**Continuous Industrial Production, The Sraffa System**

**and Leontief's Dynamic Price System**

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**ABSTRACT**

*The production equations of the Sraffa system have an agrarian point-input point-output character. This paper presents a generalization to cover continuous-input continuous-output processes which turns out to be identical with Leontief's dynamic model. It is shown that all the properties of the usual Sraffa system such as the impossibility of devising a physical measure of capital, possibility of reswitching of techniques etc. carry over to the dynamic model. A further generalization to cover fixed capital has been made to show the problems of joint utilization and transferability of fixed capital between industries and changing efficiency of machines over their lifetimes can be tackled. The general relation between the rate of profit on invested capital, the on-cost markup rate and the rate of capital turnover has been derived and some empirical observations on that relationship have been presented.*

**1. Introduction**

The production equations of the Sraffa system [Sraffa 1960] have an 'agrarian' flavor - inputs in all industries are applied at one point of time and outputs of all commodities are obtained at the end of the 'season'. These point-input point-output production processes do not properly describe modern industrial production which is characterized by continuous-input continuous-output processes. Industrial enterprises belonging to sectors like engineering, chemicals, electronics, equipment manufacturing, power, etc., typically operate continuous production processes. Their purchases and sales are not concentrated respectively at the "start of the season" and "end of the season" as is typical of agriculture. Instead in these industries enterprises buy inputs every day, produce every day and sell every day. The "stocks consumed" during a year's production may be several times the "stock held", the proportion varying depending upon the rate of stock turnover. The process of production itself is rendered continuous by the carrying of stocks of raw materials, semi-finished goods and finished goods which enable the enterprises to eliminate the time gaps between the purchase of materials and their use in production, between production at various stages of finish and between the production of finished goods and their sale respectively.

The purpose of this paper is to present a generalization of the Sraffa system that incorporates continuous industrial production and to investigate its properties. The paper is divided into nine sections. Sections 3 and 4 show that all the essential properties of the usual Sraffa system carry over to the generalized Sraffa system except one viz. the on-cost profit markup r ate and the rate of profit earned on invested capital differ depending on the capital-turnover rates in the different industries. Interestingly, the generalized Sraffa system, it turns out, is identical with the price system associated with Leontief’s dynamic system. Section 5 contains doctrinal discussion of the Leontief and Sraffa systems. Section 6 introduce fixed capital into the primal and dual Leontief-Sraffa system and shows how prices can be reduced to dated labour terms and (b) the standard system can be constructed, under the usual general conditions. Section 7 goes further to treat problems such as joint utilization of machines, transferability of fixed capital between industries and treatment of changing efficiency of machines over their lifetimes. Section 8 offers some brief but suggestive empirical evidence on the relationship of on-cost markup rates and the capital turnover rates from Indian industrial data and Section 9 concludes with brief comments and observations.

**2. The generalized Sraffa System**

It has already been remarked that continuous industrial production becomes possible only by continually replenishing stocks of raw materials, semi-finished goods and finished goods by continuous purchases, production at successive stages of finish and the production and sale of finished goods. The rate of stock turnover, that is to say, the ratio of stock consumed to stock held (as well as the ratio of stock sold to stock held of the finished goods) is different for different items of stock within each industry depending upon whether the item is of "fast" or "slow moving" variety and also across industries depending upon technology and the nature of demand. In the usual Sraffa system the ratio is 1 for all items of stock in all the industries. The generalization that is envisaged simply requires that the ratios be allowed to be different. Accordingly we write



(1)

where is the stock of commodity *j* held by the *i*th industry, (i.e. stock of *j* consumed in the production of *i*) is the flow input requirement, is the labour used,  is the output produced and *w* and *r* are the wage rate and rate of profit respectively. The flow input  arises from the following accounting formula,

Stock Consumed = Opening Stock + Purchases - Closing Stock

The throughputs from any one stage of value addition to the next are similarly defined. For a steadily growing economy the closing stock = (1+g) (opening stock) and the value of closing stock is the working capital employed in the next year. The value ratio is called the stock turnover rate. The value ratio is the gross profit markup factor which, in the absence of fixed costs, is also the net profit markup factor. Being value ratios, the stock turnover rations and the gross profit markup rates are not determined until the prices of commodities are determined. In system (1) technology will be described by three sets of coefficients, , are the usual I-O and labour coefficients and  are the stock-turnover ratios. Thus, is best understood to be "permanent working capital" on which owners earn profit at the rate prevailing under free competition. The special case  reverts us to the usual Sraffa system.

