**Time Lag Model for Input-Output Multiplier**

**Xinru LI, Peng LIU**

**Abstract:** Input- output technique has been widely used in many fields of national economy. At its core is the Leontief inverse matrix, namely input-output (I-O) multiplier. Most studies based on I-O multiplier neglect its time lag, leading to a misunderstanding that complete effect will be realized by one time immediately. The purpose of this paper is to build a time lag model for I-O multiplier. Time lag here comes from two sides: information transmission and production process. The former refers to time spent on microeconomic activities, such as finding source of goods and establishing orders. The latter is production time taken for the production of goods. In addition, we also take production lead-time into account. Using the power series expansion of Leontief inverse, it is well known that complete effect can be decomposed into direct effect and countless indirect effects. Under assumption of constant inventory, each effect corresponds to a time lag. We build a model to calculate average-weighted time lag of the complete effect. In empirical analysis, based on China’s non-competitive I-O table that reflects processing trade in 2010, we calculate the average time lag of effect on domestic value added generated by each industry’s export. The result shows that time lag of effect generated by processing export is shorter than that by non-processing export, which is meaningful for policy making.

**Keywords:** time lag; input-output multiplier; non-competitive input-output table; processing trade; domestic value added.

**Introduction**

Input-output technique, founded by an American scientist, Wassily W. Leontief, has been widely used in many fields of national economy. At its core is the Leontief inverse matrix, namely input-output (I-O) multiplier, reflecting the total demand on products generated by one unit final demand. I-O multiplier has a wide application in calculating the influence of final demand changes on value added, employment and other economic variables. For example, according to the non-competitive input-output table reflecting processing export, ten thousand dollars of goods exported from China to the U.S. in 2011 would directly and indirectly increase Chinese employment by 0.51 person-year and Chinese GDP by 6.22 thousand dollars. However, most studies based on I-O multiplier neglect its time lag, leading to a misunderstanding that complete effect will be realized by one time immediately. And many outstanding experts like Lawrence Lau have raised some questions such as: Whether would the complete effect calculated by I-O model accrue at the same year? How long is the time lag? For these reasons, this paper aims at discussing time lag of I-O multiplier.

Time lag refers to a period of time from event occurrence to having an effect. Multiplier effect is usually accompanied by time lag effect. In many cases time lag, as an indispensable complement of calculation results, is helpful for people to have a better understanding and evaluation of the event. Money multiplier is a persuasive example. Adjustment of currency in circulation and commodity price by central bank monetary policy is retarded by time. Through empirical research, American economist and Nobel laureate, Freedman [1,2], found that the change of nominal income growth after the change of money growth averagely costs 6-8 months, and it needs another 6-9 months to affect price. That is, there is a time lag of 12-18 months from the change of money growth to the change of price. Jian Chai, Jue Guo and Shouyang Wang[3] established the VARX model to estimate the influence of economic indicator M2 on macro-economy. Results showed that positive effect on economic growth exerted by increase of M2 reached maximum with a time lag of more than one quarter, and shock effect on inflation has a longer time lag, reaching its maximum in the 5-6th quarter. Time lag is an indispensable consideration when monetary policy is made. It has important significance for sticking to right direction and proper intensity, further realizing policy goal.

Leontief inverse matrix is also a kind of multiplier, so there is time lag when we use I-O model to calculate and analyze practical problems in economy, environment, population, education and other fields [4-9] as well. However, most existing researches on time lag were based on dynamic I-O model [10-13], in which time lag comes from investment process. Moreover, dynamic I-O model is limited to complex structure and tedious calculation. Time lag in this paper, differently, refers to time lag of Leontief inverse matrix, coming from information transmission and production process. The new model has distinct advantages in structure and calculation.

Concept of time lag has been presented before the nineteenth century, but studies about time lag of I-O multiplier and its transmission mechanism are relatively infrequent. Eric Dietzenbacher et al. proposed Average Propagation Length (APL) from production chain perspective. It can be used for production stage division, further to analyze interdependence between industries [14-16]. Rencheng Tong and Jian Xu [17] pointed out the requirement relationship between industries could be decomposed in time and become a time series problem. They proposed an iterative algorithm to calculate pulling function in a given time. The same modeling idea and method are used in time lag of price conduction [18]. This paper focuses on time lag of I-O multiplier with APL model theory for reference. Compared to the above iterative method, we give a formula to calculate the average length of multiplier time lag, by relaxing assumption of production cycle.

**Time lag model**

We assume inventory of each industry is reasonable, with prompt replacement when consumption takes place. Each industry keeps normal production. In this paper, we take effect on value added generated by steel export for example, analyzing its time lag using power series expansion of I-O multiplier. As shown in Figure 1, complete effect generated by steel export can be decomposed into direct effect and countless indirect effects. Direct effect refers to the value added generated by steel production. Its time lag includes two parts: time for information transmission and time for production process. The former refers to time spent on microeconomic activities, such as finding source of goods and establishing orders. The latter is production time taken for the production of goods. In addition, production lead-time, producing before receiving an order, is subtracted; the first-round indirect pulling function is the value added generated by raw material production, such as pig iron, coke, coal, equipment, etc. Its time lag is time for information transmission plus time for production process, then minus production lead-time. Similarly, the rest indirect effects occur in sequence.

