (8b). If a ($k \times q$) matrix Λ is specified, where each row i contains the position of each jth data matrix in ς in each jth equation of (8b), the ith block $\Re_i$ can be worked out by using the first of (10) below. In that formula, Λ_i denotes the ith row of the matrix Λ, and the superscript tr indicates that the same transformation as the ijth element in the scheme (8b) is to be applied to the block $\Im_j$. Each block $\Im_j$ has to experience the same algebraic sign of the jth matrix in ς in each ith equation. The second and third of (10) show the matrix Λ and the formulae to calculate each block $\Re_i$.

[4] The following are defined: 1) the sum element by element of a matrix A and a column vector $\mathbf{b}$, the matrix $C = A + \mathbf{b}\ \mathbf{i}^{'}$; 2) the sum element by element of a matrix A and a row vector $\mathbf{b}$, the matrix $C = A + \mathbf{i}\ \mathbf{b}$; 3) the sum element by element of a matrix A and a scalar b, the matrix $C = A + b\ (\mathbf{i}\ \mathbf{i}^{'})$; 4) the sum element by element of a vector $\mathbf{a}$ and a scalar b, the vector $\mathbf{c} = \mathbf{a} + b\ \mathbf{i}$.

$$\Re_i = \left(\sum_{j \in \Lambda_i} \Im_j^{tr} \right) \quad i = 1, \ldots, k$$

$$\Lambda = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \tag{10}$$

$$\begin{cases} \Re_1 = +\Im_1 - \Im_2 - \Im_3 \\ \Re_2 = +\Im_1 \mathbf{i}_m - \Im_2 \mathbf{i}_m - \Im_3 \mathbf{i}_m \\ \Re_3 = +\mathbf{i'}_n \Im_1 - \mathbf{i'}_n \Im_2 - \mathbf{i'}_n \Im_3 \\ \Re_4 = +\mathbf{i'}_n \Im_1 \mathbf{i}_m - \mathbf{i'}_n \Im_2 \mathbf{i}_m - \mathbf{i'}_n \Im_3 \mathbf{i}_m \end{cases}$$

The sequential vectorization of the k blocks yields $\gamma^{''}$.

3 A Method for the Simultaneous Balancing of an Accounting System at Current and Constant Prices

The main problem of simultaneously balancing an accounting system at current and constant prices is related to the resolution of a nonlinear system of simultaneous equations. Let A^c indicate a generic matrix of accounting data at current prices, A^k the same matrix at constant prices, and A^i the corresponding matrix of deflators. It is now necessary to add to the above system (8b) a set of equations which permits the simultaneous balancing of all the matrices. Considering only the first row in (8b), the simultaneous system of equations is defined as follows:

$$\begin{array}{c} A^c - B^c - C^c = \mathit{0} \\ A^k - B^k - C^k = \mathit{0} \\ \left(A^c \cdot A^i\right) - A^k = \mathit{0} \\ \left(B^c \cdot B^i\right) - B^k = \mathit{0} \\ \left(C^c \cdot C^i\right) - C^k = \mathit{0} \end{array} \tag{11}$$

where $(A^c{\cdot}A^i)$, $(B^c{\cdot}B^i)$ and $(C^c{\cdot}C^i)$ are Hadamard products.

Clearly, the above system cannot be resolved with by the method described in Section 2 because it is nonlinear. Given a general accounting system like (8), its simultaneous balancing at current and constant prices can be worked out by using a procedure based on the predictor-corrector method. For the sake of brevity, only the system (11) is considered and rewritten as follows:

$$\left\{ A^c - B^c - C^c = \mathit{0} \right. \qquad (\mathit{Step}\, 1)$$

$$\begin{cases} \log\left(A^c\right) + \log\left(A^i\right) - \log\left(A^k\right) = \mathit{0} \\ \log\left(B^c\right) + \log\left(B^i\right) - \log\left(B^k\right) = \mathit{0} \\ \log\left(C^c\right) + \log\left(C^i\right) - \log\left(C^k\right) = \mathit{0} \end{cases} \qquad (\mathit{Step}\, 2) \tag{12}$$

$$\left\{ A^k - B^k - C^k = \mathit{0} \right. \qquad (\mathit{Step}\, 3)$$

where each group of equations represents a linear system. The general solution to the problem, therefore, lies in applying a suitable iterative method of estimation for *Steps 1-3* until a balanced accounting system is obtained, at both current and constant prices. At each step, the starting values are the balanced estimates worked out in the previous step. In the practical application of the iterative procedure, *Step 2* must be utilized twice: the first time to obtain the estimates of the flows at constant prices to use in *Step 3*, the second to obtain the estimates of the flows at current prices to use in *Step 1* of the next iteration.

