**What matters in measuring domestic value added in exports by international or single country model[[1]](#footnote-2)**

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**Abstract** This paper proposes a method to compute the domestic value added in exports based on international input-output model, and examines it with the method based on single-country model using world input-output table. It shows that for any country, in total, the results of domestic value added in exports by international IO model equal to that by single country IO model. However, in decomposition, the method on international IO model gives the effects of feedbacks among countries, originating from the inter country division and the international industrial chains. Yet the results of single country model cannot provide this kind of decomposition. Then by using WIOTs, we compute the domestic value added in exports by the method in this paper, and analyze the results.

**Key words** international input-output model; the domestic value added in exports; feedbacks

**1.Introduction**

Nowadays, with the increasingly global integration of production process, a certain product is the outcome of the cooperation among many countries, and thus it has resulted in increasingly fragmentation of the production among these countries. Therefore, the index of gross trade statistics requires compensation, and many related indices are proposed. For example, the concept of trade in value added (TiVA) is considered as an very important one. (TRADE IN VALUE-ADDED: CONCEPTS, METHODOLOGIES AND CHALLENGES (JOINT OECD-WTO NOTE), 2012; Koopman, R. et al, 2010; Johnson & Nogurea (2012), Stehrer (2012), etc). Many plausible method of computing factor content of trade can also be used in the related issue (Reimer (2006), Trefler & Zhu (2010)).

 Domestic value added in exports (DVA), which means the domestic value added occurred in the production process of a country’s gross exports, has also been explored deeply and extensively. In literature, there are two ways to measure domestic value added in exports (DVA): (1) using input-output model and input-output table; (2) using firm level data (Hiau Looi Keey, Heiwai Tangz, 2013; Richard Upward, Zheng Wang , Jinghai Zheng, 2013).

When using input-output model to calculate DVA, there are two kinds of methods: by international input-output model (KWW (2014), Bart Los, Marcel Timmer and Gaaitzen de Vries, ,2015 Johnson & Nogurea (2012), Stehrer (2012), etc.), and by national noncompetitive input-output model (single country model) (Chen, et al, 2012; Duan, 2012). Based on international input-output model, Koopman, Wang and Wei (2014, henceforward KWW) presented a decomposition of a country’s gross exports into various components, with key distinction between domestic and foreign content of exports. Bart Los, Marcel Timmer and Gaaitzen de Vries (2015) examined the decomposition and proposed their method of computing domestic value added in exports by hypothetical extraction technique. By the method of single country model, Chen, et al (2012) developed a methodology to compute domestic value added in exports based on national non competitive input-output model with distinction between processing and non processing exports.

 For the two ways of computing domestic value added in exports by input-output technique, international and single country model, what are their relationships and differences? To answer this question, firstly, this paper proposes a method to compute the domestic value added in exports based on international input-output model, and shows that the result of our method is equal to that ofLos et al (2015). Secondly, we examine our method on international IO model with the method based on single-country model, and prove that in aggregate level, the results of the two kinds of model are equivalent. However, they still have great differences. It is that, in decomposition, the method on international IO model gives the effects of feedbacks and spillovers among countries, originating from the inter country division and the international industrial chains, while the results of single country model cannot provide this kind of decomposition.

 The second section of this paper reviews the method of computing domestic value added in exports by single country model. Then a method based on international model is proposed. In the following fourth section, we prove the equivalence of the two ways in aggregate level, and the fifth section investigates their key differences. The empirical results based on WIOTs are shown in the sixth section. Finally, a conclusion is given.

**2. The Introduction of the method of computing domestic value added in exports by single country model**

The main equation of computing domestic value added in exports by non competitive national input-output model (Chen, et al., 2012) is

$DVA\_{si}=V\_{i}(I-A\_{D})^{-1}E\_{x}$ (1)

where $DVA\_{si}$ is country i’s domestic value added in exports by single country model, $V\_{i}$ the vector of domestic value added coefficients in country i, $A\_{D}$ the matrix of domestic input coefficients in country i, and $E\_{x}$ the column vector of gross exports of country i.

Table 1 is a simplified international input-output table with 3 countries. For the convenience of comparison, we can rewrite equation (1) by the same signs in Table 1.

**Table 1 A simplified international input-output table with 3 countries**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Intermediate demand | Sub total | Final demand | Total Output |
| 1 | 2 | 3 | 1 | 2 | 3 |
| Intermediate input | 1 | *Z*11 | *Z*12 | *Z*13 |  | *y*11 | *y*12 | *y*13 | *q*1 |
| 2 | *Z*21 | *Z*22 | *Z*23 |  | *y*21 | *y*22 | *y*23 | *q*2 |
| 3 | *Z*31 | *Z*32 | *Z*33 |  | *y*31 | *y*32 | *y*33 | *q*3 |
| Primary input | *v*1 | *v*2 | *v*2 |  |  |  |  |  |
| Total input | *q*1 | *q*2 | *q*3 |  |  |  |  |  |

In table 1, Z denotes the intermediate flow matrix, with superscript denoting the three countries respectively. Any column in Z provides the information of the inputs in the production of a country, including the inputs of domestic products and the intermediate inputs imported from other countries. For any row in Z, it shows how the products of a given country are used as intermediate inputs in the production of itself and other countries. For final demand matrix Y, any column in it shows the final demand of a given country for all countries, including its final demand for domestic products, and its final demand for the final products from other countries (final imports). For any row in Y, it gives how the final products of a given country are used by itself and by other countries (final exports).

In table 1, the exports of a country are divided into two parts, intermediate exports and final exports. In international input-output model, the intermediate product export is endogenous, while final product export is exogenous.

