

# The Joint Use of Technology and Know-how. What are the Effects on Unemployment and Innovation?

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## Abstract

This paper explores the relationship between endogenous growth, driven by technological progress, and technological unemployment. Knowledge formation is the source of growth, for which two dimensions of knowledge are considered: Economy-wide spread knowledge, such as technologies, and specific individual skills. Implementing technological change requires the complementary increase of both types of knowledge. From this follows a vintage-type production pattern, which establishes the employment-growth link. Creative destruction causes the obsolescence of some vintages of knowledge accompanied by a loss of current jobs. Subsequent search on the labor market includes frictions due to inadequate skill supply and may elude an adequate job creation. We analyze under which condition unemployment occurs, which comes from a technology-skill mismatch, and how this produces a feedback on technological progress. The evaluation of innovation policy then reveals the ambiguous relationship between productivity and scale of research via the emergence of a skill mismatch.

*Keywords:* endogenous growth, knowledge diffusion, unemployment, skill mismatch

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## 1 Introduction

In a knowledge-based economy, unemployment and growth should be considered as outcomes of mutual economic impacts. Both empirical and theoretical evidence confirm that knowledge has positive effects on individual employment probability and on aggregate output growth (see, for example: Griliches, 1997, for an overview; Benhabib and Spiegel, 1994, and Barro and Lee, 1993, for the knowledge-growth relationship; OECD, 1992 chap.2, for the link between knowledge and employment). In this paper we address the question of whether knowledge still keeps its positive role if we consider jointly growth and unemployment.

Knowledge is a multi-dimensional input. It includes different facets such as codified and tacit knowledge or specific know-how (e.g. programming skills) and general knowledge (e.g. reading skills). In this paper, we focus on the separation between codified knowledge in the form of technologies and related tacit knowledge in the form of technological skills. Both interact in such a way that it is necessary to use them jointly in order to implement technological change. Technologies determine the general productivity frontier which is characterized, for example, by the quantity and quality of available blue-prints. In contrast to this, individual skills set the degree by which this knowledge frontier can be used because only appropriate skilled workers are able to utilize the state-of-the-art technologies. In this paper, we examine the employment and growth effects that follow from heterogeneities in individual skills in a so-specific knowledge-based economy. Jobs represent particular technologies that demand certain skills. Thus, jobs and workers only match if workers have the relevant skills. Moreover, technological progress changes the job characteristics and the skill demand. A skill mismatch then occurs as a divergence of skill demand and skill supply. Technological unemployment may emerge as a possible consequence which has a feedback on growth as its limits the use of innovative technologies.

Several ideas have been offered to account for the impact of growth on employment and vice versa. An early one is the approach of bounded factor substitution by Harrod (1939) and Domar (1946). Recent literature (see Aricó, 2003, for an overview) focuses rather on imperfections on the labor market such as search costs (Pissarides, 1990, and Aghion and Howitt,

1994), efficiency wages (van Schaik and de Groot, 1998, and Meckel, 2001) and coordination failures (King and Welling, 1995, and Acemoglu, 1997 and 1999). Şener (2000) considers the effects of knowledge formation in the case of skill-biased technological change, in which unemployment of less skilled workers results from the emergence of innovative technologies.

In this paper, we develop a model of endogenous growth, which puts forward innovation as a possible source of unemployment. Two ingredients determine the growth-employment link in the model. First, creative destruction caused by technological change introduces a job turnover which makes search necessary for both employees and jobs (Aghion and Howitt, 1994). Second, similar to Chaira and Hopenhayn (1991) and Stokey (1991) we assume that capital goods in the form of economy-wide available technologies and individual skills are complements in the production of a final good. An example would be that no computer works without the corresponding application skills. On the other hand, software knowledge is also useless without the respective hardware.

In the case of heterogenous skills, the success of the search for appropriate employees and vacancies depends on the ratio of skill demand to skill supply. An inadequate ratio implies a skill mismatch, which involves consequences for growth and employment. This aspect is used to extent the search model based on Pissarides (1990). Job-worker matches are constrained because innovative firms only hire workers if they are currently skilled. Hence, new technologies represent the current skill demand. However, skill supply may be inadequate if technological knowledge diffuses at a low rate. Rapid technological change then obstructs the full use of labor, which represents obsolete skills to a certain extent. It follows from this perception that joblessness cannot be reduced by more innovation-based growth as unemployment results from a skill mismatch that is caused by too many innovations.

The solution of the model reveals ambiguous effects of knowledge formation as a central result. As regards the two aspects of creating new knowledge, namely the development of innovations and the process of acquiring know-how, it appears that a policy on knowledge expansion is an objective that is not straightforward to achieve. This is because the creation of economy-wide knowledge in the form of technological innovations may cause a depreciation of individual technological know-how. Unemployment and

less abilities to implement new technologies oppose the positive growth effect of innovations via the occurrence of a skill mismatch. As a consequence, the growth-employment relationship becomes ambiguous which is in line with the contradictory empirical results (see, for example, Caballero, 1993, Davis and Haltiwanger, 1992, and Tonti and Tanda, 1998).

The employment and skill based approach used in this paper contributes to the debate about the possibility of promoting growth by expanding the scale of the R&D. Early endogenous growth models (for example, Romer, 1990, Grossman and Helpman, 1991, and Aghion and Howitt, 1992) can be interpreted in the way that it should be a policy concern to support private innovation efforts. This is a result of the properties of the growth equation, which is linear in the scale of inputs to R&D. Hence, any policy that leads to a higher share of employment in R&D also increases total growth. Subsequent literature of non-scale growth models based on Jones (1995a, 1995b) eliminate this effect of R&D scale by introducing a counteracting factor such as increasing difficulties in research over time (see Dinopoulos and Thompson, 1998, for an overview). In this paper, we argue that it is not the necessity of increasing R&D efforts over time but an increasing skill mismatch which works against the scale effects in research and produces an inverse relationship between productivity and scale of R&D. As a result, the growth effects of innovation policies are rather ambiguous.

This paper is organized as follows: Section 2 introduces the model. First, it addresses the technology and skill complementarity and after this it derives the optimum conditions of the production decisions in the sectors R&D and manufacturing. Section 3 develops the equilibrium labor allocation that defines simultaneously steady-state growth and unemployment. Section 4 discusses the consequences of knowledge formation under a skill mismatch scenario with technological unemployment. Finally, Section 5 concludes the paper.

## 2 The model

### 2.1 The Set-up

There are three classes of tradable objects: labor, a consumption good, and an intermediate good. Furthermore two dimensions of knowledge are con-

sidered: ...rst, economy-wide used knowledge, denoted as technology; second, individual speci...c knowledge labeled as know-how. A joint use of the two dimensions is necessary in order to implement technological change. The two sectors R&D and manufacturing are associated with the relevant economic activities. Di¤erent competing manufacturing vintages produce a homogenous consumption good and create know-how as a by-product. A number of research units in the R&D sector do research in order to develop a new quality of the intermediate good. However, each innovation creates an economy-wide monopoly in the production of the intermediate goods. Thus, we assume a quality-ladder model, in which the last innovator rules the market.

