

*Convenient policy for Health care expenditure
in a multisectoral extended model*

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As well as a policy variable that has the potential to affect economic development, a reform of health care expenditure involves the change of GDP because of its role played inside the processes of generation and distribution of income. In this paper an effort is made to verify, through the Macro Multipliers approach, the possibility to design a convenient policy for health care expenses. Such a policy permits to rule the incidence of health's expenses with respect to total output and without neglecting the effects that it originates on the main macroeconomic variables like as GDP. The empirical analysis is built on an SAM framework developed for the United States economic system. The convenient policy differs from selective policy for health sector. The first one implies a complex redistribution of the resources in order to achieve the best result in terms of reduction of the ratio between health expenditure and GDP but without depressing total industrial output and income generation.

Keywords: Health care expenditure, Macro Multipliers, Multisectoral extended model.

JEL classification: C67, D31, D57, R15.

1 Introduction

Debate on U.S. health policy focuses on limiting the growth of health spending that is now around 18 percent of GDP and its public share accounts for almost half of the total¹. In this debate many of the important questions related to rising health expenses involve the institutional arrangements that preside over its financing and often empirical researches give advice on how to depress public demand for healthcare. Nevertheless the attempt to develop an understanding on better public policies that have to bear able either to ensure society's efficient consumption of health and prevent increasing expenditures should take into account the driving force of health care services in determining the total output of U.S. economy.

Rising health care spending is a topic of absolutely general concern also emerged after the passage of the "Affordable Care Act", but unlike the past increasingly literature focus on the

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¹From 5.2 percent in 1960 to 16.2 percent in 2008 and the expectation that the share will increase to more than 19 percent by 2019 (BEA 2009).

positive relation between health demand, income growth and better health interventions (Hall and Jones, 2007). This viewpoint consider the Health sector as a leading activity whose expenditure has the potential to pull forward a wide array of other industries including the traditional sector of manufacturing, education, financial services, communications, and construction². These typologies of analysis suggest a fundamental repositioning of the debate about health care from how governments can limit spending to how to obtain economic positive direct and indirect effects from health spending that is undertaken.

The U.S. Health system has more than a few peculiarities. They are directly consequences of a series of incremental reforms not always inspired by public authorities. These modifications impacted from time to time on partial aspects of the Health care system improving contradictions. The U.S. Health care system is considered a very expensive system showing the higher fraction of GDP devoted to health care services. More than a few international report assign to U.S. Health system the worst performance in health between industrialized countries but nevertheless it is undoubtedly the most advanced system in the field of biomedical research and innovation. It is probably because of its paradoxes that is often considered a "non-system", especially for the lack of central national strategy concerning the approach to financing.

But the size and the scope of the recently passed "Patient Protection and Affordable Care Act" demonstrate the economic importance of Health care spending in U.S. economy and perhaps a new thought on this direction. There is no doubt that the Health expenditures are able to stimulate, as for the public and the private share, an articulated production process where various types of market and nonmarket inputs are consumed. While many questions were raised by Economists around the issue of rising costs of U.S. health care system and its financing method, none study conducted a preliminary analysis about the role of health care in the income generation.

This paper offers a first look at this large question by focusing and further developing the issue of economic relevance related to U.S. Health product inside the processes of generation, distribution and redistribution of national income. The effort is made in order to evaluate the potential economic impact of health among all component of total output in a multisectoral framework (Ciaschini et al., 2010). The new perspective of researches on health care services that this paper aims to enhance should be founded on the quantification of the relative force of the health sector in driving total output. Rather than being considered erroneously an indefinitely "squeezable" sector of public expenditure the economic analysis can gains insights into the economic consequences of health sector and its role into public policy design³.

We develops the Macro Multipliers analysis in order to study the macroeconomic relevance of Health product either as a policy variable and a policy target (Ciaschini et al. 2011). This type of methodology allows to verify the prominence of health output inside the key policy structures that are underlie by the extended circular flow of income that inspire the SAM scheme (Ciaschini and Socci 2007). All key policy structures, as for policy variable and for policy target, are revealed by the decomposition of the reduced form of the multisectoral extended model that is implemented on the U.S. Social Accounting matrix (SAM) which describes all phases of income generation. Of

²"Just as electricity and manufacturing were the industries that stimulated the growth of the rest of the economy at the beginning of the 20th century, healthcare is the growth industry of the 21st century." (Fogel, 2008).

³The Multisectoral analysis is an appropriate tool in order to analyse the articulated description of the income/value added generation process which, starting from the final demand of goods and services, takes account of the contribution of each producing sector to the income formation (Leontief 1965).

particular importance is the analysis develop on the reduced form of the model that is suitable for policy simulation with the innovative approach of Macro Multiplier (MM) (Ciaschini et al. 2009).

In particular the analysis of MM focuses on the inner composition, or key-structures, determined by the structural matrix, of the policy control (exogenous final demand) and of the policy objective (total output), as the compositions that rule the magnitude of the policy effect. The relevance of health within these key structures reveals the weight of health product as value-added generator and as total-output stimulator with respect to all other outputs. Income generation, technologic and labour-force stimulus are, in fact, the threefold features of the design a policy of reform⁴. Once determined all key structures are scheduled according the value of the key index for the health product, the health-key-index. In this respect we are able to compare the relevance of the U.S. health product both in terms of policy variable and policy object.

For this purpose the second section of this paper describes some characteristics of the institutional framework of U.S. Health care system. The third section shows the multisectoral extended model and the SAM framework. The fourth section shows the results of the traditional dispersion analysis performed on the structural matrix. The fifth section describes the Macro Multipliers approach and the health-key-indices of key structures that the decomposition reveals. The last section presents the major results stressing the role played by the U.S. Health product as an economic policy variable.

2 Multisectoral extended model and SAM

2.1 *Multisectoral extended model*

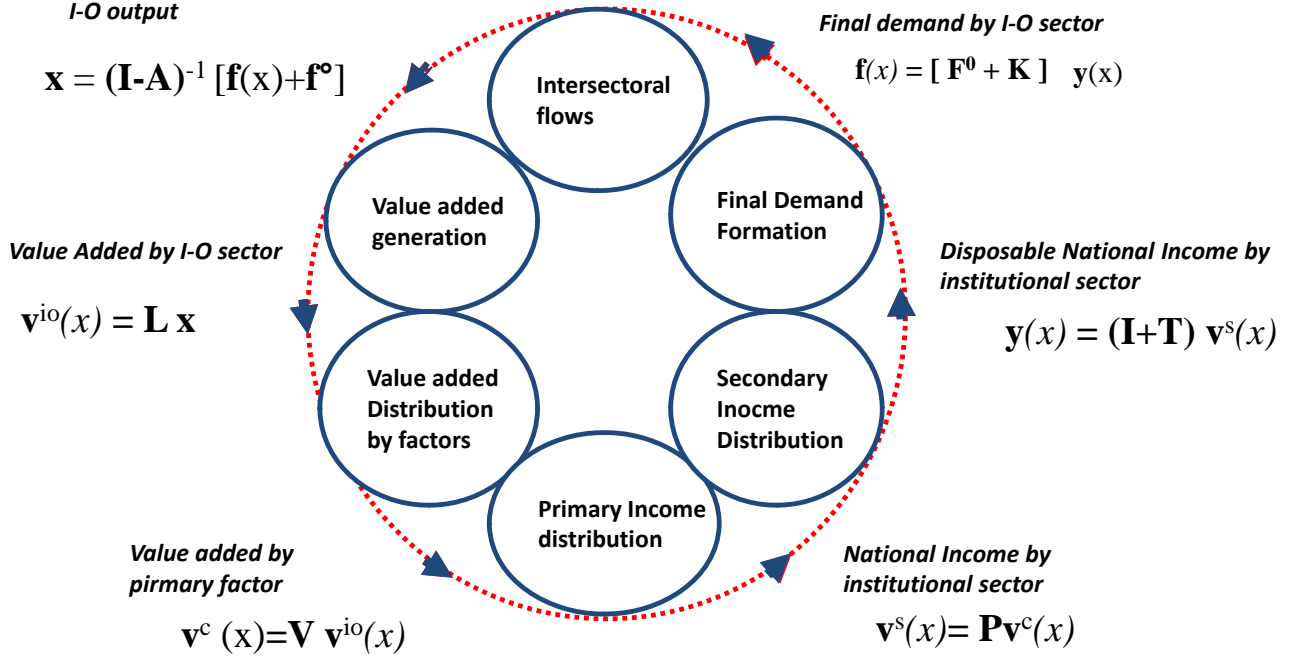
In our analysis we will use a multi-industry, multi-factor and multi-sector model following a Miyazawa approach (Miyazawa 1976, Ciaschini 1988), but in our case using extended income circular flow based on SAM scheme (Ciaschini and Socci 2007, 2003). For the extended income-output model we adopt fixed prices and constant all coefficients and shares.