However, for single country input-output model, intermediate exports, as well as final exports, are considered as exogenous. Therefore, taking country 1 as example, equation (1) can also be written as

$$DVA\_{s1}=V\_{1}\left(I-A\_{11}\right)^{-1}(Z\_{12}+Z\_{13}+y\_{12}+y\_{13})$$

Similarly, we can give the equation for country 2 and country 3.

$$DVA\_{s2}=V\_{2}\left(I-A\_{22}\right)^{-1}(Z\_{21}+Z\_{23}+y\_{21}+y\_{23})$$

$$DVA\_{s3}=V\_{3}\left(I-A\_{33}\right)^{-1}(Z\_{31}+Z\_{32}+y\_{31}+y\_{32})$$

Or

$$DVA\_{s}=\left(\begin{matrix}DVA\_{s1}\\DVA\_{s2}\\DVA\_{s3}\end{matrix}\right)=V\left(\begin{matrix}B\_{1}&&\\&B\_{2}&\\&&B\_{3}\end{matrix}\right)\left(y^{t}+z\right)=B\left(y^{t}+z\right)$$

where $B\_{ii}=\left(I-A\_{ii}\right)^{-1}$, z$=\left(\begin{matrix}0&Z\_{12}&Z\_{13}\\Z\_{21}&0&Z\_{23}\\Z\_{31}&Z\_{32}&0\end{matrix}\right)\left(\begin{matrix}1\\1\\1\end{matrix}\right)$, and z is the vector of intermediate exports of the countries.

Then the domestic value added in exports computed by single country model is

$DVA\_{s}=\hat{V}B\left(y^{t}+z\right)$ (2)

where vi is the vector of value added rate of country i. The equation for n countries case is the same as equation (2).

**3. The method to compute the domestic value added in exports by international input-output model**

We can compute the value added in exports through different paths, and they can confirm each other. The other two paths and the generalization from two to N countries are given in Appendix A.

First, the following points should be clarified:

 (1) Intermediate product trade and final product trade are handled in different ways, in that in international input-output model they are endogenous and exogenous respectively.

(2) There are two steps to compute domestic value added embodied in trade: first, calculate the output induced by trade; second, use direct factor requirement coefficients and value added rate to compute the factor content and value added embodied in trade. The key point is step 1, so in this and the following sections we will discuss it detailed.

A country’s final export is exogenous, and it is easy to get the output pulled by this kind of export:

$q^{t}=(I-A)^{-1}y^{t}$ (4)

As endogenous variables, intermediate product trade is incurred by the production processes of final demand, including final product trade and domestic final demand. Therefore, it can be divided into two parts correspondingly, the intermediate trade for the production of final product trade, and the intermediate trade for the production of domestic final demand. The effect of intermediate for production of final trade is already contained in equation (4). What we need to do is to compute the output induced by the intermediate trade for the production of domestic final demand. It means that it requires intermediate trade in order to produce the domestic final demand, which will cause the output increase of concerned countries.

For a particular country i, its output incurred by the intermediate trade used for domestic final demands of all countries can be divided into two parts: the first part is the effect of intermediate trade used for other countries’ domestic final demands except for country i; the second part is the effect of intermediate trade used for the production of domestic final demand of country i itself.

(1)The first part: the effect of intermediate trade used for other countries’ domestic final demands

Suppose there are two countries, country 1 and country 2. The production process of country 1’s domestic final products will bring the outputs of the sectors in country 1 by using the domestic inputs, while bring country 2’s outputs increases by using the intermediate products imported from country 2. That is, country 2’s output will go up because of the production of country 1’s domestic final demand products through intermediate trade, and vice versa. Let qz1 be the output of country 1 led by country 2’s domestic final demand, and qz2 be the output of country 2 led by country 1’s domestic final demand. Taking all rounds into consideration, we can get qz1 & qz2 by

$\left(\begin{matrix}q^{z1}\\q^{d2}\end{matrix}\right)=(I-A)^{-1}\left(\begin{matrix}0\\y^{22}\end{matrix}\right)$ (5)

$\left(\begin{matrix}q^{d1}\\q^{z2}\end{matrix}\right)=(I-A)^{-1}\left(\begin{matrix}y^{11}\\0\end{matrix}\right)$ (6)

Let$(I-A)^{-1}=\left(\begin{matrix}L^{11}&L^{12}\\L^{21}&L^{22}\end{matrix}\right)$, and $q^{1z}=\left(\begin{matrix}q^{z1}\\q^{z2}\end{matrix}\right)$, then

$q^{1z}=\left(\begin{matrix}0&L^{12}\\L^{21}&0\end{matrix}\right)\left(\begin{matrix}y^{11}\\y^{22}\end{matrix}\right)$ (7)

(2)The second part: the effect of the effect of intermediate trade used for the production of domestic final demand of country i itself

The production of country 1’s final products requires intermediate products imported from country 2. The feedback is that in country 2, it requires the intermediate input from country 1, for the production of this part of intermediate export to country 1. Therefore, country 1’s output will increase further because of this type of feedback. The feedbacks will continue infinitely, shown as follows.

First round: it requires the inputs imported from country 2 to produce one unit of final product of country 1, and to produce this part of export in country 2 needs the intermediate inputs imported from country 1, then there will be the output increase in country 1: $B^{1}A^{12}B^{2}A^{21}B^{1}$, where $B^{1}=\left(I-A^{11}\right)^{-1}$,$B^{2}=\left(I-A^{22}\right)^{-1}$;

The second round: to produce the output of country 1, $B^{1}A^{12}B^{2}A^{21}B^{1}$, it requires imports from country 2, and the production of the imports from country 2 requires exports of country 1, thus the output of country 1 will increase further. That is $B^{1}A^{12}B^{2}A^{21}B^{1}A^{12}B^{2}A^{21}B^{1}$;

And so on and so forth.