The economy is populated by a continuous mass  $L$  of in...nite living individuals. Each individual is uniformly endowed with one unit of labor per time unit. Furthermore, workers di¤er in their technological skills in such form that individual  $i$  supplies know-how  $A_i$ . Labor is used in the two sectors R&D, with  $L^R$  denoting the concerned amount of labor, and manufacturing,  $L^M$ . We consider a non-cleared labor market. Hence, unemployment is taken into account in the following labor market equation, where  $u$  symbolizes the unemployment rate<sup>1</sup>:

$$L = L^R + L^M + uL: \quad (1)$$

All individuals  $i \in [1; L]$  share the same linear intertemporal preferences during a in...nite lifetime. Their utility  $v$  is generated by consuming the individual amount of the consumption good,  $Y_{i,t}$ , at date  $t$ :

$$v(Y_i) = \int_0^R Y_{i,t} e^{-rt} dt, \quad (2)$$

where  $r > 0$  is the interest rate, which is also equal to time preference. We assume a frictionless Walrasian credit market in which future expected income can be discounted at the constant rate  $r$ .<sup>2</sup>

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<sup>1</sup> Most variables such as  $L^M$ ,  $L^R$  and  $u$  depend on time. However, we neglect time indices when possible, i.e whenever we only consider steady-state values. Time indices become necessary, though, when we explore the vintage structure, which produces a limited lifetime of technologies and ...rms.

<sup>2</sup> No investments take place. A credit market is however needed because R&D workers earn no income unless their ...rm innovates. (See also Aghion and Howitt, 1992.)

Production of the consumption good takes place in the manufacturing sector by means of a variable amount of the intermediate good,  $x$ , labor,  $L^M$ , and the use of a certain technology  $\zeta$  which sets the productivity level  $A_\zeta$  itself.<sup>3</sup> Technological progress, i.e. increases in  $A_\zeta$ , is embodied and manifests in the arising of new manufacturing vintages. Thus, total output is created by different vintages, which indicate several levels of technology in the endogenous interval  $\zeta \in [\zeta^{\min}, \zeta^{\max}]$ , which is derived in Section 2.3. The vintages therefore differ in their productivity and only the leading one,  $\zeta^{\max}$ , exploits the technology frontier. Due to the limited lifetime of a vintage, a certain technology  $\zeta$  represents  $\zeta^{\max}$  at the birth of the technology and equals  $\zeta^{\min}$  just before the technology becomes obsolete. The start-up of a firm, which represents  $\zeta$ , needs implementation costs  $F_\zeta$ , which grow at the rate  $g_A$ . Consequently, total flow of profits  $\pi_{\zeta,t}^M$ , which vintage  $\zeta$  can earn after its implementation must equal the implementation costs at the minimum. Market entry is therefore restricted by the condition  $e^{rt} \pi_{\zeta,t}^M \geq F_\zeta$ . We assume that implementation costs prevent competitors from market entry because a shared market generates insufficient profits. One firm therefore monopolizes the vintage output and produces with technology  $\zeta$ . As a result, the difference  $\zeta^{\max} - \zeta^{\min}$  also equals the number of manufacturing firms. The market for the homogenous consumption good is served by many firms, which operate competitively although market entry is limited. Potential competitors discipline established firms in such a way that zero profits can be earned during the lifetime of a firm.

The productivity level of a firm depends on the age of its technology set-up. As a result, long existing firms have a relative productivity disadvantage. The joint use of technology and skills establishes the productivity level  $A_\zeta$  of a particular vintage subject to the following restrictions:

- (a) The implementation of innovative technologies requires hiring highly skilled workers. Hence, the technological level of innovative manufacturing firms equals the maximum individual know-how, i.e.  $A_i = A_{\zeta^{\max}}$ .
- (b) The technology sets the firm's productivity at any stage. Firms choose

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<sup>3</sup> We use  $A$  for both the economy-wide technology and individual know-how in order to make the common base clear: know-how is the individual ability to deal with a particular technology.

the highest productivity level at date  $t$ , when the firm is established.

No technological upgrading is possible afterwards, i.e.  $A_\varepsilon \cdot A_{\varepsilon,t}$ .

Furthermore, the current technology is backwards-compatible, i.e.  $A_{\varepsilon}^{\max}$  also includes the previous sets of  $A_\varepsilon$  with  $\varepsilon < \varepsilon^{\max}$ . Assumptions (a) and (b) imply that productivity is restricted in two ways. The minimum of both know-how and fixed technology determines the productivity level.

A manufacturing vintage produces the amount  $Y_\varepsilon$  of consumption goods by means of labor and a fixed technology. An intermediate good supplied by R&D firms represents the licence that gives access to the technology. Suppose that a firm has to buy one licence per worker so that the amount of intermediate goods equals the number of employees, i.e.  $x = L^M$  in a fixed  $(x; L^M)$ -bundle. Moreover, some overhead costs exist measured in a forgone number of intermediate goods  $x^{\min}$ . The production function therefore yields:

$$Y_\varepsilon = \begin{cases} A_\varepsilon L_\varepsilon^{M^\circ} & \text{if } x^{\min} = A_\varepsilon x^\circ \\ 0 & \text{otherwise} \end{cases}; \quad (3)$$

Assuming  $0 < \beta < 1$  indicates the usual diminishing marginal rate of return to the  $(x; L^M)$ -bundle. Only vintages with a sufficiently productive technology  $\varepsilon > \varepsilon^{\min}$  produce total output  $Y = \sum_{\varepsilon=\varepsilon^{\min}}^{\varepsilon^{\max}} Y_\varepsilon d_\varepsilon$ .

Research in the R&D sector develops higher qualities of the intermediate good, which indicate higher productivity  $A_\varepsilon$ . Innovative technologies  $\varepsilon$  which increase the quality of  $x$ , are gradually developed and therefore arise along a line from zero to infinity. The current last innovation  $\varepsilon^{\max}$  thus also measures the number of all innovations that have shifted the productivity frontier so far. Let  $\gamma$  denote the constant size of an innovation. As a result, the productivity gain enabled by an innovative technology is:

$$A_{\varepsilon^{\max}+1} = \gamma A_{\varepsilon^{\max}}, \quad \text{with } \gamma > 1; \quad (4)$$

The R&D firm that has developed  $\varepsilon^{\max}$  also supplies a flow of the intermediate good. Skilled labor is used to search for innovative technologies, but the production of an additional license to use the technology produces nearly zero costs.<sup>4</sup> We therefore assume that only fixed costs of  $C$  per pe-

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<sup>4</sup> An example for such skill intensive production would be the information and communica-

riod must be covered in order to establish a production unit to create  $x$ . Those R&D units, which produce a non state-of-the-art intermediate good with  $A_i < A_{i}^{\max}$ , cannot serve the vintage that uses  $A_{i}^{\max}$ . However, it is easy to show that total market coverage is necessary in order to guarantee  $\eta_i^R > C_i$  if fixed costs or the size of innovations are high. We therefore assume that the market of the intermediate good is monopolistic as it is standard in quality ladder models with one industry (based on Romer, 1990). The amount supplied of the intermediate good,  $x$ , is thus not technologically restricted but equals the profit maximizing amount  $x^*$ , which will be derived in the next section:

$$x = x^*; \quad x^* = \arg \max_x \eta_i^R(x; \cdot) : \quad (5)$$

The minimum of either the growth rate of  $A_i$  or the degree by which  $A_i$  is acquirable limits productivity increases. We therefore develop, first, the endogenous innovation rate of  $A_i$  and second, endogenous skill supply, which determines the scarcity of appropriate  $A_i$ .