A worthwhile aspect of this method is that, through the definition of a suitable matrix of variances for the deflators, it is able to control the variations induced in the price system by the balancing procedure. Comparing (12) and (8b), it is clearly seen how the block balancing procedure described in Section 2 of this paper can be utilized to balance the accounting system at every single step of the simultaneous procedure.

4 The Variance Structure System

In the application of the balancing procedure, a relevant aspect that needs to be faced is the *calibration* of the system of variances. It is, in fact, very easy to demonstrate that the balancing procedure converges very slowly or does not converge at all when the system of variances is not well structured. In particular, the most critical phase of the balancing procedure is the definition of the variance matrix in *Step 2* in (12).

In order to clarify the problem, it is important to recall that the matrices of variances contain values in the same unit of measure as the accounting values, and residuals are apportioned among the unbalanced data in proportion to the relative values of variances within the overall system of equations (Nicolardi, 2000). Therefore, as the values in the data matrices at current and constant prices are higher than the values in the price index matrix, at *Step 2* in (12) the values in the latter will be revaluated less than those in the data matrices at current and constant prices. An appropriate normalisation of the whole system of variances in the balancing procedure can solve the problem. In this work, a normalisation based on the ratio between the maximum log-values in each complete set of data, that is the current price and constant price data sets, and the implied deflator data set has been applied.

In particular, let M^c, M^k, M^i and M, indicate respectively the whole set of data at current prices, the whole set of data at constant prices, the whole set of implied deflators and the data set encompassing all the previous data sets. A vector of three elements, say $\mathbf{v}^r$, is thus defined as follows:

$$\mathbf{v}^r = \left(\frac{_{ml}M}{_{ml}M^c}; \frac{_{ml}M}{_{ml}M^k}; \frac{_{ml}M}{_{ml}M^i} \right) \tag{13}$$

where the subscript *ml* indicates the maximum log value of the corresponding data set. The three ratios in $\mathbf{v}^r$ can be used to normalise the variances of M^c, M^k and M^i respectively.[5]

[5]One of the ratios in $\mathbf{v}^r$ will be equal to 1. This depends on the formula that is used to calculate the data at constant prices based on the data at current prices. If scheme (11) is used, the element equal to 1 is $\mathbf{v}^r_1$.

A second aspect that needs to be considered in the balancing procedure is related to the computation of the matrices of variances themselves. In fact, at each step of any iteration of the balancing procedure the values in each data matrix change because of the balancing process itself. It is, therefore, necessary to re-estimate the matrices of variances at each step. In this work, a very flexible method to calculate variances has been adopted, and it provides a suitable solution to accelerate the re-estimation process. A matrix of weights between 0 and 1 has to be defined. The weights express the relative degree of reliability of each single item in the data matrices, where 0-weighting is given to the reliable data and 1 to the unreliable data, and intermediate values to the remaining data according to the relative reliability of the different aggregates. Variances are thus easily estimated by multiplying each element of data matrices by its own weight. The balancing process will not change the values that have been given 0-weighting, while it will vary the other elements according to the relative value of their corresponding weights, and calculate the 1-weighted values residually (Nicolardi, 2000). The weight matrices can be worked out once at the beginning of the balancing process and at each step the variances of the input data matrices are obtained multiplying the output data matrices from the previous step by the weight matrices as defined before.

5 An Application to the Italian 2006 SUT

The simultaneous balancing procedure at current and constant prices was tested on the Italian 2006 SUT (ISTAT, 2010)[6]. The Italian National Institute of Statistics (ISTAT, hereafter) currently publishes the SUT both at 30 NACE (Rev. 1.1) branches and 30 CPA products, and at 59 NACE branches and 59 CPA products. The 30 and 59 NACE branches classifications actually correspond to the A31 and A60 ESA classifications where, respectively, group Q, i.e. *Extra-territorial organizations and bodies*, and group 99, i.e. *Services provided by extra-territorial organizations and bodies*, are not considered by ISTAT. The 30 and 59 CPA products classifications correspond to the P31 and P60 ESA classifications where, respectively, group Q, i.e. *Services provided by extra-territorial organizations and bodies*, and group 99, i.e. *Services provided by extra-territorial organizations and bodies*, are not considered.

