The Macroeconomic Social Accounting Matrix of Tunisia in 1996

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Abstract

This paper shows the characteristics of the macroeconomic Social Accounting Matrix (SAM) of Tunisia in 1996. It is established that the SAM is one of the data bases of the Computable General Equilibrium (CGE) Models. These last years, the researchers have intensified the use of the SAM within the framework of the modelling of CGE within their work. The SAM is regarded as the general data base because it contains the whole relationship between the economic agents of a given economy, including the existing relationship on the level of table input – output. The SAM can be presented in two forms: the aggregate form (or macroeconomic) and desegregated form (or microeconomic). In this paper, we will focus on the first form of SAM corresponding to Tunisia during 1996. This study is made up of two sections. In the first section, we will present the construction of the unbalanced Macroeconomic Social Accounting Matrix (SAMmac) of Tunisia in 1996. This section includes three sub-sections. The first sub-section contains the accounts of our SAMmac in 1996. Then, the second sub-section will be devoted to the statistical sources used for the construction of our SAMmac. Finally, the last sub-section will concentrate on our SAMmac which is in imbalance because of the various sources used. We present, in the second section, the balancing of SAMmac of Tunisia in 1996 in basing upon the method known as "Cross-Entropy". Initially, in a first sub-section, we will define the "Cross-Entropy" approach. Then, in a second sub-section, we will use this last approach to balance our SAMmac. Conclusions follow.

Keywords: Social Accounting Matrix, Cross-Entropy, Tunisia, balanced of the SAM
Introduction

This paper deals with a methodology of the construction of a Macroeconomic of Social Accounting Matrix (SAMmac) of Tunisia in 1996. Firstly, we build SAMmac unbalanced by relying on the various sources of the statistics available. Then, we will present the "Cross-Entropy" approach to balance or to equalise the sums in lines and columns of each account on the level of SAMmac. Moreover, this approach can be applied, including in the case of more desegregated SAM. Finally, we will present SAMmac of Tunisia in 1996 as an application of the above selected approach.

1 the construction of an unbalanced SAMmac of 1996

1.2 Definitions of the accounts of SAMmac

The SAM is the database of the CGE models. Generally, it occurs in the shape of a square matrix, which traces flows of the exchanges between the economic agents. The SAM is founded on the principle of balance between expenses (in columns) and the receipts (in lines) on the level of each account, but also on the level of the whole accounts. It can be presented in two forms: the aggregate form and the desegregated form. In this paper, we will be interested in the aggregate form of the Tunisia SAM in 1996. Besides, the SAM of a small open economy has five types of accounts:

- account of the activities
- account of the products
- accounts of the factors
- accounts of the institutions
- account of the saving-investment

In the case of Tunisia, the SAMmac comprises nine accounts on line and nine accounts in column: the account of the activities, the account of the products, two accounts of the factors (Labour and Capital), four accounts of institutions (Household, Companies (firms), State, Rest Of the World (ROW)) and one account of the Saving-Investment. Table 1 draws up the structure of our SAMmac of Tunisia in 1996. A diagrammatic presentation is proposed in figure 1 in order to make readable the relationship between the various accounts of the SAMmac.

Account of the activities (1st line, 1st column)

The first column describes the uses of the account of the activities whose total corresponds to the total cost of the production. In other words, this account translates the behaviour of the producers in the Tunisian economy. They carry out payments with the account of the products for their intermediate consumption, that of the payments to the accounts of the factors in the form of values-added, of the payments to the account of the State constituting the value-added taxes. As for the first line, it describes the resources of the account of the activities which corresponds to the total domestic production. These resources consist of the value of the Tunisian products sold on the domestic and foreign markets, poured by the account of the products.
Table 1: The structure of the Tunisian Social Accounting Matrix (in 1996)

<table>
<thead>
<tr>
<th>Expenses</th>
<th>Activities</th>
<th>Products</th>
<th>Factors</th>
<th>Institutions</th>
<th>Savi- Inves.</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>receipts</td>
<td>Activities</td>
<td>Products</td>
<td>Factors</td>
<td>Institutions</td>
<td>Savi- Inves.</td>
<td>Total</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Labour</td>
<td>Capital</td>
<td>Households</td>
<td>Firms</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>Activities</td>
<td>Products</td>
<td>Factors</td>
<td>Institutions</td>
<td>Savi- Inves.</td>
<td>Total</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Labour</td>
<td>Capital</td>
<td>Households</td>
<td>Firms</td>
</tr>
<tr>
<td></td>
<td>Products</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Intermediate</td>
<td>Marketing margins</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Consumption</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Labour</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Added value to labour</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Capital</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Added value to capital</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Institutions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Households</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Firms</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>State</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tax on the products</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(indirect tax)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ROW</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Savi- Inves.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: IEQ and some modifications done by the author
Figure 1 : the circular flow of the income in an open economy