While the size of an innovation is exogenous, the frequency with which innovations occur depends on the number of workers in the R&D sector,  $L^R$ . The arrival probability is Poisson-distributed, with " $L^R$ " denoting the arrival rate of a unit time, where " $L^R$ " measures the productivity of research. In a steady state  $L^R$  is constant and depends on equilibrium labor allocation. Transforming (4) into real time units yields the average rate of innovation in equilibrium  $g_A$ :

$$g_A = \frac{\partial A / \partial t}{\partial A} = L^R \ln(\cdot) : \quad (6)$$

Know-how creation is a non-directed process of learning-by-using, which arises as a by-product of manufacturing. Employed workers can acquire know-how when technological knowledge is diffusing through the economy. However two classes of differently skilled workers arise and only the highly skilled part of the labor force can get employed in the innovative manufacturing vintage or in research. Regarding manufacturing workers, we assume an individual probability of  $\gamma$  to get skilled, namely  $A_i = A_{i}^{\max}$ . In total, a

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tion technology industry.

share  $\gamma$  of the employees acquires the maximum level of technological know-how according to the law of great numbers. Let  $D$  denote the number of highly skilled workers, who are able to fill a vacancy in the currently arising manufacturing firm that uses the maximum technological level:

$$D = \gamma L^M: \quad (7)$$

In contrast to this, low skilled workers can just be employed in the non-innovative manufacturing vintages.

## 2.2 The Intermediate Good Monopolist's Decision Problem

Research workers receive no income unless their firm innovates, at which time they are paid by a flow of products  $\pi_{\zeta}^R$  from producing the intermediate good as a monopoly. Thus, a number  $L_{\zeta+1}^R$  of R&D units try to replace the incumbent by developing innovation  $(\zeta + 1)$ . The objective of these R&D units is to maximize the expected value  $V_{\zeta+1}$  from research generated by the next innovation. The following section shows how this value is determined.

The next incumbent rules the market until innovation  $(\zeta + 2)$  arises. Hence,  $V_{\zeta+1}$  includes of a flow of future products  $\pi_{\zeta+1}^R$  generated by the  $(\zeta + 1)^{th}$  innovation over an interval whose length is exponentially distributed with  $L_{\zeta+1}^R$ , namely the arrival rate of the  $(\zeta + 2)^{th}$  innovation. Innovation  $(\zeta + 2)$  establishes the so-called business-stealing effect, when the incumbent is replaced by a new one. The following asset equation includes this effect:

$$rV_{\zeta+1} = \pi_{\zeta+1}^R + L_{\zeta+1}^R V_{\zeta+1}: \quad (8)$$

The profit-maximization behavior of the  $(\zeta + 1)^{th}$  incumbent determines  $\pi_{\zeta+1}^R$  by solving

$$\pi_{\zeta+1}^R = \max [p_{\zeta+1}(x)x_{\zeta+1} - C_{\zeta+1}] \quad (9)$$

in order to decide on optimal output  $x^* = x_{\zeta+1}$  and price  $p^* = p_{\zeta+1}$  at which the innovator can sell the flow of the intermediate good to the final sector. Since manufacturing firms operate competitively,  $p_{\zeta+1}$  must equal marginal revenues of  $x$  in producing the consumption good. From the pro-

duction function (3) we can derive the profit equation for  $\pi_{\zeta+1}^M$  of the future manufacturing vintage:

$$\pi_{\zeta+1}^M = A_{\zeta+1} (x_{\zeta+1})^{\alpha} - (p_{\zeta+1} + w_{\zeta+1}) x_{\zeta+1} - p_{\zeta+1} x^{\min}; \quad (10)$$

where  $w_{\zeta}$  denotes wages in manufacturing. Variable costs of  $(p_{\zeta+1} + w_{\zeta+1})$  arise because of the joint use of labor and the intermediate good in the  $(x; L^M)$ -bundle. First-order condition of  $\max \pi_{\zeta+1}^M$  yields the inverse demand curve,<sup>5</sup> which the  $(\zeta + 1)^{th}$  incumbent faces:

$$p_{\zeta+1} = {}^{\alpha}A_{\zeta+1} (x_{\zeta+1})^{\alpha-1} - w_{\zeta+1}; \quad (11)$$

The maximization program of (11) in (9) leads immediately to the following optimal values of output and price. The quality of a  $(x; L^M)$ -bundle implies that  $x^a$  is equal to labor demand in manufacturing  $L_{\zeta+1}^M$ :

$$x_{\zeta+1} = x^a = L_{\zeta+1}^M = \frac{\mu^{\alpha-2}}{I_{\zeta+1}} \frac{\Pi}{\Pi^{\alpha-1}}; \quad (12)$$

with  $I = w_{\zeta} = A_{\zeta}$  denoting the productivity-adjusted wage. The profit maximizing price yields:

$$p_{\zeta+1} = p^a = \frac{\mu}{\alpha} \frac{1}{I} \frac{\Pi}{\Pi^{\alpha-1}} - w_{\zeta+1} \quad (13)$$

The expected value of innovation  $(\zeta + 1)$  is now specified. It can be derived from solving (8) and (9) using the values for  $x^a$  and  $p^a$ :

$$V_{\zeta+1} = \frac{w_{\zeta+1} L_{\zeta+1}^M \frac{1}{\alpha} \frac{1}{I} \frac{\Pi}{\Pi^{\alpha-1}} - C_{\zeta+1}}{r + {}^{\alpha}L_{\zeta+1}^R} \quad (14)$$

### 2.3 Consumption Good Production

A number of  $i_{\zeta}^{\max}$   $i_{\zeta}^{\min}$  firms produce the homogenous consumption good. Each firm represents a certain vintage, which differ in their productivity. Innovative technologies, developed in the R&D sector, create new vintages. This causes, however, a fall in relative productivity of the current

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<sup>5</sup> R&D firms consider the distribution of technology in manufacturing, which is a linear transformation of the leading technology. We therefore ignore the distribution of  $A$  in manufacturing for the sake of illustration.

ones. Therefore, technological progress gradually decreases relative productivity of a technology after it has been established in a manufacturing firm. A relative productivity below minimum indicates the obsolescence of the respective technology and the related vintage disappears. Let  $S_\ell$  denote a such defined lifetime of a firm. As a result, technological progress restricts the lifetime of firms and causes a turnover of vintages. The lifetime of a firm representing vintage  $\ell$  is endogenous and goes from market entry at  $\tilde{t}$ , when relative productivity is the highest  $\zeta = \zeta_{t=\tilde{t}}^{\max}$ , until its closure at  $T$ , when relative productivity is minimum  $\zeta = \zeta_{t=T}^{\min}$ . This section derives the endogenous values of  $\zeta^{\max}$  and  $\zeta^{\min}$  and  $S_\ell$  and determines the relative decline in input demand of manufacturing firms.

