Bayesian analysis of product-level global CO_2 emission multipliers from 1995 to 2009*

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Abstract

We present the regression-form equations of basic transformation models based on different technology or sales structure assumptions in terms of supply and use tables (SUTs) for the purpose of deriving various input-output (IO) multipliers. Using a world SUTs data constructed by the World Input-Output Database project, we estimate and analyze the development of product-level global CO_2 emission multipliers for 40 countries and 59 products for the period of 1995-2009. For this purpose, we adopt a Bayesian approach in order to take into account the inherent uncertainty of SUTs data and to avoid a usual practice of products aggregation in published SUTs, which, in our view, may lead to severe loss of information on the aggregated product-industry links.

Keywords: Stochastic input-output multipliers, supply and use tables, carbon dioxide multipliers, Bayesian econometric analysis

JEL Classification Codes: R11, R15, C11, C67, Q53

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1 Introduction

Input-output (IO) impact analysis, widely used for various policy relevant issues, is mostly based on the so-called Leontief inverse matrix (Leontief 1936, 1941). In particular, the issue of computing sectoral or product multipliers is an important one because multipliers indicate the extent of an economy-wide impact of an exogenous shock in categories of final demand (consisting of consumption, government expenditures, investments and exports) on an economic or environmental factor of interest, such as income, employment and pollutant emissions (see e.g., Miller and Blair 2009). However, the procedure of direct application of the Leontief inverse matrix in quantifying various multipliers has two disadvantages. First, only the point estimates are computed while it is evident that for a robust estimation the uncertainty inherent to the IO data compilation process should be taken into account. Second, a transformation of supply and use tables (SUTs) into symmetric IO tables (SIOTs) as an analytical step towards modeling may lead to a senseless problem of negatives in SIOTs. It should be noted that while SIOTs can be only product-by-product or industry-by-industry types, SUTs provide more detailed and useful information as they distinguish between products and industries. This feature of SUTs allows an analyst to appropriately consider secondary products besides the main products of industries, and also provides a natural link to the additional important data sets such as international trade and employment statistics.

Since SIOTs are analytical constructs of SUTs, it seems quite reasonable to use directly SUTs for the analyses of the IO-related issues. Ten Raa and Rueda-Cantuche (2007a) were first to realize that for the case of the so-called product technology assumption it is possible to obtain the estimates of product IO multipliers (and their confidence intervals) directly from SUTs by running an appropriate ordinary least-squares (OLS) regression. This study in its empirical application uses a firm-level data. Exactly the same methodology has been applied to the widely available SUTs data at the industry level in Rueda-Cantuche and Amores (2010) and Rueda-Cantuche (2011). The last study refers to the method as the supply-use based econometric (SUBE) approach. In all these studies it is argued that the Leontief inverse-based multipliers *overestimate* the "true" values of the multipliers. This conclusion is based on the empirical studies of output and employment multipliers for the Andalusian economy and of carbon dioxide emission multipliers for the Danish economy and the EU.[1] However, one might raise the following concerns about

[1]Quantification of uncertainties of technical coefficients within the classical IO analysis through

these papers. First and most importantly, aggregation that is often required for OLS implementation may lead, in our view, to severe loss of information. With product technology assumption, the number of observations and of parameters are equal to the number of industries and products, respectively. Since usually the number of products in SUTs is (much) larger than that of industries, to have sufficient degrees of freedom for OLS approach, the number of products has to be reduced. For instance, in both Rueda-Cantuche and Amores (2010) and Rueda-Cantuche (2011) a total of 64.4% of products (i.e., 38 out of 59 products) are aggregated which reduces the number of products to 21. However, such aggregation may very well lead to severe loss of information, in particular, in SUTs framework because a crucial information on the heterogeneities of aggregated product-industry relationships is practically ignored.[2] We, however, believe that the huge human, time and financial efforts of national statistical offices that are put into the construction of SUTs with more products than industries *must* be used effectively in the practical applications of these tables.

Second, in the mentioned papers there are cases where the estimates of IO multipliers are lower than their economically plausible lower bounds. It is a problem because such results are, in fact, equivalent to having negative elements in the input (or technology) matrix that is *implicitly* estimated as a result of the SUBE outcome. For example, in ten Raa and Rueda-Cantuche (2007a, Table 2, pp. 328-330) there are seven cases out of 89 with the output multiplier estimates less than one, which in one case is even negative. Negative multipliers have no economic justification within the IO framework, because the lower bound of the output multipliers is unity by construction. There are much more cases when the lower bounds of the reported multipliers' confidence intervals cross the unity limit.[3] Therefore, we think that for

cross-sectional econometric approach was suggested by Gerking (1976). This author also refers to Klein (1974) for a time-series econometric approach to estimation of technical coefficients. The literature on stochastic IO analysis is large, where the effect of imposing some explicit distributional assumptions with respect to input coefficients or intersectoral transactions on the bias of the Leontief inverse are studied analytically or through Monte Carlo techniques. See e.g., Simonovits (1975), West (1986), Jackson (1986), Roland-Holst (1989), Jackson (1989), Kop Jansen (1994), ten Raa and Steel (1994), Dietzenbacher (1995) and Diáz and Morillas (2011). While this literature imposes stochasticity on IO technology or transaction matrices, in this paper following ten Raa and Rueda-Cantuche (2007a) the source of uncertainty of IO multipliers is assumed to come from SUTs. We believe this is a more reasonable starting point, because SUTs are the building block of SIOTs and make explicit distinction between products and industries.

[2] Alternatively, one can disaggregate industries, but this "would require more detailed information on inputs and outputs that NSIs [national statistical institutes] very rarely report" (Rueda-Cantuche and Amores 2010, p. 992).

[3] In Rueda-Cantuche (2011, Table 2, p. 272) there are two cases (out of 21) in which the estimates of CO_2 multipliers are less than the corresponding direct emission coefficients, which by

the SUBE approach it seems more reasonable to use constrained OLS in order to avoid the mentioned problem of negatives in the implicitly estimated input matrix.

In this paper we, first, derive the SUTs-based regression-form systems for three IO transformation models that are based on the assumptions of industry technology, fixed industry sales structure and fixed product sales structure. Together with the product technology assumption of the SUBE system of ten Raa and Rueda-Cantuche (2007a), they make the complete list of all basic IO transformation models expressed in terms of SUTs in a regression-form setting. We find that the regression-form frameworks cannot be reasonably used for estimating *stochastic* IO multipliers for the first three transformation models. This is due to the fact that the derived systems are exactly identified, i.e., the number of unknown parameters is equal to the number of equations in each of these SUTs frameworks, which represents either only product or industry dimension. This is, however, not the case with the product technology model, which was already discovered by ten Raa and Rueda-Cantuche (2007a), that allows one to apply sensibly various statistical techniques to the corresponding system. We note that, in terms of the derived IO multipliers, the product technology model is exactly equivalent to the so-called by-product method of Stone (1961), which has been shown recently by Suh et al. (2010).

Second, using a unique dataset of international SUTs constructed by the World Input-Output Database project (for details, see Timmer 2012), we estimate and analyze the development of product-level global carbon dioxide emission multipliers for 40 countries and 59 products for the period of 1999-2009. We use the product technology assumption, which is advocated by Eurostat (2008) and underlies

construction make their theoretical lower bounds. For instance, commodity 10 (Metallurgy and fabricated metal products) direct emission coefficient is 416.1 CO_2 tonnes per million Euro, while the corresponding estimated emission multiplier is much smaller and equals 206.8 CO_2 tonnes per million Euro. One can argue that the lower bound may not be exactly equal to the direct factor coefficient due to stochasticity in the data, but such a big difference in the multipliers estimates and their direct coefficients, to our view, is simply implausible on theoretical grounds. There are many more cases in the study where the lower bounds of the reported confidence intervals are less than their direct CO_2 coefficients. This implies that the "unbiased and consistent OLS estimates" might very well be biased themselves. In fact, Rueda-Cantuche (2011) states: "... the typical results in which [output] multipliers must always be greater than one may lose meaning ... since, in a rectangular framework, it is not necessarily the primary industry that will fully satisfy the new demand for a product. Other sectors may produce it secondarily even leading to reductions in the output of some industries" (p. 267). Yes, it is true that sectoral outputs may reduce, but the multipliers of interest have product dimension, not an industry dimension. In fact, in the SUBE setting the same product technology assumption is imposed as in its underlying IO framework. Namely, the net output of *a* particular product is assumed to have exactly the same effect on any factor of interest for *all* industries, irrespective of whether these industries are the primary or secondary producers of that product.

the SUBE approach.[4] However, in contrast to the SUBE approach, we adopt a Bayesian method in order to take into account the inherent uncertainty of SUTs data. Some of the advantages of the Bayesian approach are: (a) it does not require product aggregation as is the case for the SUBE approach, hence we use all the available information in published SUTs, (b) results are presented in terms of intuitively meaningful posterior densities, and (c) non-sample information can be easily incorporated in the analysis via prior distributions specification. Bayesian econometric techniques are based on a sound probability theory and in comparison to the frequentist econometrics still tend to require more computing efforts. However, Bayesian methods are becoming quite popular as their implementation is becoming more and more accessible due to the rapid development of computer technology.

The rest of this paper is organized as follows. In Section 2 the regression-form equations in terms of SUTs of basic alternative IO models are presented. The methodology is discussed in Section 3. Section 4 presents the results of our empirical application. Concluding remarks are given in Section 5.

2 Transformations of SUTs into SIOTs and the corresponding regression-type models

Constructing symmetric input-output tables (SIOTs) from supply and use tables (SUTs) requires certain assumptions. Following the terminology used in Eurostat (2008, Chapter 11), product-by-product SIOTs are based on *technology* assumptions, while industry-by-industry SIOTs are based on *sales structure* assumptions. The different assumptions used in the literature lead to four basic alternative IO models, which are listed in Table 1. This table indicates that some transformations may result in negative elements in the derived SIOTs. Because of this and the plausibility of the imposed assumptions, model A (product technology assumption) and model D (fixed product sales structure assumption) are widely used by statistical offices and advocated by Eurostat (2008).[5] For further explanation of the assumptions used

[4]As mentioned, the studies implementing SUBE approach argue that the Leontief-inverse-based multipliers overestimate their "true" values. But if one looks closer to the fewer reverse cases, one observes much severe underestimation cases. In fact, we should note that such comparison of the Leontief-inverse-based and OLS-based multipliers is not quite fair, since the two estimates are based on essentially different data sets. That is, for OLS one has much more industries than products, but to find the corresponding Leontief-inverse-based products multipliers one has to further reduce the number of industries to that of products.

[5]It states that "the types of tables that best fulfils the standard quality criteria is the industry-by-industry table based on the assumption of fixed product sales structures and the product-by-

Table 1: Basic transformation models

Technology assumption	*Product-by-product IO table*	*Negatives*
Product technology	Model A: Each product is produced in its own specific way, irrespective of the industry where it is produced.	Yes
Industry technology	Model B: Each industry has its own specific way of production, irrespective of its product mix.	No
Sales structure assumption	*Industry-by-industry IO table*	*Negatives*
Fixed industry sales structure	Model C: Each industry has its own specific sales structure, irrespective of its product mix.	Yes
Fixed product sales structure	Model D: Each product has its own specific sales structure, irrespective of the industry where it is produced.	No

Note: The column "Negatives" indicates the possibility of occurrence of negative elements in the derived SIOTs. Source: Figure 11.3, Eurostat (2008, p. 310).

in the transformation process, the reader is referred to ten Raa and Rueda-Cantuche (2007b) and Rueda-Cantuche and ten Raa (2009).

Following the convention in input-output (IO) analysis, let us define an industry-by-product make matrix by $\mathbf{V}$ (whose transpose is a supply table) and a product-by-industry domestic use table by $\mathbf{U}$. Hence, the vectors of product output, $\mathbf{q}$, and industry output, $\mathbf{x}$, are derived as[6]

$$\mathbf{q} = \mathbf{V}'\boldsymbol{\imath}, \tag{1}$$

$$\mathbf{x} = \mathbf{V}\boldsymbol{\imath}, \tag{2}$$

where $\boldsymbol{\imath}$ is a summation vector of appropriate dimension.

In what follows we discuss how gross output and factor multipliers can be estimated directly from SUTs presented in a regression-form systems for each transformation model listed in Table 1. The well-known open Leontief model states that the vector of *factor multipliers* $\boldsymbol{\beta}$ is obtained from

$$\boldsymbol{\beta}' = \boldsymbol{\mu}'(\mathbf{I} - \mathbf{A})^{-1} \text{ or, equivalently, } \boldsymbol{\mu} = (\mathbf{I} - \mathbf{A}')\boldsymbol{\beta}, \tag{3}$$

where $\boldsymbol{\mu}$ is the direct factor coefficient vector (i.e., factor per unit of industry/product

product input-output table based on the product technology assumption. These types of tables reflect the accumulated experience and current practice of those countries most permanently involved in the compilation of symmetric input-output tables" (p. 340).