In fact, the results attained in social accounting encourage the attempt to build an extended version of the income circular flow where the interactions between industries and institutions could be specified and evaluated.

Figure 1 shows a diagram where the fundamental mechanism of production and distribution is shown in terms of interaction between industries, institutional sectors and primary factors (value added components). In figure each arrow identifies an expenditure flow while each box a matrix transformation of a flow variable into another. In the upper part the inter industry demand loop in figure can be recognized. The extended output-income loop emerging in figure allows for an extension of the study of the propagation. We can choose, in fact, on which flow variable to act with a unit shock and on which variable to observe the effects. For each flow variable we need to specify an order of magnitude, such as the scale and a composition, or the structure. If we want to impose a unit shock on final demand and observe its propagation on domestic output we need

⁴Although the traditional tools of multisectoral analysis can reveal the relevance of a commodity in the production process it cannot face the problem concerning the composition of the shock to be conveyed as policy control on final demand (Skolka 1986).

Figure 1. Extended income-output model



to refer to equation 14, but other arrangements of structural matrices are easily found if we need to impose a shock on, say, income redistribution and observe it on value added by factor.

As shown in figure 1, with the dotted arrow, the income distribution process creates a feedback loop between industry output and final demand. This loop is built through various logical phases (UNSO 1994). The production process, that takes place at industry level, generates total output, \mathbf{x} , and gross value added by the m I-O industries, $\mathbf{v}(x)$, (Gross value added generation). Value added by I-O industry is then allocated to the c value added components (factors), $\mathbf{v}^c(x)$ (Gross value added allocation). Value added by components is then allocated to the s institutional sub-sectors, $\mathbf{v}^s(x)$ (Primary distribution of income). Value added by institutional sectors is then redistributed among them through taxation to generate disposable incomes by the s institutional sub-sectors, $\mathbf{y}(x)$ (Secondary distribution of income). Finally disposable income will generate final demand by institutional sub-sectors which will be transformed into final demand by I-O industries, $\mathbf{f}(x)$ (Final demand formation).

The extended I-O model starts from following fundamental equation

$$\mathbf{x} + \mathbf{m} = \mathbf{B} \cdot \mathbf{i} + \mathbf{f} \quad (1)$$

where \mathbf{x} is the output vector by industry, \mathbf{m} is imports vector, matrix \mathbf{B} is intermediates consumption and \mathbf{f} finally is final demand vector. Our extended I-O model have a great part of final demand endogenous. For this reason we will determinate the our distributive structural matrices and this is topic for endogenous final demand analysis.

Gross value added generation(by I-O sectors)

$$\mathbf{v}(x) = \mathbf{L} \cdot \mathbf{x} \quad (2)$$

where $\mathbf{L}[m,m]$ gives the shares value added by industry starting from the output vector and technical coefficients matrix.

Gross value added allocation(by VA components)

$$\mathbf{v}^c(x) = \mathbf{V} \cdot \mathbf{v}(x) \quad (3)$$

where $\mathbf{V}[c,m]$ represents the distribution of value added to the factors (components).

Primary distribution of income(by Institutional sectors)

$$\mathbf{v}^s(x) = \mathbf{P} \cdot \mathbf{v}^c(x) \quad (4)$$

where $\mathbf{P}[s,c]$ represents the distribution factors' value added income to the sectors.

Secondary distribution of income(by Institutional sectors)

$$\mathbf{y}(x) = (\mathbf{I} + \mathbf{T}) \cdot \mathbf{v}^s(x) \quad (5)$$

where $\mathbf{T}[s,s]$ represents net income transfers among sectors.

Final demand formation(by I-O sectors)

$$\mathbf{f}(x) = \mathbf{F}^0 \cdot \mathbf{y}(x) + \mathbf{K} \cdot \mathbf{y}(x) + \mathbf{f}^0 \quad (6)$$

where \mathbf{F}^0 provide the consumption demand structure by industry and is given by the product of two matrices, $\mathbf{F}^0 = \mathbf{F}^1 \cdot \mathbf{C}$, where $\mathbf{F}^1 [m,s]$ transforms the consumption expenditure by institutional sector into consumption by industry and $\mathbf{C}[s,s]$ represents the consumption propensities by institutional sector.

The matrix \mathbf{K} represents the investment demand shares and is given by $\mathbf{K} = \mathbf{K}^1 \cdot s \cdot (\mathbf{I} - \mathbf{C})$ where $\mathbf{K}^1[m,s]$ represents the investment demands to I-O industry and scalar s represents the share of private savings which is transformed into investment i.e. "active savings"; \mathbf{f}^0 is a vector of m elements which represents exogenous demand (exports).

If we put $\mathbf{F} = [\mathbf{F}^0 + \mathbf{K}]$ equation 6 becomes

$$\mathbf{f}(x) = \mathbf{F} \cdot \mathbf{y}(x) + \mathbf{f}^0 \quad (7)$$

substituting through the equations 1-6 in 7 we get

$$\mathbf{f}(x) = \mathbf{F} \cdot [\mathbf{I} + \mathbf{T}] \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L} \cdot \mathbf{x} + \mathbf{f}^0 \quad (8)$$

We now turn to the output generation process shown in equation 1. *Output generation*

$$\mathbf{x} + \mathbf{m} = \mathbf{A} \cdot \mathbf{x} + \mathbf{f}(x) \quad (9)$$

where \mathbf{m} represents imports, \mathbf{A} the technical coefficients matrix, $\mathbf{f}(x)$ represents the demand vector.

Substituting the equations 8 in 9 we finally get:

$$\mathbf{x} = [\mathbf{I} - \mathbf{A} - (\mathbf{F}) \cdot (\mathbf{I} + \mathbf{T}) \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L}]^{-1} \cdot (\mathbf{f}^0 - \mathbf{m}) \quad (10)$$

2.2 USA Social Accounting Matrix

The basic organization of the data base that has been built, is inspired by the SAM scheme and follows the matrix presentation of national T-Accounts (Socci 2004). The income circular flow is quantified and connects data on the production process (final demand, total output and value added generation) gathered by activities which play the role of industries, with data on the distribution process (factor allocation of value added, primary and secondary distribution of incomes) collected by institutional sectors.

The Social Accounting Matrix for the United States, year 2009 and market price⁵, is obtained through link between the IO table and the national accounts by institutional sectors (BEA 2009).

The matrix can be broken up into quadrants which can be further divided into blocks. A brief sketch of blocks in each of the six sub matrices, as shown in table 4, can be easily described as follows:

- quadrant I - production and final demand formation;
- quadrant II - primary allocation;
- quadrant III - secondary distribution and capital formation blocks;
- quadrant IV - operations with the rest of the world block.

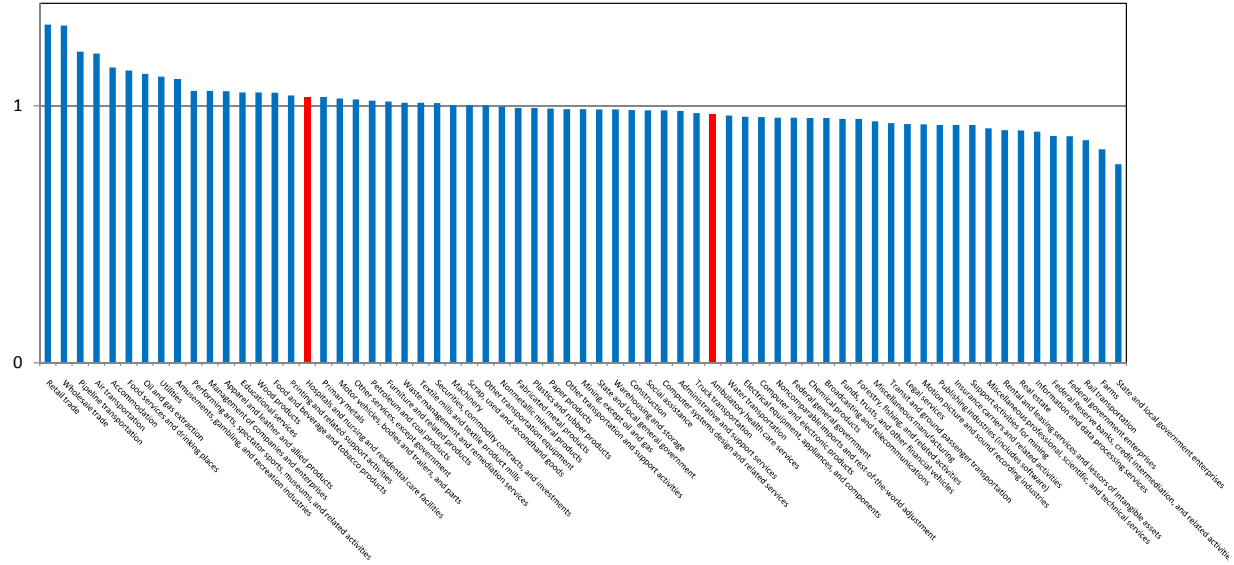
Accounts are given in rows and columns corresponding to eight denominations namely Output, Compensation of employees, Other Incomes, Households, Business, Capital formation, Government and Rest of the World.