Remember that despite one firm monopolizing one vintage, competition arises among different vintages and the potential market entry of further competitors disciplines the incumbents. This implies zero-profits for vintage  $\ell$  during its production period  $S_\ell = T - \tilde{t}$ . Over this time, the used technology has a sufficiently high relative productivity  $A_\ell$  to make production profitable. The free entry condition therefore yields:

$$\int_{\tilde{t}}^T e^{r(t_1 - t)} A_\ell x_{\ell,t}^\alpha p_{\ell,t} x^{\min} + (p_{\ell,t} + w_{\ell,t}) x_{\ell,t}^\alpha dt = F_\ell. \quad (15)$$

The left-hand side of (15) indicates the discounted value of profits until market exit. The present value of profits must equal fixed costs  $F_\ell$ ; which are paid to establish production, and which grow from vintage to vintage at the rate  $g_A$ .

While  $A_\ell$  is fixed, costs of  $x_\ell$  depend on time. Prices  $p_\ell$  and wages  $w_\ell$ , which a firm of vintage  $\ell$  has to pay, are the current prices  $p_t$  and wages  $w_t$  determined by the productivity of the leading technology.<sup>6</sup> From (12) and (13) we can see that a steady state demands constant productivity-adjusted wages and a fixed price-wage ratio. Hence,  $(p_{\ell,t} + w_{\ell,t})$  increases at the rate of innovation  $g_A$  and makes the use of  $x$  more costly for all

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<sup>6</sup> All vintages of consumption goods are homogeneous and sell at the same zero-profit price. However, skilled and unskilled workers are paid differently. Hence, all wages except  $w_{\ell}^{\max}$  are the same (namely  $w_{\ell}^{\max} = s$ ) because skilled workers are only employed in  $\zeta^{\max}$ . Shifts of  $\zeta^{\max}$  therefore increase wages of all vintages. Furthermore  $p_\ell$  grows with innovations as the intermediate good producer is not able to undertake price-discrimination between the different manufacturing vintages.

vintages. Since costs grow, but the vintage productivity remains the same, increasing variable expenditures  $(p_{\ell,t} + w_{\ell,t})x_{\ell,t}$  equal the decreasing revenues  $A_{\ell}x_{\ell,t}^{\alpha} \cdot p_{\ell,t}x^{\min}$  at  $T$ . At this point  $(p_{\ell,t} + w_{\ell,t})$  attains its maximum,  $(p_{\ell,T} + w_{\ell,T}) = (p + w)^{\max}$ , and the input of  $x_{\ell,T}$  is at its minimum. Put differently, increasing input prices shift the cost function until it is tangent to the fixed revenue function. The slope of the cost function then is  $(p + w)^{\max}$ . Further cost increases lead to the immediate market exit since the necessity of covering overhead costs  $x^{\min}$  excludes a continuing production. At date  $T$  prices and wages have grown a total period of  $T = t + S_{\ell}$  at rate  $g_A$ . Hence, the maximum variable costs, which the firm is able to pay, can be written as a function of the initial values,  $p_0$  and  $w_0$ :

$$(p + w)^{\max} = (p_0 + w_0)e^{g_A(t+S_{\ell})} = (p_{\ell,t} + w_{\ell,t})e^{g_AS_{\ell}}; \quad (16)$$

As a result of symmetry, lifetime of all vintages is constant in steady state, i.e.  $S_{\ell} = S$ . Solving (16) for  $S$  yields:

$$S = \frac{1}{g_A} = \frac{1}{\ln(p^{\max}/(p^{\max} + w^{\max}))}, \quad (17)$$

where  $j$  denotes the lifetime in technological time units, with  $j = \ln(p^{\max}/(p^{\max} + w^{\max})) \approx \ln(p_{\ell,t}/(p_{\ell,t} + w_{\ell,t}))$ . As a result, a higher  $g_A$  cuts the lifetime of a firm because technologies become obsolete more rapidly.

Non-leading vintages produce less and less, until production becomes unprofitable after  $S$  periods. This is because innovations cause a permanent fall in relative productivity of these firms. Output decline is accompanied by a reduction in the input of  $x$  and  $L^M$ . The intermediate good demand for any productivity level is given by (12). The loss of relative productivity caused by an innovation is the same as the productivity differential between vintages  $\ell$  and  $\ell+1$  as the former  $\zeta^{\max}_{\ell}$  turns out to become  $\zeta^{\max}_{\ell+1}$  and so forth. Thus, a new technology leads to an input reduction of a non-innovative vintage, which is equal to the differential between two succeeding productivity levels,  $x_{\ell} - x_{\ell+1}$ :

$$x_{\ell} - x_{\ell+1} = (\zeta_{\ell} - \zeta_{\ell+1}) \frac{\alpha^2}{\alpha - 1} \frac{1}{T_{\ell}}; \quad (18)$$

In equilibrium, the manufacturing sector includes ( $\zeta^{\max} \mathbf{i} \zeta^{\min}$ ) vintages. From the arrival rate we know that " $L^R$  vintages arise per period. As every single vintage exists  $S$  periods, the total number of vintages is:

$$\zeta^{\max} \mathbf{i} \zeta^{\min} = "L^R S = \frac{1}{\ln(\zeta)} \quad (19)$$

### 3 Steady State Growth and Unemployment

#### 3.1 Endogenous Intersectoral Labor Allocation

Flows on the labor market arise between the two sectors and between employment and unemployment. Skilled labor can be allocated freely between R&D and the innovative vintage of manufacturing. As long as income differentials between the two sectors exist, workers move to the one that promises higher income. Furthermore, restricted lifetime of manufacturing firms causes flows into and out of unemployment. Innovations lead to a reduction in employment of non-innovative firms, but they create employment in the innovative ones. Equilibrium requires constant flows on the labor market in such a way that the intersectoral labor allocation and unemployment are fixed. To achieve this, no-arbitrage between the two sectors according to the production decisions must hold. In addition, skill demand and skill supply must be balanced in order to avoid further increases in unemployment.