The second line corresponds to the resources of the account of the products which is composed of the intermediate consumption (paid by the account of the activities), the marketing margins (paid by the account of the products), the final consumption (paid by the account household), the public consumption (paid by the State account), the external consumption in the form of exports (paid by the account of the ROW) and the investment demand (Gross fixed capital formation (GFCF) and variations of stocks) stemming from the account of the Saving–Investment. This account can also receive subsidies coming from the State account.

The second column corresponds to the uses of the account of the products, it is composed of the payments bound for the accounts of the activities (i.e. by eliminating the value of the indirect taxes from the value of the gross output, extracted from the Input-Output table in 1996. Thus, this result provides us with the obtained what is called the matrix activity – product), the value of the imports (paid with account ROW), the payments to the account of State (indirect taxes: consumer taxes, Added Value Tax (AVT), tariffs (customs duty) and other indirect taxes) and the payments to the account of the products (marketing margins). As far as the Tunisian SAMmac, only the goods pour marketing margins with the account of the trades (according to the 1996 Input-Output table of Tunisia).

Account of the factors : Labour (3\textsuperscript{rd} line, 3\textsuperscript{rd} column)

The third line constitutes the value-added versed to the labour factor in the form of gross salaries. Whereas, the third column corresponds to the payment poured with the households which hold the labour factor.

Account of the factors : Capital (4\textsuperscript{th} line, 4\textsuperscript{th} column)

The fourth line constitutes the value-added versed to the Capital factor in the form of Gross Exploitation Surplus (GES). This value-added is versed directly in column and partly with the households (for the capital of the individual firms) then partly with the account of the companies (for the capital held by the companies).
Account of the institutions: Household (5th line, 5th column)

The households receive their income (in lines) in the form of the total of sectoral remunerations of the work poured by the Labour account, of the capital yield poured by the capital account and of the transfers from the companies, from the State and from the ROW. These incomes are broken down (in columns) with the purchase of the goods and the services (final consumption), with the direct taxes on the income (paid to the State account), with the transfers related on the Companies and the Rest of the world, like the saving (paid to the account of the Saving–Investment).

Account of the institutions: Companies (6th line, 6th column)

The sixth line represent the resources of the companies (firms) account. They are composed of the capital yield held by the companies (in opposition to households or with the individual firms) and of the transfers coming from the households, from the State and from the Rest of the world. While the sixth column represents the uses of the companies account, which constitute the transfers towards the other institutions (Households, State, Rest of the World), and the income taxes poured to the State account and the saving poured to the saving–Investment account.

Account of the institutions: State (7th line, 7th column)

The financial resources of the state (line 7) consist of revenues from taxes (value-added tax paid by the activities account, indirect taxes, tariffs on products, other taxation of the imports, taxation of exports, income tax paid by the households and income taxes paid by the companies), as well as transfers coming from the other institutions. The State breaks down its uses (column 7) in the form of purchases of finished goods (non-commercial public services which are produced by the public administration activity), subsidies paid to the product accounts, transfers to the households, to the companies, to the Rest of the world and of the public saving (corresponding to the current balance of the State) versed to the Saving–Investment account.

Account of the institutions: Rest of the World (8th line, 8th column)

Tunisia is an small open economy. In other words, it carries out transactions with the outside world (the EU, the Arab Maghreb Union (AMU) and other countries of the Rest of the world). These flows correspond to the flows of products in the form of imports and exports, transfers and flow of saving. Thus, the resources of ROW account, on the level of line 8, are composed of imports paid (they are CIF (including freight and insurance)) by the products account and the transfers poured by the households, the companies and the State. Whereas on the level of column 8, the uses of ROW account consist of payments related to the exports carried out by the products account, the transfers poured to the interior institutions and by the pouring of the external saving to the Saving-Investment account.