In this work, the Supply Table (ST, hereafter) at basic prices, including its transformation into purchasers' prices, and the Use Table (UT, hereafter) at purchasers' prices, both at 30 NACE 1.1 branches by 30 CPA products, were used. Obviously, the ST and UT published by ISTAT are already balanced and, therefore, the first step was to obtain a non-balanced SUT at current prices by modifying the data by means of a random algorithm. The modifications obtained were held within a maximum limit of 5%. A 2006 SUT at constant prices with 2000 as base year was then worked out. The data at current prices were, therefore, turned into constant prices by means of the chain-linked implied deflators calculated on the SUT data time series at both current and previous-base year prices. It is, however, quite problematic to obtain a complete and coherent set of deflators from the series of the chain-linked implied deflators, and this is mainly true for the intermediate sections of both tables. In fact, it is possible that some cells of the intermediate sections of the annual SUT contain

[6] All the data used in this work are from the online ISTAT database.

values for some years, while for the remainder the corresponding cells are empty. In order to solve this last problem, a time-series linear interpolation or the price index of similar products or the average index of the same CPA Groups or NACE branches was used. Attention was also focused on the estimation at constant prices of the SUT flows that do not have price and quantity dimensions of their own, such as, for example, *Trade and transport margins* and *Taxes less subsidies on products*.

Table 1 shows the unbalanced 2006 ST at current and constant prices in an aggregated form, according to the branches from 1 to 6 of the A6 NACE 1.1 classification, and the groups from 1 to 6 of the P6 CPA classification. The contents of the table are organized as follows: Output by products and by branches at basic prices in columns 2-8, Imports by products (I) in column 9, Total supply by products at basic prices (TSBP) in column 10, Trade and transport margins (TTM), and Taxes less subsidies on products (NT) respectively in columns 11-12, Total supplies at purchasers' prices (TSPP) in column 13.

The unbalanced 2006 UT at current and constant prices is shown in Table 2, where the Intermediate consumption at purchasers' prices by product and by branch is reported in columns 2-8, and all the final uses are in columns 9-14. More specifically, column 9 demonstrates the Household final consumption expenditure at purchasers' prices (HFC), column 10 shows the Non-Profit Institutions Serving Households final consumption expenditure at purchasers' prices (NPISHFC), column 11 contains the Government final consumption expenditure at purchasers' prices (GFC), column 12 demonstrates the Gross capital formation at purchasers' prices (GCF), and column 13 shows Exports (E). The Total Uses at purchasers' prices (TU) are reported in column 14. Table 2 reports also the Value added by branch at basic prices (rows 11 and 21) and the Production by branch at basic prices (rows 12 and 22).[7]

The balancing procedure for the SUT at current and constant prices was divided in 3 steps, as in (12), in each of which the block balancing method was used. The system of balancing equations was defined in a very analytical way, using a scheme similar to that described by Nicolardi (2000). For *Steps 1* and *3*, 2194 equations were utilized with a vector of 2480 elements, while for *Step 2* the number of equations was 2543 with a vector of 7440 elements. In the same iteration, as stop criterion of the iterative procedure for the accounting system at both current and constant prices the condition $max(|\varepsilon|)<0.1$ was placed for *Step 1* and *Step 3*, while for *Step 2* the stop criterion was $max(|\varepsilon|)<0.001$.

In accordance with the method depicted in Section 4, weights between 0 and 1 were attributed to the initial estimates by considering the different level of reliability of each item. Specifically, the weights assigned to the estimates, which are all the items at current prices and some deflators, and are obtained directly from the national accounts, were lower than the weights assigned to the estimates worked out by means of indirect methods that are the estimates at constant prices and the estimates of the remaining implied deflators. Therefore, the variances were obtained through the product of each estimate for the respective weight. The vector of normalisation $\mathbf{v}^r$ was subsequently calculated as depicted in Section 4.

[7] In Appendix, Table 6 contains the A31 and P31 1995 ESA classification codes (column 1), which are used in the SUT published by ISTAT, and the corresponding description (column 2), while the A6 and P6 1995 ESA classification codes, which are used in Tables 1-4, and the corresponding descriptions are shown in Table 7 (respectively, columns 1 and 2).