Skilled labor  $D$  is able to do research and to replace the current incumbent, which provides the flow  $x$  of the intermediate good. The development of the next quality level ( $\zeta + 1$ ) of the intermediate good generates future profits of  $V_{\zeta+1}$ . However, the use of one labor unit in research just causes a unit time probability of " $< 1$ " of being the next innovator. The alternative use of skilled labor is in the innovative vintage of manufacturing, which provides the wage rate  $w_\zeta$ . Hence, no-arbitrage between working in R&D and manufacturing requires:

$$w_\zeta = "V_{\zeta+1}. \quad (20)$$

Substituting  $V_{\varepsilon+1}$  by (14) and some rearranging yields:

$$L^R = \frac{\mu_1}{\mu_1 + 1} L^M - \frac{r}{w} C \quad (\text{AE})$$

(AE) implies a positive relationship between  $L^M$  and  $L^R$ . This indicates that labor input in the two sectors produces mutual income gains. First, research output which depends on  $L^R$  increases labor productivity in manufacturing. Second, the size of the manufacturing sector determines demand and thereby profits of the intermediate good production.

The condition which equation (AE) implies, eliminates possible labor flows between the sectors. The flows between employment and unemployment arise from the vintage turnover and the assumption that technological progress is embodied in both technologies and individual know-how. The vintage turnover results in a job-turnover, which is not free of frictions since skill requirements change during the turnover. The setting of the production technology according to (??) and (3) establishes these frictions. Increases in the productivity level  $A_\varepsilon$  must be accompanied by a sufficiently high number of workers endowed with the corresponding  $A_i$  in order to realize the full employment. Otherwise, skill demand caused by new technologies exceeds skill supply. As a consequence of such a scenario, a part of the non-skilled workers ( $L \setminus D$ ) becomes unemployed.

Remember that no updating of technologies is possible. Technological progress embodied in the innovative vintage therefore leads to a loss in relative productivity of non-innovative vintages. Accordingly, firms, affected in such a way, reduce their labor demand. The quality of a  $(x; L^M)$ -bundle implies that  $x_\varepsilon$  is equal to labor demand in manufacturing  $L_\varepsilon^M$ . Thus,  $x_\varepsilon \in x_{\varepsilon+1}$  of (18) is equal to the number of layoffs of one vintage. In addition to this, one vintage shuts down and dismisses  $x_\varepsilon^{\min}$  employees from their jobs. As  $\zeta^{\max} \in \zeta^{\min}$  of (19) is the number of different vintages and  $"L^R$  innovations arrive per period,

$$U^+ = "L^R \left( \zeta^{\max} - \zeta^{\min} \right) (x_\varepsilon \in x_{\varepsilon+1}) + x_\varepsilon^{\min} \quad (21)$$

workers go into unemployment.<sup>7</sup> From (18), (19), and a constant  $x^{\min}$ , we

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<sup>7</sup> Researchers are skilled automatically by developing the innovations. Hence, they cannot

can see that the number of layoffs per period is fixed in a steady state. We can therefore simplify the analysis by considering that a constant fraction  $\bar{A} = (\zeta^{\max} - \zeta^{\min})(x_{\ell} - x_{\ell-1}) + x_{\ell^{\min}} = (L^M + L^R)$  of the employed labor force ( $1 - u)L$ ) becomes unemployed per innovation. Hence, equilibrium flow into unemployment is:

$$U^+ = "L^R \bar{A}(1 - u)L \quad (22)$$

New vacancies only arise in the innovative vintage. Hence, the recruiting success of this vintage determines the flow out of unemployment. However, the filling of vacancies is restricted by the supply of know-how. Only skilled labor, which supplies  $A_i = A_i^{\max}$ , can be considered for the innovative vintage. Hence, the number of skilled workers according to (7) establishes the equilibrium flow out of unemployment,<sup>8</sup>  $U^i = D$ . If skill supply is inadequate,  $U^+$  exceeds  $U^i$  during the transition towards equilibrium unemployment. Adjustments in the intersectoral labor allocation produce equilibrium unemployment  $u$ , when the flows into and out of unemployment according to  $U^+$  and  $U^i$  are the same:

$$u = \begin{cases} 1 - \frac{U^i}{"AL^R L} & \text{if } u > 0 \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

Unemployment arises due to technological reasons as innovative technologies require skills, which are supplied by a too small number of workers.<sup>9</sup> With the specification of the unemployment rate, we can now devise the employment equation by inserting (23) into the simple labor market equation (1):

$$L^R = \begin{cases} \frac{8}{3} & \text{if } u > 0 \\ : & L^M \end{cases} \quad (EE)$$

The  $L^R=L^M$  combinations of (EE) produce such a balance between tech-

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<sup>8</sup> become unemployed.

<sup>9</sup> Note that it is not necessary to be highly skilled to find a new job because some jobs for low skilled labor become vacant when former employees move to the innovative vintage.

<sup>9</sup> A fall in the wages increases the demand of low-skilled labor by expanding the number of non-innovative manufacturing firms. However, according to the no-arbitrage condition wages are set by the productivity of R&D and are therefore a market outcome.

nological progress and technology di<sup>x</sup>usion that no further skill mismatch arises. The relationship between  $L^M$  and  $L^R$ , which follows from (EE), depends on whether unemployment exists or not and can be either positive or negative. In the full employment case,  $L^M$  and  $L^R$  are related negatively because labor is completely engaged and can be used in either sector. However, we achieve a positive relationship if  $u > 0$ . This indicates the complementary relationship between technological innovation and technological di<sup>x</sup>usion represented by the two di<sup>x</sup>erent sectors. High employment in manufacturing is accompanied by high technology di<sup>x</sup>usion making many innovations possible without producing a scarcity of know-how. As a result, a high number of researchers can work in R&D.

### 3.2 The Steady-State Solution

The solution of the model is speci...ed by equilibrium intersectoral labor allocation,  $L^M=L^R$ , and equilibrium unemployment,  $u$ . A steady state provides both endogenous growth and endogenous unemployment. The following proposition summarizes the properties of the corresponding equilibrium.