In total, country 1’ s output caused by the intermediate trade used for the production of country 1’s domestic final demand is

$$B^{1}A^{12}B^{2}A^{21}B^{1}+B^{1}A^{12}B^{2}A^{21}B^{1}A^{12}B^{2}A^{21}B^{1}+B^{1}A^{12}B^{2}A^{21}B^{1}A^{12}B^{2}A^{21}B^{1}A^{12}B^{2}A^{21}B^{1}+…$$

$=B^{1}A^{12}B^{2}A^{21}B^{1}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}$

Likewise, we can obtain country 2’s output caused by the intermediate trade used for the production of country 2’s final demand: $B^{2}A^{21}B^{1}A^{12}B^{2}\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}$.

To add up the above two parts of effects, the vector of output induced by the intermediate trade used for all countries’ domestic final demands is

$q^{z}=\left(\begin{matrix}B^{1}A^{12}B^{2}A^{21}B^{1}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}&L^{12}\\L^{21}&B^{2}A^{21}B^{1}A^{12}B^{2}\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}\end{matrix}\right)\left(\begin{matrix}y^{d1}\\y^{d2}\end{matrix}\right)$ (8)

For $L=(I-A)^{-1}=\left(\begin{matrix}L^{11}&L^{12}\\L^{21}&L^{22}\end{matrix}\right)$*,* it is easy to get

$L^{11}=B^{1}+B^{1}A^{12}B^{2}A^{21}B^{1}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}$ (9)

$L^{12}=B^{1}A^{12}B^{2}\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}$ (10)

$L^{21}=B^{2}A^{21}B^{1}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}$ (11)

$L^{22}=B^{2}+B^{2}A^{21}B^{1}A^{12}B^{2}\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}$ (12)

where, $B^{1}=\left(I-A^{11}\right)^{-1}$，$B^{2}=\left(I-A^{22}\right)^{-1}$. By equation (9） and equation(12）, we know that

 $B^{1}A^{12}B^{2}A^{21}B^{1}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}=L^{11}-B^{1}$

$B^{2}A^{21}B^{1}A^{12}B^{2}\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}=L^{22}-B^{2}$

then

$q^{z}=\left(\begin{matrix}L^{11}-B^{1}&L^{12}\\L^{21}&L^{22}-B^{2}\end{matrix}\right)\left(\begin{matrix}y^{d1}\\y^{d2}\end{matrix}\right)$ (13)

To plus equation (4)and equation(13), we get the total output induced by trade, including final trade and intermediate trade: $q^{t}+q^{z}=Ly^{t}+(L-B)y^{d}$

**4. The relationship of the method in this paper and that of LTV**

Bart Los et.al (2015) proposed a method to compute the domestic value added in exports (DVA) using hypothetical extraction technique (represented by “LTV” in the following sections). It corresponds to the sum of the first five elements in KWW’s equation (36).

By using the method proposed in this paper, for country i’s domestic value added in exports, we have

$$DVA\_{si}=\left(\begin{matrix}0&\cdots &\begin{matrix}v\_{i}&\begin{matrix}0&\begin{matrix}\cdots &0\end{matrix}\end{matrix}\end{matrix}\end{matrix}\right)(Ly^{t}+\left(L-B\right)y^{d})$$

Since the rule of the transformation from gross trade to net trade that all trade can be treated as final trade, from equation (17), we have that

$$Ly^{t}+\left(L-B\right)y^{d}=\left(\begin{matrix}\begin{matrix}L\_{11}&L\_{12}\\L\_{21}&L\_{22}\end{matrix}&\begin{matrix}\cdots &L\_{1N}\\\cdots &L\_{2N}\end{matrix}\\\begin{matrix}\cdots &\cdots \\L\_{N1}&L\_{N2}\end{matrix}&\begin{matrix}&\cdots \\\cdots &L\_{NN}\end{matrix}\end{matrix}\right)\left(\begin{matrix}\begin{matrix}0&y\_{12}+A\_{12}B\_{2}y\_{22}\\y\_{21}+A\_{21}B\_{1}y\_{11}&0\end{matrix}&\begin{matrix}\cdots &y\_{1N}+A\_{1N}B\_{N}y\_{NN}\\\cdots &y\_{2N}+A\_{2N}B\_{N}y\_{NN}\end{matrix}\\\begin{matrix}\cdots &\cdots \\y\_{N1}+A\_{N1}B\_{1}y\_{11}&y\_{N2}+A\_{N2}B\_{2}y\_{22}\end{matrix}&\begin{matrix}&\cdots \\\cdots &0\end{matrix}\end{matrix}\right)\left(\begin{matrix}1\\1\\\begin{matrix}\vdots \\1\end{matrix}\end{matrix}\right)$$