**3. Properties of System (1)**

In this section, we show that the generalized system (1) possesses all the essential properties of the usual Sraffa system. To start with there are *n* equations in *n+1* unknowns, viz., *n-1* relative prices, the real wage rate and the rate of profit. Also the behavior of relative prices due to changes in the distribution of income between wages and profits depends only on the capital to labour ratios in the industries - it is independent of the or ratios. To see this consider a two-goods economy and suppose

 to be the *numeraire,*



(2)

where and. The solution for the relative price is



Differentiating with respect to *r* we find that

 ≷  as ≷ 

or

 ≷  (3)

i.e. the price of commodity 1 increases (decreases) with an increase (decrease) in the rate of profit if it is more (less) capital intensively produced.

Further, a unique standard system for system (1) can be constructed just like the usual Sraffa system: Expressing (1) for unit outputs in matrix notation,



(4)

(5)

At *w=0* the price system is



With  it can be expressed as

 (6)

If  is positive (Hawkins-Simon conditions), must be non-negative. So by the Perron-Frobenius theorem it has a dominant eigenvalue with which is associated a non-negative eigenvector . Thus for values of *r < R* the matrix has positive minors so that the price solution of (6) is strictly positive.

Multiplying (4) by the eigenvector  we get,

 (7)

At *w=0*

 (8)

If , the standard net product is set equal to 1 and further if  then substituting (8) into (7) gives the linear wage-profit frontier

 (9)

where is the share of wages in the standard net product.

Finally, we can expand (5) in a matrix power series which must be convergent for *r < R*,



(10)

where  In other words system (1) is amenable to a reduction to dated labour terms just like the usual Sraffa system.

It is easily possible to construct numerical examples using equation (10) in which it can be shown

(i) that relative prices of two commodities show a non-monotonic behavior, i.e. first a rise, then a fall, then a rise again, etc. with respect to a monotonic rise in the rate of profit, thus showing the impossibility of measuring capital independently of the distribution of income [Sraffa (1960), Chapter 6, Section 48].

(ii) that the choice between two alternative technologies of producing the same commodity may switch two or more times at different rates of profit [Sraffa (1960), Chapter 12].

**4. Profit Markups and Profit Rates**

The only respect in which system (1) differs from the Sraffa system is in the relationship of the on-cost profit markup rate and the rate of profit on invested capital. It is well-known that most industries follow the full cost pricing principle, i.e. they apply a gross profit markup on prime cost to arrive at the price. The prime cost includes the cost of stocks consumed, energy costs, wage costs and other variable costs. If unit prime cost isthe price charged is

(11)



The rate of profit on invested capital is then

 (12)

where is the output sold,  is the fixed cost and is the equity capital,  the capital turnover ratio is the prime cost to capital ratio. Long run considerations mean that ; all costs are variable so that the formula is simply

 (13)

For the usual Sraffa system, the input cost is also the invested capital in each industry so the markup on material cost is charged to recover the wage cost and net profit and the relationship becomes

 (14)

Even if, in the Sraffa system, the wage is assumed to be paid pre-factum the invested capital equals the input plus wage costs and the relation becomes

 (15)

In either case, , i.e. the capital turnover ratio is equal to 1 in all industries.

However, for the generalized Sraffa system, even when the wage is paid post-factum,



which is in accord with the usual full-cost pricing method. The markup rate varies inversely with the capital turnover ratio; industries having high turnover ratios (e.g. retail, FMCG) need to apply a lower markup as compared to industries with low turnover ratios (e.g. shipbuilding, industrial erection) to attain the uniform competitive rate of profit.

**5. Dual of System (4)**

Readers would have undoubtedly noticed that system (1) which is expressed for unit outputs in matrix notation in equation (4) is nothing but the price system corresponding to Leontief's open dynamic model. First formulated by Georgescu-Roegen (1951), its properties have been extensively studied by Morishima (1958), Dorfman, Samuelson and Solow (1958), Solow (1960), Jorgenson (1960) and Zaghini (1971).