[6]Matrices are given in bold capital letters, vectors in bold lower case letters, and scalars in italicized lower case letters. Vectors are columns by definition, thus row vectors are obtained by transposition, indicated by a prime. $\hat{\mathbf{x}}$ denotes the diagonal matrix with elements of vector $\mathbf{x}$ along its main diagonal and zero otherwise.

output), $\mathbf{A}$ is the input matrix, and $\mathbf{I}$ is the identity matrix of appropriate dimension. The Leontief inverse matrix, $(\mathbf{I}-\mathbf{A})^{-1}$, takes into full account all direct and indirect interindustry or interproduct input linkages (see e.g., Miller and Blair 2009). If in (3) we have $\boldsymbol{\mu} = \boldsymbol{\imath}$, then the resulting multiplier $\boldsymbol{\beta} \equiv \boldsymbol{\beta}^o$ represents the gross output multiplier vector.

Model A: Product technology assumption. The corresponding product-by-product input matrix $\mathbf{A}$ and the direct factor coefficient vector $\boldsymbol{\mu}$ are defined as

$$\mathbf{A} = \mathbf{U}\mathbf{V}'^{-1}, \tag{4}$$

$$\boldsymbol{\mu}' = \mathbf{e}'\mathbf{V}'^{-1}, \tag{5}$$

where $\mathbf{e}$ is the vector of sectoral factor use/generation (such as pollutant emissions or employment figures by industries). Plugging (4)-(5) in the second expression of (3) yields $\mathbf{V}^{-1}\mathbf{e} = (\mathbf{I} - \mathbf{V}^{-1}\mathbf{U}')\boldsymbol{\beta} = \mathbf{V}^{-1}(\mathbf{V} - \mathbf{U}')\boldsymbol{\beta}$, which if premultiplied by $\mathbf{V}$ gives the regression-form system of the model as

$$\mathbf{e} = (\mathbf{V} - \mathbf{U}')\boldsymbol{\beta}. \tag{6}$$

Thus, adding an error term to the right-hand side of (6) gives us a regression equation, where the dependent variable is the sectoral factor use, $\mathbf{e}$, and the independent variables are *products' net sectoral outputs*, $\mathbf{V} - \mathbf{U}'$ (i.e., the kth independent variable is net sectoral output of product k, $v_{jk} - u_{kj}$ for all industries j). Since these variables are all observed one can use the regression analysis to estimate the total factor multipliers $\boldsymbol{\beta}$.

For the gross output multipliers calculation, define $\boldsymbol{\mu} = \boldsymbol{\imath}$. Together with (4), (3) becomes $\boldsymbol{\imath} = (\mathbf{I} - \mathbf{V}^{-1}\mathbf{U}')\boldsymbol{\beta}^o = \mathbf{V}^{-1}(\mathbf{V} - \mathbf{U}')\boldsymbol{\beta}^o$. Premultiplication by $\mathbf{V}$ and using (2) gives

$$\mathbf{x} = (\mathbf{V} - \mathbf{U}')\boldsymbol{\beta}^o, \tag{7}$$

where now the dependent variable is the vector of sectoral gross outputs. Equations (6)-(7) were first derived by ten Raa and Rueda-Cantuche (2007a).

Using (6)-(7), in comparison to the standard open Leontief model (3) with the input matrix and the direct coefficient vector defined as in (4)-(5), has two important advantages. First, the numbers of industries and products do not have to be the same. This is required for the Leontief-inverse-based multiplier estimation as one needs to quantify the inverse matrix $\mathbf{V}'^{-1}$ that would not be *uniquely* defined with unequal number of industries and products. Second, the derived regression forms

(6)-(7) allow one to take uncertainty in the SUTs data into account by employing various statistical techniques.

Observe that the number of observations in (6)-(7) is equal to the number of industries, while the number of parameters equals the number of products. Thus, given that in published SUTs it is often the case that the number of products is larger than that of industries, simply running OLS on (6) and (7) is unreasonable because of the lack of sufficient degrees of freedom. Therefore, to use OLS one has to aggregate (often many) products in SUTs in order to get the necessary degrees of freedom.

Model B: Industry technology assumption. Under this assumption the product-by-product input matrix $\mathbf{A}$ and the direct factor coefficient vector $\boldsymbol{\mu}$ are derived from (for matrix notations see fn. 6)

$$\mathbf{A} = \mathbf{U}\hat{\mathbf{x}}^{-1}\mathbf{V}\hat{\mathbf{q}}^{-1}, \tag{8}$$

$$\boldsymbol{\mu}' = \mathbf{e}'\hat{\mathbf{x}}^{-1}\mathbf{V}\hat{\mathbf{q}}^{-1}. \tag{9}$$

Plugging (8)-(9) in the second expression of (3) yields $\hat{\mathbf{q}}^{-1}\mathbf{V}'\hat{\mathbf{x}}^{-1}\mathbf{e} = (\mathbf{I} - \hat{\mathbf{q}}^{-1}\mathbf{V}'\hat{\mathbf{x}}^{-1}\mathbf{U}')\boldsymbol{\beta}$, which if premultiplied by $\hat{\mathbf{q}}$ gives

$$\mathbf{V}'\hat{\mathbf{x}}^{-1}\mathbf{e} = (\hat{\mathbf{q}} - \mathbf{V}'\hat{\mathbf{x}}^{-1}\mathbf{U}')\boldsymbol{\beta}. \tag{10}$$

The interpretation of the dependent and independent variables in (10) in a regression setting becomes clear once one notices that the product-by-industry matrix $\mathbf{V}'\hat{\mathbf{x}}^{-1} \equiv \mathbf{C}$ is the *industry output proportions* matrix, which in the literature is known as the *product mix* matrix. That is, c_{ij} denotes the fraction of total industry output j that is in the form of product i.[7] Therefore, premultiplication of $\mathbf{e}$ by the product mix matrix $\mathbf{C}$ transforms sectoral factor uses into product factor uses. The amount of factor use/generation associated with product i is equal to the 'weighted' average of sectoral factor uses, $\sum_l c_{il}e_l$, where sectoral output proportions of product i are taken as weights. Strictly speaking, these are not weights as such since, in general, $\sum_l c_{il} \neq 1$. Similarly, industries (or product use destinations) in $\mathbf{U}'$ are transformed into products based on the product mix matrix, $\mathbf{C}$. Note that the ij-th element of $\mathbf{V}'\hat{\mathbf{x}}^{-1}\mathbf{U}'$ is the amount of product j that is used in the production of commodity i and is equal to $\sum_l c_{il}u_{jl}$. Hence, the dependent variable in (10) is factor use by product and the regressors are commodities' net product outputs.

[7] Hence, by construction the columns in $\mathbf{C}$ sum up to one, i.e., $\sum_k c_{kj} = 1$ for all industries j.

Consequently, as in (6), we derived an equation where the factor use is a function of products' net outputs.

The regression-type equation for gross output multipliers derivation can be easily obtained by plugging $\boldsymbol{\mu} = \boldsymbol{\imath}$ and (8) in (3), which results in $\boldsymbol{\imath} = (\mathbf{I} - \hat{\mathbf{q}}^{-1}\mathbf{V}'\hat{\mathbf{x}}^{-1}\mathbf{U}')\boldsymbol{\beta}^o$. Premultiplication by $\hat{\mathbf{q}}$ yields[8]

$$\mathbf{q} = (\hat{\mathbf{q}} - \mathbf{V}'\hat{\mathbf{x}}^{-1}\mathbf{U}')\boldsymbol{\beta}^o. \tag{11}$$

Note that in (10) and (11) the number of both observations and parameters are identical and equal the number of products. Hence, using any statistical technique in order to estimate *stochastic* factor and output multipliers in this case makes a little sense as the system is exactly identified.

Model C: Fixed industry sales structure. Under this assumption the industry-by-industry input matrix $\mathbf{A}$ and the direct factor coefficient vector $\boldsymbol{\mu}$ are given by[9]

$$\mathbf{A} = \hat{\mathbf{x}}\mathbf{V}'^{-1}\mathbf{U}\hat{\mathbf{x}}^{-1}, \tag{12}$$

$$\boldsymbol{\mu}' = \mathbf{e}'\hat{\mathbf{x}}^{-1}. \tag{13}$$

Substituting (12)-(13) in the second expression of (3) yields $\hat{\mathbf{x}}^{-1}\mathbf{e} = (\mathbf{I} - \hat{\mathbf{x}}^{-1}\mathbf{U}'\mathbf{V}^{-1}\hat{\mathbf{x}})\boldsymbol{\beta}$, which if premultiplied by $\hat{\mathbf{x}}$ gives

$$\mathbf{e} = (\hat{\mathbf{x}} - \mathbf{U}'\mathbf{V}^{-1}\hat{\mathbf{x}})\boldsymbol{\beta}, \quad \text{or} \quad \mathbf{e} = (\mathbf{V} - \mathbf{U}')\mathbf{V}^{-1}\hat{\mathbf{x}}\boldsymbol{\beta}. \tag{14}$$

In comparison to the product technology model (6), the fixed industry sales structure model in a regression setting uses *industries'* net sectoral outputs, $(\mathbf{V} - \mathbf{U}')\mathbf{V}^{-1}\hat{\mathbf{x}}$, as independent variables in explaining sectoral factor uses instead of the products' net sectoral outputs, $\mathbf{V} - \mathbf{U}'$. The last are transformed into the former using the transformed product mix matrix $\mathbf{V}^{-1}\hat{\mathbf{x}} = \mathbf{C}'^{-1}$.[10] The regression-type equation for output multipliers can be similarly derived and has the form

$$\mathbf{x} = (\hat{\mathbf{x}} - \mathbf{U}'\mathbf{V}^{-1}\hat{\mathbf{x}})\boldsymbol{\beta}^o, \quad \text{or} \quad \mathbf{x} = (\mathbf{V} - \mathbf{U}')\mathbf{V}^{-1}\hat{\mathbf{x}}\boldsymbol{\beta}^o. \tag{15}$$

Notice that in (14) and (15) the number of both observations and parameters are equal and represent industries. Hence, similar to model B, here also using any

[8]This can be also derived from (10) simply by substituting $\mathbf{x}$ for $\mathbf{e}$.

[9]The second definition follows from $\boldsymbol{\mu}' = \mathbf{x}'\mathbf{V}'^{-1}\hat{\mathbf{e}}\hat{\mathbf{x}}^{-1} = \boldsymbol{\imath}'\mathbf{V}'\mathbf{V}'^{-1}\hat{\mathbf{e}}\hat{\mathbf{x}}^{-1} = \boldsymbol{\imath}'\hat{\mathbf{e}}\hat{\mathbf{x}}^{-1} = \mathbf{e}'\hat{\mathbf{x}}^{-1}$ where we used (2).

[10]Note that $\boldsymbol{\imath}'\mathbf{C} = \boldsymbol{\imath}'$ implies that the identity $\mathbf{C}'^{-1}\boldsymbol{\imath} = \boldsymbol{\imath}$ holds.

statistical techniques to estimate stochastic IO multipliers would make little sense as the two systems are exactly identified. Also note that to implement model C one actually needs to have the same number of industries and products, otherwise the regressors in (14) and (15) due to existence of the inverse matrix $\mathbf{V}^{-1}$ are not (uniquely) defined.

Model D: Fixed product sales structure. Under this assumption the industry-by-industry input matrix $\mathbf{A}$ is derived from

$$\mathbf{A} = \mathbf{V}\hat{\mathbf{q}}^{-1}\mathbf{U}\hat{\mathbf{x}}^{-1}, \tag{16}$$

while the direct factor coefficients vector $\boldsymbol{\mu}$ is given in (13), thus exactly matches that of model C. Using this information together with (3) yields

$$\mathbf{e} = (\hat{\mathbf{x}} - \mathbf{U}'\hat{\mathbf{q}}^{-1}\mathbf{V}')\boldsymbol{\beta}, \tag{17}$$

$$\mathbf{x} = (\hat{\mathbf{x}} - \mathbf{U}'\hat{\mathbf{q}}^{-1}\mathbf{V}')\boldsymbol{\beta}^{o}. \tag{18}$$

The *commodity output proportions* (or market share) matrix is defined as $\mathbf{D} = \mathbf{V}\hat{\mathbf{q}}^{-1}$, whose typical element d_{ij} denotes the share of total output of commodity j that is produced by industry i. Hence, in comparison to model C, the industries' sectoral uses are obtained by weighting the sectoral product uses by the corresponding market share coefficients, $\mathbf{U}'\mathbf{D}'$. Note that for the fixed product sales structure model the number of parameters and observations coincide (which was also the case for models B and C) and represent industries.