*Proposition 1 (a) The system of no-arbitrage (AE) and employment condition (EE) establishes steady-state growth. (b) A unique equilibrium exists for a su<sup>x</sup>ciently small number of researchers and (c) it arises as a stable focus if the arrival rate of innovations is small enough.*

Proof. See Appendix ■

Of further interest is whether a skill mismatch produces unemployment. Steady state growth produces a shortage of skills if the creation of know-how through technology di<sup>x</sup>usion is inadequate. This implies for unemployment:

*Proposition 2 Unemployment emerges if intersectoral labor allocation is inappropriate, i.e. if the share of workers in manufacturing is insu<sup>x</sup>iciently small to enable adequate technology di<sup>x</sup>usion.*

Proof: A necessary condition for unemployment is that workers are inadequately skilled. According to (23) a positive unemployment rate exists

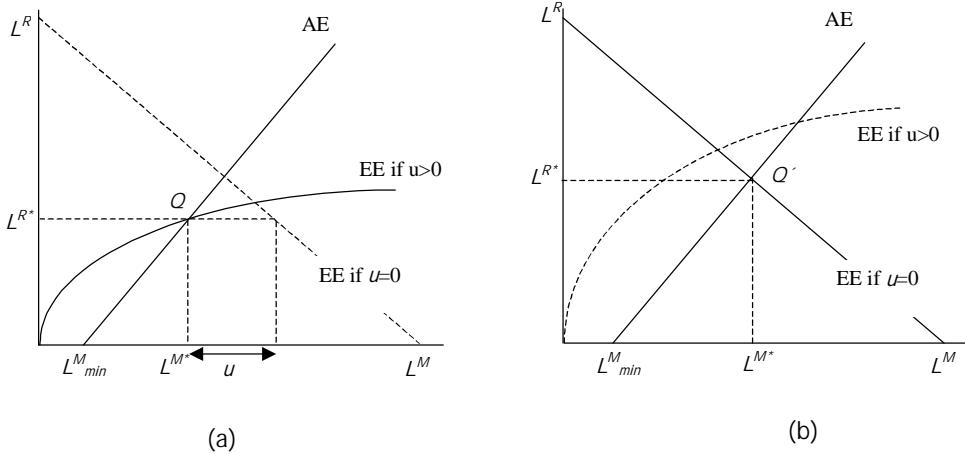


Figure 1: Steady state solution

if:

$$\frac{L^M}{L^R} < \frac{\alpha}{\gamma} L \quad (\text{UC})$$

Condition (UC) attributes unemployment to an inappropriate intersectoral labor allocation. A low ratio  $L^M/L^R$  indicates an economy in which knowledge creation exceeds knowledge diffusion. Too few workers in manufacturing allow only an insufficient formation of individual know-how as a by-product of manufacturing. As a consequence, the arising skill mismatch produces unemployment. ¥

Figure 1 gives a graphical illustration of a steady-state solution, where  $AE^{10}$  and  $EE$  represent the equilibrium conditions. We distinguish between (a) the unemployment and (b) the full employment case, where (UC) is the relevant hinge, which determines whether the positively or the negatively sloped  $EE$  is the crucial restriction. The intersections of  $AE$  and  $EE$ , namely  $Q$  and  $Q^*$  establish equilibrium labor allocations which determine the size of  $L^M$  and  $L^R$ . According to (6), the size of the R&D sector generates the equilibrium innovation rate  $g_A = L^R \ln(\cdot)$ . From the production function

<sup>10</sup> R&D does not occur until a minimum number of workers  $L_{\min}^M$  are employed in manufacturing. This property can be attributed to the necessity of a minimum demand for new products in order to set costly research. This familiar outcome of Romer-type models (1990) is discussed, for example, by Garcia-Castrillo and Sanso (2002).

(3) follows that this rate is equal to output growth. This is because equilibrium input of  $L^M$  is constant and therefore vintage production increases with  $g_A$ . Furthermore, according to (19) a constant size of innovations leaves the number of vintages unchanged and aggregate output is only driven by increases in the vintage production.

A steady state solution also determines the employment size. In case (a) the intersection of  $AE$  and  $EE_{u>0}$  lies within the employment space, which is bounded by the employment curve  $EE_{u=0}$ . Hence  $EE_{u>0}$  constrains the outcome. The space between  $Q$  and  $EE_{u=0}$  measures the level of unemployment. In contrast,  $EE_{u=0}$  limits labor demand in case (b). Sufficient skill supply precludes technological unemployment and implies a total labor demand that exceeds labor supply. No-arbitrage condition and the size of the labor force then establish equilibrium intersectoral labor allocation.

## 4 Consequences of Knowledge Formation

This section is devoted to the debate about the benefits of R&D scale, comparing equilibrium outcomes of growth and unemployment for variations in monetary and technological returns to research. A critical evaluation of the so-called scale effect of early innovation models led to the development of non-scale growth models. In accord with this literature, Arnold (1998) showed how human capital accumulation and innovation may be inversely affected by certain policy measures. From his model follows a full policy invariance with respect to growth. In this section we argue that changes in individual knowledge and technologies do not cancel out each other, but that there might be a partly crowding-out of one type of knowledge by the expansion of another one. Thus a policy that promotes growth is feasible but not straightforward to implement. Via changes in employment, productivity of research is only ambiguously related to total research output.

The ambiguity in assessing publicly provided subsidies for R&D can be seen from comparative static analysis, which reveals how intersectoral labor allocation and total employment change. Suppose that some policy measures, such as publicly financed basic research or changes in patent law, augment the productivity of private research,  $\pi$ . As a result, the arrival rate of innovations increases. Another popular instrument of policy makers

is to provide subsidies for innovative industries. We introduce this tool by extending the profit equation (9) by a share  $\eta$  of the fixed costs covered by subsidies:

$$\eta V_{\varepsilon+1}^R = p_{\varepsilon+1}(x) x_{\varepsilon+1} + (1 - \eta) C_{\varepsilon+1}; \quad (24)$$

In general, expenditures for subsidies are financed by taxes. Let  $\mu$  denote the lump sum tax per capita. The public budget constraint therefore is:

$$\eta C_{\varepsilon} = \mu L; \quad (25)$$

A tax system defined in such a way does not change the main equilibrium properties. To see this, consider that an equal tax on labor income applies to both sectors, manufacturing as well as R&D. The no-arbitrage condition (??) changes into  $V_{\varepsilon} + \mu = V_{\varepsilon+1} + \mu$ . It is obvious that taxes themselves are not distortionary. Furthermore, the extent of the skill mismatch according to (EE) does not depend on taxes. However, subsidies change the value of an innovation  $V_{\varepsilon}$  as it depends on  $V_{\varepsilon}^R$ . This modification slightly changes the no-arbitrage equation (AE) to:

$$L^R = L^M \left( \frac{1}{\alpha} + 1 \right) + \frac{r}{w} + \frac{(1 - \eta) C}{w}; \quad (\text{AE}')$$

The effects of measures with the intention to increase the scale of research are summarized as it follows in the proposition:

**Proposition 3** *Consider increasing incentives to do research because of a rise in the arrival rate of innovations, or as a result of subsidies for innovative intermediate goods. In the case of a skill mismatch (i.e.  $u > 0$ ), unemployment further increases and the size of the R&D sector decreases. The effect on growth therefore tends to be negative. Contrary to this, the number of researchers increases, which involves a positive effect on growth, if the emergence of a skill mismatch can be excluded.*