It is well known that the dual of (4) is Leontief's dynamic open model and its solution is obtained by the Leontief dynamic inverse,

 (16)

where C and X are the *n x 1* vectors of final consumption and gross output vectors respectively. At C = 0 the economy attains its maximum growth rate G = R. So for values of g < G it can be shown that (1b) has a strictly positive solution provided the principal minors of I-A are positive. Symmetrically with (5) the solution may be written as,

 (17)

The foregoing analysis has some bearing on some doctrinal aspects of a) the interpretation of the Sraffa system itself, (b) the relation between the Leontief and Sraffa systems and (c) the capital controversies of the sixties.

Sraffa (1960) himself presented his system as being a purely static system in which no changes in output take place, and that approach the has been adopted by the Sraffian literature. However, it appears that this view will need to be reconsidered in the light of the fact that the generalized Sraffa price system is identical with the Leontief dynamic price system so that their dual and its solution, Leontief's dynamic inverse, are identical too. And this remark holds whether or not S = A; S = A is a special case for which the dual is

 (18)

Kurz and Salvadori (2008) found that the works of Leontief and Sraffa had common sources of inspiration and striking similarities of approach. They pointed out however that Leontief's assumption of an exogenously given value-added vector was not theoretically sustainable. It should of course be understood the Leontief wished to apply his model empirically to real world economies. Any assumption that he might have made about an exogenous r (or *w/*for that matter) would have been just as unsustainable. So he simply chose as an approximation to plug in the empirically observed value-added vector to solve the prices and indicate how prices could be split into direct and indirect value-added components without attempting to explain how the value-added vector came to be what it was observed to be. The compromise was made purely for the purpose for empirical application. Be that as it may, it is preusely because no one knows which value of r (or *w/*) to plug that the Sraffa system has not yet been empirically estimates. Kurz and Salvadori's remark about the theoretical unsustainability of Leontief's assumption is even more true of the price system (4); assuming an exogenous value-added vector in (4) amounts to assuming away completely the term and reverting the dynamic system back to the static open system making it impossible to capture the independent influence of the stock coefficients on relative prices and defeating the very purpose of formulating the dynamic model. In other words, if the dynamic model is employed, whether for theoretical or empirical purposes, the problem of the determination of income distribution along with relative prices will have to be squarely faced, it cannot be bypassed.

It has been shown in section (3) that all the important properties of the usual Sraffa system viz., the impossibility of measuring capital independently of the distribution and prices, the possibility of reswitching of techniques, the existence of the standard commodity, the general inapplicability of marginal productivity theory are shared by Leontief's dynamic price system. It is somewhat strange that Samuelson and Solow who made a deep study of the Leontief dynamic model never encountered any of them. Equally strange it is that when they were confronted with those properties [Sraffa (1960)], they were stunned into incredulity and entered into a long debate, the famous capital controversy of the sixties, which has caused an irreconcilable division among economic theorists, ever since. What if they had independently discovered some of those properties themselves? I hope that readers will not find this counterfactual question entirely pointless!

**6. Fixed capital**

The elements in consist of inventory items that are continually used up in production (their depreciation rate is 100%). Fixed capital items like equipment, machinery and plant (depreciation <100%) need to be treated differently. Considering that products of machines of different ages are priced uniformly the annuity method to the only appropriate method to be used [Sraffa, 1960, Chapter X]. Then the book-value of a machine of various area (t=0, m) is given by,



(19)

where  (20)

Suppose M0, M1, …, Mk-1 be the numbers of machines of ages 0…, k-1 used in production of commodity *i* (along with other inputs and stocks) during a year. They emerge one year older at the end of the year; the value terms can be expressed as

 (21)

Substituting into these the book values above and rearranging gives the value  where . The method of applying book prices to the old machines and collapsing them into a single term can be called as "reduction to new machines". This method can always be applied when (a) machines work with constant efficiency over their lives or (b) when they are made to work with constant efficiency by incurring repairs and maintenance expenses which are included in the matrix. By reducing all machines to their new machine equivalent the technical coefficient of the fixed capital items can be simply defined as irrespective of the age of that item. The effect of reducing all old machines to their new machine equivalents whether by the use of book-values or by Sraffa’s more general method is to eliminate all the processes that use or produce old machines to one single process with inputs on one-side and the principal product of that industry on the other, thus replicating the simplicity of single-product industries. The non-substitution thereon can be readily applied to this system. Accordingly the Leontief-Sraffa dynamic price system can be written as

(22)



where is in general an *n x n* matrix containing the elements  where  is the annuity factor for machine j whose life in industry i is  and 0’s in those cells that are not durable machines. The relationship between the rate of profit on invested capital and the on-cost markup rate becomes more complicated in the presence of fixed capital,

 (23)

where  is the life of the *jth* machine in the *ith* industry.

where 

(24)

i.e. the capital turnover ratio itself varies inversely with the rate of profit. In other words an increase in the rate of profit lowers the capital turnover ratio and causes the markup rate to increase.