Each Quadrant in Table 4, then, gives account of the national flows and their allocation in different blocks in order to describe the whole circular flow. Table 4 gathers data from 67 Input-Output sectors, 5 institutional sectors⁶, 3 value added components [Compensation of employees, Taxes on production and imports, less subsidies, Gross operating surplus], Last Quadrants (V and VI) describe flows between regions and the public administration and the rest of the world.

⁵The flows are expressed in Million of dollars.

⁶The Households, Business, Federal Government, State and Local Government and Rest of the World

Figure 5. Backward dispersion: Health care services relevance

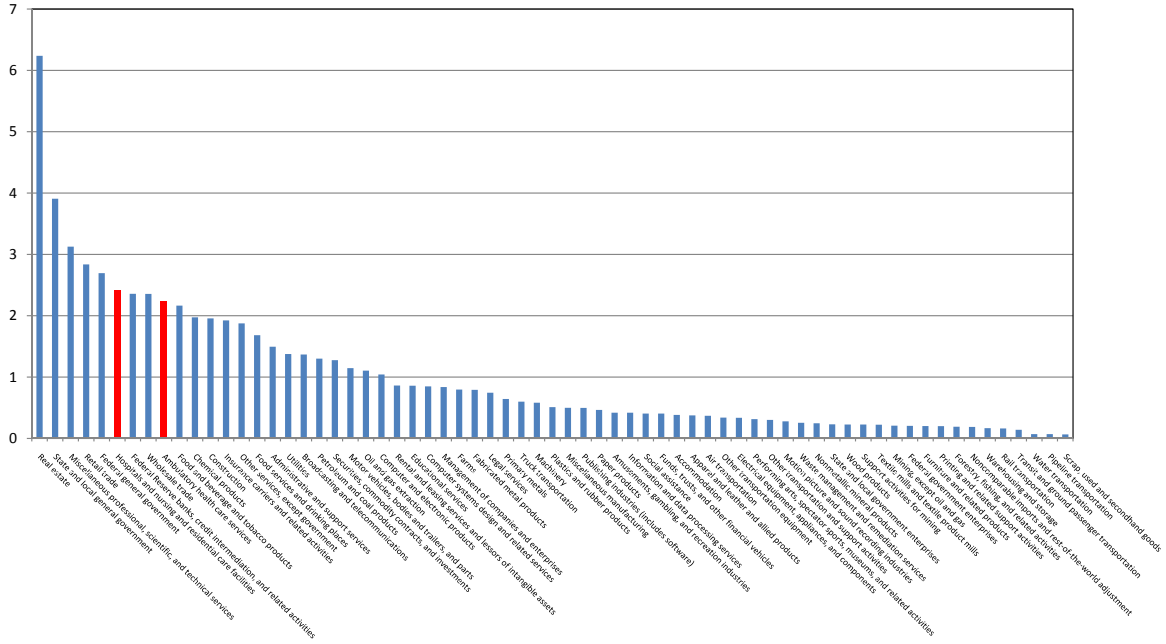


3 Backward and Forward dispersion for Health care services

Inside the phases of creation, distribution and redistribution of income that the SAM for the U.S. economy describe, Health care services can be twice identified. First the Health care services derive from the consumption programmed by Households and Institutions. This part of private Health care output can be generated by two type of activities: "*Ambulatory health care services*" (54) and "*Hospitals and nursing and residential care facilities*" (55). That amounts are respectively 823.703 and 889.594 million of dollars. The remaining part is qualified as Federal Government production (884.400 million of dollars) and State and Local government (431.200 million of dollars). Matrix R, the reduced form of the model, has the potential to underline the direct and indirect effects on the disaggregated output generated. This is possible performing an exogenous shock trough a predetermined final demand or trough any other macroeconomic variables described in the model. Starting from the reduced form we can build two types of indexes of dispersion that are able to point out the role of any products in terms of backward and forward dispersion. The first type of index can appreciate the relevance of a good to activate the production chain or, to put it better, the index evaluate an increase of a unit final demand shock of the i th good in terms of a change of the output of the other commodities. The second type of index evaluate the relevance of a good when a unit final demand shock of all commodities is performed. The inverse matrix allows to build the indexes of dispersion focusing on the good "*Ambulatory health care services*" and "*Hospitals and nursing and residential care facilities*" in order to determine its role as a key commodity. Once determined the Rasmussen dispersion indexes (Rasmussen 1956) it is possible give a rank of all goods in term of power and sensitivity of dispersion. As in figure 5 the two typologies of private Health care services are differently ranked. "*Hospitals and nursing and residential care facilities*" has a backward index equal to 1.03, is ranked at the 17th position while "*Ambulatory health care services*" index is equal to 0.97 (42th position).

The figure 6 shoes the forward index. In this case both goods are ranked in the first positions: "*Hospitals and nursing and residential care facilities*" is the 6th (2.41) while "*Ambulatory health*

Figure 6. Forward dispersion: Health care services relevance



care services" is the 9th (2.23).

4 The health care services as policy objective and policy control

The set of key structures (both for objective variable and control variable) is easily rearranged according the constant absolute change. This phases is needed in order to build two type of index trough which each structure can be measured. In particular, these indexes can be focused on each singular commodity of which can reveal its relative role among all key structures with the specific aspire to quantify its importance both in terms of objective variable and control variable.

As it is for the key objective structures, given matrix \mathbf{Z} , it is possible to define the index:

$$\mu_{ij} = \frac{|m_i \cdot z_{ij}|}{\frac{\frac{1}{m} |m_i \cdot z_{.j}|}{\frac{1}{m^2} \sum_{j=1}^m |m_i \cdot z_{.j}|}} \quad (11)$$

that quantify the importance of the i th good in all the m key objective structures. In particular, the index can reveal the role played by the selected good inside the key objective structures (\mathbf{z}_i) when the correspondent Macro Multipliers (m_i) are activated⁷. As for the key control structures, it is possible to define the index starting from matrix \mathbf{P} :

$$\gamma_{ij} = \frac{|p_{ij}|}{\frac{\frac{1}{n} |p_{.j}|}{\frac{1}{n^2} \sum_{j=1}^n |p_{.j}|}} \quad (12)$$

The index quantify the importance of the i th good in all the n key control structures. In particular,

⁷When the index assume a value smaller than 1 that means the good has a low importance inside both the key objective and control structures.