**Proof:** All of these results can be established graphically, using Figure 2. The emergence of income differentials between R&D and manufacturing causes labor flows between the two sectors. This effect refers to AE. Changes

in the skill mismatch leads to flows between employment and unemployment. This affects the equilibrium represented by  $EE$ . Consider first the case of a present skill mismatch, i.e.  $u > 0$ . Via changes in the relative sector income, increasing  $\alpha$  or  $\beta$  shift the  $AE$ -curve to the left. Additionally, a relative increase in the technological level to individual skills, caused by a rise in  $\gamma$ , shifts the  $EE_{u>0}$ -curve downwards. It appears that a new equilibrium involves less total employment and therefore less researchers. If there exists no productivity effect, namely in the case of production subsidies, this result implies that growth slows down. The expected favorable productivity effect of research by  $\gamma$  is neither growth enhancing anyhow because it creates an extra skill mismatch. Employment in both sectors therefore declines. As a result, scale and productivity of R&D are inversely related and it is not clear whether the rate of technological progress increases or decreases. To put it differently: increases in  $\alpha$  and  $\beta$  only raise the ability or the incentives to develop innovations, whereas the ability to implement the innovations into manufacturing units has been relatively weakened. This effect might argue for the observation that a rise in global R&D activities over time is not accompanied by an equal increase in output growth. All these effects via the labor market disappear if skill supply is adequate indicating that no skill mismatch and no unemployment emerges. The  $AE$ -curve then shifts to the left again, while the intersection with  $EE_{u=0}$  determines the labor allocation between the two sectors. This case reveals the known positive relationship between R&D scale and output growth. Innovation policy with clear outcomes is therefore feasible in the full employment setting only.  $\diamond$

The undertaken analysis also offers some explanation for the ambiguous relationship between growth and employment which is found empirically. Tonti and Tanda (1998) give some empirical evidence for a negative impact of technological progress on aggregate employment. As regards general output growth, Davis and Haltiwanger (1992) find a standard negative relationship, whereas Caballero (1993) argues that it is rather positive. As it has been shown, unemployment can be associated with a decreasing scale of the research sector, but it is also related to an increasing productivity in R&D. Hence, it is unclear whether the innovation rate increases or decreases when joblessness arises via the inadequate ratio between technology and skills.

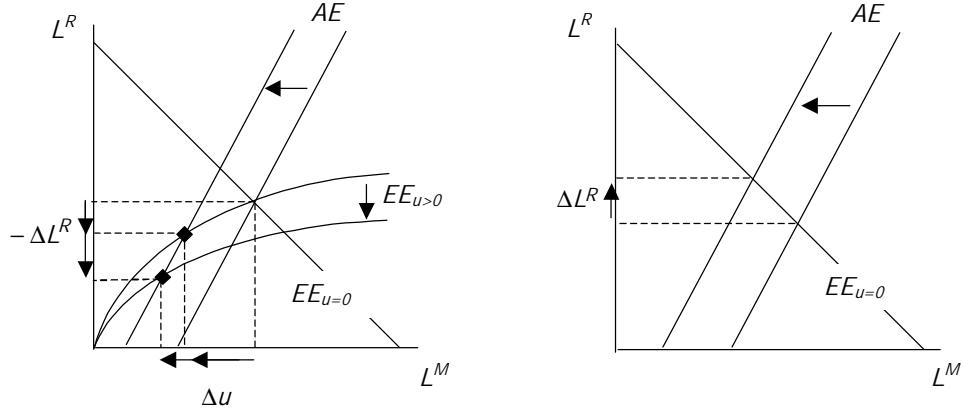


Figure 2: Increases in "and %":

## 5 Conclusions

This paper developed a model, in which innovation-based growth and technological unemployment are co-determined. Growth results exclusively from technology improvements. The employment-growth link comes from the joint use of individual skills and technologies, which is necessary to implement technological change. Overall growth can therefore be constrained by either the research output or the insufficient supply of technological skills by the workers. We identify the second case as a skill mismatch. In addition to the growth effect, a skill mismatch causes technological unemployment. This outcome is valid whenever skills supplied by a considerable part of the workers do not match the running state-of-the-art technologies.

The model investigates the conditions of the emergence of a skill mismatch. We find that the ratio of knowledge creation to knowledge diffusion determines the employment outcome. Considering that technological change requires the implementation of technologies besides their invention, it appears that low employment and a skill shortage have negative effects on output growth. An equilibrium that avoids the skill mismatch, and that is therefore optimal for growth and employment, requires a balance between the formation of different aspects of knowledge, such as technologies and skills.

As regards the empirical evidence, the relationship between growth and

unemployment is only ambiguous. Our model supports this ambiguity as a possible result. Knowledge formation is supposed to be a good instrument to increase growth. However, considering different dimensions of knowledge can produce different relationships to employment.

From the evaluation of possible consequences for growth and employment, it appears that policies on innovation produce ambiguous effects. Assessing such policies must take into account that individual skills adjust to innovative technologies just with delay. As a consequence, technological unemployment is a by-product of innovation-based growth. As regards the growth effect, subsidies and increasing productivity in R&D only enhance the research output, which shifts the technology frontier. However, the degree declines by which the economy benefits from these innovations if the supply of individual technological skills cannot keep the pace of their demand driven by the innovations. The negative employment effect might lower the overall innovation rate. As a result, scale and productivity of research are inversely related via the emergence of a skill mismatch.

## Appendix

Proof of Proposition 1:

Proof of Part (a):

Equilibrium growth,  $g_A = \frac{d}{dt} \ln(L)$ , depends on the equilibrium intersectoral labor allocation  $L^R = L^M$ . The solution of the system of (AE) and (EE) includes the following two cases:

*The full employment case:*

Equilibrium intersectoral labor allocation is given by the system of no-arbitrage and employment condition for  $u = 0$ . Substituting (AE) into (EE) for  $u = 0$  yields:

$$L_{u=0}^{R^a} = \frac{\frac{1}{\alpha} L(1 - \frac{r}{\alpha}) + \frac{C}{\alpha}}{\frac{1}{\alpha} L(1 - \frac{r}{\alpha}) + \frac{C}{\alpha}} \quad (A1)$$

*The unemployment case*

Equilibrium intersectoral labor allocation is given by the system of (AE) and (EE) for  $u > 0$ . Substituting (AE) into (EE) for  $u > 0$  yields:

$$L_{u>0}^{R^a} = \frac{\frac{1}{\alpha} L(1 - \frac{r}{\alpha}) + \frac{C}{\alpha}}{\ln(\frac{1}{\alpha} L(1 - \frac{r}{\alpha}) + \frac{C}{\alpha})^2} \quad (A2)$$

where:

$$\begin{aligned}\mathcal{C}_1 &= 2^{\otimes} C^R i^{-1} (4 \cdot i^{-\otimes}) \\ \mathcal{C}_2 &= {}^{\otimes} \ln(\cdot) C^R'' + 2 \ln(\cdot) / r i^{-1} \\ \mathcal{C}_3 &= {}^{\otimes} \ln(\cdot) / r + 2^{\otimes} i^{-1} \cdot + 4 / i^{-1} \cdot^2 (1 i^{-\otimes}) i^{-1} \\ \mathcal{C}_4 &= \frac{{}^{\otimes} i^{-1} \cdot (1 + {}^{\otimes} i^{-1} \cdot)}{\ln(\cdot)}\end{aligned}$$

It is not straightforward to see, that (A2) produces a smaller value of  $L^R$  compared to (A1), because the total effect of  $\mathcal{C}_1; \mathcal{C}_2; \mathcal{C}_3$  and  $\mathcal{C}_4$  is indefinite. This comes from intersection of  $AE$  and  $EE$  outward full employment space. However, this solution is excluded by (UC) such that  $L_{u>0}^{R^a} < L_{u=0}^{R^a}$ .

