A Dynamic Input-Output Model with Explicit New and Old Technologies

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In traditional dynamic I-O models progress is reflected in terms of improvement of average characteristics of economic sectors from year to year: productivity growth, reduction of technical coefficients (of inputs required per unit of output), reduction of capital intensities etc, see Los (2000). At that the attention is focused on the question – how the next year is better than the previous one? Meanwhile, a deeper question - how within the same year new technology, introduced in it, is better than the old one; how new and old technologies coexist and interplay within the same year – could make modelling more general and more close to reality.

Further, capital investments are traditionally considered as a mere addition to capital stock, in which they dissolve as a part of some homogenous entity. Meanwhile progress is primarily associated just with this new capital that embodies new technologies. Consequently adequate modelling of progress requires that outputs produced by new and old technologies and their I-O matrixes should be considered separately and explicitly. It means that the Gross Fixed Capital Formation in National Accounts should be attributed by own technological parameters so that it could be seen what Capital Formation does form.

In not input-output approaches, separate production on new and old equipment, working side by side in the same year, is already shown explicitly in general equilibrium models based on the Cobb-Douglas function, see Fougeyrollas et all (2001). But Cobb-Douglas function assumes labour-capital substitution (i.e. any increase of either of these factors would lead to an increase in output) which does not comply with reality. While the Leontieff's model, on the contrary, regards the labour force as a separate powerful restraint to growth, which in certain circumstances cannot be overcome by increasing any of the other factors. So passing to explicit considering new technologies in I-O approach[1] will substantially foster its efficiency and overcome shortcomings of the aforesaid general equilibrium models.

Within input-output approach a model close to these concepts was proposed by Idenburg and Wilting (2000), who observed that a standard technological matrix depicts a mix of technologies implemented during a set of years, and who used a separate matrix for output on new investments. But the core model was still based on a standard average matrix, using matrix for new investments only as a component for calculating the standard one. At that it was implicitly assumed that the structure of output was identical to the structure of capacities, which hindered reflection unused capacities and the Schumpeterian 'creative distraction' of progress.

This paper attempts to overcome 'addiction' to standard average technological matrix by splitting it in the advanced and old parts, to create qualitatively new, more flexible dynamic I-O model. It afforded to get saw-edged development paths much resembling real statistical reports and to deal with cycles, employment and capacity utilization issues. The paper develops the earlier results of the author, see Ryaboshlyk (2000, 2002, 2003).

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[1] It should be noted that alternative technologies had been introduced in I-O models long ago, see Morishima (1964), but it didn't account for capital stock constrains and afforded arbitrary usage of any technology.

Description of the model

Initial uni-technology dynamic closed I-O model with capital, labour, and inventories is shown below.

Uni-Technology I-O Model:

$$X^{\circ}(t) = A^{\circ} * X^{\circ}(t) + C * x_c(t) + A^{\circ}_k * X^{\circ}_k(t) + X_{\Delta S}(t) \qquad (1)$$

$$l^{\circ} * X^{\circ}(t) \le L \qquad (2)$$

$$[k^{\circ}] * X^{\circ}(t) \le K^{\circ}(t-1) \qquad (3)$$

$$A^{\circ}_s * X^{\circ}(t) \le S(t-1) \qquad (4)$$

$$K^{\circ}(t) = K^{\circ}(t-1) + X^{\circ}_k(t) - SCR^{\circ}(t) \qquad (5)$$

$$S(t) = S(t-1) + X_{\Delta S}(t) \qquad (6)$$

where

$X^{\circ}(t)$ - vector of output by industries in the year t;

A° - i-o matrix;

'$^{\circ}$' - a mark denoting attribution to old, initial technology, against '^{n}' attributing to new technology, the latter mark will be used later;

C - vector of household consumption structure (normalised proportions between products in which the sum of components equals to one);

$x_c(t)$ - scalar level of consumption, amount of assortments C;

$X^{\circ}_k(t)$ - vector of investments, of fixed capital formation by industries;

A°_k - matrix of coefficients of capital formation, each column shows inputs of different products required for creation of a unit of fixed capital in a definite industry;

$SCR^{\circ}(t)$ – vector of scrapping of fixed capital in year t. It is assumed that capital is equally productive throughout its lifetime τ until scrapped at the end of this period, so that scrapping is equal to investments which took place a lifetime ago;

$X_{\Delta S}$ - vector of changes of inventories (stocks);

l° - labour/output ratios, a row vector;

L - total labour force, is assumed to be constant;

$K^{\circ}(t)$ – vector of capital stock by industries at the end of year t;

$[k^{\circ}]$ - capital intensity diagonal matrix, its diagonal elements show capital/output ratios for corresponding industries;

S (t) – vector of inventories (stocks) held at the end of year t;

A°_s - inventory/output ratio matrix shows how many inventories (stocks) of each product should be accumulated at the beginning of year per unit of output.