Therefore, the domestic value added in exports of country i (DVA) is

$$DVA\_{s}=\left(\begin{matrix}0&\cdots &\begin{matrix}v\_{s}&\begin{matrix}0&\begin{matrix}\cdots &0\end{matrix}\end{matrix}\end{matrix}\end{matrix}\right)\left(\begin{matrix}\begin{matrix}L\_{11}&L\_{12}\\L\_{21}&L\_{22}\end{matrix}&\begin{matrix}\cdots &L\_{1N}\\\cdots &L\_{2N}\end{matrix}\\\begin{matrix}\cdots &\cdots \\L\_{N1}&L\_{N2}\end{matrix}&\begin{matrix}&\cdots \\\cdots &L\_{NN}\end{matrix}\end{matrix}\right)\left(\begin{matrix}\begin{matrix}0&y\_{12}+A\_{12}B\_{2}y\_{22}\\y\_{21}+A\_{21}B\_{1}y\_{11}&0\end{matrix}&\begin{matrix}\cdots &y\_{1N}+A\_{1N}B\_{N}y\_{NN}\\\cdots &y\_{2N}+A\_{2N}B\_{N}y\_{NN}\end{matrix}\\\begin{matrix}\cdots &\cdots \\y\_{N1}+A\_{N1}B\_{1}y\_{11}&y\_{N2}+A\_{N2}B\_{2}y\_{22}\end{matrix}&\begin{matrix}&\cdots \\\cdots &0\end{matrix}\end{matrix}\right)\left(\begin{matrix}1\\1\\\begin{matrix}\vdots \\1\end{matrix}\end{matrix}\right)=V\_{s}\sum\_{j\ne s}^{N}L\_{ss}y\_{sr}+V\_{s}\sum\_{j\ne s}^{N}L\_{sj}y\_{jj}+\sum\_{j\ne s}^{N}\sum\_{t\ne j,s}^{N}V\_{s}L\_{sj}y\_{jt}+V\_{s}\sum\_{j\ne s}^{N}L\_{sj}y\_{js}+V\_{s}\sum\_{j\ne s}^{N}L\_{sj}A\_{js}B\_{s}y\_{ss}$$

It also corresponds to the sum of the first five items in KWW’s equation (36), and exactly equal to the result of LTV method.

**5. The equivalence and differences between the results of international IO model and that of single country IO model**

**5.1 The equivalence**

For the domestic value added in exports by international IO model, we have

$$DVA\_{I}=V[Ly^{t}+(L-B)y^{d}]$$

And for DVA by single country IO model, we have

$$DVA\_{S}=VB\left(y^{t}+z\right)$$

**Theorem 1** In total level, the result of domestic value added in exports computed by international IO method is equal to that computed by single country model.

Proof. z is the vector of intermediate exports, and we have that

$$z=\overbar{A}q$$

for $q=L(y^{t}+y^{d})$, it comes that

$$z=\overbar{A}q=\overbar{A}L(y^{t}+y^{d})$$

Then

$$DVA\_{S}=VB\left(y^{t}+z\right)=VB\left(y^{t}+\overbar{A}L(y^{t}+y^{d})\right)=VBy^{t}+VB\overbar{A}Ly^{t}+VB\overbar{A}Ly^{d}$$

We know that

$$B\overbar{A}L=\left(I-\hat{A}\right)^{-1}\left(I-\hat{A}-\left(I-A\right)\right)L=\left(I-B\left(I-A\right)\right)L=L-B$$

Therefore

$$DVA\_{S}=VBy^{t}+VB\overbar{A}Ly^{t}+VB\overbar{A}Ly^{d}=V\left(Ly^{t}+\left(L-B\right)y^{d}\right)=DVA\_{I}$$

**5.2 The difference**

Although in aggregate level, the DVA by international IO method is equal to DVA by single country IO method,

 International input-output model describes the inter-country transaction and international cooperation in detail, and thus can reflect the world production connections clearly.

The production relations originating from international production division are shown in the matrix L-B. For the case of two countries, country 1 consumes the intermediate imports from country 2, and vice versa. Therefore, there are the effects of spillovers and feedbacks in the production of final products of the two countries. These spillovers and feedbacks are reflected in the matrix of L-B. However, the single country model does not require the intermediate transactions among countries, and computes DVA in exports by multiplying domestic multiplier matrix *Bi* with gross exports directly. Therefore it cannot analyze the inter-country relations detailed as the international IO model.

Based on international input-output model, for the case of two country, we can decompose the multiplier matrix L-B into the effects of spillovers and feedbacks (M. Sonis, G.D. Hewings, 1993; Round, 1985). This paper mainly focuses on the economic relationships shown in DVA in exports in the case of more than two countries. Define the matrix of DVA in exports as

$$MDVA\_{I}=\hat{v}\left[L\hat{y}^{t}+\left(L-B\right)\hat{y}^{d}\right]=\left(\begin{matrix}\begin{matrix}\hat{v}^{1}(L^{11}\hat{y}\_{}^{t1}+\left(L^{11}-B^{1}\right)\hat{y}\_{}^{d1})&\hat{v}^{1}(L^{12}\hat{y}\_{}^{t2}+L^{12}\hat{y}\_{}^{d2})\\\hat{v}^{2}(L^{21}\hat{y}\_{}^{t1}+L^{21}\hat{y}\_{}^{d1})&\hat{v}^{2}(L^{22}\hat{y}\_{}^{t2}+\left(L^{22}-B^{2}\right)\hat{y}\_{}^{d2})\end{matrix}&\begin{matrix}\cdots &\hat{v}^{1}(L^{1N}\hat{y}\_{}^{tN}+L^{1N}\hat{y}\_{}^{dN})\\\cdots &\hat{v}^{2}(L^{2N}\hat{y}\_{}^{tN}+L^{2N}\hat{y}\_{}^{dN})\end{matrix}\\\begin{matrix}\cdots &\cdots \\\hat{v}^{N}(L^{N1}\hat{y}\_{}^{t1}+L^{N1}\hat{y}\_{}^{d1})&\hat{v}^{N}(L^{N2}\hat{y}\_{}^{t2}+L^{N2}\hat{y}\_{}^{d2})\end{matrix}&\begin{matrix}&\cdots \\\cdots &\hat{v}^{N}(L^{NN}\hat{y}\_{}^{t1}+\left(L^{NN}-B^{N}\right)\hat{y}\_{}^{t1})\end{matrix}\end{matrix}\right)$$

Obviously, the ith row block of matrix $MDVA\_{I}$ reflects the “origins” of the DVA in exports in country i, that is, the information of “whom I am affected by”. For instance, the elements in the block of the jth column in the ith row indicate the DVA in exports in country i for the production of the final products in country j, or the DVA in exports of country i that is absorbed in country j. The row sums of the ith row block are the DVA in exports by industry in country i. The jth column block in *MDVAI* shows the DVA in exports in other countries except for country j, for the production of final products in country i, and it means “whom I affect”.