Finally, we want to bring to the reader's attention the recent finding of Suh et al. (2010, pp. 341-342) that the product technology assumption and the so-called *by-product method* of Stone (1961) have identical IO multipliers. According to the Stone's method, a secondary product of an industry is considered as by-product "which is related technically to its main production and which forms the principal product of another industry. ... [Hence,] it may be shown as a negative input into the industry in which it is actually produced and as a negative output of the industry in which it is normally produced" (Stone 1961, pp. 39). The input matrix $\mathbf{A}$ and the direct factor coefficient vector $\boldsymbol{\mu}$ according to Stone's method are defined as:

$$\mathbf{A} = (\mathbf{U} - \widetilde{\mathbf{V}}')\widehat{\mathbf{V}}'^{-1}, \tag{19}$$

$$\boldsymbol{\mu}' = \mathbf{e}'\widehat{\mathbf{V}}'^{-1}, \tag{20}$$

where $\mathbf{V} = \widehat{\mathbf{V}} + \widetilde{\mathbf{V}}$ and $\widehat{\mathbf{V}}$ (resp. $\widetilde{\mathbf{V}}$) contains only the values of principal (resp. secondary) products of all industries from $\mathbf{V}$. Now plugging (19)-(20) in (3) yields $\widehat{\mathbf{V}}^{-1}\mathbf{e} = (\mathbf{I} - \widehat{\mathbf{V}}^{-1}(\mathbf{U}' - \widetilde{\mathbf{V}}))\boldsymbol{\beta}$. Premultiplication by $\widehat{\mathbf{V}}$ gives $\mathbf{e} = (\widehat{\mathbf{V}} - (\mathbf{U}' - \widetilde{\mathbf{V}}))\boldsymbol{\beta} = (\mathbf{V} - \mathbf{U}')\boldsymbol{\beta}$ which is exactly equivalent to the corresponding product technology model (6). For the gross output multipliers estimation, instead of the factor use vector $\mathbf{e}$ one has to use the vector of gross outputs $\mathbf{x}$.

Table 2: SUTs framework of the form $\mathbf{y} = \boldsymbol{\Gamma}\boldsymbol{\beta}$ for estimation of IO multipliers $\boldsymbol{\beta}$

	Model A		Model B		Model C		Model D	
	Factor	Output	Factor	Output	Factor	Output	Factor	Output
$\mathbf{y}$	$\mathbf{e}$	$\mathbf{x}$	$\mathbf{V}'\hat{\mathbf{x}}^{-1}\mathbf{e}$	$\mathbf{q}$	$\mathbf{e}$	$\mathbf{x}$	$\mathbf{e}$	$\mathbf{x}$
$\boldsymbol{\Gamma}$	$\mathbf{V} - \mathbf{U}'$		$\hat{\mathbf{q}} - \mathbf{V}'\hat{\mathbf{x}}^{-1}\mathbf{U}'$		$\hat{\mathbf{x}} - \mathbf{U}'\mathbf{V}^{-1}\hat{\mathbf{x}}$		$\hat{\mathbf{x}} - \mathbf{U}'\hat{\mathbf{q}}^{-1}\mathbf{V}'$	
Dimension of $\boldsymbol{\beta}$	Products		Products		Industries		Industries	
Dimension of $\mathbf{y}$	Industries		Products		Industries		Industries	
Symmetric SUTs?	No		No		Yes		No	

Note: For each model the systems for factor and gross output multipliers derivation have different $\mathbf{y}$'s but the same 'regressors' matrix $\boldsymbol{\Gamma}$. The regression-type systems of Model A and by-product method of Stone (1961) are exactly equivalent.

All the four basic transformation models written in terms of SUTs in the 'regression' form of $\mathbf{y} = \boldsymbol{\Gamma}\boldsymbol{\beta}$ are summarized in Table 2. Given that the corresponding systems of models B, C and D have equal number of unknowns and observations, it makes little sense to apply statistical techniques to these models in order to estimate stochastic IO multipliers. If such estimation is done, it will give in any case very narrow uncertainty ranges for the parameters of interest, which will not capture adequately the uncertainty in the SUTs data. This is, however, not the case with model A, hence it provides a nice framework of applying various statistical techniques in order to take into account the data uncertainty problem. Moreover, the parameters of interest refer to products that are more homogenous in nature than industries. Therefore analyses based on model A's SUTs framework provide more insights at the more homogenous product level rather than industry level.

3 A Bayesian approach

One can, of course, run OLS on (6) and (7) by adding an error term to their right-hand sides, but then in the published SUTs (many) products need to be aggregated in order to obtain sufficient degrees of freedom for OLS regression. However, aggregation might very well lead, in our view, to a significant loss of information on the heterogeneities of aggregated industry-product links reported in SUTs. We believe that huge human, time and financial efforts of national statistical offices that are put into the construction of SUTs with more products than industries *must* be used effectively in the practical applications of these tables. We choose Bayesian approach as our estimation philosophy, because Bayesian methods generally allow for the number of unknowns to be larger than the number of observations and are based on a sound probability theory. We argue that by using the SUTs framework and the corresponding Bayesian approach both the inherent uncertainty (stochasticity) of SUTs and the related data and the individual heterogeneities of specific product-industry interrelationships are adequately taken into account. The presented below Bayesian approach is also used by Temurshoev (2012) in estimating gross output feedback and spillover effects for forty economies of the world for the period of 1995-2009.

Consider the linear regression model

$$\mathbf{y} = \mathbf{\Gamma}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \tag{21}$$

where $\boldsymbol{\varepsilon}$ denotes the vector of regression errors. To compute $\boldsymbol{\beta}$ in (21), we make the following assumptions:[11]

1. For all observations i, $\varepsilon_i \sim N(0, \sigma^2\omega_i)$.
2. All elements in $\mathbf{\Gamma}$ are either fixed, or they are random variables that are independent of all elements of $\boldsymbol{\varepsilon}$, that is, the parameters $\boldsymbol{\eta}$ of the probability density function of $\mathbf{\Gamma}$, $p(\mathbf{\Gamma}|\boldsymbol{\eta})$, do not include $\boldsymbol{\beta}$, σ^2 and ω_i's.

In Bayesian literature it is convenient to work with error precisions rather than variances. Hence, in what follows we work with the constant and varying components of the error precision of $h = 1/\sigma^2$ and $\lambda_i = 1/\omega_i$. That is, the covariance matrix is $h^{-1}\mathbf{\Lambda}^{-1}$, where $\mathbf{\Lambda}$ is a diagonal matrix with λ_i's along its diagonal and zeros otherwise. Given our assumptions and using the properties of the multivariate Normal distribution, the *likelihood function* can be written as[12]

$$p(\mathbf{y}|\boldsymbol{\beta}, h, \boldsymbol{\lambda}) = \left(\frac{h}{2\pi}\right)^{n/2} |\mathbf{\Lambda}|^{1/2} \exp\left[-\frac{h}{2}(\mathbf{y} - \mathbf{\Gamma}\boldsymbol{\beta})'\mathbf{\Lambda}(\mathbf{y} - \mathbf{\Gamma}\boldsymbol{\beta})\right], \tag{22}$$

[11] For detail discussion of the approach used in this paper, see Koop (2003, Chapter 6).

[12] To be more precise, the likelihood function is $p(\mathbf{y}, \mathbf{\Gamma}|\boldsymbol{\beta}, h, \boldsymbol{\lambda}, \boldsymbol{\eta})$. However, the second assumption above implies that the likelihood can written as $p(\mathbf{y}, \mathbf{\Gamma}|\boldsymbol{\beta}, h, \boldsymbol{\lambda}, \boldsymbol{\eta}) = p(\mathbf{\Gamma}|\boldsymbol{\eta})p(\mathbf{y}|\mathbf{\Gamma}, \boldsymbol{\beta}, h, \boldsymbol{\lambda})$, hence without loss of information we can simply work with the likelihood function conditional on $\mathbf{\Gamma}$, $p(\mathbf{y}|\mathbf{\Gamma}, \boldsymbol{\beta}, h, \boldsymbol{\lambda})$. For notational convenience, we suppress the dependence on $\mathbf{\Gamma}$ throughout the paper.

where n is the number of observations.

Next, we need to define the *prior density* $p(\boldsymbol{\beta}, h, \boldsymbol{\lambda})$. We use the widely used independent Normal-Gamma prior for $\boldsymbol{\beta}$ and h, that is the prior density is

$$p(\boldsymbol{\beta}, h, \boldsymbol{\lambda}) = p(\boldsymbol{\beta})p(h)p(\boldsymbol{\lambda}), \tag{23}$$

with

$$\begin{aligned} p(\boldsymbol{\beta}) &= f_N(\boldsymbol{\beta}|\underline{\boldsymbol{\beta}}, \underline{\mathbf{V}}), && (24) \\ p(h) &= f_G(h|\underline{s}^{-2}, \underline{v}), && (25) \end{aligned}$$

where $f_N(\boldsymbol{\beta}|\underline{\boldsymbol{\beta}}, \underline{\mathbf{V}})$ indicates that $\boldsymbol{\beta}$ has multivariate Normal distribution with mean $\underline{\boldsymbol{\beta}}$ and covariance matrix $\underline{\mathbf{V}}$, and $f_G(h|\underline{s}^{-2}, \underline{v})$ defines h having Gamma distribution with mean $\underline{s}^{-2} > 0$ and degrees of freedom $\underline{v} > 0$.

We consider heteroscedasticity of an *unknown* form, thus assume that the λ_is are independent and identically distributed (i.i.d.) draws from the Gamma distribution with mean 1 and degrees of freedom v_λ,

$$p(\boldsymbol{\lambda}|v_\lambda) = \prod_{i=1}^{n} f_G(\lambda_i|1, v_\lambda). \tag{26}$$

This implies that the error variances are different from each other, but they are taken from the same distribution. “Thus, we can have a very flexible model, but enough structure is still imposed to allow for statistical inference” (Koop 2003, p. 125). Following the literature, the prior distribution for the degrees of freedom v_λ is chosen to be the Gamma density with two degrees of freedom,

$$p(v_\lambda) = f_G(v_\lambda|\underline{v}_\lambda, 2), \tag{27}$$

which is the exponential density. Note that the prior for $\boldsymbol{\lambda}$ is specified in two steps, (26) and (27). In the literature this is called *hierarchical prior.* Alternatively, using probability rules the prior for $\boldsymbol{\lambda}$ can be simply written as $p(\boldsymbol{\lambda}|v_\lambda)p(v_\lambda)$.

It is important to mention that such a treatment of heteroscedasticity is equivalent to the so-called *scale mixture of Normals* models. That is, the assumption that ε_i are independent $N(0, h^{-1}\lambda_i^{-1})$ with prior for λ_i given in (26) is equivalent to the assumption that the distribution of ε_i is a mixture (or weighted average) of different Normal distributions with different variances (i.e., different scales) but the same means (i.e., zero means). A crucial result due to Geweke (1993) is that when such mixing is performed using $f_G(\lambda_i|1, v_\lambda)$ densities, the linear regression model with the mixture of Normals errors is exactly equivalent to a linear regression model with i.i.d. Student-t errors with mean zero and v_λ degrees of freedom. Hence, the presented model allows for *more flexible* error distribution, because the Normal distribution is a special case of the Student-t distribution when $v_\lambda \to \infty$. Therefore, the above two-step error prior specification allows us to free up the assumption of Normal errors.