Account of the Saving – Investment (9th line, 9th column)

Generally, this account is called Capital-account by the majority of the economists. But in our case, the capital account corresponds to an account of factor. To avoid ambiguities in our work, and generally on the level of a static framework, we use the name of the Saving – Investment account (like the researchers of the I.E.Q., Tunis (Tunisie)). In addition, on the level of a dynamic framework, the economists generally call it the account of accumulation. In our SAMmac in 1996, the receipts of the Savings-Investment account contain the saving of the households, the companies, the State and the Rest of the world (corresponding to the deficit of the current account of the Rest of the world). Column 9 presents the uses of this account which consist of the investment demand (GFCF and the variations of stocks) addressed to the products account.
1.2 The origin of SAMmac:

Within the framework of the construction of the Tunisian SAMmac in 1996, it was necessary for us to gather the statistical data available through the various sources of information. Among the principal countable and statistical sources, we will present the following ones:

- the Input-Output table in 1996 (IOT 96): it is presented in the form of three blocks: resources, uses and production.
- the Overall Economic Table (OET) results from the national accounts of the Tunisian INS (1983 base, years 1996-2000). It is composed of six accounts: account of the production, account of the exploitation, account of the income, account of the use of the income, capital account and financial account.
- the accounts of the institutional sectors: these accounts are subdivided into two parts: uses and resources. Each part divided itself into five accounts: account of the production, account of the exploitation, account of the income, capital account and financial account.

From this various information collected within various sources, we noticed that there was an inequality between some sums in lines and some sums in corresponding columns. Consequently, this has obliged us to proceed to the "Cross-Entropy" approach, used by certain economists, in the case of a lack or not-availability of the data or even of an inequality of the totals of the lines with those of the corresponding columns.

1.3 The initial SAMmac: unbalanced

The Tunisian SAMmac in 1996 (table 2) is very aggregate. It is composed of 9 accounts (9 in lines, 9 in columns). From this matrix, we can detect macroeconomic aggregates, for example:

The sum of the value-added (D 16,660.6 million) is equal to the sum of the value-added versed to Labour factor (D 6,989 million) and to the value-added versed to the Capital factor (D 9,671.6 million). However this value corresponds to the GDP of 1996 at factor cost (c.f.). It is noticed that 42 % of the GDP at the factor costs are poured to the account of the Labour factor, whereas in the same time, the remainder is versed with the capital account. In the column of the account of the activities, the negative value corresponds to the subsidy of exploitation paid by the State. The marketing margins are estimated at D 2,339.8 million. The indirect taxes (D 2,847.2 million) are subdivided in indirect taxes related to the production (D 1,519.5 million) and in indirect taxes on the imports (D 1,327.7 million). The capital factor income (D 9,671.6 million) is shared between the households (62.8 %) and the companies (37.2 %) while the factor income Labour is entirely versed with the households (D 6,989 million).
Table 2:

The Macro Social Accounting Matrix of Tunisia in 1996 (in Million Dinars)

<table>
<thead>
<tr>
<th>activities</th>
<th>products</th>
<th>labour</th>
<th>capital</th>
<th>household</th>
<th>firms</th>
<th>state</th>
<th>ROW</th>
<th>saving-Investment</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>activities</td>
<td>33,402</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>33,402</td>
</tr>
<tr>
<td>products</td>
<td>1,7182.9</td>
<td>2,339.8</td>
<td>11,618.4</td>
<td>2,965</td>
<td>8,028.7</td>
<td>4,769.2</td>
<td></td>
<td></td>
<td>46,904</td>
</tr>
<tr>
<td>labour</td>
<td>6,989</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6,989</td>
</tr>
<tr>
<td>capital</td>
<td>9,671.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9,671.6</td>
</tr>
<tr>
<td>household</td>
<td>6,989</td>
<td>6,073.5</td>
<td>2,957.6</td>
<td>1,032.6</td>
<td>131.8</td>
<td></td>
<td></td>
<td></td>
<td>17,184.5</td>
</tr>
<tr>
<td>firms</td>
<td></td>
<td>3,598.1</td>
<td>363.8</td>
<td>705.5</td>
<td>878.7</td>
<td></td>
<td></td>
<td></td>
<td>5,546.1</td>
</tr>
<tr>
<td>State</td>
<td>(-441.5)</td>
<td>2,847.2</td>
<td>1,974</td>
<td>334.2</td>
<td>44.7</td>
<td></td>
<td></td>
<td></td>
<td>4,758.6</td>
</tr>
<tr>
<td>ROW</td>
<td>8,315</td>
<td></td>
<td>498.4</td>
<td>719.4</td>
<td>14.4</td>
<td></td>
<td></td>
<td></td>
<td>9,547.2</td>
</tr>
<tr>
<td>Saving-Investment</td>
<td>1,599.4</td>
<td>1,805.8</td>
<td>873.4</td>
<td>499.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4,778</td>
</tr>
<tr>
<td>Total</td>
<td>33,402</td>
<td>46,904</td>
<td>6,989</td>
<td>9,671.6</td>
<td>16,054</td>
<td>5,817</td>
<td>5,590.9</td>
<td>9,583.3</td>
<td>4,769.2</td>
</tr>
</tbody>
</table>