We cannot deduce the matrix of the DVA in exports by using the single country IO model. Therefore, there is no information on “whom I am affected by” and “whom I affect”, and it can only propose the DVA in exports in aggregate level. But the single country model requires much less data than the international IO model.

**5. Conclusion**

When using input-output model to calculate DVA, there are two kinds of methods: by international input-output model and by national noncompetitive input-output model (single country model). This paper explores the equivalence and differences of the two methods. We first propose a new method to compute DVA in exports, and compare it with the method given by Los et al (2015) (LTV). It shows that for the total DVA in exports of a country, the results by the method in this paper are equal to that of LTV method. But the method in this paper can give the DVA in exports by industry, and the equation is more simplified. Then we prove that in aggregate level without considering the sources of a country’s DVA in exports, the result by international IO model is equivalent to that by single country IO model.However, the method based on international IO model reflects the production connections and the effects of spillovers and feedbacks. It can give the sources of a country’s DVA in exports, that is, “whom I am affected by”. We can also analyze “whom I affect” by the model.

When we are trying to compute the DVA in exports of a country, the method based on single country model just requires non competitive IO table of the country, and does not need to compile the comprehensive international IO table. Therefore, one can choose the appropriate model according to the available data and the goal of the analysis.

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**Appendix A**

**A.1 The other two paths of deriving the method of computing the output induced by exports and the domestic factor content in exports**

A.1.1 path 2

The output caused by final trade is the same as that in path 1 (see section 3, equation (3)). Therefore, similarly to that in path 1, we need to measure the effect of intermediate trade used for all countries’ domestic final demands. In this path, we compute the amount of intermediate trade used for domestic final demand first, and then find a way to compute the output induced by this amount of intermediate trade.

 First, the output induced by domestic final demands is

$q^{d}=(I-A)^{-1}y^{d}$ (A.1)

where yd denotes the vector of domestic final demands, for two country case, $y^{d}=\left(\begin{matrix}y^{11}\\y^{22}\end{matrix}\right)$.Then, the intermediate trade used for the above output $q^{d}$ is

$z^{d}=\left(\begin{matrix}z^{d1}\\z^{d2}\end{matrix}\right)=\left(\begin{matrix}0&A^{12}\\A^{21}&0\end{matrix}\right)q^{d}=\left(\begin{matrix}0&A^{12}\\A^{21}&0\end{matrix}\right)(I-A)^{-1}y^{d}$ (A.2)

Next, we measure the outputs of the two countries caused by $z^{d}$. This cannot be treated in the same way as final trade. Here, we use the idea of the single country model, and consider $z^{d1} \& z^{d2}$ as exogenous variables of the single production systems of country 1 and country 2. Then, we have

 $q\_{}^{zd1}=\left(I-A^{11}\right)^{-1}z^{d1}$ (A.3a)

 $q\_{}^{zd2}=\left(I-A^{22}\right)^{-1}z^{d2}$ (A.3b)

 Let $q^{zd}=\left(\begin{matrix}q^{zd1}\\q^{zd2}\end{matrix}\right)^{}$. Then,

 $q^{zd}=\left(\begin{matrix}\left(I-A^{11}\right)^{-1}&0\\0&\left(I-A^{22}\right)^{-1}\end{matrix}\right)\left(\begin{matrix}0&A^{12}\\A^{21}&0\end{matrix}\right)q^{d}$

$= \left(\begin{matrix}(I-A^{11})^{-1}&0\\0&(I-A^{22})^{-1}\end{matrix}\right)\left(\begin{matrix}0&A^{12}\\A^{21}&0\end{matrix}\right)(I-A)^{-1}y^{d}$

This result is consistent with that of the first path. We prove it as follows.

$$q^{zd}=\left(\begin{matrix}\left(I-A^{11}\right)^{-1}&0\\0&\left(I-A^{22}\right)^{-1}\end{matrix}\right)\left(\begin{matrix}0&A^{12}\\A^{21}&0\end{matrix}\right)q^{d}$$

$=\left(\begin{matrix}(I-A^{11})^{-1}&0\\0&(I-A^{22})^{-1}\end{matrix}\right)\left(\begin{matrix}0&A^{12}\\A^{21}&0\end{matrix}\right)(I-A)^{-1}y^{d}$

$=\left(\begin{matrix}0&B^{1}A^{12}\\B^{2}A^{21}&0\end{matrix}\right)\left(\begin{matrix}L^{11}&L^{12}\\L^{21}&L^{22}\end{matrix}\right)\left(\begin{matrix}y^{11}\\y^{22}\end{matrix}\right)$

$$=\left(\begin{matrix}B^{1}A^{12}L^{21}&B^{1}A^{12}L^{22}\\B^{2}A^{21}L^{11}&B^{2}A^{21}L^{12}\end{matrix}\right)\left(\begin{matrix}y^{11}\\y^{22}\end{matrix}\right)$$

Using $\left(\begin{matrix}A^{11}&A^{12}\\A^{21}&A^{22}\end{matrix}\right)\left(\begin{matrix}L^{11}&L^{12}\\L^{21}&L^{22}\end{matrix}\right)=\left(\begin{matrix}I&0\\0&I\end{matrix}\right)$, we can deduce that

 $B^{1}A^{12}L^{21}=B^{1}A^{12}B^{2}A^{21}B^{1}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}$

$B^{1}A^{12}L^{22}=B^{1}A^{12}\left(B^{2}+B^{2}A^{21}B^{1}A^{12}B^{2}\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}\right)$