Direct consumption coefficients in value I-O table are non-negative numbers less than 1. Therefore, as number of intermediate consumption increases, indirect effect weakens. As time goes on, pulling function generated by steel export weakens. Time lag we focus on is an average completion time of complete pulling function.

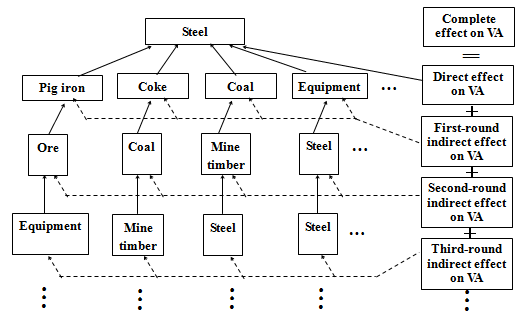


Figure 1: Diagram of pulling effect on value added (VA) generated by steel export

For simplicity sake, we assume direct consumption coefficient matrix , value added coefficient vector  and direct occupation coefficient matrix  remain unchanged. To calculate effect on value added generated by steel export, we use a two-step process: first, figure out export-driven total output; second, calculate export-driven value added using value added coefficient.

Let  be the steel export volume. Utilize I-O model to figure out export-driven total output as follows:





Complete effect on value added generated by steel export is:





For the industry , value added created by its production is the direct pulling function, denoted by . It is well known that production of industry  requires raw materials produced by various industries. Value added created by its raw materials production, denoted as ( is the element of direct consumption coefficient matrix , similarly hereinafter. ), is the first-round indirect pulling effect with once intermediate consumption. Likewise, these raw materials productions also need intermediate inputs. Value added, expressed as and created by these inputs productions, is the second-round indirect pulling effect with twice intermediate consumption. It is an endless process that can be expressed by an infinite power series. The sum  can explain the complete effect on value added generated by a unit demand on industry , and that is value added multiplier of industry ,denoted by . Its matrix expression shows as follows:





Considering time for information transmission and production process, complete effect won’t be realized by one time immediately. There is time lag for value added multiplier mentioned above.

In this paper, time for information transmission plus time for production process then minus production lead-time, is defined as production cycle. To make calculation simple, we assume the production cycle is the same in various industries, denoted as . Time lag of , direct pulling effect on value added created by industry , is equal to one production cycle . Time lag of , first-round indirect pulling effect, is equal to two production cycles . Time lag of , second-round indirect pulling effect, is equal to three production cycles . Therefore, analysis of time lag is based on the infinite power series.

As mentioned above, direct and indirect pulling effects weaken exponentially over time. As a consequence, when we estimate time lag of complete effect, it is reasonable to give a greater weight to the earlier effect and a smaller weight to the later effect. Using the share of each round effect in complete effect as weight, calculate weighted mean time lag of different round effect and take it as time lag of the complete effect. Denote time lag of  as . According to our definition, we have:





Denote value added multiplier vector by , its time lag vector by , and Leontief inverse matrix by . Then Eq. (4) can be expressed in matrix equation as follows:



We further deduce calculation formula for time lag of value added multiplier:





**Empirical study**

Based on China’s non-competitive I-O table that reflects processing trade in 2010, we calculate the average time lag of effect on domestic value added generated by each industry’s final demand. For simplicity, production cycle  is set to 2 months. We get the following results by the formulae (5): Industries producing the same product but for different purpose (domestic use, processing export or non-processing export), have different time lags (see Appendix A, B); Time lag of value added generated by processing export is shorter than that by non-processing export, mainly in the range of 4-6 months and 5-7 months respectively, as shown in Figure 2.

Figure 2: Time lag of value added generated by each industry’s one unit final demand.

(D for domestic use, P for processing export, N for non-processing export, similarly hereinafter)

**Conclusion**

Input-output multiplier is the core of input-output technique. Its time lag research has important implication for empirical study and policy advice. For example, the government should be selective to protect or support certain industries in order to achieve anticipated goal faster or keep policy effective time longer. For China, it is necessary to differentiate processing export from non-processing export when we calculate time lag of effect generated by export. Results shows that time lag of effect generated by processing export is shorter than that by non-processing export. If the government wants to achieve anticipated goal faster, it should provide more protection to processing export enterprises. But if it wants to keep policy effective time longer, non-processing export enterprises should be supported with emphasis.

Although this paper just calculates time lag of value added multiplier, time lag model we mentioned also apply to other multipliers. Moreover, it is worth noticing that value of production cycle is crucial for result accuracy.