Source: Calculated by the author

GDP (at factor cost) = sum of AV = D 16,661 million
GDP (at market prices) = D 19,066 million
The inequality, noticed in our SAMmac, appears on the level of the under matrix of the transfers. This is explained by the variations of the accounting of flows between the four institutions in our SAMmac (we eliminated the flows between the institutions themselves (i.e. between the household account in line and the household account in column) because of the difficulty of introducing them on the level of the process of our later modelling "according to the researchers of the IEQ") and by the totals calculated starting from the accounts of the institutions in the National Accounting of Tunisia.

The gross income of the households consist of the payments of the factors of production (Labour and Capital) and the received transfers of the other institutions. This income is broken down for final consumption and for transfers addressed to the companies and to the State (including the direct taxes). The remainder, as for it, is turned towards the saving. The household final consumption represents 72.37% of the total expenditure of the households, whereas the saving represents 10%.

The companies pour dividends to the households and to the Rest of the world, taxes and dividends to the State, and the remainder is preserved like a saving. The saving is 31% of the total expenditure of the companies.

For our case, the State is composed of the Police headquarters, the Local Communities and the Organisations of Social security. The public revenue consists of indirect taxes (59.83%), direct taxes (income and benefit tax) and other received transfers from the other institutions. The public expenditure is distributed between subsidies of exploitation (7.9%), final consumption (53%), and transfers poured to the other institutions. The rough saving is 15.62% of the total public expenditure.

The balance of the current account of the Rest of the world corresponds to the saving of this account. In 1996, this balance was estimated at D 499.4 M (according to the “Comptes de la nation et le budget économique 2001”, INS (Tunisie)), that is to say 3% of the GDP at the c.f.

2 the balancing of SAMmac : " Cross-Entropy "

Previously, we have showed the inconsistency of our SAMmac due to the various sources of the statistical data. This obliges us to use an effective and recent approach in order to solve this kind of problem, namely the approach of the " Cross-Entropy ".

2.1 The " Cross-Entropy " Approach:

The method of the " Cross-Entropy " is an approach originating from the information theory which is developed by Shannon (1948) and is taken again by Theil (1976) within an economic framework (according to Kapur and Kesavan (1992), and Golan and Al. (1996)). This approach is the used most by Sherman Robinson and these associates in group IFPRI (International Food Policy Research Institute, Washington D.C.) with the aim of applying estimates to the Social Accounting Matrices (Robinson & Al. (1998), Robinson & El – Said (2000) and Robinson, Cattaneo & El – Said (2001)). The "Entropy" technique is a method of resolution of the problems under given. The problem is under given because, for a matrix $n*n$, we seek to identify $n^2$ unknown, of the not-negative parameters (it is with saying the cells of the SAM). However, there are only $(2n-1)$ independent restrictions in line and column. In other words, these restrictions must exist in the problem of estimate so that one can have enough information to obtain a single solution and to provide enough degrees of freedom. The economist of the " Entropy " estimate uses only all information available of the problem corresponding: the procedure of estimate should be neither unaware of information available nor to add false information.