$=B^{1}A^{12}B^{2}\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}$

$=L^{12}$

$B^{2}A^{21}L^{11}=B^{2}A^{21}\left(B^{1}+B^{1}A^{12}B^{2}A^{21}B^{1}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}\right)$

$=B^{2}A^{21}B^{1}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}$

$=L^{21}$

$B^{2}A^{21}L^{12}=B^{2}A^{21}B^{1}A^{12}B^{2}\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}$

 Then

$$q^{zd}=\left(\begin{matrix}B^{1}A^{12}B^{2}A^{21}B^{1}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}&L^{12}\\L^{21}&B^{2}A^{21}B^{1}A^{12}B^{2}\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}\end{matrix}\right)\left(\begin{matrix}y^{11}\\y^{22}\end{matrix}\right)=q^{z}$$

$q^{zd}=q^{z}=\left(\begin{matrix}L^{11}-B^{1}&L^{12}\\L^{21}&L^{22}-B^{2}\end{matrix}\right)\left(\begin{matrix}y^{11}\\y^{22}\end{matrix}\right)$ (A.4)

 After adding equation (3) and（A.4）, we have $q^{t}+q^{zd}=Ly^{t}+(L-B)y^{d}$, which is the same as that in path 1.

**A.1.2 Path 3**

In path 3, we start with the single country model, and consider final trade and intermediate trade as exogenous for a particular country’s production system. Then, we deduce the method based on the international model through the production connections of the countries.

Without loss of generality, we discuss country 1 first. Let yt1 be country 1’s vector of final export, then in the simplified two country case, $y^{t1}=y^{12}$; let z1 denotes the intermediate export of country 1, and yd1 represents the vector of country 1’s domestic final demand, $y^{d1}=y^{11}$.For country 1, the three kinds of final demands resulting in the following output

$q^{ft1}=\left(I-A^{11}\right)^{-1}y^{t1}$ (A.5)

$q^{fz1}=\left(I-A^{11}\right)^{-1}z^{1}$ (A.6)

$q^{d1}=\left(I-A^{11}\right)^{-1}y^{d1}$ (A.7)

The total of these three parts is the vector of total output of country 1. Then, the sum of the former two equations is the vector of output induced by country 1’s exports based on the single country input-output model. Furthermore, the production process of country 1 requires the imports from country 2, therefore we have the following production connections between the two countries.

The vector of import from country 2 used for the production of country 1’s final export is

$z\_{2}^{t1}=A^{21}q^{ft1}=A^{21}\left(I-A^{11}\right)^{-1}y^{t1}$ (A.8)

The vector of import from country 2 used for the production of country 1’s intermediate export is

$z\_{2}^{z1}=A^{21}q^{fz1}=A^{21}\left(I-A^{11}\right)^{-1}z^{1}$ (A.9)

The vector of import from country 2 used for the production of country 1’s domestic final demand is

$z\_{2}^{d1}=A^{21}q^{d1}=A^{21}\left(I-A^{11}\right)^{-1}y^{d1}$ (A.10)

Then, the sum of the above three parts is the intermediate exports of country 2 to country 1, that is

$z^{2}=z\_{2}^{t1}+z\_{2}^{z1}+z\_{2}^{d1}$ (A.11)

We have

 $z^{2}=z\_{2}^{t1}+z\_{2}^{z1}+z\_{2}^{d1}=A^{21}\left(I-A^{11}\right)^{-1}(y^{t1}+z^{1}+y^{d1})$ (A.12)

Similarly, for country 2, we have

$z^{1}=z\_{1}^{t2}+z\_{1}^{z2}+z\_{1}^{d2}=A^{12}\left(I-A^{22}\right)^{-1}(y^{t2}+z^{2}+y^{d2})$ (A.13)

Next, we deduce the method based on the international model using the above production connections of the two countries.

The vectors of output incurred by final export and intermediate export by using the single country model are

$q^{ft}=\left(\begin{matrix}B^{1}&\\&B^{2}\end{matrix}\right)\left(\begin{matrix}y^{t1}\\y^{t2}\end{matrix}\right)$ (A.14)

$q^{fz}=\left(\begin{matrix}B^{1}&\\&B^{2}\end{matrix}\right)\left(\begin{matrix}z^{1}\\z^{2}\end{matrix}\right)$ (A.15)

According to the production connections of the two countries, and considering intermediate trade as unknown variables, we can calculate them by solving the following equations

 $z^{1}=z\_{1}^{t2}+z\_{1}^{z2}+z\_{1}^{d2}=A^{12}B^{2}(y^{t2}+z^{2}+y^{d2})$ (A.16a)

$z^{2}=z\_{2}^{t1}+z\_{2}^{z1}+z\_{2}^{d1}=A^{21}B^{1}(y^{t1}+z^{1}+y^{d1})$ (A.16b)

Then, we have

$$z^{1}=\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}(A^{12}B^{2}y^{t2}+A^{12}B^{2}A^{21}B^{1}y^{t1}+A^{12}B^{2}y^{d2}+A^{12}B^{2}A^{21}B^{1}y^{d1})$$

$$z^{2}=\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}(A^{21}B^{1}y^{t1}+A^{21}B^{1}A^{12}B^{2}y^{t2}+A^{21}B^{1}y^{d1}+A^{21}B^{1}A^{12}B^{2}y^{d2})$$

Or

$$\left(\begin{matrix}z^{1}\\z^{2}\end{matrix}\right)=\left(\begin{matrix}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}A^{12}B^{2}A^{21}B^{1}&\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}A^{12}B^{2}\\\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}A^{21}B^{1}&\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}A^{21}B^{1}A^{12}B^{2}\end{matrix}\right)\left(\begin{matrix}y^{t1}+y^{d1}\\y^{t2}+y^{d2}\end{matrix}\right)$$