Given the data, what and how can we learn about the parameters of interest? The core of Bayesian analysis in answering this crucial question states that "the posterior is proportional to the likelihood times the prior", i.e., $p(\boldsymbol{\beta}, h, \boldsymbol{\lambda}|\mathbf{y}) \propto p(\mathbf{y}|\boldsymbol{\beta}, h, \boldsymbol{\lambda})p(\boldsymbol{\beta}, h, \boldsymbol{\lambda})$. If we perform this multiplication, the joint posterior turns out not to take the form of any well-known and understood density, and thus it cannot be directly used for simple posterior inference. Therefore, we need to use posterior simulation methods. If it turns out that draws can be taken from the so-called *full conditional posterior densities* of the parameters of interest, then an appropriate Markov Chain Monte Carlo (MCMC) algorithm can be used such that these draws will be the valid draws from the joint posterior distribution (see e.g., Gilks et al. 1996). Without going into the details, the posterior conditional densities of interest to us can be shown to have the following forms:

$$\begin{aligned} p(\boldsymbol{\beta}|\mathbf{y}, h, \boldsymbol{\lambda}) &= f_N(\boldsymbol{\beta}|\overline{\boldsymbol{\beta}}, \overline{\mathbf{V}}), && (28)\\ p(h|\mathbf{y}, \boldsymbol{\beta}, \boldsymbol{\lambda}) &= f_G(h|\overline{s}^{-2}, \overline{v}), && (29)\\ p(\lambda_i|\mathbf{y}, \boldsymbol{\beta}, h, v_\lambda) &= f_G\left(\lambda_i \middle| \frac{v_\lambda + 1}{h\varepsilon_i^2 + v_\lambda}, v_\lambda + 1\right), && (30)\\ p(v_\lambda|\mathbf{y}, \boldsymbol{\beta}, h, \boldsymbol{\lambda}) &\propto \left(\frac{v_\lambda}{2}\right)^{nv_\lambda/2} \Gamma\left(\frac{v_\lambda}{2}\right) \exp(-\eta v_\lambda), && (31) \end{aligned}$$

where $\Gamma(a) \equiv \int_0^\infty t^{a-1} \exp(-t)dt$ is the Gamma function and

$$\begin{aligned} \overline{\mathbf{V}} &= (\underline{\mathbf{V}}^{-1} + h\mathbf{X}'\boldsymbol{\Lambda}\mathbf{X})^{-1}, \\ \overline{\boldsymbol{\beta}} &= \overline{\mathbf{V}}(\underline{\mathbf{V}}^{-1}\underline{\boldsymbol{\beta}} + h\mathbf{X}'\boldsymbol{\Lambda}\mathbf{y}), \\ \overline{v} &= n + \underline{v}, \\ \overline{s}^2 &= \left[(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'\boldsymbol{\Lambda}(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) + \underline{v}\underline{s}^2\right]/\overline{v}, \\ \eta &= \frac{1}{\underline{v}_\lambda} + \frac{1}{2}\sum_{i=1}^n \left[\ln\left(\lambda_i^{-1}\right) + \lambda_i\right]. \end{aligned}$$

We will not delve into the interpretations of the above results, since the interested reader can find these in any book on Bayesian econometric methods (see e.g., Koop 2003). We only mention that Bayesian theory makes it possible to combine the prior and data information in an intuitively appealing and sensible way, which is probability theory-based approach. The densities (28)-(30) have well-known forms, hence empirically it is easy to take draws from them. In such cases, Bayesians use a popular posterior simulator called *Gibbs sampler*, whose strategy is taking draws from the full conditional posterior distributions of parameters conditional on the previous draws of all the remaining parameters. After discarding initial replications of all parameters draws, the so-called *burn-in replications*, it can be shown that under mild conditions the remaining draws are valid draws from the corresponding joint posterior distribution (see e.g., Geweke 1999). However, we cannot use only Gibbs sampler in our case because the density (31) is a non-standard density. We will use a posterior simulator called *random walk chain Metropolis-Hastings* algorithm to take draws from (31). The details of this simulator are not given here due to space limitation, and the interested reader is referred to Chib and Greenberg (1995) and Geweke (2005, Chapter 4). So we use Gibbs sampler as posterior simulator for (28)-(30), and the Metropolis-Hastings algorithm as posterior simulator for the conditional density (31). Such mixture of posterior simulators is perfectly acceptable, and in the literature is referred to as *Metropolis-within-Gibbs* algorithm. The derived posterior conditional densities tell us everything about the distribution of the parameters of interest. For example, the mean of the posterior distribution of a parameter is considered to be its estimate.

4 Empirical application

Using the international supply and use tables (SUTs) dataset constructed by the World Input-Output Database (WIOD) project, in this section we present the results of application of the Bayesian methodology discussed in Section 3 to the SUTs framework of the product technology model (see Section 2). The database includes time series of national and international SUTs, world input-output (IO) tables, and various socio-economic and environmental accounts for 40 major economies of the world at the level of 35 industries and 59 products (for details, see Timmer 2012). It provides harmonized data for 15 years, from 1995 to 2009, for which we want to analyze the development of *global* carbon dioxide emission multipliers. The unit of measurement of CO_2 emissions data is kilotonnes (kt). The international SUTs, which distinguish between the origin and destination countries of the intermediate and final uses, are valued at basic prices and expressed in previous year prices. The world SUTs with $n = 40$ countries used in our empirical application have the form

$$\mathbf{U} = \begin{bmatrix} \mathbf{U}^{11} & \mathbf{U}^{12} & \cdots & \mathbf{U}^{1n} \\ \mathbf{U}^{21} & \mathbf{U}^{22} & \cdots & \mathbf{U}^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{U}^{n1} & \mathbf{U}^{n2} & \cdots & \mathbf{U}^{nn} \end{bmatrix} \text{ and } \mathbf{V} = \begin{bmatrix} \mathbf{V}^{1} & \mathbf{O} & \cdots & \mathbf{O} \\ \mathbf{O} & \mathbf{V}^{2} & \cdots & \mathbf{O} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{O} & \mathbf{O} & \cdots & \mathbf{V}^{n} \end{bmatrix}, \tag{32}$$

where the ijth element of $\mathbf{U}^{rs}$ indicates the amount of intermediate input of product i from country r used by industry j in country s, $\mathbf{V}^r$ is the industry-by-product make matrix of country r, and $\mathbf{O}$ is the null matrix of appropriate dimension.[13]

We start with the elicitation of the prior hyperparameters $\underline{\boldsymbol{\beta}}$, $\underline{\mathbf{V}}$, $\underline{s}^{-2}$, $\underline{v}$ and $\underline{v}_\lambda$. It is highly desirable to choose theory-based priors for global CO_2 multipliers, $\boldsymbol{\beta}$. The prior multipliers are computed on the base of the industry technology assumption (model B) discussed in Section 2. This is an alternative technology assumption that can be used as an analytical device to transform SUTs into the product-by-product IO matrices. Mathematically, the priors are derived from $\underline{\boldsymbol{\beta}}' = \underline{\boldsymbol{\mu}}'(\mathbf{I} - \mathbf{A})^{-1}$ where $\mathbf{A}$ and $\underline{\boldsymbol{\mu}}$ are given in (8) and (9), respectively. Note that computation of the Leontief-inverse-based factor multipliers under the industry technology assumption is always feasible irrespective of whether the underlying SUTs are square or rectangular.

We note that for the considered SUTs framework we need to impose linear constraints on the coefficients of the model, $\boldsymbol{\beta}$. This is due to the underlying IO theory which states that the global factor multipliers cannot be less than the corresponding direct factor coefficients (i.e., (3) implies that $\boldsymbol{\beta} \geq \boldsymbol{\mu}$). Since we cannot compute $\boldsymbol{\mu}$ for the product technology model (because SUTs are rectangular), we use instead the direct factor coefficients $\underline{\boldsymbol{\mu}}$ from the industry technology model. Although the two direct factor coefficients should not be too different from each other, they are not exactly equal either. Hence, we take the constraints to be of the form $\boldsymbol{\beta} \geq 0.8\underline{\boldsymbol{\mu}}$, i.e., model A's CO_2 emission multipliers are constrained below by values equal to 80% of model B's direct CO_2 coefficients. Imposing linear inequality constraints of any kind is quite simple within the Bayesian analysis, since they can be imposed

[13]For an alternative make matrix regionalized world SUTs framework, see Jackson and Schwarm (2011). However, as mentioned by the authors the preference for the use-regionalized SUTs framework (32) "is based on the foundation of production behavior consistent with the demand-driven IO model rather than market share behavior, which appears to be more consistent with a supply-driven IO model" (p. 195).

through the prior. In our empirical study instead of (24) we use the prior given by

$$p(\boldsymbol{\beta}) = f_N(\boldsymbol{\beta}|\underline{\boldsymbol{\beta}}, \underline{\mathbf{V}})1(\boldsymbol{\beta} \geq 0.8\underline{\boldsymbol{\mu}}),$$

where $1(\boldsymbol{\beta} \geq 0.8\underline{\boldsymbol{\mu}})$ is the indicator function which equals one if global CO_2 multipliers are at least 80% of model B's direct CO_2 coefficients, and zero otherwise. Using this prior, the conditional posterior of $\boldsymbol{\beta}$ can be derived as

$$p(\boldsymbol{\beta}|\mathbf{y}, h, \boldsymbol{\lambda}) = f_N(\boldsymbol{\beta}|\overline{\boldsymbol{\beta}}, \overline{\mathbf{V}})1(\boldsymbol{\beta} \geq 0.8\underline{\boldsymbol{\mu}}), \tag{33}$$

which is used in our empirical application instead of (28). Thus, the conditional posterior of $\boldsymbol{\beta}$ used in our Metropolis-within-Gibbs algorithm is the truncated multivariate Normal distribution. To take draws from this truncated distribution we use an efficient simulation strategy proposed by Geweke (1991).

Apparently, the multipliers under the two different technology assumptions are different: while it will be the case that under the industry technology model all products representing one industry will have more or less similar multipliers, that should not definitely be the case with the product technology model. Hence, we define the prior variance $\underline{\mathbf{V}}$ such that 95% of the probability in the prior density is located within the interval that allows the value of the prior coefficient to be $a \geq 2$ times larger or smaller than $\underline{\beta}_j$s computed under the industry technology assumption. We use the useful rule-of-thumb that states that approximately 95% of the outcomes of a random variable β_j will fall within two standard deviations of its mean $\underline{\beta}_j$. Thus, if we want to have the corresponding upper "bound" to be as large as a times its mean, then from $\underline{\beta}_j + 2\sigma_{\beta_j} = a\underline{\beta}_j$ we derive the prior standard deviation of β_j as $\sigma_{\beta_j} = 0.5(a-1)\underline{\beta}_j$. Thus, the prior variance of β_j is chosen to be

$$var(\beta_j) = \frac{(a-1)^2}{4}\underline{\beta}_j^2 \tag{34}$$

for all products j. The prior covariance matrix $\underline{\mathbf{V}}$ is then defined as a diagonal matrix with jj-th element equal to $var(\beta_j)$ and zero otherwise. That is, following the common practice, we set all the prior covariances to zero because it is usually hard to make reasonable guesses about the covariance values. In our empirical application we set $a = 2.5$ in (34) being confident that such choice of prior information does not miss any reasonable value of β_j under the product technology assumption. That is, in the majority of cases IO multipliers under the product technology model cannot be larger or smaller than those under the industry technology model by more than

250%. This, in fact, implies that we have a our relatively non-informative prior for $\boldsymbol{\beta}$. In the setting of this model the prior can be interpreted as arising from a fictitious data set, where, for example, the prior degrees of freedom of the constant error precision h, $\underline{v}$, can be interpreted as a prior sample size. Given that our prior for $\boldsymbol{\beta}$ is somewhat non-informative, we set $\underline{v} = 0.01n = 14$, where the number of observations is $n = 1400$ $(= 35 \times 40)$. Strictly speaking, we are assuming that our prior information about h has 1% of the weight as the data information. This means that we want our results to be driven mainly by the data information rather than the priors.[14] Further, given that $\underline{s}^{-2}$ is the prior mean of h, we take the variance of $\mathbf{y} - \boldsymbol{\Gamma}\underline{\boldsymbol{\beta}}$ as a reasonable prior guess for $\underline{s}^2$. Finally, the degrees of freedom hyperparameter for the prior of varying error precisions is set to $\underline{v}_\lambda = 25$, "a value which allocates substantial prior weight both to very fat-tailed distributions (e.g., $\underline{v}_\lambda < 10$), as well as error distributions which are roughly Normal (e.g., $\underline{v}_\lambda > 40$)" (Koop 2003, p. 129). We do not give here the details of the random walk chain Metropolis-Hastings algorithm with Normal increment random variable used for simulations from distribution (31) in our Metropolis-within-Gibbs simulator, and refer the interested reader to Koop (2003, p. 129) whose presented steps we closely follow here.[15]

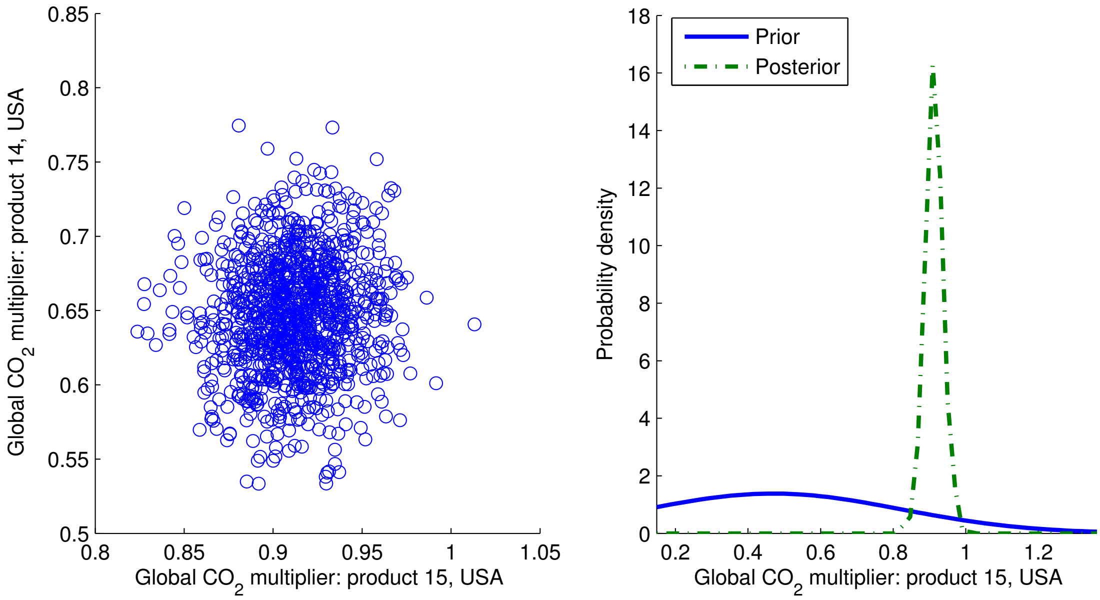
We discard an initial 200 burn-in replications and retain the subsequent 1100 replications for deriving the estimates of the parameters of our model for each year separately. The derived Markov Chain Monte Carlo (MCMC) diagnostics of the parameters (such as numerical standard errors and Geweke's (1992) convergence diagnostic) confirmed the convergence of our MCMC algorithms. Due to space constraints, we leave out the details of the MCMC diagnostics. To give a flavor of Bayesian analysis, in Figure 1 we illustrate two arbitrary chosen examples of the posterior results for 2009. In the first subplot we show the 1100 retained outcomes of our MCMC simulations of global CO_2 multipliers for two US products: Wood

[14] Our sensitivity analyses confirm the robustness of our posterior mean results to choosing higher values of a and $\underline{v}$. Of course, when we choose rather large values for $\underline{v}$, the priors will have more influence on the derived results (but we would like to minimize this effect and let the data speak for themselves). For example, when we choose $\underline{v} = 0.1n = 140$, the posterior means are practically robust, but the corresponding uncertainty ranges are somewhat wider than those with $\underline{v} = 14$. This is not surprising because the priors' variances are already quite large, and if we give more weight to priors, we should have larger uncertainty ranges of the parameters of interest.