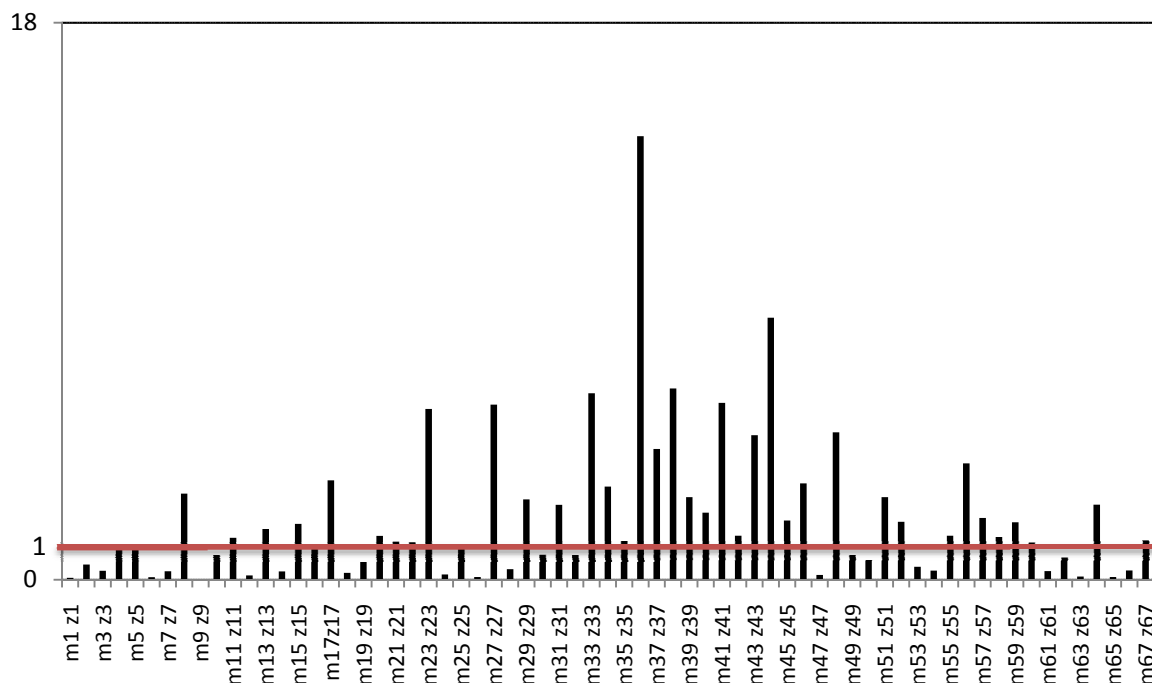
the index can reveal the role played by the selected good inside the key objective structures (\mathbf{p}_i).

4.1 The key-objective-structures index

The importance of Health care output that is detected with reference to the economic policy design is quantified among the structures revealed by the MM approach. All indexes to which we refer in the previous section let to stress the relevance of private Health. They allow to quantify how important is private health care output in influencing output change when an exogenous shock is performed on final demand components. If the economic policy will develop through the objected represented by the disaggregated output for the instance of U.S. economy it is necessary to verify the weight of "Ambulatory health care services" (54) and *Hospitals and nursing and residential care facilities* (55) inside the composition of the key object variable. More than a few structural differences can be stressed considering the value of the key-objective-structures index following the equation (11) with respect to good "Ambulatory health care services" and *Hospitals and nursing and residential care facilities*.

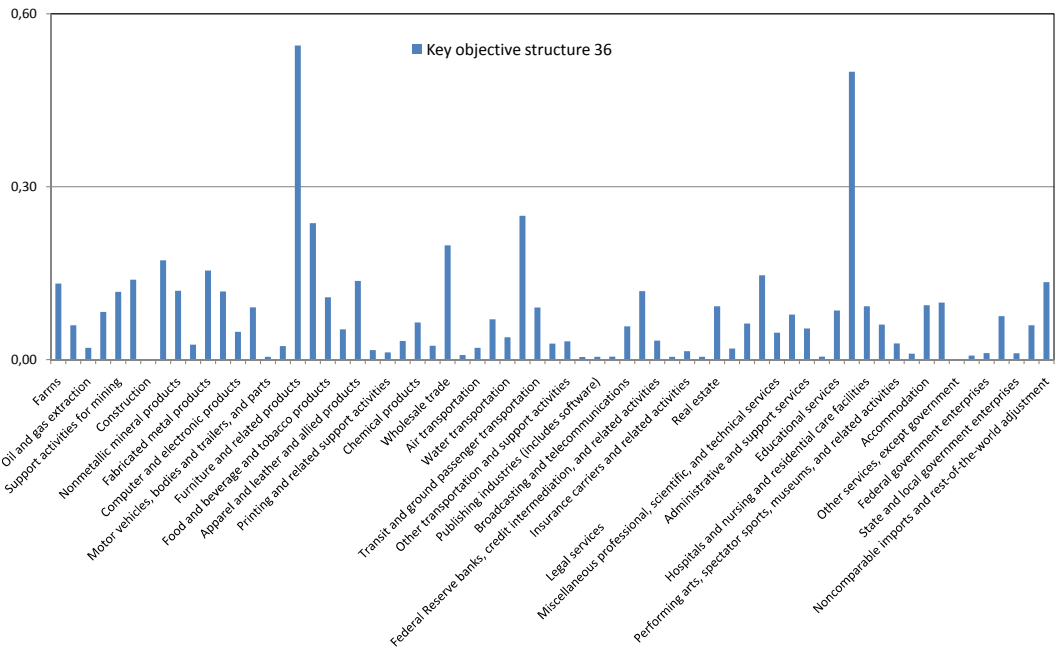
The results regarding the key-objective-structures index for the USA instance are shown in figures 7 and 9. We can see that good "Ambulatory health care services" expresses an important influence into 40 key objective structures among 67 ($\mu_{ij} > 1$). From figure 7 it can be seen how good *Ambulatory health care services*" demonstrate the most influence in structure 36.

Figure 7. The index of key policy for the objective variable ("Ambulatory health care services")



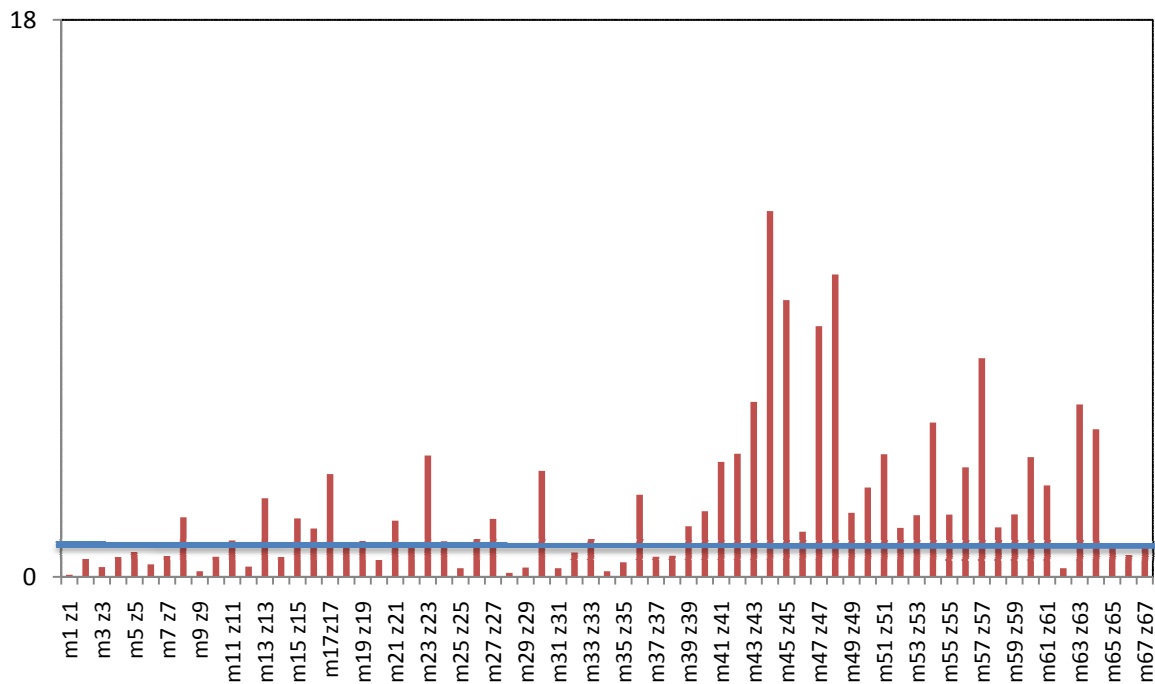
Into detail, structure 36 whose composition is showed in figure 8 manifests an important change of this type of private health output. The structure also enhance the good "Furniture and related products" (17), "Truck transportation" (32) and "Miscellaneous manufacturing" (18). We can see that the good "*Hospitals and nursing and residential care facilities*" (55) has an important