The output brought by intermediate trade will be

$\left(\begin{matrix}q^{fz1}\\q^{fz2}\end{matrix}\right)=\left(\begin{matrix}B^{1}&\\&B^{2}\end{matrix}\right)\left(\begin{matrix}z^{1}\\z^{2}\end{matrix}\right)$

$$=\left(\begin{matrix}B^{1}&\\&B^{2}\end{matrix}\right)\left(\begin{matrix}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}A^{12}B^{2}A^{21}B^{1}&\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}A^{12}B^{2}\\\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}A^{21}B^{1}&\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}A^{21}B^{1}A^{12}B^{2}\end{matrix}\right)\left(\begin{matrix}y^{t1}+y^{d1}\\y^{t2}+y^{d2}\end{matrix}\right)$$

$$=\left(\begin{matrix}B^{1}A^{12}B^{2}A^{21}B^{1}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}&L^{12}\\L^{21}&B^{2}A^{21}B^{1}A^{12}B^{2}\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}\end{matrix}\right)\left(\begin{matrix}y^{t1}+y^{d1}\\y^{t2}+y^{d2}\end{matrix}\right)$$

$=\left(\begin{matrix}B^{1}A^{12}B^{2}A^{21}B^{1}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}&L^{12}\\L^{21}&B^{2}A^{21}B^{1}A^{12}B^{2}\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}\end{matrix}\right)\left(\begin{matrix}y^{t1}\\y^{t2}\end{matrix}\right)$

$+\left(\begin{matrix}B^{1}A^{12}B^{2}A^{21}B^{1}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}&L^{12}\\L^{21}&B^{2}A^{21}B^{1}A^{12}B^{2}\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}\end{matrix}\right)\left(\begin{matrix}y^{d1}\\y^{d2}\end{matrix}\right)$ (A.17)

That is, the output caused by the intermediate trade is decomposed into two parts, attributed to the effects of final trade and domestic final demand respectively. The former is the effect of intermediate trade used for the production of final trade, and the latter is the effect of intermediate trade used for the production of domestic final demand. After adding the former to equation (A.14) , which is the output induced by final exports, we have

$$\left(\begin{matrix}B^{1}+B^{1}A^{12}B^{2}A^{21}B^{1}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}&L^{12}\\L^{21}&B^{2}+B^{2}A^{21}B^{1}A^{12}B^{2}\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}\end{matrix}\right)\left(\begin{matrix}y^{t1}\\y^{t2}\end{matrix}\right)$$

$=\left(\begin{matrix}L^{11}&L^{12}\\L^{21}&L^{22}\end{matrix}\right)\left(\begin{matrix}y^{t1}\\y^{t2}\end{matrix}\right)$ (A.18)

This is the same result obtained from equation (3).

The second part of equation (A.17) is the output induced by the intermediate trade used for the domestic final demand products, which is

$$\left(\begin{matrix}B^{1}A^{12}B^{2}A^{21}B^{1}\left(I-A^{12}B^{2}A^{21}B^{1}\right)^{-1}&L^{12}\\L^{21}&B^{2}A^{21}B^{1}A^{12}B^{2}\left(I-A^{21}B^{1}A^{12}B^{2}\right)^{-1}\end{matrix}\right)\left(\begin{matrix}y^{d1}\\y^{d2}\end{matrix}\right)$$

$=\left(\begin{matrix}L^{11}-B^{1}&L^{12}\\L^{21}&L^{22}-B^{2}\end{matrix}\right)\left(\begin{matrix}y^{d1}\\y^{d2}\end{matrix}\right)$ (A.19)

The result is consistent with equation (12). Thus it is proved that the result of the third path is just the same as that of the former two paths. Therefore, they are equivalent, and can be verified by each other.

**A.2 Generalizing the method: From two countries to N countries**

In this section, we generalize the method from two countries to N countries. That is, the outputs induced by final trade and by intermediate trade are

$q^{t}=Ly^{t}=\left(\begin{matrix}\begin{matrix}L^{11}&L^{12}\\L^{21}&L^{22}\end{matrix}&\begin{matrix}\cdots &L^{1N}\\\cdots &L^{2N}\end{matrix}\\\begin{matrix}\cdots &\cdots \\L^{N1}&L^{N2}\end{matrix}&\begin{matrix}\cdots &\cdots \\\cdots &L^{NN}\end{matrix}\end{matrix}\right)\left(\begin{matrix}y^{t1}\\\begin{matrix}y^{t2}\\\cdots \\y^{tN}\end{matrix}\end{matrix}\right)$ (A.20)

and

 $q^{z}=\left(L-B\right)y^{d}=\left(\begin{matrix}\begin{matrix}L^{11}-B^{1}&L^{12}\\L^{21}&L^{22}-B^{2}\end{matrix}&\begin{matrix}\cdots &L^{1N}\\\cdots &L^{2N}\end{matrix}\\\begin{matrix}\cdots &\cdots \\L^{N1}&L^{N2}\end{matrix}&\begin{matrix}\cdots &\cdots \\\cdots &L^{NN}-B^{N}\end{matrix}\end{matrix}\right)\left(\begin{matrix}y^{d1}\\\begin{matrix}y^{d2}\\\cdots \\y^{dN}\end{matrix}\end{matrix}\right)$ (A.21)

where N denotes the number of the countries, and $B^{i}=\left(I-A^{ii}\right)^{-1}$, $B=\left(\begin{matrix}\begin{matrix}B^{1}&\\&B^{2}\end{matrix}&\begin{matrix}&\\&\end{matrix}\\\begin{matrix}&\\&\end{matrix}&\begin{matrix}\ddots &\\&B^{N}\end{matrix}\end{matrix}\right)$, $y^{ti}=\sum\_{j\ne i}^{}y^{ij}$, and $y^{di}=y^{ii}$. Here, $y^{ij}$ denotes the exports of country i to country j, and $y^{ii}$ is country i’s domestic final demand. The proof follows the idea of path 3. First, obtain the vector of intermediate trade by solving the following equations:

$$z^{1}=A^{12}B^{2}z^{2}+A^{13}B^{3}z^{3}+\cdots +A^{1N}B^{N}z^{N}+(A^{12}B^{2}(y^{t2}+y^{d2})+A^{13}B^{3}(y^{t3}+y^{d3})+\cdots +A^{1N}B^{N}(y^{tN}+y^{dN}))$$

$$z^{2}=A^{21}B^{1}z^{1}+A^{23}B^{3}z^{3}+\cdots +A^{2N}B^{N}z^{N}+(A^{21}B^{1}(y^{t1}+y^{d1})+A^{23}B^{3}(y^{t3}+y^{d3})+\cdots +A^{2N}B^{N}(y^{tN}+y^{dN}))$$

*…*

$$z^{N}=A^{N1}B^{1}z^{1}+A^{N2}B^{2}z^{2}+\cdots +A^{N,N-1}B^{N}z^{N}+(A^{N1}B^{1}(y^{t1}+y^{d1})+A^{N3}B^{3}(y^{t3}+y^{d3})+\cdots +A^{N,N-1}B^{N-1}(y^{t,N-1}+y^{d,N-1}))$$

Or in matrix

$$\left(\begin{matrix}I&-A^{12}B^{2}&\begin{matrix}\cdots &-A^{1N}B^{N}\end{matrix}\\-A^{21}B^{1}&I&\begin{matrix}\cdots &-A^{2N}B^{N}\end{matrix}\\\begin{matrix}\vdots \\-A^{N1}B^{1}\end{matrix}&\begin{matrix}\vdots \\-A^{N2}B^{2}\end{matrix}&\begin{matrix}\begin{matrix}&\vdots \end{matrix}\\\begin{matrix}\cdots &I\end{matrix}\end{matrix}\end{matrix}\right)\left(\begin{matrix}z^{1}\\z^{2}\\\begin{matrix}\vdots \\z^{N}\end{matrix}\end{matrix}\right)=\left(\begin{matrix}0&A^{12}B^{2}&\begin{matrix}\cdots &A^{1N}B^{N}\end{matrix}\\A^{21}B^{1}&0&\begin{matrix}\cdots &A^{2N}B^{N}\end{matrix}\\\begin{matrix}\vdots \\A^{N1}B^{1}\end{matrix}&\begin{matrix}\vdots \\A^{N2}B^{2}\end{matrix}&\begin{matrix}\begin{matrix}&\vdots \end{matrix}\\\begin{matrix}\cdots &0\end{matrix}\end{matrix}\end{matrix}\right)\left(\begin{matrix}y^{t1}+y^{d1}\\y^{t2}+y^{d2}\\\begin{matrix}\vdots \\y^{tN}+y^{dN}\end{matrix}\end{matrix}\right)$$

Let

$z=\left(\begin{matrix}z^{1}\\z^{2}\\\begin{matrix}\vdots \\z^{N}\end{matrix}\end{matrix}\right), \overbar{A}=\left(\begin{matrix}\begin{matrix}0&A^{12}\\A^{21}&0\end{matrix}&\begin{matrix}\cdots &A^{1N}\\\cdots &A^{2N}\end{matrix}\\\begin{matrix}\cdots &\cdots \\A^{N1}&A^{N2}\end{matrix}&\begin{matrix}&\cdots \\\cdots &0\end{matrix}\end{matrix}\right), \hat{A}=\left(\begin{matrix}\begin{matrix}A^{11}&\\&A^{22}\end{matrix}&\begin{matrix}&\\&\end{matrix}\\\begin{matrix}&\\&\end{matrix}&\begin{matrix}\ddots &\\&A^{NN}\end{matrix}\end{matrix}\right)$. Then,$A=\hat{A}+\overbar{A}, and B=(I-\hat{A})^{-1}$.

Therefore,

$\left(I-\overbar{A}B\right)z=\overbar{A}B(y^{t}+y^{d})$ (A.22)

Solving it, we have

$z=\left(I-\overbar{A}B\right)^{-1}\overbar{A}B(y^{t}+y^{d})$ (A.23)

The output induced by the intermediate trade is

$q^{ze}=B\left(I-\overbar{A}B\right)^{-1}\overbar{A}B\left(y^{t}+y^{d}\right)=B\overbar{A}B\left(I-\overbar{A}B\right)^{-1}y^{t}+B\overbar{A}B\left(I-\overbar{A}B\right)^{-1}y^{d}$ (A.24)

From the above equation, the output induced by the intermediate trade used for the domestic final demand is

$q^{zd}=B\overbar{A}B\left(I-\overbar{A}B\right)^{-1}y^{d}$ (A.25)

And the output brought by final exports is

$q^{t}=By^{t}+B\overbar{A}B\left(I-\overbar{A}B\right)^{-1}y^{t}=\left(B+B\overbar{A}B\left(I-\overbar{A}B\right)^{-1}\right)=(I-B\overbar{A})^{-1}By^{t}$ (A.26)

Since

 $L=\left(I-\hat{A}-\overbar{A}\right)^{-1}=\left(I-\left(I-\hat{A}\right)^{-1}\overbar{A}\right)^{-1}\left(I-\hat{A}\right)^{-1}$

$=(I-B\overbar{A})^{-1}B$

we have

$q^{zd}=(L-B)y^{d}$ (A.27)

$q^{t}=Ly^{t}$ (A.28)

1. This research is funded by National Natural Science Foundation of China (project No. 70903071) [↑](#footnote-ref-2)