**Proof of Part (b):**

**Lemma** Since  $L_{EE}^M j_{L^R=0} = 0$  and  $L_{AE}^M j_{L^R=0} > 0$ , a unique equilibrium exists if the employment curve  $EE$  is increasing monotonically but decreasing in its slope.

**Proof** Rearranging (EE) yields:

$$L^R = \frac{1}{2} \left( \frac{P}{\frac{(L^M \bar{A})^2 + 4 \bar{A}' L^M}{\bar{A}}} \right)^{\frac{1}{2}} \quad (A3)$$

$EE$  is increasing monotonically considering partial differentiating of (A1) with respect to  $L^M$

$$\frac{\partial L^R}{\partial L^M} = \frac{2 L^M \bar{A}^2 \cdot^2 + 4 \bar{A}'' \cdot}{4 \bar{A}'' \frac{(L^M \bar{A}'')^2 + 4 \bar{A}''' L^M}{\bar{A}}} \cdot^{\frac{1}{2}} > 0 \quad (A4)$$

After simplifying

$$(2 \cdot)^2 > 0 \text{ Q.E.D.} \quad (A5)$$

The slope is decreasing if

$$\frac{\partial^2 L^R}{\partial L^M^2} = \frac{1}{2 \bar{A}''} \left( \frac{2 L^M \bar{A}^2 \cdot^2 + 4 \bar{A}'' \cdot}{4(L^M \bar{A}'')^2 + 4 \bar{A}''' L^M} \right)' \cdot^{\frac{1}{2}} \frac{(2 L^M \bar{A}^2 \cdot^2 + 4 \bar{A}'' \cdot)^2}{4(L^M \bar{A}'')^2 + 4 \bar{A}''' L^M}^{\frac{3}{2}} < 0 \quad (A6)$$

Simplifying yields:

$$L^M < \frac{1}{\bar{A}''} \quad (A7)$$

Thus, a unique equilibrium exists for a sufficiently small size of the man-

ufacturing sector. This result can be aCirmed for just slight di¤erences in productivity of technology and know-how creation.

### Proof of part (c)

Equilibrium is given by the system of (AE) and (EE). Analyzing the dynamics yields the stability of the system. The equilibrium of the no-arbitrage curve AE implies that  $L^R = \partial L^R / \partial t < 0$ , if  $L^R > L_{AE}^R$ , because more researchers reduce pro...ts from research and make manufacturing for the better alternative. Let  $\tilde{A}_1$  denote the speed of this adjustment to write:

$$L^R = \tilde{A}_1 \frac{6}{4} \frac{L^M (\frac{1}{\alpha} i - 1) i - \frac{r}{\alpha} i - \frac{C}{W} i}{\underbrace{\{z\}}_{L_{AE}^R}} L^R \frac{7}{5}; \text{ with } \tilde{A}_1 > 0 \quad (\text{A8})$$

The employment condition (EE) implies that  $L^M = \partial L^M / \partial t > 0$ , if  $L^M > L_{EE}^M$ , because more employment in manufacturing increases know-how creation and reduces unemployment in both sectors. Let  $\tilde{A}_2$  denote the speed of this move to write:

$$L^M = \tilde{A}_2 \frac{6}{4} \frac{i - \frac{\tilde{A}}{\tilde{A} L^R} L^{R^2} i - L^M \frac{7}{5}}{\underbrace{\{z\}}_{L_{EE}^M}}; \text{ with } \tilde{A}_2 > 0 \quad (\text{A9})$$

According to Olech's theorem<sup>11</sup> the equilibrium is asymptotically stable; if:

$$(1) \quad \frac{\partial L^M}{\partial L^M} + \frac{\partial L^R}{\partial L^R} < 0 \quad (\text{A10})$$

$$(2) \quad \frac{\partial L^M}{\partial L^M} \frac{\partial L^R}{\partial L^R} i - \frac{\partial L^M}{\partial L^R} \frac{\partial L^R}{\partial L^M} > 0 \quad (\text{A11})$$

$$\text{ad (1)} \quad i \tilde{A}_1 i \tilde{A}_2 ! < 0 \text{ is true because } \tilde{A}_1, \tilde{A}_2 > 0 \quad (\text{A12})$$

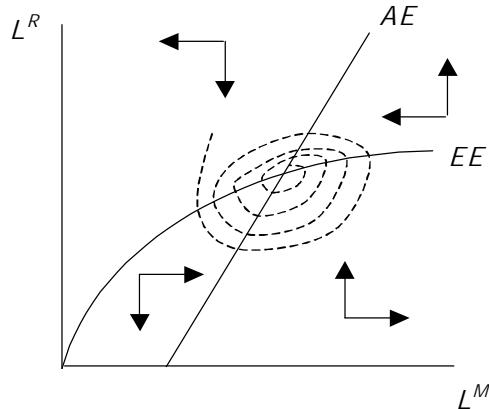
$$\text{ad (2)} \quad (i \tilde{A}_1)(i \tilde{A}_2) i \frac{\tilde{A}_1 \tilde{A}_2 (1 i ^{\alpha}) "AL^R 2 i "AL^R}{\underbrace{(1 i ^{\alpha}) "AL^R)^2}_{3}} > 0 \quad (\text{A13})$$

is true<sup>12</sup> if  $"L^R < \frac{\alpha}{\alpha A(1 i ^{\alpha})} \frac{(1 i ^{\alpha}) "AL^R)^2}{2 i "AL^R}$  )  $"L^R < \frac{\alpha}{\alpha A(1 i ^{\alpha})} 1 + \frac{P_2}{2} i$ . The term  $"L^R$  indicates the expected research output. Hence equilibrium is stable for a moderate innovation rate.

<sup>11</sup> See Olech (1963)

<sup>12</sup> The solution  $"AL^R > 1L$  is excluded by (EE), which would produce negative values for  $L^R$  otherwise.

Analyzing the trajectories, considering  $\partial L^R / \partial t = i \tilde{A}_1 < 0$  and  $\partial L^M / \partial t = i \tilde{A}_2 < 0$ , attributes the solution of the differential system to a stable focus. (See figure A 1 for an illustration.)



A1 Properties of the equilibrium

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