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Appendix A

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| --- | --- | --- | --- | --- |
| **Time lag of value added generated by each sector's one unit final demand (Unit: month)** | | | | |
| **Industry Number** | **Industry Name** | **D** | **P** | **N** |
| 1 | agriculture, forestry, animal husbandry and fishery | 3.9 | NA | 3.9 |
| 2 | coal mining and washing | 4.6 | NA | 4.6 |
| 3 | petroleum and natural gas exploitation | 4.0 | NA | 3.8 |
| 4 | ferrous metal mining-selection | 5.4 | NA | 5.3 |
| 5 | non-ferrous metal mining-selection | 5.4 | NA | 5.3 |
| 6 | nonmetal and other minerals mining-selection | 5.4 | NA | 5.4 |
| 7 | food and alcoholic beverages | 5.6 | 4.3 | 5.7 |
| 8 | tobacco | 3.8 | 4.8 | 3.6 |
| 9 | textile materials | 6.1 | 3.3 | 6.1 |
| 10 | textile, knitted products | 6.2 | 5.7 | 6.4 |
| 11 | textiles and apparels and shoes | 6.5 | 5.8 | 6.6 |
| 12 | leather, fur and feather products | 6.4 | 5.6 | 6.6 |
| 13 | wood processing and furniture manufacturing | 6.3 | 6.0 | 6.3 |
| 14 | paper making and printing | 6.2 | 3.8 | 6.2 |
| 15 | educational and sports goods | 6.4 | 5.9 | 6.6 |
| 16 | petroleum and nuclear fuel | 5.1 | 2.2 | 5.1 |
| 17 | basic chemical materials | 6.1 | 4.1 | 6.2 |
| 18 | fertilizer and pesticides | 6.3 | 5.9 | 6.3 |
| 19 | synthetic materials | 5.9 | 3.6 | 6.0 |
| 20 | specialty chemical products | 6.6 | 4.1 | 6.4 |
| 21 | other chemical products | 5.9 | 2.5 | 6.0 |
| 22 | plastics and rubber | 6.3 | 2.2 | 6.3 |
| 23 | nonmetal mineral products | 6.1 | 5.6 | 6.2 |
| 24 | black metal smelting | 5.7 | 4.1 | 5.7 |
| 25 | steel pressing | 6.2 | 4.1 | 6.3 |
| 26 | colored metals smelting and pressing | 6.3 | 4.4 | 6.1 |
| 27 | metal products | 6.5 | 3.4 | 6.6 |
| 28 | general purpose machinery | 6.4 | 4.0 | 6.3 |
| 29 | special purpose machinery | 6.2 | 4.2 | 6.2 |
| 30 | railroad transportation equipment | 6.6 | 4.5 | 6.6 |
| 31 | motor industry | 6.8 | 4.0 | 6.9 |
| 32 | shipbuilding and floating devices | 5.8 | 5.2 | 6.0 |
| 33 | other transport equipment | 6.6 | 5.3 | 6.5 |
| 34 | electrical equipment | 6.8 | 2.9 | 6.8 |
| 35 | electric transmission, distribution and control equipment | 6.7 | 5.7 | 6.5 |
| 36 | household electric and non-electric appliances | 6.7 | 6.3 | 6.8 |
| 37 | other electrical machinery and equipment | 6.5 | 6.2 | 6.7 |
| 38 | communication equipments and radar | 6.8 | 6.3 | 6.7 |
| 39 | electronic computer manufacturing | 5.8 | 5.0 | 5.3 |
| 40 | electronic components | 6.6 | 5.2 | 6.5 |
| 41 | household audio-visual equipment | 6.5 | 5.3 | 6.7 |
| 42 | other electrical equipments | 6.0 | 5.5 | 6.1 |
| 43 | instrumentation manufacturing | 6.2 | 4.1 | 6.2 |
| 44 | culture and office machinery manufacturing | 6.5 | 6.9 | 6.8 |
| 45 | crafts and other manufacturing ( scrap waste included) | 4.7 | 2.0 | 4.8 |
| 46 | electricity, heat production and supply | 5.8 | 5.9 | 5.8 |
| 47 | gas production and supply | 5.3 | NA | 5.4 |
| 48 | water production and supply | 4.8 | NA | 4.9 |
| 49 | construction | 6.0 | NA | 6.2 |
| 50 | transport and storage | 4.7 | 4.7 | 4.7 |
| 51 | postal service | 4.8 | NA | 4.9 |
| 52 | information transmission, computer services and software | 4.1 | NA | 4.3 |
| 53 | wholesale and retail sales | 3.2 | 3.3 | 3.3 |
| 54 | accommodation and catering | 5.0 | NA | 5.0 |
| 55 | finance | 3.4 | NA | 3.6 |
| 56 | real estate | 3.0 | NA | 3.2 |
| 57 | leasing and business services | 5.0 | NA | 5.3 |
| 58 | R&D | 5.2 | NA | 5.1 |
| 59 | integrated technological services | 4.3 | NA | 3.9 |
| 60 | water, environment and public facilities management | 4.9 | NA | 4.3 |
| 61 | residential and other services | 4.3 | NA | 4.6 |
| 62 | education | 3.1 | NA | 3.9 |
| 63 | health, social protection and social welfare | 5.4 | NA | 5.1 |
| 64 | culture, sport and entertainment | 4.3 | 2.7 | 4.7 |
| 65 | public administration and social organization | 3.6 | NA | 4.1 |

Appendix B