[15] We use MATLAB software in performing the Bayesian approach for this study and adopt for our purposes the relevant programs of Gary Koop's Bayesian Econometrics (Koop 2003) and James LeSage's Econometrics Toolbox (LeSage 1999). We made use of the advanced high performance computing facility of Millipede cluster offered by the Center for High Performance Computing and Visualisation of the University of Groningen. We used one node in our computations that has 24 GB memory.

and products of wood and cork (except furniture), articles of straw and plaiting materials (WIOD code: 14) and Pulp, paper and paper products (15). The space of this figure represents the range of possible values of global CO_2 multipliers for the two mentioned products (from the multivariate parameter $\boldsymbol{\beta}$) in the mature stage of the MCMC simulation. That is, the graph depicts the common stationary distribution of the global CO_2 multipliers for the US products 14 and 15 that is equal to the target distribution.

Figure 1: Illustration of two examples of the posterior results for 2009



The second graph of Figure 1 illustrates the prior and posterior of the multipliers of interest for Pulp, paper and paper products (15) of the US in 2009. The standard Leontief IO approach gives the multiplier value of 0.4694 CO_2 kt per million USD under the industry technology assumption, which is chosen as the mean of the (truncated) Normal prior for this parameter. The graph reveals that this prior has rather large variance. However, the corresponding posterior (derived from the retained 1100 replications) having significantly lower variance is much more informative. Our estimate of the global CO_2 multiplier of the product of interest is the mean of this posterior which equals 0.9122 CO_2 kt per million USD. The corresponding 95% *highest posterior density interval* (HPDI) is [0.8699, 0.9574]. Hence, we observe that the derived estimate is 94.34% larger than its prior mean representing model B's estimate (which is the largest difference found for the two models estimates for 2009), and, moreover, the last is not included in the 95% HPDI of our global CO_2

emission multiplier estimate that is based on the product technology assumption.

In terms of the WIOD product and industry classifications, we find that industry "Pulp, paper, printing and publishing" includes two types of products: Pulp, paper and paper products (15) and Printed matter and recorded media (16). The estimates of the 2009 global CO_2 emission multipliers of the second product are also shown in Table 3. We observe that the Leontief-inverse-based global CO_2 multipliers of products 15 and 16 are, respectively, 0.4694 and 0.3684. These estimates are based on the industry technology assumption, which in this case states that the industry Pulp, paper, printing and publishing has its own specific way of production irrespective of its product mix, i.e., irrespective of weather it produces Pulp, paper and paper products or Printed matter and recorded media.

Table 3: Global multipliers and the number of production ties, US, 2009

Products	Industry technology	Product technology		Number of significant direct linkages in SUTs*		
		Estimate	95% HPDI	$> 0.1\%$	$> 0.01\%$	$> 0.001\%$
Pulp, paper and paper products (15)	0.4694	0.9122	[0.8699, 0.9574]	42	146	424
Printed matter and recorded media (16)	0.3684	0.5963	[0.5384, 0.6565]	19	79	278
Electrical energy, gas, steam and hot water (32)	4.045	5.0768	[5.0638, 5.0898]	31	36	127
Collected & purified water, its distribution services (33)	4.045	4.8082	[4.6715, 4.9484]	26	32	38

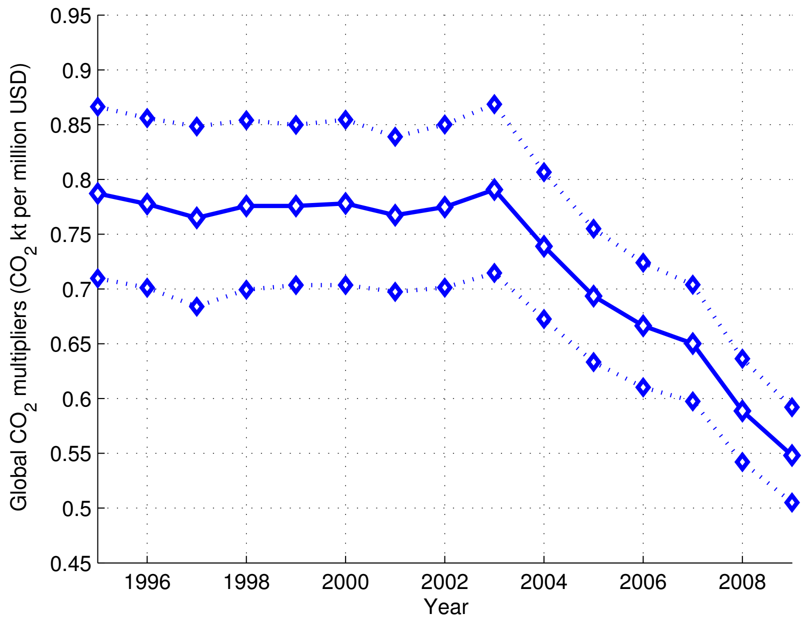
Note: *The number of production ties in the SUTs system of the considered products are equal to the number of significant positive entries in the corresponding column of $|\mathbf{V} - \mathbf{U}'|$. For example, for case "$> 0.1\%$" we count such positive entries only when the absolute values of the net outputs are larger than 0.1% of the overall sum of the absolute values of the net outputs for each product.

Although the Leontief-inverse-based CO_2 multiplier for product 15 is 27% higher than that for product 16, the corresponding difference for the Bayesian estimates is twice as large and equals 53%. Bayesian results are based on the product technology assumption, which treats each product in a separate way irrespective of the industry where it is produced. The last three columns of Table 3 explain this difference. For the two products we count the number of significant production linkages within the 'world' production structure. For example, if we count all the linkages with absolute values of net outputs greater than 0.01% of the overall sum of the absolute values of the net outputs for a particular product in our 40 country framework, we find that the net output vector of Pulp, paper and paper products (i.e., the corresponding column of $\mathbf{V} - \mathbf{U}'$) has significant links to 146 sectors. The corresponding number for the second product Printed matter and recorded media is only 79. That is, we observe that Pulp, paper and paper products has more extensive linkages within the forty-country SUTs system than Printed matter and recorded media. Therefore, the product specific global CO_2 emission multipliers must be larger (resp. lower) for the product with more (resp. less) extensive production-usage linkages, and that is exactly what we arrive at. Note also that model B's estimate of the global CO_2 multiplier for product 16, 0.3684, is lower than the corresponding Bayesian estimate of 0.5963 and is not contained in the derived 95% HPDI.

The last two rows of Table 3 show similar information for the US Electrical energy, gas, steam and hot water (32) and Collected and purified water, distribution services of water (33), which make the sector Electricity, gas and water supply. The industry technology model derives a global CO_2 multiplier of 4.045 for both products, but the product-specific characteristics are again taken fully into account by the Bayesian estimates, which are different from the first estimates and again consider the extensiveness of products linkages in the production system. Note also that the Bayesian estimates of the multipliers of interest for products 32 and 33 are, respectively, 25.5% and 18.9% larger than their industry technology estimates which are again not included in the 95% HPDIs of the product technology estimates.

Next, we analyze the development of the global CO_2 multipliers over the period of 1995-2009. First, we derive the overall results for all countries and all products on an annual basis. There are 2360 ($= 59 \times 40$) estimates of the multipliers for each year, thus we take their weighted average as an overall indicator of the multipliers of interest, where the weights are the shares of product outputs in our forty-country setting. Using the corresponding posterior distributions, such commodity-weighted results of the global CO_2 multipliers and their corresponding 90% HPDIs were derived, which are graphed in Figure 2. From this graph we observe that the mean of the weighted average global CO_2 multipliers within our forty-country setting was 0.79 CO_2 kt per million USD in 1995, stayed more or less stable at this level up until 2003, and consequently steadily decreased over time reaching the value of 0.55 in 2009. The corresponding 90% HPDIs are roughly 0.08 of magnitude far away from the means from 1995 to 2003, and consequently the uncertainty range starts decreasing and reaches the value of ± 0.04 at the end of the considered period. In the IO parlance, a million USD increase in average final demand within our forty-country system generated, on average, an extra of 0.79 kt CO_2 emissions by all forty countries in 1995. However, in 2009, on average, 0.55 kt CO_2 emissions per million USD average final demand were generated. Thus, we see a huge decrease of -30.4% of the mean average global CO_2 multipliers in 2009 relative to 1995. The corresponding

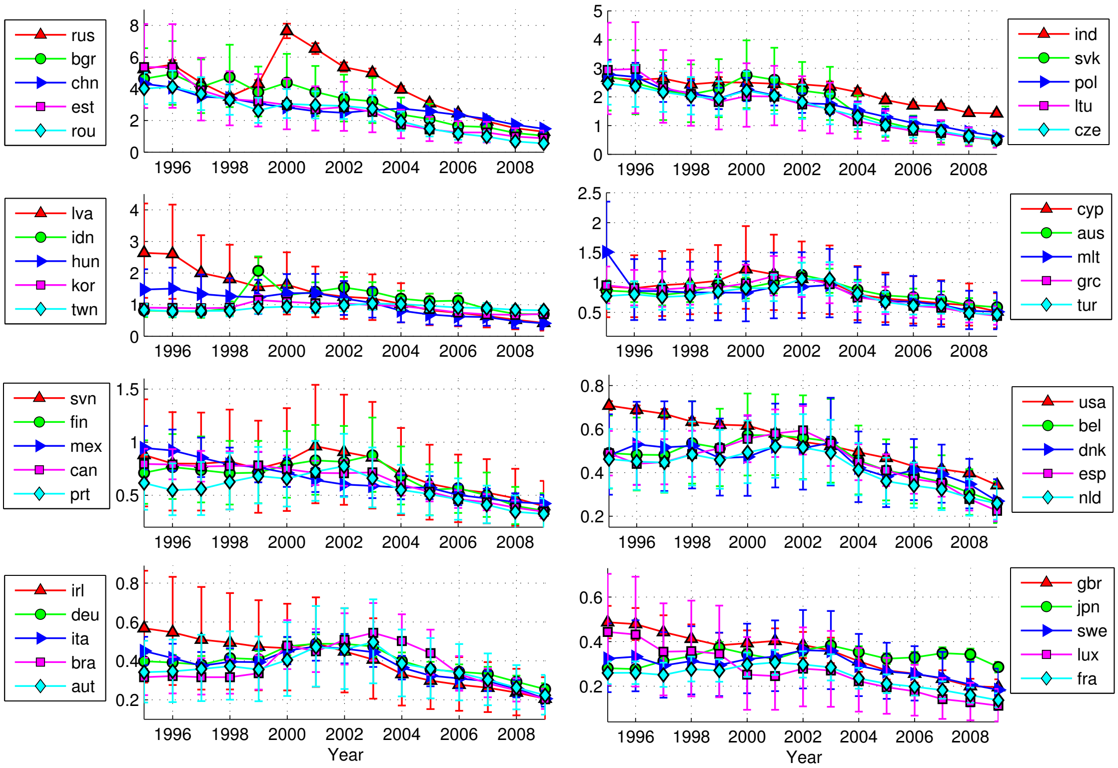
Figure 2: Product output-weighted aggregate results, 1995-2009



changes for the lower and upper 90% HPDIs bounds are -28.8% and -31.7%. These findings imply that consumer responsibility for generating CO_2 emissions declined dramatically over the period under study. In Temurshoev (2012) it has been found that the overall degree of production interdependencies among the considered forty countries largely increased over the same period. Hence, the decline in the global CO_2 emission multipliers observed in Figure 2 must be due to a significant decrease in the *direct* carbon dioxide emission intensities. Otherwise, if these direct intensities would not decrease and at least stayed unchanged, then an increase in the degree of production interdependencies would automatically imply an increase in the global CO_2 multipliers over time, which is not observed in Figure 2.