Equation (1) includes main components of national accounts. The other equations are constraints or dynamic interlinks between adjacent years.

Development of such economy is strictly limited by labour force. At that characteristic features of steady state, when the highest output is attained and could be supported infinitely, are full employment and the balance between investment and scrapping. The latter condition could be conveyed in the following way:

$$X^{\circ}_{k}(t) = [\,k^{\circ}\,] * X^{\circ}(t)/\,\tau \qquad (7)$$

where

τ – fixed capital's lifetime;

$[\,k^{\circ}\,] * X^{\circ}(t)$ – capital stock required for providing output $X^{\circ}(t)$.

From (7) and some other formulas stated afore, output and consumption at saturation phase are determined as below:

$$\begin{bmatrix} X^{\circ}(t) \\ x_c(t) \end{bmatrix} = \begin{bmatrix} I - A^{\circ} - A^{\circ}_{k} * [\,k_o\,]/\tau & -C \\ 1^{\circ} & 0 \end{bmatrix}^{-1} * \begin{bmatrix} 0 \\ L \end{bmatrix} \qquad (8)$$

where I - is identity matrix.

The other variables could be easily derived basing on (8).

After that the uni-technology model becomes of no interest for continual growth analysis for it can't exceed its upper limit.

One of the ways out is to neglect labour, whereby opening possibilities for various paths of development on account of some diversion from reality. For in this case it is implicitly assumed that economic growth is somehow backed by corresponding growth of labour, see Blanc and Carmen (2000), Aulin-Ahmavaara (2000), Idenburg et all (2000).

The other, now most widespread, way is to turn technological parameters A°, A°_{k}, 1°, $[\,k^{\circ}\,]$ in the variables, see Los (2000).

Here it is suggested to introduce advanced technology explicitly without touching the old parameters. One of the reasons in favour of this approach is that it shows how investments create capital with its own advanced parameters, different from old ones; at that changes of overall, average parameters will be generated endogenously. After all innovative growth is driven just by such potential difference between new and old technologies and explicit reflection of it could give further insight and push modelling closer to reality.

Qualitatively it is clear that if the economy is proposed to use an advanced technology affording higher labour productivity and other advantages, but not having any capital stock yet, then the development will get a pattern of intensive investments in alternative technology with gradual decline of the old one, which will be being scrapped without replacement; and in result a new level of saturation will be attained.

The initial model now could be modified from uni- to multi-technology type so that some of its components will be 'doubled' according to old and new possibilities. Below such 'doubled' components are highlighted by curly brackets.

Multi-Technology I-O Model:

$$\underbrace{X^{\circ}(t)+X^{n}(t)}=\underbrace{A^{\circ}X^{\circ}(t)+A^{n}X^{n}(t)}+C\,x_c(t)+\underbrace{A^{\circ}_{k}X^{\circ}_{k}(t)+A^{n}_{k}X^{n}_{k}(t)}+X_{\Delta S}(t) \qquad (9)$$

$$\underbrace{l^{\circ}X^{\circ}(t)+l^{n}X^{n}(t)}\le L \qquad (10)$$

$$\begin{cases}[k^{\circ}]\,X^{\circ}(t)\le K^{\circ}(t\text{-}1) & (11)\\ [k^{n}]\,X^{n}(t)\le K^{n}(t\text{-}1) & (12)\end{cases}$$

$$\underbrace{A^{\circ}_{S}X^{\circ}(t)+A^{n}_{S}X^{n}(t)}\le S(t\text{-}1) \qquad (13)$$

$$\begin{cases}K^{\circ}(t)=K^{\circ}(t\text{-}1)+X^{\circ}_{k}(t)-SCR^{\circ}(t) & (14)\\ K^{n}(t)=K^{n}(t\text{-}1)+X^{n}_{k}(t)-SCR^{n}(t); \qquad K^{n}(0)=0 & (15)\end{cases}$$

$$S(t)=S(t\text{-}1)+X_{\Delta S}(t) \qquad (16)$$

the target function:

$$\sum_{t} x_c(t)\,(1+d)^{1-t} \Rightarrow \max \qquad (17)$$

where:

A^{n}, A^{n}_{k}, l^{n}, $[k^{n}]$, A^{n}_{S} - parameters of new technology;

$X^{n}(t)$, $X^{n}_{k}(t)$, $K^{n}(t)$, $SCR^{n}(t)$ output, investments, capital stock and scrapping of new technology;
d – discount coefficient.