Figure 8. "Ambulatory health care services" - Key-objective-structure 36



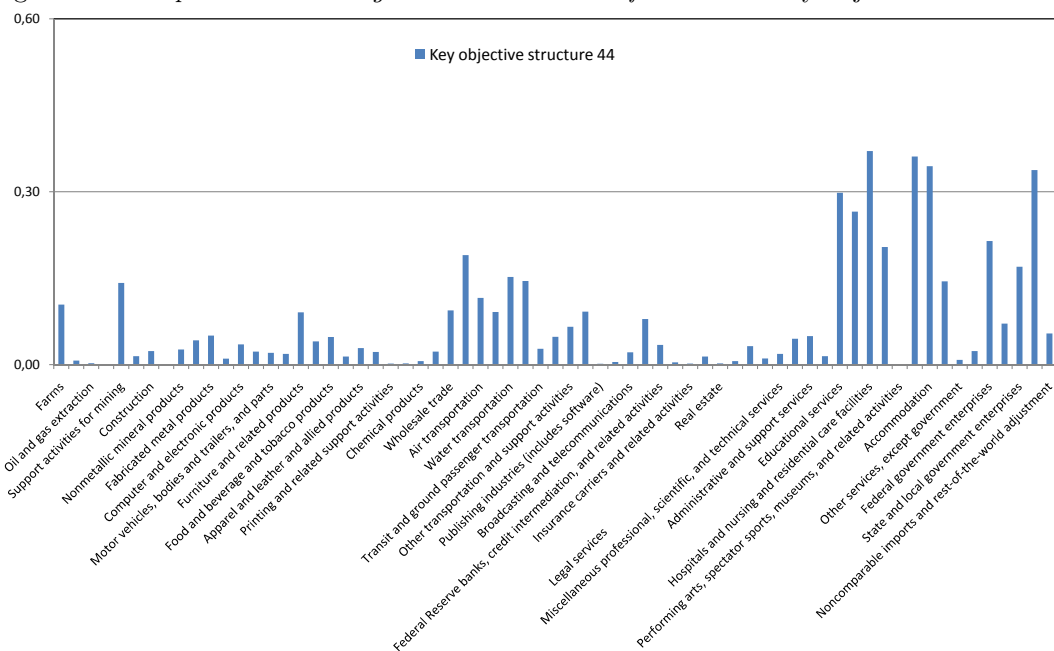
position into structure 43 and 67 ($\mu_{ij} > 1$). From figure 9 it can be stressed the key-objective-structures 44.

Figure 9. The index of key policy for the objective variable ("Hospitals and nursing and residential care facilities")



The composition of structure 44 besides enhancing output "Hospitals and nursing and residen-

Figure 10. "Hospitals and nursing and residential care facilities" - Key-objective-structure 44



tial care facilities" shows the high importance of all health goods ⁸. Goods that get the major change are "Amusements, gambling, and recreation" (58), "Accommodation industries" (59), "Scrap, used and secondhand goods" (66) and Educational services (53).

4.2 The key control structures index

According to equation 12 it is possible to calculate the index of the key control structures for the good "Ambulatory health care services" and "Hospitals and nursing and residential care facilities". That indexes allow to identify the key structures of final demand in which the health good plays a significant role. In this respect the key-control-structures index has the potential to reveal which type of good is favoured by the key policies choose according the value of the index.

We can see that the good "Ambulatory health care services" (54) get an important role into 22 key-control-structures among 67 structures. Figure 11 shows the value of the index γ_{ij} .

Between the key control structures the structure 36 whose composition is showed in figure 12 is the more interesting. In order to activate this structure is necessary to enhance good "Furniture and related products" (17), "Truck transportation" (32), Wholesale trade (27) and Miscellaneous manufacturing (18).

We can see that good "Hospitals and nursing and residential care facilities" (55) get an important role into 25 key-control-structures ($\gamma_{ij} > 1$) among 67 (see figure 13). the Structure 44 is the more interesting among the 25 key-control-structures (its composition is showed in figure 14).

In activating this key policy, the goods "Amusements, gambling, and recreation" (58), "Accommodation industries" (59), "Scrap, used and secondhand goods" (66), Educational services (53), "Social Assistance" (56), "Federal Government" (62) and "State and Local Government"

⁸The composition of structure is showed in figure 10.

Figure 11. The index of key policy for the control variable ("*Ambulatory health care services*")

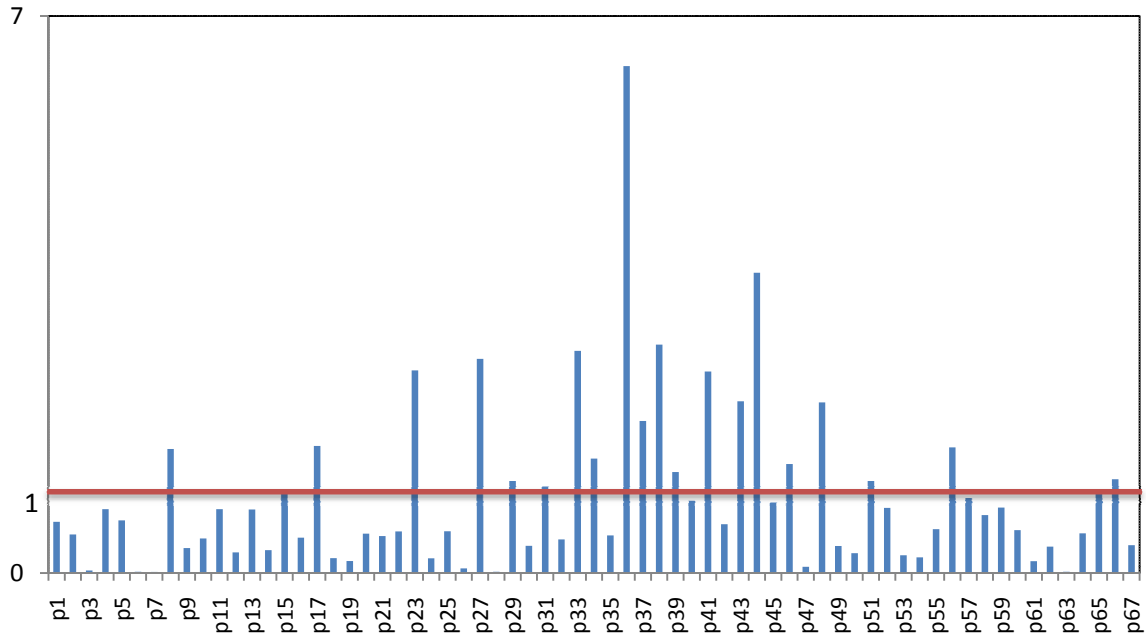
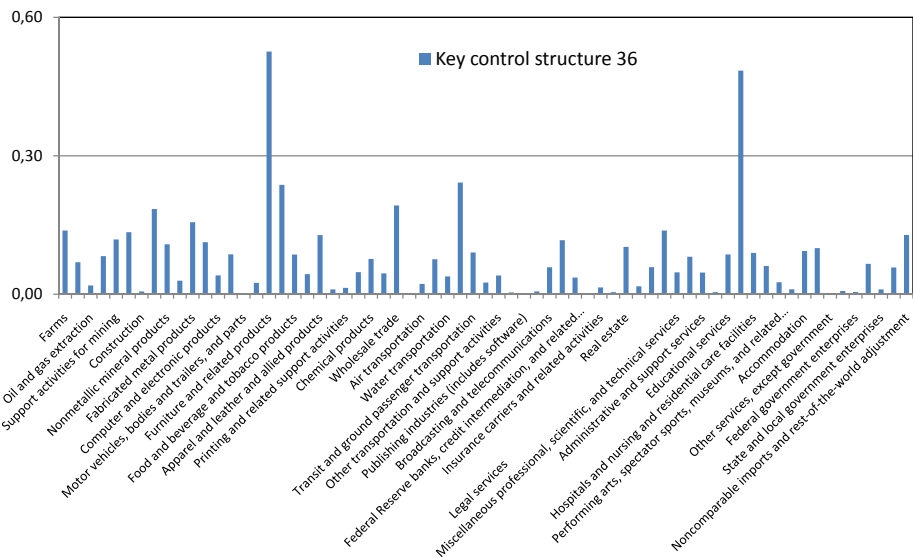


Figure 12. "*Ambulatory health care services*" - Key-control-structure 36



(64) are important to activate the structure.