From the overall average figures discussed above we cannot say anything about the country-specific global CO_2 multipliers. Hence, next we compute commodity output-weighted CO_2 multipliers for each country separately. The derived means and corresponding 90% HPDIs (represented by bars) of these multipliers are given in Figure 3. To make the plots readable, we graph the results for five countries within one subplot and include countries according to the size of their 1995-2009 average of the product output-weighted global CO_2 multipliers in descending order in the subplots from the top to the bottom of Figure 3. The list of WIOD country acronyms and the description of products and industries are given in Appendix 1.

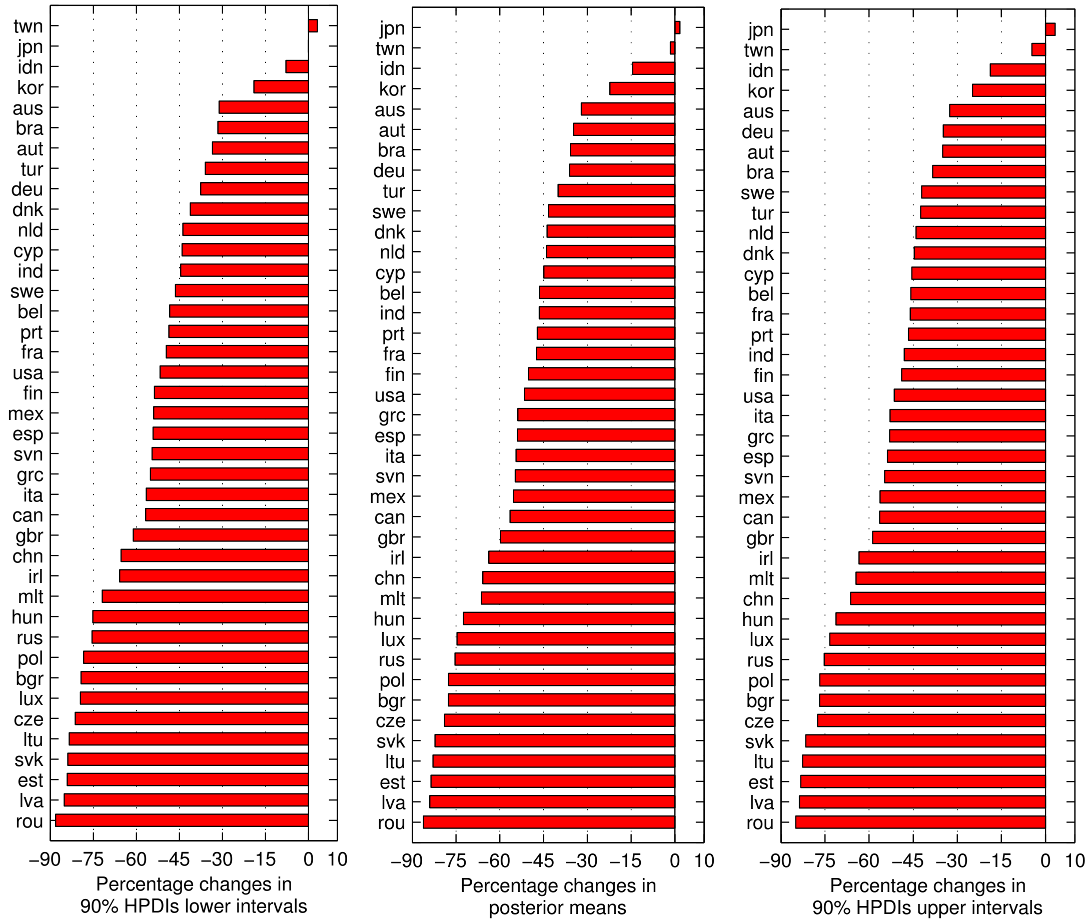
Figure 3: Country-specific global CO_2 multipliers, 1995-2009



Countries with the largest 1995-2009 average of product-level global CO_2 multipliers are (the multipliers averages are given in parentheses): Russia (4.1163), Bulgaria (3.1282), China (2.7931), Estonia (2.6542), Romania (2.4152), India (2.1940), Slovak Republic (1.7829), Poland (1.7318), Lithuania (1.6199), and Czech Republic (1.5847). On the other hand, countries with the smallest average CO_2 multipliers include Ireland (0.4010), Germany (0.3932), Italy (0.3747), Brazil (0.3731), Austria (0.3653), United Kingdom (0.3477), Japan (0.3313), Sweden (0.2913), Luxembourg (0.2638) and France (0.2415). We clearly observe a lot of heterogeneity in terms of the multipliers values across countries, but a finding common to almost all considered countries observed from Figure 3 is the overall decreasing trend of the development of the global CO_2 multipliers during the period of 1995-2009. The percentage changes of the corresponding posterior means and the 90% HPDIs' bounds for the year of 2009 relative to 1995 are shown in Figure 4.

Figure 4 shows that only for Japan the change in the global CO_2 multipliers is positive, and the corresponding change in posterior means is 1.71% (these changes for the 90% HPDIs lower and upper bounds are -0.08% and 3.27%, respectively). For Taiwan we see a positive change in the lower 90% HPDIs bounds of 3.06%, however

Figure 4: Changes in the global CO_2 multipliers (%), 2009 vs. 1995



the corresponding figures for the means and upper 90% HPDIs intervals are -1.57% and -4.63%, respectively. For the rest of the countries, the changes in posterior means and 90% HPDIs intervals of the global CO_2 multipliers are all negative. Countries that achieved a dramatic decrease of more than -75% (in absolute terms) in the average means and 90% HPDIs bounds of the multipliers of interest are Romania, Latvia, Estonia, Lithuania, Slovak Republic, Czech Republic, Bulgaria, Poland and Russia.

The main advantage of using SUTs framework is estimating IO multipliers at the product level, thus in what follows we discuss our results for products. Table 4 gives the mean of the average global CO_2 multipliers, including their 70% HPDIs, over the period of 1995-2009 (in descending order of the reported Bayesian estimates). The underlying corresponding annual estimates were derived as weighted averages of the posterior means and 70% HPDIs, where we have used country-level total commodity output shares (that vary from year to year) as the corresponding weights.

Table 4: Product-level average global CO_2 multipliers for 1995-2009

Product code	Average global CO_2 multipliers			Model B		Difference (in %)	Outside 70% HPDIs?
	Mean	70% HPDI	Rank	Estimate	Rank		
32	6.4855	[6.4439, 6.5255]	1	5.7543	1	12.7	Yes
33	6.1635	[5.8363, 6.4121]	2	5.6244	2	9.6	Yes
20	2.5043	[2.4423, 2.5616]	3	2.3402	3	7.0	Yes
40	2.1906	[2.0350, 2.3164]	4	2.0629	4	6.2	No
41	1.8745	[1.7707, 1.9643]	5	1.7817	5	5.2	No
17	1.5753	[1.5239, 1.6233]	6	1.4973	6	5.2	Yes
7	1.4822	[1.2346, 1.6610]	7	1.3201	11	12.3	No
22	1.4717	[1.4079, 1.5307]	8	1.3464	8	9.3	Yes
21	1.4619	[1.4142, 1.5074]	9	1.3730	7	6.5	Yes
5	1.4403	[1.2961, 1.5794]	10	1.3329	10	8.1	No
4	1.3556	[1.1657, 1.4940]	11	1.2279	12	10.4	No
8	1.3516	[1.1814, 1.4835]	12	1.3431	9	0.6	No
18	1.1743	[1.1451, 1.2014]	13	1.1219	13	4.7	Yes
39	0.8690	[0.8392, 0.8955]	14	0.8490	14	2.4	No
15	0.8334	[0.7638, 0.8948]	15	0.6901	16	20.8	Yes
16	0.7482	[0.6725, 0.8163]	16	0.6564	21	14.0	Yes
19	0.7371	[0.6879, 0.7820]	17	0.7216	15	2.1	No
34	0.6986	[0.6865, 0.7102]	18	0.6839	17	2.1	Yes
11	0.6972	[0.6199, 0.7654]	19	0.6583	20	5.9	No
14	0.6880	[0.6132, 0.7535]	20	0.6778	18	1.5	No
23	0.6626	[0.6272, 0.6944]	21	0.6679	19	-0.8	No
3	0.6410	[0.4847, 0.7589]	22	0.5764	30	11.2	No
12	0.6402	[0.5478, 0.7203]	23	0.6446	22	-0.7	No
2	0.6352	[0.5141, 0.7305]	24	0.5799	28	9.5	No
29	0.6281	[0.5494, 0.6966]	25	0.6105	24	2.9	No
10	0.6221	[0.5044, 0.7166]	26	0.5790	29	7.4	No
28	0.6121	[0.5819, 0.6399]	27	0.6016	25	1.7	No
30	0.6107	[0.5492, 0.6659]	28	0.6248	23	-2.3	No
1	0.6102	[0.5856, 0.6331]	29	0.5838	26	4.5	Yes
9	0.5982	[0.5810, 0.6147]	30	0.5806	27	3.0	Yes
13	0.5697	[0.4235, 0.6873]	31	0.5463	32	4.3	No
27	0.5423	[0.4468, 0.6226]	32	0.5438	33	-0.3	No
26	0.5315	[0.4590, 0.5927]	33	0.5278	34	0.7	No
25	0.5279	[0.4667, 0.5827]	34	0.5561	31	-5.1	No
24	0.5213	[0.4394, 0.5914]	35	0.5155	35	1.1	No
42	0.4973	[0.4482, 0.5384]	36	0.5016	36	-0.9	No
38	0.4705	[0.4461, 0.4929]	37	0.4552	37	3.4	No
57	0.4582	[0.4140, 0.4956]	38	0.3860	41	18.7	Yes
31	0.4510	[0.3300, 0.5474]	39	0.4216	38	7.0	No
53	0.4211	[0.3993, 0.4408]	40	0.3931	40	7.1	Yes
54	0.4120	[0.3918, 0.4306]	41	0.4039	39	2.0	No
56	0.3486	[0.2692, 0.4105]	42	0.3228	45	8.0	No
55	0.3439	[0.2845, 0.3919]	43	0.3066	47	12.2	No
58	0.3375	[0.2904, 0.3744]	44	0.3433	43	-1.7	No
50	0.3203	[0.2600, 0.3691]	45	0.3149	46	1.7	No
37	0.3150	[0.2976, 0.3305]	46	0.3250	44	-3.1	No
48	0.2947	[0.2206, 0.3551]	47	0.2716	52	8.5	No
52	0.2928	[0.2789, 0.3052]	48	0.3627	42	-19.3	Yes
35	0.2709	[0.2325, 0.3034]	49	0.2776	51	-2.4	No
51	0.2700	[0.2475, 0.2891]	50	0.3021	49	-10.6	Yes
43	0.2581	[0.2291, 0.2824]	51	0.3050	48	-15.4	Yes
36	0.2496	[0.2344, 0.2628]	52	0.2896	50	-13.8	Yes
49	0.2118	[0.1787, 0.2386]	53	0.2030	53	4.3	No
47	0.1968	[0.1843, 0.2078]	54	0.1992	54	-1.2	No
45	0.1774	[0.1438, 0.2066]	55	0.1739	56	2.0	No
44	0.1767	[0.1600, 0.1918]	56	0.1782	55	-0.8	No
59	0.1233	[0.1025, 0.1387]	57	0.1217	57	1.3	No
46	0.1159	[0.0845, 0.1413]	58	0.1063	58	9.0	No
6	0.0315	[0.0224, 0.0378]	59	0.0246	59	28.0	No