The dual of equation (22) is given by

 (25)

Where F is an nxn matrix containing the elements  or 0 where



The solution  gives the multipliers that must be applied to production equation of commodity in equation (22). In particular, for the fixed capital items denotes the number of machines that will be employed in industry . Once that is known it is an easy matter to find the age distribution of the machine employed in industry that is consistent with a balanced rate of growth for all industries. This is obtained from

 (26)

Where *M* is the number of machines of age . The machines of lower ages are obtained by multiplying successively by the factor (*1+g*) until we reach brand new machines whose number should be . The gross output of new machines produced each year by the machine producing industries must be (*1+g*) times the new machines that are employed by all the industries in the economy during that year, i.e.  summed over , for each machine producing industry *j*.

Observe that (22) is amenable to a reduction to dated labour terms in the usual way.



(27)

And likewise for the dual system (25) can be expanded to show gross output required to produce the consumption demand vector C. Secondly, for C=0, system (25) represents the standard system with an all positive solution for *X* provided only that the Hawkins-Simon conditions are satisfied.



(28)

in which the standard ratio *G = R* is the lowest positive root of the detrimental equation.

**7. Notes on Fixed Capital Systems**

The technique of reducing all old machines to their new machine equivalents by applying to them their book values (Schefold (1989) calls it centering) allows the system to handle the problems of joint utilization and transferability of any number of machines in any number of industries. For example, if lorries are used half the time to transport bread and the remaining time to transport medicines then ½  would be the annual value charged in the bread and pharmaceutical industries respectively. Also old machines can be traded and transferred between industries. If a uniform rate of profit prevails then the book-values should be the market prices otherwise either buyer or the seller would stand to lose. If the rate of profit is not uniform, the demand prices for machines that industries enjoying a higher rate of profit would pay stand at a higher level than the book-values an ascribed to them by low profit industries and there is scope for mutually profitable trade. The industry that buys the old machine would of course apply to it the annuity charge for its remaining life calculated at the higher rate of profit that it is enjoying.

Machines that work with changing efficiency over their lifetimes poses a problem. For instance if a machine works with decreasing efficiency over it life then the technical coefficient corresponding to its use must rise over time. To avoid this problem the following procedure may be used. Consider a machine that has a life of 3 years. It works with full efficiency when it is brand new, but produces 80% of the output when it is 1 year old and 75% when it is 2 years old. Then the production equations for the three processes are



Using the book values to eliminate the old machines and adding up the equations would give

  

i.e. 

In this manner all the processes can be reduced one single average process over the lifetime of the machine.

**8. Empirical Evidence on markups and turnover rates**

An idea of the magnitudes and variabilities of the capital turnover ratios and gross profit markup factors in different industries can be obtained from the data reported in the table below. The data pertain to listed companies in India. The definitions employed are (a) capital turnover ratio is cost of goods sold divided by total assets (current plus net fixed assets) and (b) the gross profit markup factor is net sales divided by cost of goods sold.