Figure 13. The index of key policy for the control variable ("*Hospitals and nursing and residential care facilities*")

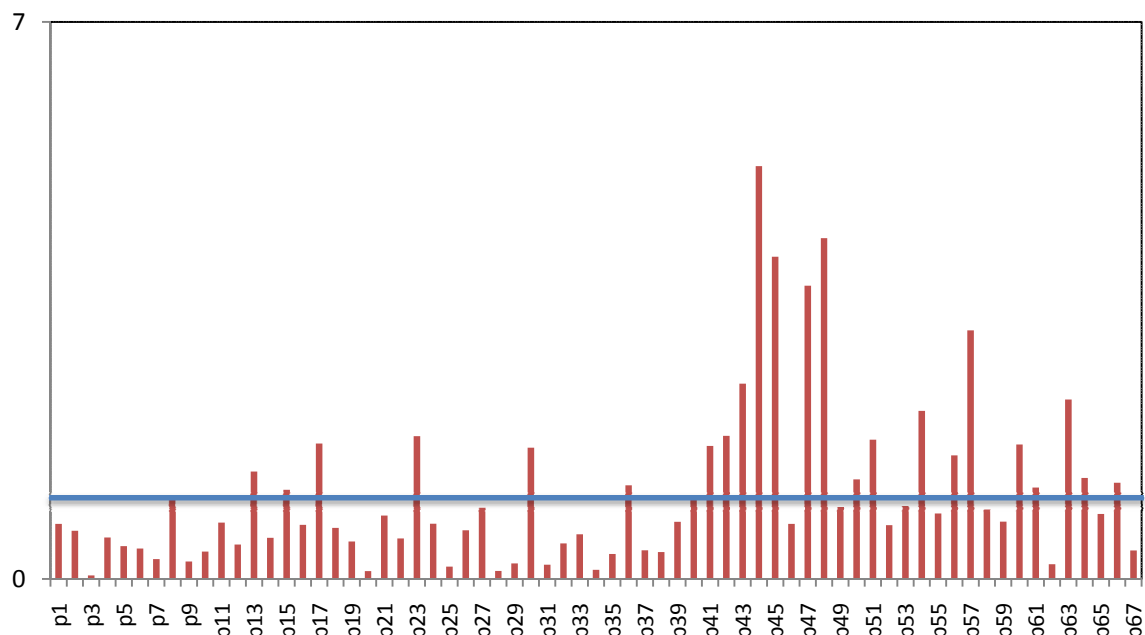
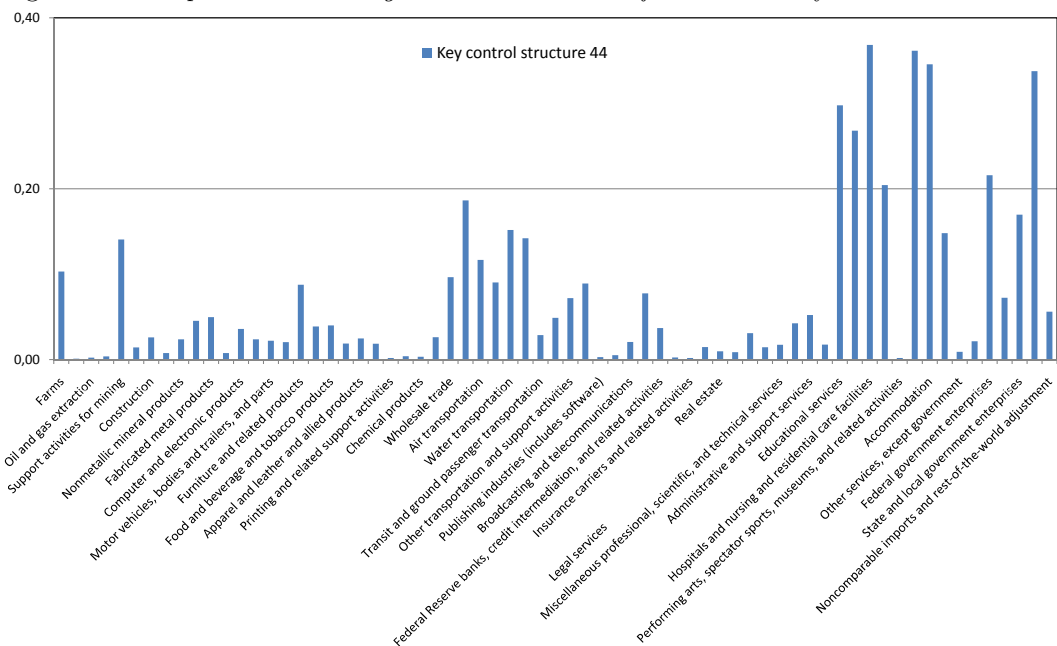


Figure 14. "*Hospitals and nursing and residential care facilities*" - Key-control-structure 44



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Appendix A: Three aggregate measures for a multisectoral economic policy

In order to allocate the policy problem in a multisectoral extended framework to that emerging from the SAM. In these models the macroeconomic variables are defined and evaluated as sectoral flows with lesser attention paid to their aggregate role, which is determined as a summation of the flow over all sectors. The macro variables implied are total output vector \mathbf{x} , industry demand vector \mathbf{b} , final demand vector \mathbf{f} and, in an ancillary role of accounting check, value added vector \mathbf{va} .

The equilibrium relationships that connect intermediate demand expenditure and final demand expenditure to total output value - verified at the industry level- can be written, according the well known formula, as equation 1:

$$\mathbf{x} = [\mathbf{I} - \mathbf{A} - (\mathbf{F}) \cdot (\mathbf{I} + \mathbf{T}) \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L}]^{-1} \cdot (\mathbf{f}^0 - \mathbf{m}) \quad (\text{A1})$$

Speaking in terms of policy models, equation A1 represents the structural form of the extended multisectoral model. This model has a corresponding reduced form in which variables that play a strategic role in defining the policy problem are put in evidence. These are the endogenously determined policy target, $\mathbf{z} = [z_1, \dots, z_n]^T$ on which the effects of the policy control are observed, and the exogenously given policy controls, $\mathbf{p} = [p_1, \dots, p_n]^T$, which the policy maker has to determine to attained the desired or convenient configuration of the policy targets.

The structural model is then manipulated in order to put the policy targets in direct relationship with the policy controls determining the reduced form of the model:

$$\mathbf{z} = \mathbf{R} \cdot \mathbf{p} \quad (\text{A2})$$

where $\mathbf{R} = [\mathbf{I} - \mathbf{A} - (\mathbf{F}) \cdot (\mathbf{I} + \mathbf{T}) \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L}]^{-1}$, $\mathbf{z} = \mathbf{x}$ and $\mathbf{p} = \mathbf{f}$. Notably with the model the target variable, \mathbf{z} , is represented by total output, \mathbf{x} , and the policy control, \mathbf{p} , by final demand, \mathbf{f} . This means that the reduced form matrix coincides with the inverse \mathbf{R} .

In the extended model the specific policy targets \mathbf{z}_0 , i.e. changes in total outputs, are linked to the peculiar policy controls \mathbf{p}_0 , i.e. changes in final demands, through the inverse \mathbf{R} . Once determined the solutions of the policy problem, the remaining endogenous variables are then consistently determined:

$$\mathbf{m}_0 = \mathbf{x}_0 - \mathbf{f}_0 \quad (\text{A3})$$

$$y_0 = \sum_i \left[x_{0i} - \sum_j a_{ij} x_{0j} \right] \quad (\text{A4})$$

However a preliminary problem has to be discussed which regards the determination of the aggregate figure to be globally associated to the change in the policy-control vector-variable starting from the observed sectoral changes in the same vector-variable. This discussion will lead us to determine three different aggregation criteria for the same policy-vector each one with a different meaning on the aggregate behaviour of the macro variable.

This aggregation criterion gives the information on the balance of the policy manoeuvre. In this case a first definition of the scale of the policy control vector \mathbf{p} will be given by:

$$bal(\mathbf{p}) = \sum_i p_i = constant$$

The vectors that show the same net balance will be allocated along the same line. Putting together all the possible policy controls \mathbf{p} into groups according their balance, a set of lines will be determined and we define then as equal policy-contours with respect to a predetermined balance or, briefly, equal-balance policy contours. An equal-balance policy contour, then, puts together all the policy-control vectors that quantify a policy manoeuvre that has the same net balance.

The family of the equal-balance policy contours orders then all the conceivable policy vectors according their net balances: a map where, for each aggregate balance value of the macro variable chosen as policy control, a set of infinite sectoral compositions of the same variable is shown. In the policy application the zero-balance manoeuvre is of great interest because is represented by the policy control performed without making its original global level. In this case the manoeuvre is realized through changes that compensate each other.

It is however apparent that balance is not sufficient by itself to describe the order of magnitude of the net changes in the role of policy control. In particular net balance does not give information on the order of magnitude of changes involved within the policy control.