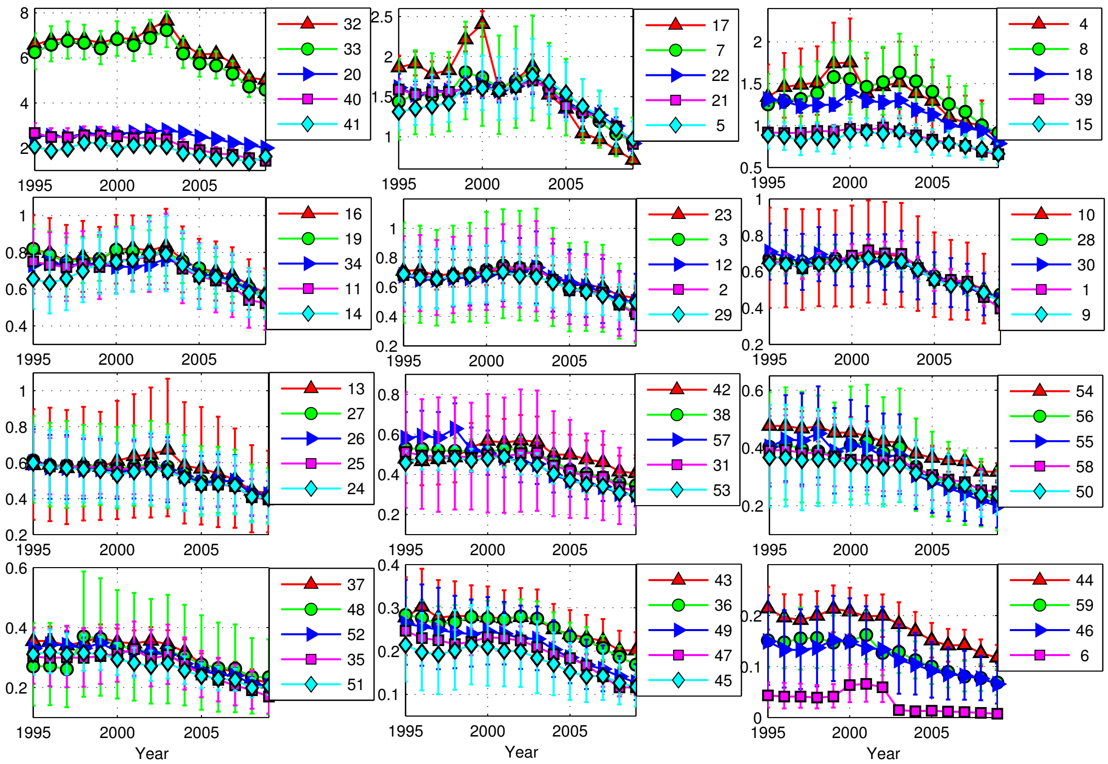
Note: For product codes see Appendix 1. Difference is defined as $100 \times (\beta_i^A - \beta_i^B)/\beta_i^B$ for all products i, where, for example, β_i^A is the global CO_2 multiplier estimate of product i under model A technology assumption.

The average estimates of the product-level global CO_2 multipliers show the amount of the average increase of CO_2 emissions produced by all forty countries as a result of one million USD increase in the countries' average final demand for particular products. Table 4 shows that demand for product Electrical energy, gas, steam and hot water (WIOD code: 32) generates the largest global CO_2 emissions across the globe and the corresponding multiplier is 6.4855 kt of CO_2 per million USD of average world final demand. The corresponding 70% HPDI is [6.4439, 6.5255]. Not surprisingly the second commodity in this list also has to do with electricity, gas and water supply, i.e., product Collected and purified water, distribution services of water (33) has the second largest global CO_2 multiplier of 6.1635 kt of CO_2 per million USD of average world final demand.

In comparison to the mentioned products, other commodities have much lower global CO_2 multipliers. Other non-metallic mineral products (20), Water transport services (40), Air transport services (41), Coke, refined petroleum products and nuclear fuels (17), Metal ores (7), Fabricated metal products, except machinery and equipment (22), Basic metals (21) and Crude petroleum and natural gas, services incidental to oil and gas extraction excluding surveying (5) join the list of top ten products with the largest global CO_2 multipliers. The corresponding multipliers estimates range from 2.5043 down to 1.4403. On the other hand, Insurance and pension funding services, except compulsory social security services (45), Financial intermediation services, except insurance and pension funding services (44), Private households with employed persons (59), Services auxiliary to financial intermediation (46) and Uranium and thorium ores (6) have the lowest global CO_2 multipliers. It is not surprising that products of financial intermediation and services of households are responsible for the lowest CO_2 emissions. As far as Uranium and thorium ores is concerned, it turns out to be produced in small amounts only in Czech Republic and there are only a few links to this product from other commodities of all the considered countries.

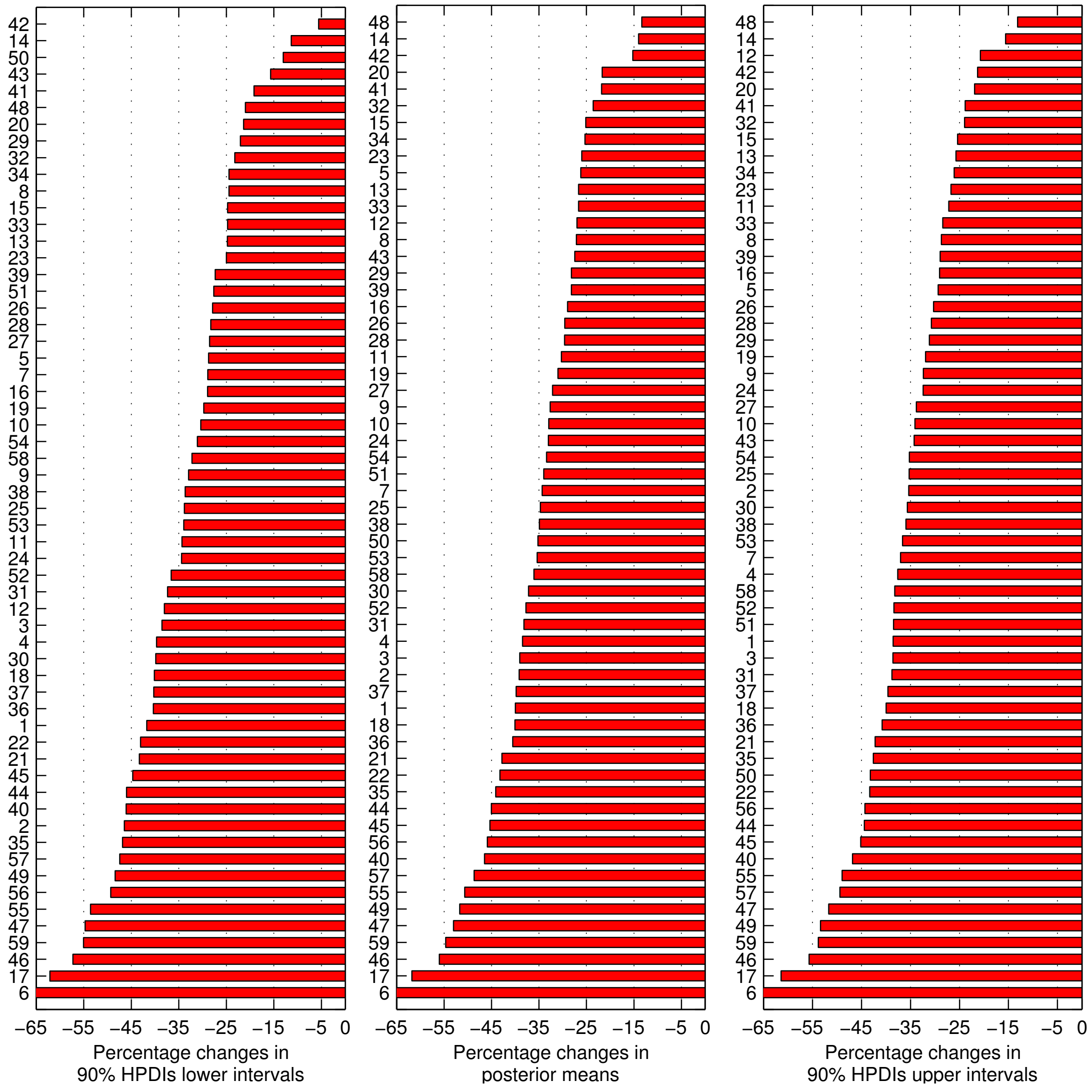
For comparison purposes in column five of Table 4 we provide the estimates of the global CO_2 multipliers based on the industry technology assumption (model B). These are again weighted averages obtained from the derived 2360 multipliers, where country-level total commodity output shares were used as weights. The percentage differences of the Bayesian estimates of the global CO_2 multipliers from the corresponding model B's estimates range from -19.3% to 28.0%, as is shown in the seventh column of Table 4. Further, from the last column of the table we observe that 31% of model B's estimates fall outside the corresponding reported 70% HPDIs of the Bayesian estimates. The corresponding figures (not reported) are 100% for 50% HPDIs, 68% for 55% HPDIs, 49% for 60% HPDIs, 19% for 80% HPDIs and 7% for 90% HPDIs. Hence, although in our Bayesian analysis the multipliers priors were chosen to be equal to the estimates based on the industry technology assumption, the final posterior estimates based on the product technology assumption turn out to be, by and large, quite different from their priors. This is due to the fact that the product technology assumption takes adequately into account products' specificities (i.e., their degree of interrelatedness with other products and their CO_2 emission generating ability) irrespective of the industries where the products are produced. Also note that the rankings of the products differ between the two technology assumptions results, although the corresponding lists of the top six products with the largest global CO_2 multipliers exactly match each other.

Figure 5: Product-level average global CO_2 multipliers, 1995-2009



The annual developments of the average global CO_2 multipliers and their 90% HPDIs are illustrated in Figure 5. The products are again given in descending order of the mean value of the multipliers of interest for the entire period, as reported in Table 4. We observe that all the posterior means and the 90% HPDIs of the global CO_2 multipliers generally show a decreasing trend. Thus, all the considered products' total carbon dioxide intensities due to final demand stimulus have definitely decreased over the considered 15 years. The degree of these changes is illustrated in Figure 6, which graphs the percentage changes of the posterior means and the 90% HPDIs lower and upper intervals of the average global CO_2 multipliers for the year of 2009 relative to 1995.

Figure 6: Changes in the global CO_2 multipliers, 2009 vs. 1995



Note: The corresponding changes for (an outlier) product 6 are -80.7%, -83.2% and -83.7%, respectively.

Figure 6 shows that products that experienced the lowest downward change in their carbon dioxide intensities (the corresponding percentage change in the posterior means is given in parenthesis) are Renting services of machinery and equipment without operator and of personal and household goods (-13.4), Wood and products of wood and cork, articles of straw and plaiting materials (-14.0), Supporting and auxiliary transport services, travel agency services (-15.2), Other non-metallic mineral products (-21.7) and Air transport services (-21.8). Given that the lowest change is larger (in absolute value) than a 10% change, it becomes clear that there has been indeed a huge decrease in the global CO_2 multipliers for all products regardless of the fact that the world production structure at the same time became much more interconnected. Commodities that show the largest decrease in the multipliers of interest include financial intermediation products (products 44, 45, 46), other community, social and personal services (55, 56, 57), Water transport services (40), Computer and related services (49), Real estate services (47), Private households with employed persons (59) and Coke, refined petroleum products and nuclear fuels (17). The range of these changes is -45.0% to -61.8%. We do not analyze further what factors could explain the cross-country differences in the global carbon dioxide emission multipliers. We consider this important topic in a different paper as its discussion falls outside the scope of the current paper.

Finally, a short note on the use of a more flexible than Normal distributional assumption on the regression errors. Recall from Section 3 that in our specification of the errors hierarchical prior we had a crucial parameter v_λ indicating the degrees of freedom for the distribution of the regression errors. Since this parameter is univariate, we can easily plot its posterior. As an example, Figure 7 shows three posterior densities of v_λ for years 1995, 2001 and 2009, while the means and their 95% HPDIs of v_λ for all years are given in Appendix 2.

Figure 7: Posterior density for degrees of freedom, $p(v_\lambda|\mathbf{y})$

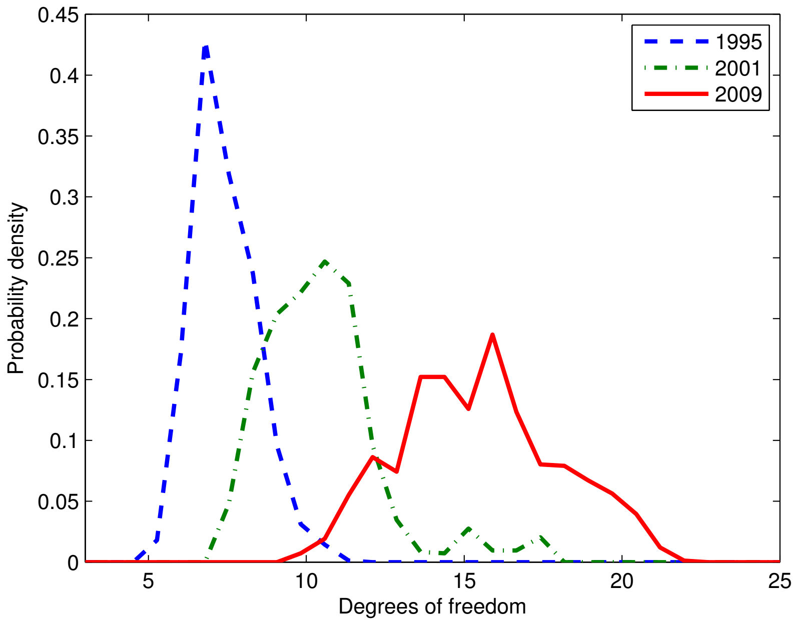


Figure 7 indicates that the posterior distributions of the degrees of freedom may be skewed. Furthermore, Figure 7 and Appendix 2 show that virtually all of the posterior probability is allocated to small values of the degrees of freedom parameter. Therefore, the errors in the SUTs framework regression (see Table 2, model A) for estimating product-level global CO_2 multipliers exhibit substantial deviations from Normality. Since the degrees of freedom estimates are always less than 25, then it was absolutely worthwhile to use a more flexible model (i.e., the scale mixture of Normals models, see Section 3) that allows for non-Normal error distribution. Note also from the posterior results that there is no support for extremely small values of v_λ which would imply extremely fat tails of the error distribution.