Table 1: 2006 Supply Table. Unbalanced flows at basic prices, including a transformation into purchasers prices (€ millions).

Prod.	Branch (NACE)							I	TSBP	TTM	NT	TSPP
(CPA)	1	2	3	4	5	6	Total					
					Current prices							
1	45889.1	-	-	1028	-	4.5	46072.6	10620.1	58055.5	30554.5	-268	90699.6
2	653.5	977398.8	-	36922	4254.2	2237.1	1023542.9	346672.6	1368630.9	288540.1	97883.7	1770209.5
3	-	1957.3	190198.2	578.9	666.4	217	188503.9	482.4	188742.3	-	13737.7	207878.3
4	594.8	25527.8	583.5	656953.5	5259.1	1494.3	681702.2	25993.6	699483.9	-315930.6	10137.2	388971.3
5	2.8	16586.5	7877	22801.5	532280.9	784.1	577997.1	28319.1	620628.8	3793.2	25234.9	647810.1
6	-	601.5	783.1	2940.1	390.3	389600.4	410983.4	1934.4	405872.1	3995.8	11610.3	415930.2
Total	48370.9	1032506.3	194029.2	715916.8	531791.7	407839.2	2991766.7	418994.9	3193780	-	155369	3496735.9
					2000 base-year prices							
1	44268.5	-	-	1294.8	-	5.8	44737.4	10184.5	56260.1	24778.7	-376.7	80886.9
2	686.8	844535.1	-	33284.4	4059.7	2041.2	887244.4	289624.2	1175271.1	256307.5	84149.1	1528177.4
3	-	1598.2	155319.7	472.7	544.3	177.2	153936	443.3	154174.1	-	11135.5	169689
4	517.7	22720.6	526.2	579033.5	4942.2	1276.8	600573.6	24030.7	617302.9	-276623	7672.6	342991.9
5	2.7	14143.7	6566.6	20341.5	438706.6	808.4	478304.7	25050.6	514942.2	4100	22221.7	539959.2
6	-	500.1	655.1	2385.6	333.5	322256	339843.9	1704.5	335647.5	2635.2	10652.4	344411.4
Total	46687.2	891477	158632.4	632292.5	439454.1	337561.2	2559784.9	355659.5	2728669.8	-	129851.2	2984778

Table 2: 2006 Use Table. Unbalanced flows at purchasers' prices (€ millions).

Prod.	Branch (NACE)							HFC	NPISHFC	GFC	GCF	E	TU
(CPA)	1	2	3	4	5	6	Total						
							Current prices						
1	6386.1	32582.5	64.9	7706.2	498.4	609.5	50541.9	35001.2	26	299.8	768.8	4448.8	87015.3
2	10509	570305.3	62713.6	147285.6	28295	40161	859466.6	414475.8	6.2	12215.3	144452.7	330408.3	1765494.9
3	221.7	7685.8	17408.2	11034.5	6808.3	4309.1	45611.4	7049.8	-	558.2	151440.5	415.2	204910.6
4	676.6	47711.1	8771.6	104349.7	23243.3	11429.4	193167	167575.9	17.2	3896.9	2966.8	24077.9	401749.4
5	1366.8	82856.2	20519.8	134331.6	121524.2	41262.1	395138.9	173247.2	81.4	7795.3	27411.7	24441.8	648171.2
6	202	9912.1	1850.2	10235.6	4243.7	26946.3	53010.8	84383.2	5571.4	277148.7	1432.1	1385.8	425555
Total	19605.4	758497.9	110551.9	405263.6	182909.2	123148.2	1559439.1	916521	5398.4	303665.1	314298.8	368969.2	3627959.8
VA	27114.5	282303.5	76497.1	296670.8	359518.2	281022.1	1349806.7						
Pr	46000	1030184.9	192347.7	725559.8	539558.7	399809.7	3027661.1						
							2000 base-year prices						
1	5883	31229.7	58.9	6681.1	446	529.9	47430.4	29126.9	23.6	241.3	685.6	3573	77560.8
2	8937.4	476993.1	52956.8	123438	23080	32602.7	717246.3	374650.5	7.3	14551.4	130692.7	286675.5	1524772.3
3	176.4	6452.5	13976.9	9834.3	4959.3	3470.2	37265.5	5888.9	-	444.4	123427	338.9	167266.5
4	523.3	42387.2	7948.3	95242.3	21441.3	11269.4	176019.4	142098.9	12	3219	2776.4	21066.9	354711.5
5	1375.5	74950.2	18096.6	116630.1	100884.4	34364.9	340550.1	132753	70.8	6546.6	23251.2	20681.1	540229.4
6	168.7	8143	1508.7	8554	3186.9	23865.5	44990.7	71372	4795.6	227015.1	1095.6	915.7	352374.4
Total	17399.7	647108.9	93941.4	352207.5	152578.2	103522.8	1332322.3	782993.1	4628.8	253138.3	270337.7	317339.3	3096789.5
VA	27593.8	253827.5	59057.3	267224	295418.8	230989.3	1157134.7						
Pr	44392.6	889993.3	157257.6	639929.6	446169.6	331249.3	2590497						