A measure more suitable for this aim is given by the sum of absolute values, that we will indicate with the abridged expression of change $Cng(\dots)$:

$$Cng(\mathbf{p}) = \sum_i |p_i| = constant$$

The change of vector \mathbf{p} quantifies the order of magnitude of the policy manoeuvre in terms of both the expansion realized and the restraints imposed to sectors. Vectors that show the same absolute change will locate along the same square with diagonal equal twice the change. Grouping all the possible policy controls \mathbf{p} according their aggregate scale defined as change, a set of squares will be generated that we define as equal-change policy contour.

The change of a vector, according our definition, has also specific properties that the previously defined balance has not. The aggregation rule given by $Cng(\dots)$ is, in fact, a vector norm. It defines a type of distance between two points, and then the vector's length, in terms of the sum of the (absolute) differences of their coordinates. Sometimes is known as Manhattan norm⁹. The notion of norm of a vector in mathematics is essential in order to define the concept of "distance" or "length" in a linear vector space as that in which the equal policy contour map is defined.

But what is more relevant the norm can give interesting suggestions for the definition of the macroeconomic aggregate value of a multisectoral vector variable. In fact the norm of a vector $\mathbf{p} \in P^n$ is a mapping that associates a real number to each element in R^n . Whatever type of norm

⁹This name alludes to the grid layout of most streets on the island of Manhattan, which causes the shortest path a car could take between two points in the city to have length equal to the sum of the (absolute) differences of their coordinates rather than the Euclidean distance.

has to fulfil some consistency requirements:

- 1) the norm of a vector different from zero is positive¹⁰;
- 2) the norm of the sum of two vectors is not greater than the sum of the norms of the two vectors;
- 3) scaling a vector by a constant the norm of the vector is scaled by the same constant.

We note that while the change satisfies the requisites to be a norm the balance doesn't satisfy these conditions, even providing valuable economic information. In fact balance can become negative and can be zero also when all his elements are not equal to zero and this is contrary to the requirements for being a norm. Of course it remains the relevance of its economic meaning; that's why *balance* will be used contextually with *change* as two criteria for aggregating our policy control vector.

Since our aim is that of operating on the multidimensional policy target through the use of the multidimensional policy control, we need to answer to a further question. Are the aggregation criteria, that we use for the definition of the policy contours, exploitable to understand something more on the transformation from policy into targets operated by the Leontief reduced form? As it will be shown further on in this paper, a matrix transformation of the vector space - the map of the equal policy contours - takes place through a process that implies three phases: rotation, scaling and counter rotation.

The main question is now whether the morphology of the equal-change policy contour is neutral with respect to the rotation and the counter-rotation phases of the matrix transformation. In these two phases no scale change has to be introduced by a rotation operation so that dimensional changes can remain confined to and completely determined by the scaling phase. In this phase the aggregated Macro Multipliers that rule the matrix transformation are determined.

Each rotation of the axes transforms the coordinates of the vectors. However an axis rotation transforms the length of the policy vector on the equal-change policy contour in a non uniform manner since the length of the policy vector on the contour is not constant. The specific geometrical features of the equal-change policy contour is, then, not neutral with respect to an axes rotation.

We then conclude that even if the two aggregation criteria described, balance and change, are sufficient under the economic profile to synthesize the characteristics of the vectors scale, a further attempt is needed to identify an aggregation criterion that can generate an equal policy contour map neutral with respect to an axes rotation. In this way we will be able to isolate the aggregated Macro Multipliers, implicit in the Leontief reduced form, that can be determined only in the scaling phase of the matrix transformation.

An aggregation criterion that overcomes these drawbacks is that of assigning to the vector's scale the value of its Euclidean norm, the *modulus*:

$$Mod(\mathbf{p}) = \sqrt{\sum_j p_j^2}$$

All the policy vectors that have constant modulus are invariant with respect to rotations of the

¹⁰The norm is a definite positive function:

$\|\mathbf{p}\| = 0 \forall \mathbf{p} \in \mathbf{P}$ and $\|\mathbf{p}\| = 0$ if and only if $p_i = 0$.

It satisfies the triangular inequality:

$\|\mathbf{p}_1 + \mathbf{p}_2\| \leq \|\mathbf{p}_1\| + \|\mathbf{p}_2\| \quad \mathbf{p}_1, \mathbf{p}_2 \in \mathbf{P}$.

It is homogeneous $\|\lambda \cdot \mathbf{p}\| = |\lambda| \cdot \|\mathbf{p}\|$ for each scalar λ .

axes and describe a circle with radius equal to a constant.

This aggregation criterion is less immediate in its economic interpretation than the two already shown. It is however useful in calculation when, operating with matrices on macroeconomic vector variables, we need to isolate the scale effects from the structure effects of a macroeconomic vector variable on another vector variable. Given the three aggregation criteria, a predetermined vector variable will show three different aggregated values. The three values provide three different indications on the three features of the scale of the variable and the possibility of switching from a value to another, when necessary, through the use of the definitions.

The relationship that exists between change and modulus is of particular interest and can be analyzed concentrating on the unit equal-change and the unit equal-modulus policy-contours.

Appendix B: Macro Multiplier approach

The direct and indirect effects of final demand on total output are then quantified in our multi-sectoral extended model from structural matrix \mathbf{R} .

$$\mathbf{R} = [\mathbf{I} - (\mathbf{A} - \mathbf{A}^m) - (\mathbf{F} - \mathbf{F}^m) \cdot (\mathbf{I} + \mathbf{T}) \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L}]^{-1} \quad (\text{B1})$$

The structural matrix¹¹ of our model can be easily decomposed in a sum of m different matrices through the Singular Value Decomposition [?].

The decomposition proposed can be applied both to square and to non-square matrices. Here the general case of square matrix \mathbf{R} will be shown. The non-square matrix case is easily developed along the same lines.

To simplify we consider 2x2 model. Let us consider matrix \mathbf{W} [2,2], for example, the square of matrix \mathbf{R} :

$$\mathbf{W} = \mathbf{R}^T \cdot \mathbf{R}$$

Matrix \mathbf{W} has a positive definite or semi definite square root. Given that $\mathbf{W} \geq 0$ by construction, its eigenvalues λ_i for $i = 1, 2$ shall be all real non negative [?].

The nonzero eigenvalues of matrices \mathbf{W} and \mathbf{W}^T coincide. The system of eigenvectors $[\mathbf{u}_i \quad i = 1, 2]$ for \mathbf{W} and $[\mathbf{v}_i \quad i = 1, 2]$ for \mathbf{W}^T are orthonormal basis.

We get then

$$\mathbf{R}^T \cdot \mathbf{z}_i = \sqrt{\lambda_i} \cdot \mathbf{p}_i \quad i = 1, 2$$

We can construct the two matrices

$$\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2] \quad \mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2]$$

As defined above, the eigenvalues of \mathbf{W} coincide with singular values of \mathbf{R} hence $s_i = \sqrt{\lambda_i}$ and

¹¹Its numerical determination is shown in table 1 section 4 \mathbf{R} .

we get

$$\mathbf{R}^T \cdot \mathbf{Z} = [s_1 \cdot \mathbf{p}_1, s_2 \cdot \mathbf{p}_2] = \mathbf{P} \cdot \mathbf{M}$$

Structural matrix \mathbf{R} in equation B1 can be then decomposed as

$$\mathbf{x} = \mathbf{Z} \cdot \mathbf{M} \cdot \mathbf{P}^T \cdot \mathbf{f} \quad (\text{B2})$$

\mathbf{P} is an [2,2] unitary matrix whose columns define the 2 reference structures for final demand:

$$\mathbf{p}_1 = \begin{bmatrix} p_{1,1} & p_{1,2} \end{bmatrix}$$

$$\mathbf{p}_2 = \begin{bmatrix} p_{2,1} & p_{2,2} \end{bmatrix}$$

\mathbf{Z} is an [2,2] unitary matrix whose columns define 2 reference structures for output:

$$\mathbf{z}_1 = \begin{bmatrix} z_{1,1} \\ z_{2,1} \end{bmatrix}, \mathbf{z}_2 = \begin{bmatrix} z_{1,2} \\ z_{2,2} \end{bmatrix}$$

and \mathbf{M} is an [2,2] diagonal matrix of the type:

$$\mathbf{M} = \begin{bmatrix} \mathbf{m}_1 & 0 \\ 0 & \mathbf{m}_2 \end{bmatrix}$$