In this paragraph, we will briefly present the procedure of estimate of the SAM by the " Cross-Entropy " method used by a great number of economists and worked out by Robinson and Al. (1998). According to Robinson and Al. (2000) and Robinson and El - Said (2000), we define a matrix $T$ with elements $T_{ij}$ which represent the payment of an account of a column $j$ to an account of a line $i$. As it is mentioned with the lower part, the matrices have a coherent countable structure. In other words, each sum of the line ($y_i$) must be equal to the sum of the corresponding column ($y_j$):

$$y_i = \sum_j T_{i,j} = \sum_j T_{j,i}$$
By dividing each input cell in the matrix by its total of the corresponding column, we obtain the matrix of the coefficients of column $A$:

$$A_{i,j} = \frac{T_{i,j}}{y_j}$$

By definition, all the sums of the column in $A$ must be equal to 1, from where a singular matrix. Being given that the sums in columns are equal to the sums on line, we can write in a matrix form:

$$y = Ay$$

It is supposed that the "Entropy" problem starts with a known matrix, $\hat{A}$ (called the "prior"), which can be a SAM of the previous year, or an unbalanced matrix. For example, in our study, the matrix $\hat{A}$ is regarded as an unbalanced matrix. The $\hat{A}$ represents the starting point of the procedure of "Cross-Entropy" balancing to derive the new matrix from coefficients $A^*$, the "Entropy" main concern is to find a new whole of coefficient $A$ which minimises Kullback-Leibler measurement (1951) of the "Cross-Entropy" (CE) distance between the matrix $\hat{A}$ (the "prior") and the new matrix of coefficient estimated $A^*$.

$$\min_{\{A\}} I = \left[ \sum \sum A_{i,j} \ln \frac{A_{i,j}}{\hat{A}_{i,j}} \right] = \left[ \sum \sum A_{i,j} \ln A_{i,j} - \sum \sum A_{i,j} \ln \hat{A}_{i,j} \right]$$

under constraint:

$$\sum_j A_{i,j} y_j^* = y_j^*$$

$$\sum_j A_{i,j} = 1 et 0 \leq A_{i,j} \leq 1$$

The solution of the preceding problem is solved by the establishment of the formula of Lagrange.

The result is composed of the information of the data and the "prior":

$$A_{i,j} = \frac{\hat{A}_{i,j} \exp(\lambda_i y_j^*)}{\sum \hat{A}_{i,j} \exp(\lambda_i y_j^*)}$$

With $\lambda_i$ is the multiplier of Lagrange associated to the information and to the sums in line and column.

The $k$ aggregates macroeconomic can be added to the whole of the constraints of the preceding problem as follows:

$$\sum \sum G_{i,j}^{(k)} T_{i,j} = \gamma^{(k)}$$

With $G$ is an aggregate matrix (n*n), on the level of the cells which represent the macro constraints taking 1 like a value, if not it takes zero. The $\gamma$ is the value of the aggregate constraint.

In the estimate of example, we have used two constraints which fix two significant macroeconomic aggregates, namely: GDP at factor cost and GDP at market price.
While referring to the stochastic approach, the model of standard regression arises as follows:

\[ Y = X \beta + e \]

With:
- \( \beta \): the coefficient of the vector to be estimated.
- \( Y \): the vector of the dependent variables.
- \( X \): the vector of the independent variables.
- \( e \): the term of the error (noise).

It is not excluded that the economist may be confronted with statistical data including errors while acting in reality. The "Cross-Entropy" problem can be also formalized as a system of "errors in the variables" which are measured with noises.

If, for example, it is supposed that the sums in columns are measured with errors, the coherence of the constraint row/column can be written as follows (while basing itself on the preceding regressive equation):

\[
y = A \left[ \bar{X} + e \right] = A \bar{X} + Ae
\]

\[ y = \bar{X} + e \]

With \( y \) is the vector of the sums in lines and \( \bar{X} \) is the known vector of the sums in columns, measured with the error \( e \). The preliminary estimator of the sums in columns may correspond to the initial sums in column, as with the average of the initial sums in columns and lines.

According to Golan, Judge and Miller (1994) and Golan and Al. (1996), the errors can be written like weighted averages of known constants \( \nu \):

\[ e_i = \sum_w w_{i,w} \nu_{i,w} \]

With \( w \) is the whole of the weights of the errors in the variables \( w_{i,1}, w_{i,2}, ... \) which carry out the following constraints:

\[ \sum_w w_{i,w} = 1 \text{ et } 0 \leq w_{i,w} \leq 1 \]

In our estimate, the weights are treated like probabilities in order to be estimated. The constants, \( \nu \), define the whole of the "support" of the errors and are always selected to produce a symmetrical distribution depending on the number of elements as the whole of \( w \). For example, we suppose that the distribution of the errors is a rectangular and symmetrical distribution compared to zero, with lower and higher limits preset or known (either three or five weights). Thus, the equation of error arises as follows:

\[ e_i = w_i \nu_i - \left( 1 - w_i \right) \nu_i \]