**Turnover Rates and Gross Profit Factors (1998-2014)1, 2**

| **Sector** |  | **Min-Max** | **Mean** | **S. D.** | **C.V.** |
| --- | --- | --- | --- | --- | --- |
| Chemicals | a) | 0.82-1.21 | 0.99 | 0.218 | 0.219 |
| b) | 1.17-1.29 | 1.16 | 0.051 | 0.043 |
| Coal & Lignite | a) | 0.37-0.53 | 0.43 | 0.080 | 0.183 |
| b) | 1.07-1.80 | 1.57 | 0.185 | 0.118 |
| Contribution & Real Estate | a) | 0.28-0.54 | 0.37 | 0.059 | 0.157 |
| b) | 1.10-1.28 | 1.19 | 0.047 | 0.040 |
| Construction materials | a) | 0.22-0.45 | 0.40 | 0.050 | 0.126 |
| b) | 1.51-1.89 | 1.68 | 0.118 | 0.070 |
| Consumer Goods | a) | 0.64-1.04 | 0.94 | 0.089 | 0.094 |
| b) | 1.19-1.30 | 1.25 | 0.032 | 0.025 |
| Crude Oils & Natural Gas | a) | 0.12-0.23 | 0.17 | 0.028 | 0.160 |
| b) | 1.40-2.69 | 1.825 | 0.308 | 0.168 |
| Electricity Generation | a) | 0.17-0.37 | 0.22 | 0.045 | 0.204 |
| b) | 1.20-1.55 | 1.34 | 0.097 | 0.072 |
| Food and Agro base Products | a) | 0.77-0.89 | 0.83 | 0.041 | 0.048 |
| b) | 1.22-1.28 | 1.24 | 0.018 | 0.014 |
| Hotel & Tourism | a) | 0.13-0.19 | 0.16 | 0.016 | 0.102 |
| b) | 1.54-2.10 | 1.76 | 0.155 | 0.088 |
| Information Technology | a) | 0.30-0.45 | 0.37 | 0.044 | 0.118 |
| b) | 1.74-2.07 | 1.97 | 0.088 | 0.044 |
| Machinery | a) | 0.52-0.72 | 0.61 | 0.056 | 0.092 |
| b) | 1.22-1.29 | 1.26 | 0.019 | 0.015 |
| Metal & Metal Products | a) | 0.35-0.61 | 0.47 | 0.082 | 0.173 |
| b) | 1.23-1.40 | 1.29 | 0.052 | 0.040 |
| Minerals | a) | 0.11-0.29 | 0.211 | 0.064 | 0.306 |
| b) | 1.60-3.34 | 2.48 | 0.589 | 0.237 |
| Textiles | a) | 0.50-0.67 | 0.58 | 0.051 | 0.088 |
| b) | 1.18-1.27 | 1.23 | 0.022 | 0.017 |
| Transport Equipment | a) | 0.58-1.02 | 0.84 | 0.112 | 0.132 |
| b) | 1.18-1.25 | 1.21 | 0.022 | 0.018 |
| Transport Services | a) | 0.25-0.43 | 0.35 | 0.058 | 0.167 |
| b) | 1.25-1.51 | 1.39 | 0.076 | 0.055 |
| Wholesale & Retail Trade | a) | 1.02-1.51 | 1.26 | 0.146 | 0.116 |
| b) | 1.07-1.14 | 1.09 | 0.021 | 0.019 |
| All sectors | a) | 0.22-0.28 | 0.26 | 0.018 | 0.072 |
| b) | 1.19-1.25 | 1.22 | 0.188 | 0.015 |

(1) Source: Centre for Monitoring Indian Economy - PROWESS Database; (2) Figures reported without removing outliers etc.

The cross-sectional regression across industries of log (*mi*) on log (*Ti*) for the year 2014 and for their weighted average across all years gives the following result



Under ideal competitive condition when a uniform rate of profit prevails across industries the equation is a rectangular hyperbola and the expected value of the slope coefficient of the regression of log  on log  is *-1*. We can hardly expected such ideal conditions to be satisfied by the sample data shown in the table above.

If the sample is divided into two groups consisting of industries greater and lesser than the median rate of return on capital and separate regressions are run on the groups the results are,



A substantial improvement in the results is obtained; *R2*s rise significantly and slope coefficients move closer to *-1*, because the variances of the returns on capital in the two subgroups in much lower than the variance of returns across the whole sample.

**9. Concluding Remarks**

This paper began by seeking a generalization of Sraffa system that incorporate continuous industrial production. The generalization turns out to be Leontief’s dynamic open price systems. Further generalization to incorporate fixed capital items has also been made. The resulting system gives a better description of the technology in terms of the usual input-output and labour coefficient as well as stock and fixed-asset turnover ratios. In so doing a more realistic description of the pricing of industrially produced commodities is obtained in terms of the role played by the turnover ratios in determining the on-cost gross and net profit makeup rates. Due to the identity between the generalized Sraffa system and Leontief’s dynamic system all the essential properties of the Sraffa system such as possibility of reswitching of the techniques, possibility of constructing an invariable measure of value, impossibility of aggregating period of production to measure capital, etc. are seen to belong to the Leontief model as well even though they have never been discussed in the context of that model. The Sraffa and Leontief systems stands unified. Finally, the generalized system helps to connect Sraffian theory with empirical work on such aspects as reswitching and reverse capital deepening that has been initiated by Han and Schefold (2006) using the framework of the Leontief dynamic model.

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