It is important to highlight that a further problem faced in *Step 2* was related to the possibility of having negative values in the SUT (e.g. Transport and trade margins and Taxes less subsidies in the ST, and Gross capital formation in the UT) and the consequential impossibility of calculating the value of the corresponding logarithms. In this work, the criterion adopted was to consider the absolute value of those negative values and then invert the sign of the operation in the corresponding balancing equation.

Table 3 shows the balanced 2006 ST at current and constant prices, while Table 4 demonstrates the balanced 2006 UT at current and constant prices.

The outcomes of the balancing procedure in terms of cumulative distribution of the relative percentage changes, in absolute value, between the amount of the items before and after the balancing process are shown in Table 5. In particular, columns 2-3 demonstrate the relative changes between the values, respectively, of the ST and UT at current prices, columns 4-5 show the relative changes between the values, respectively, of the ST and UT at constant prices, and the relative changes between the implied deflators of, respectively, the ST and UT are reported in columns 6-7.

The analysis of Table 5 demonstrates that the outcomes of the balancing process are reasonably satisfactory. In fact, 74.8% of the ST data at current prices and 76.3% of the UT data at current prices (columns 2-3) were re-valuated, in absolute value, by a maximum of 5%, while only, respectively, 8.1% and 17.6% of the values were re-valuated by an amount more than 10%. Moreover, by analyzing in detail the variations yielded by the balancing procedure, it can be noted that the greatest relative variations involve the flows with negligible values, mainly in the matrix of intermediate consumption of the UT. The significance of these results can be evaluated recalling that, as seen above, the unbalanced SUT at current prices implemented in this work was obtained by means of a random algorithm that changed the data of the original balanced SUT by a maximum of 5%. In this sense, it is possible to state that the balancing process worked out a SUT at current prices whose values are comparable to those published by ISTAT.

6 Conclusions

In this paper, a suitable procedure to balance the SUT simultaneously at current and constant prices was introduced. The balancing method was tested on the Italian 2006 ST at basic prices, including its transformation to purchasers' prices, and the Italian 2006 UT at purchasers' prices. Comparison of the balanced and unbalanced 30 NACE branches by 30 CPA products SUT demonstrates that the simultaneous balancing procedure yielded reasonable changes in the initial estimates from the economic and accounting point of view. In fact, the balancing system re-valued the flows at current prices of the UT by an average of 2.1%, while the flows of the ST were re-valued by an average of 3.7%. The balancing process also yielded the largest modifications to the flows at constant prices, as was to be expected, but it can be noted that the largest relative variations involved the flows with negligible values. The changes in the values of the implied deflators were, on average, less then 1% in absolute value. Furthermore, the convergence of the balancing procedure was reasonably rapid: 171 iterations were necessary to obtain the balancing of the accounting system.

Table 3: 2006 Supply Table. Balanced flows at basic prices, including a transformation into purchasers prices (€ millions).