5 Conclusion

In this paper we first derived the regression-form equations of three input-output (IO) transformation models (based on the assumptions of industry technology, fixed industry sales structure and fixed product sales structure) in terms of supply and use tables (SUTs). Writing the models in terms of SUTs might be useful, since these data always underly implicitly any standard IO related study. Together with the system obtained in ten Raa and Rueda-Cantuche (2007a), the derived equations make the complete list of all basic IO transformation models in a regression-form SUTs framework.

Using a new dataset of international SUTs (expressed in previous year prices) constructed by the World Input-Output Database project, we quantify and present the development of the product-level global carbon dioxide emission multipliers for 40 countries and 59 products for the period of 1999-2009. For this purpose we apply a Bayesian approach to the SUTs system of product technology assumption model, which is advocated by Eurostat (2008). Bayesian methods are based on a sound probability theory, present the results in terms of intuitively meaningful posterior densities, and can use any non-sample information sensibly via priors specification. Bayesian approach allows us to avoid the usual practice of products aggregation in published SUTs, which, in our view, may lead to severe loss of information on the aggregated product-industry links. The detail analysis of the development of the product-level carbon dioxide emissions is given at the world, country and product levels for the considered period.

References

Chib, S. and E. Greenberg (1995), 'Understanding the Metropolis-Hastings algorithm'. *American Statistician* **49**, 327–335.

Diáz, B. and A. Morillas (2011), 'Incorporating uncertainty in the coefficients and multipliers of an IO table: a case study'. *Papers in Regional Science* **90**, 845–861.

Dietzenbacher, E. (1995), 'On the bias of multipliers estimates'. *Journal of Regional Science* **35**, 377–390.

Eurostat (2008), *European Manual of Supply, Use and Input-Output Tables. Methodologies and Working Papers*. Luxembourg: Office for Official Publications of the European Communities.

Gerking, S. D. (1976), 'Input-output as a simple econometic model'. *Review of Economics and Statistics* **58**, 274–282.

Geweke, J. (1991), 'Efficient simulation from the multivariate Normal and Student-t distributions subject to linear constraints'. In: E. Keramidas (ed.): *Computer Science and Statistics: Proceedings of the Twenty-Third Symposium of the Interface*. Fairfax: Interface Foundation of North America, Inc., pp. 571–578.

Geweke, J. (1992), 'Evaluating the accuracy of sampling-based approaches to the calculation of posterior moments'. In: J. M. Bernando, J. O. Berger, A. P. Dawid, and A. F. M. Smith (eds.): *Bayesian Statistics 4*. Oxford: Clarendon Press, pp. 641–649.

Geweke, J. (1993), 'Treatment of the independent Student-t linear model'. *Journal of Applied Econometrics* **8**, S19–S40.

Geweke, J. (1999), 'Using simulation methods for Bayesian econometric models: inference, development, and communication (with discussion and rejoinder)'. *Econometric Reviews* **18**, 1–126.

Geweke, J. (2005), *Contemporary Bayesian Econometrics and Statistics*. New Jersey: John Wiley & Sons.

Gilks, W. R., S. Richardson, and D. Spiegelhalter (1996), *Markov Chain Monte Carlo in Practice*. New York: Chapman & Hall.

Jackson, R. W. (1986), 'The full-distribution approach to aggregate representation in the input-output modeling framework'. *Journal of Regional Science* **26**, 515–531.

Jackson, R. W. (1989), 'Probabilistic input-output analysis: modeling directions'. *Socio-Economic Planning Sciences* **23**, 87–95.

Jackson, R. W. and W. R. Schwarm (2011), 'Accounting foundations for interregional commodity-by-industry input-output models'. *Letters in Spatial and Resource Sciences* **4**, 187–196.

Klein, L. R. (1974), *A Textbook on Econometrics*. New Jersey: Prentice-Hall.

Koop, G. (2003), *Bayesian Econometrics*. Chichester: John Wiley & Sons.

Kop Jansen, P. S. (1994), 'Analysis of multipliers in stochastic input-output models'. *Regional Science and Urban Economics* **24**, 55–74.

Leontief, W. W. (1936), 'Quantitative input-output relations in the economic system of the United States'. *Review of Economics and Statistics* **18**, 105–125.

Leontief, W. W. (1941), *The Structure of American Economy, 1919-1929: An Empirical Application of Equilibrium Analysis*. Cambridge: Cambridge University Press.

LeSage, J. P. (1999), *Applied Econometrics using MATLAB*. Available at `http://www.spatial-econometrics.com/`.

Miller, R. E. and P. D. Blair (2009), *Input-Output Analysis: Foundations and Extensions*. Cambridge: Cambridge University Press, 2nd edition.

Roland-Holst, D. W. (1989), 'Bias and stability of multiplier estimates'. *Review of Economics and Statistics* **71**, 718–721.

Rueda-Cantuche, J. M. (2011), 'Underestimation of the performance of the EU carbon dioxide emission reductions via external trade'. *Economic Systems Research* **23**. 261–280.

Rueda-Cantuche, J. M. and A. F. Amores (2010), 'Consistent and unbiased carbon dioxide emission multipliers: performance of Danish emission reductions via external trade'. *Ecological Economics* **69**, 988–998.

Rueda-Cantuche, J. M. and T. ten Raa (2009), 'The choice of model in the construction of industry coefficients matrices'. *Economic Systems Research* **21**, 363–376.

Simonovits, A. (1975), 'A note on the underestimation and overestimation of the Leontief inverse'. *Econometrica* **43**, 493–498.

Stone, R. (1961), *Input-Output and National Accounts*. Paris: Organization for European Economic Cooperation.

Suh, S., B. Weidema, J. H. Schmidt, and R. Heijungs (2010), 'Generalized make and use framework for allocation in life cycle assessment'. *Journal of Industrial Ecology* **14**, 335–353.

Temurshoev, U. (2012), 'Intercountry feedback and spillover effects within the international supply and use framework: A Bayesian perspective'. Working paper, University of Groningen.

ten Raa, T. and J. M. Rueda-Cantuche (2007a), 'Stochastic analysis of input-output multipliers on the basis of use and make tables'. *Review of Income and Wealth* **53**, 318–334.

ten Raa, T. and J. M. Rueda-Cantuche (2007b), 'A generalized expression for the commodity and the industry technology models in input-output analysis'. *Economic Systems Research* **19**, 99–104.

ten Raa, T. and M. F. Steel (1994), 'Revised stochastic analysis of an input-output model'. *Regional Science and Urban Economics* **24**, 361–371.

Timmer, M. P. (ed.) (2012), *The World Input-Output Database (WIOD): Contents, Sources and Methods*. Available at http://www.wiod.org/publications/source_docs/WIOD_sources.pdf.

West, G. R. (1986), 'A stochastic analysis of an input-output model'. *Econometrica* **54**, 363–374.

Appendix 1: Country acronyms, product and industry descriptions

Acr.	Country	Code	Product description	Code	Industry description
AUS	Australia	1	Products of agriculture, hunting and related services	1	Agriculture, hunting, forestry and fishing
AUT	Austria	2	Products of forestry, logging and related services	2	Mining and quarrying
BEL	Belgium	3	Fish and other fishing products	3	Food, beverages and tobacco
BGR	Bulgaria	4	Coal and lignite; peat	4	Textiles and textile products
BRA	Brazil	5	Crude petroleum and natural gas	5	Leather, leather and footwear
CAN	Canada	6	Uranium and thorium ores	6	Wood and products of wood and cork
CHN	China	7	Metal ores	7	Pulp, paper, printing and publishing
CYP	Cyprus	8	Other mining and quarrying products	8	Coke, refined petroleum and nuclear fuel
CZE	Czech Republic	9	Food products and beverages	9	Chemicals and chemical products
DEU	Germany	10	Tobacco products	10	Rubber and plastics
DNK	Denmark	11	Textiles	11	Other non-metallic mineral
ESP	Spain	12	Wearing apparel; furs	12	Basic metals and fabricated metal
EST	Estonia	13	Leather and leather products	13	Machinery, nec
FIN	Finland	14	Wood and products of wood and cork (except furniture)	14	Electrical and optical equipment
FRA	France	15	Pulp, paper and paper products	15	Transport equipment
GBR	United Kingdom	16	Printed matter and recorded media	16	Manufacturing, nec; recycling
GRC	Greece	17	Coke, refined petroleum products and nuclear fuels	17	Electricity, gas and water supply
HUN	Hungary	18	Chemicals, chemical products and man-made fibres	18	Construction
IDN	Indonesia	19	Rubber and plastic products	19	Sale, maintenance and repair of motor vehicles and motorcycles
IND	India	20	Other non-metallic mineral products	20	Wholesale trade and commission trade, exc. of motor vehicles and motorcycles
IRL	Ireland	21	Basic metals	21	Retail trade; repair of household goods
ITA	Italy	22	Fabricated metal products, exc. machinery and equipment	22	Hotels and restaurants
JPN	Japan	23	Machinery and equipment n.e.c.	23	Inland transport
KOR	Korea	24	Office machinery and computers	24	Water transport
LTU	Lithuania	25	Electrical machinery and apparatus n.e.c.	25	Air transport
LUX	Luxembourg	26	Radio, television and communication equipment and apparatus	26	Other supporting and auxiliary transport activities
LVA	Latvia	27	Medical, precision and optical instruments, watches and clocks	27	Post and telecommunications
MEX	Mexico	28	Motor vehicles, trailers and semi-trailers	28	Financial intermediation
MLT	Malta	29	Other transport equipment	29	Real estate activities
NLD	Netherlands	30	Furniture; other manufactured goods n.e.c.	30	Renting of M&Eq and other business activities
POL	Poland	31	Secondary raw materials	31	Public admin and defence; compulsory social security
PRT	Portugal	32	Electrical energy, gas, steam and hot water	32	Education
ROU	Romania	33	Collected and purified water, distribution services of water	33	Health and social work
RUS	Russia	34	Construction work	34	Other community, social and personal services
SVK	Slovak Republic	35	Trade, maintenance and repair services of motor vehicles and motorcycles	35	Private households with employed persons
SVN	Slovenia	36	Wholesale trade and commission trade services		
SWE	Sweden	37	Retail trade services, except of motor vehicles and motorcycles		
TUR	Turkey	38	Hotel and restaurant services		
TWN	Taiwan	39	Land transport; transport via pipeline services		
USA	United States	40	Water transport services		
		41	Air transport services		
		42	Supporting and auxiliary transport services; travel agency services		
		43	Post and telecommunication services		
		44	Financial intermediation services, exc. insurance and pension funding services		
		45	Insurance and pension funding services, exc. compulsory social security services		
		46	Services auxiliary to financial intermediation		
		47	Real estate services		
		48	Renting services of machinery and equipment without operator and of personal and household goods		
		49	Computer and related services		
		50	Research and development services		
		51	Other business services		
		52	Public administration and defence services; compulsory social security services		
		53	Education services		
		54	Health and social work services		
		55	Sewage and refuse disposal services, sanitation and similar services		
		56	Membership organisation services n.e.c.		
		57	Recreational, cultural and sporting services		
		58	Other services		
		59	Private households with employed persons		

Appendix 2: Posterior results for degrees of freedom, v_λ

Year	Mean	95% HPDI
1995	7.41	[5.94, 9.15]
1996	9.04	[7.13, 11.18]
1997	9.03	[7.27, 11.19]
1998	8.90	[7.14, 11.04]
1999	8.51	[6.44, 10.40]
2000	8.25	[6.64, 10.01]
2001	10.46	[8.05, 14.96]
2002	9.42	[7.03, 11.76]
2003	11.12	[8.94, 13.97]
2004	13.72	[11.04, 17.00]
2005	13.39	[10.48, 17.92]
2006	13.93	[10.22, 17.63]
2007	14.31	[10.82, 19.03]
2008	12.72	[9.82, 17.17]
2009	15.48	[11.53, 19.82]