Scalars m_i are all real and positive and can be ordered as $m_1 > m_2$. Now we have all the elements to show how this decomposition correctly represents the MM that quantify the aggregate scale effects and the associated structures of the impact of a shock in disposable income on total output. In fact if we express the actual vector \mathbf{f} in terms of the structures identified by matrix \mathbf{P} , we obtain final demand vector, \mathbf{f}^0 , expressed in terms of the structures suggested by the \mathbf{R} :

$$\mathbf{f}^0 = \mathbf{P} \cdot \mathbf{f} \quad (\text{B3})$$

On the other hand we can also express total output according the output structures implied by matrix \mathbf{R} :

$$\mathbf{x}^0 = \mathbf{Z}^T \cdot \mathbf{x} \quad (\text{B4})$$

Equation B2 then becomes through equations B3 and B3:

$$\mathbf{x}^0 = \mathbf{M} \cdot \mathbf{f}^0 \quad (\text{B5})$$

which implies:

$$\mathbf{x}_i^0 = m_i \cdot \mathbf{f}_i^0 \quad (\text{B6})$$

where $i = 1, 2$. We note that matrix \mathbf{R} hides 2 fundamental combination of the outputs. Each of them is obtain multiplying the corresponding combination of final demand by a predetermined scalar which has in fact the role of aggregated Macro Multiplier.

The complex effect on the output vector of final demand shocks can be reduced to a multiplication by a constant m_i .

The structures we have identified play a fundamental role in determining the potential behavior of the economic system, i.e. the behavior of the system under all possible shocks. We can in fact evaluate which will be the effect on output of all final demand possible structures. This is easily done imposing in equation 16 a vector whose modulus is constant, say equal to one, but whose structure can assume all possible configurations. If vector \mathbf{f} in equation B2 is such that

$$\sqrt{\sum_{j=1}^{\Sigma} j} = 1 \quad (\text{B7})$$

then geometrically we mean that the final demand vector describes a sphere of unit radius (the unit circle).

It rotates around the origin assuming all the possible structures, including those implied by the columns of matrix \mathbf{P} . Correspondingly the vector of total output will describe an ellipsoid with semi-axes of length m_1, \dots, s_m , oriented according the directions designated by the columns of matrix \mathbf{Z} . This ellipsoid is sometimes called the isocost of final demand control.

When final demand vector crosses a structure in \mathbf{P} , the vector of total output crosses the corresponding structure in \mathbf{Z} and the ratio between the moduli of the two vectors is given by the corresponding scalar m . Singular values \mathbf{m}_i , then, determine the aggregated effect of a final demand shock on output. For this reason we will call them macro multipliers. These MM are aggregated, in the sense that each of them applies on all components of each macroeconomic variables taken into consideration, and are consistent with the multi-industry specification of the model.

In our original $[m, m]$ extended model, we can than say that, given our matrix \mathbf{R} , we are able to isolate impacts of different (aggregate) magnitude, since that MM present in matrix \mathbf{R} , \mathbf{m}_i can be activated through a shock along the demand structure \mathbf{p}_i and its impact can be observed along the output structure \mathbf{z}_i .

Appendix C: Table and figures

Figure C1. I-O commodities, Primary factors, Institutional Sectors and Capital Formation classification

1 Farms	40 Information and data processing services
2 Forestry, fishing, and related activities	41 Federal Reserve banks, credit intermediation, and related activities
3 Oil and gas extraction	42 Securities, commodity contracts, and investments
4 Mining, except oil and gas	43 Insurance carriers and related activities
5 Support activities for mining	44 Funds, trusts, and other financial vehicles
6 Utilities	45 Real estate
7 Construction	46 Rental and leasing services and lessors of intangible assets
8 Wood products	47 Legal services
9 Nonmetallic mineral products	48 Computer systems design and related services
10 Primary metals	49 Miscellaneous professional, scientific, and technical services
11 Fabricated metal products	50 Management of companies and enterprises
12 Machinery	51 Administrative and support services
13 Computer and electronic products	52 Waste management and remediation services
14 Electrical equipment, appliances, and components	53 Educational services
15 Motor vehicles, bodies and trailers, and parts	54 Ambulatory health care services
16 Other transportation equipment	55 Hospitals and nursing and residential care facilities
17 Furniture and related products	56 Social assistance
18 Miscellaneous manufacturing	57 Performing arts, spectator sports, museums, and related activities
19 Food and beverage and tobacco products	58 Amusements, gambling, and recreation industries
20 Textile mills and textile product mills	59 Accommodation
21 Apparel and leather and allied products	60 Food services and drinking places
22 Paper products	61 Other services, except government
23 Printing and related support activities	62 Federal general government
24 Petroleum and coal products	63 Federal government enterprises
25 Chemical products	64 State and local general government
26 Plastics and rubber products	65 State and local government enterprises
27 Wholesale trade	66 Scrap, used and secondhand goods
28 Retail trade	67 Noncomparable imports and rest-of-the-world adjustment
29 Air transportation	VA1 Compensation of employees
30 Rail transportation	VA2 Taxes on production and imports, less subsidies
31 Water transportation	VA3 Gross operating surplus
32 Truck transportation	I Households and institutions
33 Transit and ground passenger transportation	II Business
34 Pipeline transportation	III Federal Government
35 Other transportation and support activities	IV State and Local Government Current
36 Warehousing and storage	V Rest of World
37 Publishing industries (includes software)	S1 Private investment
38 Motion picture and sound recording industries	S2 National Gross investment
39 Broadcasting and telecommunications	S3 State and local government gross investment

Figure C2. Key-objective-structures

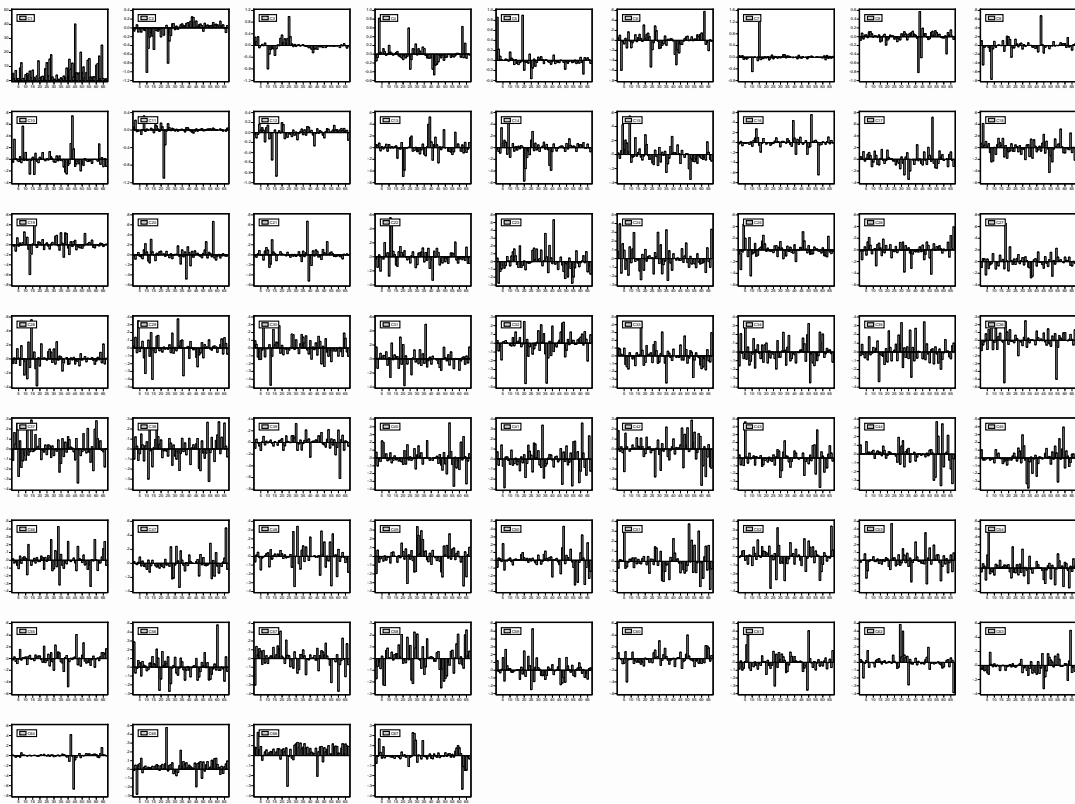


Figure C3. Key-control-structures