Prod.	Branch (NACE)							I	TSBP	TTM	NT	TSPP
(CPA)	1	2	3	4	5	6	Total					
						Current prices						
1	45977.1	-	-	1077.1	-	4.7	47059	10801.6	57860.6	31166.2	-256.8	88770
2	654.3	988689.6	-	38124.9	4082.1	2332.9	1033883.5	342232.3	1376115.9	284905.9	97611.2	1758633
3	-	2137.3	185409.6	617.1	682.2	240	189086.1	493.7	189579.9	-	13847.1	203427
4	658.1	24396.3	498.9	649172.6	4775.1	1560.9	681062.1	25562.3	706624.3	-323647.7	10030.1	393006.7
5	2.8	17661	7575.5	24387.6	532960.1	824	583410.9	28590.1	612001	3735.5	25068.3	640804.9
6	-	586.6	691.2	2952.6	403.5	402337.3	406971	1861.9	408832.9	3840	11375.2	424048.2
	47292.4	1033470.8	194175.1	716332	542902.9	407299.5	2941472.5	409541.9	3351014.5	-	157675	3508689.5
						2000 base-year prices						
1	44327.7	-	-	1325.5	-	5.8	45658.9	10269	55928	23971.2	-378.8	79520.4
2	689.8	854885.7	-	34447.3	3907.3	2132	896061.9	286520.6	1182582.6	253653	82645	1518880.5
3	-	1750.1	151499.2	508.8	560.8	197.4	154516.2	454	154970.2	-	11015.5	165985.7
4	579.3	21685.7	450	571492.2	4492.5	1328.7	600028.1	23519.7	623547.7	-284202.6	7409.7	346754.8
5	2.8	15021.8	6290.2	21775.9	439214	844.6	483149.2	25237.4	508386.6	4036	21535.9	533958.4
6	-	489	578.9	2389.2	346.6	332713.3	336516.9	1639.7	338156.5	2542.4	10304	351002.9
Total	45599.5	893832.2	158818.3	631938.6	448521.1	337221.7	2515931.3	347640.2	2863571.5	-	132531.1	2996102.6

Table 4: 2006 Use Table. Balanced flows at purchasers' prices (€ millions).

Prod.	Branch (NACE)							HFC	NPISHFC	GFC	GCF	E	TU
(CPA)	1	2	3	4	5	6	Total						
Current prices													
1	6558.4	33028.4	67.8	7739.3	495.6	608.2	48497.5	35038.6	24.5	296.3	720.1	4192.9	88770
2	10580.6	568458.9	63981.9	146576.9	27646.4	39531.1	856775.7	426472.2	6.1	12625.5	139303.2	323450.4	1758633
3	213.3	7372.4	17208.7	10654.2	6407.2	4072.9	45928.7	7502.1	-	588.8	148993.9	413.4	203427
4	665.6	47064.6	8936.7	102682.4	22705.7	11091.3	193145.7	169527.8	16.3	3993.7	2864.7	23458.4	393006.7
5	1359.5	83062.3	21146.3	133135.1	120295.4	41006.8	400005.4	181854.8	81.3	8135.3	26729.4	23998.7	640804.9
6	201.8	9926.6	1902.9	10108.6	4168.7	26268.4	52576.4	86330.3	5435	277019.1	1356.8	1330.4	424048.2
Total	19578.9	748912.9	113244.3	410896.1	181719.2	122578.2	1596929.4	906725.9	5563.1	302658.8	319968	376844.3	3508689.5
VA	27713.4	284558	80930.7	305435.8	361183.8	284721.5	1344543.1						
Pr	47292.4	1033470.8	194175.1	716332	542902.9	407299.5	2941472.5						
2000 base-year prices													
1	6122	31817.8	62.9	6797.8	451.8	532.5	45784.6	29370.6	22.6	242.2	655.7	3444.7	79520.4
2	9020.6	474898.4	54030.8	122348.5	22403.2	31511.7	714212.8	384130.7	6.9	14963.1	126143.1	279423.9	1518880.5
3	171.4	6177.2	13801.5	9510.1	4659.5	3236.8	37556.6	6247	-	466.5	121381.9	333.7	165985.7
4	526.9	41945.2	8117.8	93880.6	20986.2	10918.3	176375.1	143823.2	11.3	3307	2686.1	20552.3	346754.8
5	1374.1	75166.9	18676.3	115822.2	99965.4	34033	345037.7	139052	70.5	6824.3	22745.4	20228.5	533958.4
6	170.8	8145.3	1551.7	8453.6	3128	23239.5	44688.7	72915	4667.3	226815.7	1043.8	872.6	351002.9
Total	17385.7	638150.6	96241.1	356812.4	151594.1	103471.7	1363655.4	775538.2	4778.6	252618.7	274656	324855.8	2996102.6
VA	28213.8	255681.7	62577.2	275126.2	296927.1	233750.1	1152276						
Pr	45599.5	893832.2	158818.3	631938.6	448521.1	337221.7	2515931.3						

Table 5: Relative percentage changes, in absolute value, between the values before and after the balancing process. Cumulative distribution.