In this case, the variance is fixed. In general, one can introduce several \( \nu \) and \( w \) to incorporate more information in connection with the distribution of the errors.
At the sight of the terminals of the errors, the problem of minimisation arises in the following form (moreover, the
equation $y = \bar{x} + e$ replaces the equation $y = Ax$):

$$\min_{\{A, w, x\}} I = \left[ \sum_{i} \sum_{j} A_{i,j} \ln A_{i,j} - \sum_{i} \sum_{j} A_{i,j} \ln \bar{A}_{i,j} + \sum_{i} \sum_{w} w_{i,w} \ln w_{i,w} \right]$$

under constraint:

$$\sum_{j} A_{i,j} \bar{x}_j + \sum_{j} A_{i,j} e_j = y_i^*$$

$$e_j = \sum_{w} w_{i,w} \bar{v}_{i,w}$$

$$\sum_{w} w_{i,w} = 1 \quad \text{et} \quad 0 \leq w_{i,w} \leq 1$$

$$\sum_{j} A_{i,j} = 1 \quad \text{et} \quad 0 \leq A_{i,j} \leq 1$$

Now, the equation of the SAM is non-linear, because of the product of $A$ and $e$.

The objective of the problem of minimisation is to find the whole of $A$ and $w$ which minimises the "Cross-Entropy" including the terms of errors ($n$ is the number of the elements of the unit $w$):

$$\min_{\{A, w, x\}} I = \left[ \sum_{i} \sum_{j} A_{i,j} \ln A_{i,j} - \sum_{i} \sum_{j} A_{i,j} \ln \bar{A}_{i,j} \right] +$$

$$\left[ \sum_{i} \sum_{w} w_{i,w} \ln w_{i,w} - \sum_{i} \sum_{w} w_{i,w} \ln \frac{1}{n} \right]$$

It should be noted that the macros aggregates (which represent constraints in the problem of estimate) can be also measured with errors, like the sums in line and column. In order to detect them, we consider two sets of weight in distinguishing between the weights $w_1$ for the weight and errors in the sum in columns, and the weights $w_2$ for the errors of the macro aggregate. Now, the problem of optimisation in the formulation "of the errors of the variables" is to find the variables $A_i$, $w_1$ and $w_2$ which minimises "Cross-Entropy" measurement including the terms of the weights of the errors:

$$\min_{\{A, w_1, w_2\}} I = \left[ \sum_{i} \sum_{j} A_{i,j} \ln A_{i,j} - \sum_{i} \sum_{j} A_{i,j} \ln \bar{A} \right] +$$

$$\left[ \sum_{i} \sum_{w} w_{1,i,w} \ln w_{1,i,w} - \sum_{i} \sum_{w} w_{1,i,w} \ln \bar{W}_{1,i,j} \right] +$$

$$\left[ \sum_{i} \sum_{w} w_{2,i,w} \ln w_{2,i,w} - \sum_{i} \sum_{w} w_{2,i,w} \ln \bar{W}_{2,i,j} \right]$$
After having presented the various estimate procedures of the SAM by the "Cross-Entropy" method; we can conclude that "Cross-Entropy" measurements return any number of information which one introduced with the aim of modifying the estimated solution of the inconsistency, as much as the inaccuracy from the moments supposed to be measured with errors. If the constraints of information are close, the distance compared to the precedent (the prior) increases. If they are not dependent, the distance from "Cross-Entropy" will be equal to zero.

2.2 The balanced SAMmac of Tunisia in 1996:

Upon basing on the article of Robinson S. and El-Said M. (2000) which is entitled “GAMS code for estimating a social accounting matrix (SAM) using Cross-Entropy (CE) methods” (Trade and Macroeconomics Division of the International Food Policy Research Institute, Washington, D.C.), we have been able to estimate the Tunisian SAMmac in 1996. Within this framework, we used the GAMS programming which was applied in the case of Mozambique. On the level of GAMS, there are two "solvers" for the non-linear programs: MINOS and CONOPT. If the sums in column are known without errors, then the equations of all the constraints are linear. In this case, it is preferable to use the MINOS "solver" because it optimises the non-linear problems of programming with linear constraints. If it is supposed that the sums in columns have errors, then the constraints are non-linear and the CONOPT "solver" is the best "solver" to use. For the Tunisian SAMmac, we applied the two "solvers" to assure the exactitude and the precision of our statistical data and their sources. The result of the two "solvers" is almost identical (the values differ only on the level of the second, or third figure (digit) after the comma), which means that our statistics are exact and coherent. Table 3 presents the balanced macroeconomic Social Accounting Matrix of Tunisia in 1996. It is noticed, starting from this balanced SAMmac, that the macros aggregates preserve the same values as those of the initial SAMmac (unbalanced) since we have introduced constraints into GAMS program. Moreover, these aggregates are collected from the relevant Tunisian statistical sources (source: INS, Tunisia).
### Table 3:

The balanced Macro Social Accounting Matrix of Tunisia in 1996 (in Million Dinars)

<table>
<thead>
<tr>
<th></th>
<th>activities</th>
<th>products</th>
<th>labour</th>
<th>capital</th>
<th>household</th>
<th>firms</th>
<th>state</th>
<th>ROW</th>
<th>saving-invest.</th>
<th>Total</th>
</tr>
</thead>
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<td>activities</td>
<td>33,433.005</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>33,410.936</td>
</tr>
<tr>
<td>products</td>
<td>17,181.852</td>
<td>2,725.270</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8,470.154</td>
<td>4,773.487</td>
<td>46,886.825</td>
</tr>
<tr>
<td>labour</td>
<td>6,992.995</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2,914.920</td>
<td>8470.154</td>
<td></td>
<td></td>
<td>6992.995</td>
</tr>
<tr>
<td>capital</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9,677.589</td>
</tr>
<tr>
<td>household</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6,992.995</td>
<td></td>
<td>775.505</td>
<td>1,346.186</td>
<td>125.571</td>
<td>15,782.483</td>
</tr>
<tr>
<td>firms</td>
<td></td>
<td></td>
<td></td>
<td>3,135.363</td>
<td>813.274</td>
<td>833.795</td>
<td>106.077</td>
<td></td>
<td></td>
<td>4,888.509</td>
</tr>
<tr>
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<td>2,813.132</td>
<td></td>
<td>1,603.847</td>
<td>1,160.958</td>
<td></td>
<td></td>
<td>872.116</td>
<td></td>
<td>6008.553</td>
</tr>
<tr>
<td>ROW</td>
<td>7,913.417</td>
<td></td>
<td></td>
<td>990.071</td>
<td>1,125.020</td>
<td>66.008</td>
<td></td>
<td></td>
<td></td>
<td>10,111.516</td>
</tr>
<tr>
<td>saving-invest.</td>
<td></td>
<td></td>
<td></td>
<td>7,913.417</td>
<td>990.071</td>
<td>1,125.020</td>
<td>66.008</td>
<td></td>
<td></td>
<td>4,773.487</td>
</tr>
<tr>
<td>Total</td>
<td>33,410.936</td>
<td>46,886.825</td>
<td>6,992.995</td>
<td>9,677.589</td>
<td>15,782.483</td>
<td>4,888.509</td>
<td>6,008.553</td>
<td>10,111.516</td>
<td></td>
<td>10,111.516</td>
</tr>
</tbody>
</table>

Source: calculated by the author with the Cross Entropy with the help of GAMS

GDP (at factor cost) = D 16,661 million  
GDP (at market prices) = D 19,066 million
Conclusion

At the beginning, the problem of imbalance is posed on the level of the sums in lines and in columns which correspond only to the four accounts of institutions (that is the households, the companies, the State and the Rest of the world) and to the Saving-Investment account, within the Tunisian SAMmac in 1996. To try to solve this imbalance, we have used the "Cross-Entropy" technique of estimate, under constraint of the preset values of the macroeconomic aggregates. After the completion of the balancing of Tunisian SAMmac in 1996, we notice that the "Cross-Entropy" procedure is relevant owing to the fact that it provided these functions of balance between the totals in columns and the totals in lines. In fact, this does not make it possible to omit within the theory of other relevant methods of balancing (for example the RAS method, the Stone-Byron method, etc.), but is realised thanks to the availability and the exactitude of the statistical data used to the reliability of their sources. In this paper, we have applied the "Cross-Entropy" approach in the case of a Macro SAM, but it can be also used within the framework of a Micro SAM. Which might be the object of another paper.

References


Documents of the Tunisian statistics

- The Tunisian System of National Accounting