Rel. ch.	Current prices		Constant prices		Implied deflators	
(%)	Supply Table	Use Table	Supply Table	Use Table	Supply Table	Use Table
0 - 1	22.7	25.3	18.6	23.1	83	63.7
1 ⊢ 5	74.8	76.3	74.8	73.6	99.8	80.6
5 ⊢ 10	91.9	82.4	92.8	81.5	100	82.1
> 10	100	100	100	100		100
n. of iter.			171			

The obtained outcomes suggest that the procedure proposed in this paper is suitable for solving the still relevant problem of simultaneous balancing at current and constant prices of national accounting systems, and, in particular, of SUT.

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Appendix: Classifications

Table 6: A31 NACE 1.1 Branches and P31 CPA Groups classifications.

Code	Description
	A31 Classification
A	Agriculture, hunting and forestry
B	Fishing
CA	Mining and quarrying of energy producing materials
CB	Mining and quarrying except energy producing materials
DA	Manufacture of food products; beverages and tobacco
DB	Manufacture of textiles and textile products
DC	Manufacture of leather and leather products
DD	Manufacture of wood and wood products
DE	Manufacture of pulp, paper and paper products; publishing and printing
DF	Manufacture of coke, refined petroleum products and nuclear fuel
DG	Manufacture of chemicals, chemical products and man-made fibres
DH	Manufacture of rubber and plastic products
DI	Manufacture of other non-metallic mineral products
DJ	Manufacture of basic metals and fabricated metal products
DK	Manufacture of machinery and equipment n.e.c.
DL	Manufacture of electrical and optical equipment
DM	Manufacture of transport equipment
DN	Manufacturing n.e.c.
E	Electricity, gas and water supply
F	Construction
G	Wholesale and retail trade; repair of motor vehicles, motorcycles and personal and household goods
H	Hotels and restaurants
I	Transport, storage and communication
J	Financial intermediation
K	Real estate, renting and business activities
L	Public administration and defence; compulsory social security
M	Education
N	Health and social work
O	Other community, social and personal service activities
P	Private households with employed persons
Q	Extra-territorial organizations and bodies

Continue on the next page

Table 6: A31 NACE 1.1 Branches and P31 CPA Groups classifications.

Continue from the previous page

Code	Description
	P31 Classification
A	Products of agriculture, hunting and forestry
B	Fish
CA	Coal and lignite; peat; crude petroleum and natural gas; uranium and thorium
CB	Metal ores and other mining and quarrying products
DA	Food products, beverages and tobacco
DB	Textiles and textile products
DC	Leather and leather products
DD	Wood and wood products
DE	Pulp, paper and paper products; recorded media; printing services
DF	Coke, refined petroleum products and nuclear fuel
DG	Chemicals, chemical products and man-made fibres
DH	Rubber and plastic products
DI	Other non metallic mineral products
DJ	Basic metals and fabricated metal products
DK	Machinery and equipment n.e.c.
DL	Electrical and optical equipment
DM	Transport equipment
DN	Other manufactured goods n.e.c.
E	Electrical energy, gas, steam and hot water
F	Construction work
G	Wholesale and retail trade services; repair services of motor vehicles, motorcycles and personal and household goods
H	Hotel and restaurant services
I	Transport, storage and communication services
J	Financial intermediation services
K	Real estate, renting and business services
L	Public administration and defence services, compulsory social security services
M	Education services
N	Health and social services
O	Other community, social and personal services
P	Private households with employed persons
Q	Services provided by extra-territorial organizations and bodies

Table 7: A6 NACE 1.1 Branches and P6 CPA Groups classification.

Code	Description
	A6 Classification
1	Agriculture, hunting and forestry; fishing and operation of fish hatcheries and fish farms
2	Industry, including energy
3	Construction
4	Wholesale and retail trade, repair of motor vehicles and household goods, hotels and restaurants; transport and communications
5	Financial, real-estate, renting and business activities
6	Other service activities
	P6 Classification
1	Products of agriculture, forestry, fisheries and aquaculture
2	Products from mining and quarrying, manufactured products and energy products
3	Construction work
4	Wholesale and retail trade, repair services, hotel and restaurant services, transport and communication services
5	Financial intermediation services, real estate, renting and business services
6	Other services