Equation (9) shows that new and old output are summed in unified 'pool' $X^{\circ}(t)+X^{n}(t)$ which could be used both for new or old intermediate consumption, or for investments in new or old capital. As to personal consumption and change of inventories their nature remained the same and they could use resources from the 'pool' in the same way as previously.

Total labour force and inventories could be distributed freely between the two technologies, (10) and (13).

Capital is used (11-12) and accumulated (14-15) individually.

The development path should maximise personal consumption taken as discounted sum of all years (17).

So the multi-technology I-O Model (9-17) is a Linear Programming task.

Numerical results

Properties of such multi-technology I-O model with explicit new and old technologies are illustrated on the example of a prototype three-industry economy (the detailed description is given in the Appendix).

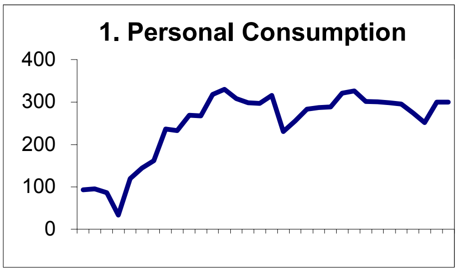
The new technology, which is proposed to the economy, previously using only old technology, has the following properties.

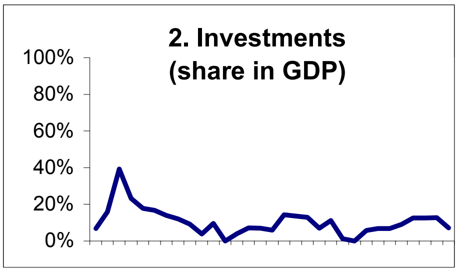
Labour productivity rise in 1.75; 3.5 and 2.5 times by the three industries respectively.

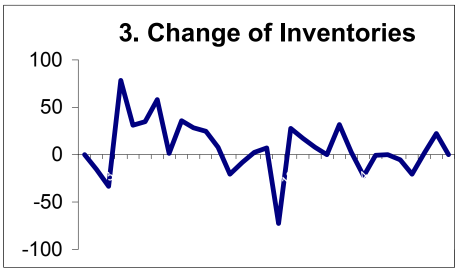
Inputs required per unit of output mainly decrease in 1.25-2 times by different components of the technological matrix.

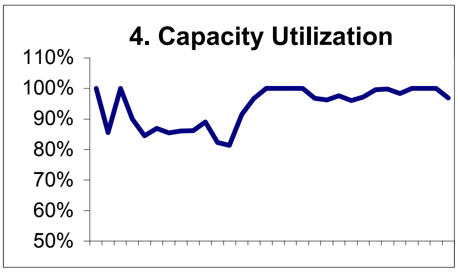
Still, capital intensities (capital/output ratios) are supposed to become higher in 1.5; 1.5 and 1.75 times respectively

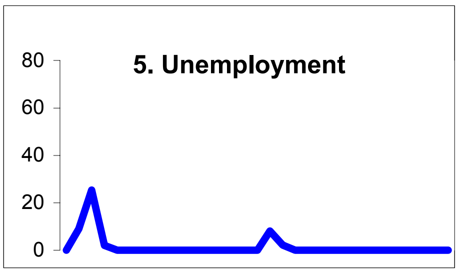
The new technology opens possibility for threefold growth of consumption, but for that new capital should be accumulated and a certain development path of gradual substitution of old technology by the new one via investments, accompanied by redistribution of the labour force, should be traversed.

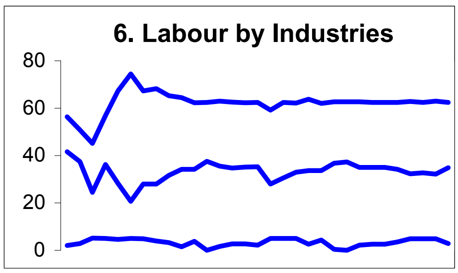












The solution of a task of linear programming (9-17) with more than 1000 variables gave distinctive saw-edged time series much more resembling real statistical reports than smooth mathematical curves of traditional models, see Fig. 1-6.

At that endogenous cycles had been generated - Fig. 1. The main reason for it is that in the early stages of implementation of new technology physical scrapping of new capital is insignificant, thus permitting temporary accumulation of capital stock (i.e. manufacturing capacities) exceeding the level of the mature stage.

Labour constraint is also important for cycle's emergence. As it could be seen at Fig. 1, before beginning of the cyclical wave the economy was growing along an almost straight path, as if all its resources were unlimited. Finally one of resources had been exhausted, in our case - the supply of labour previously working on old technology, and this point of exhaustion turns out to be a starting point for slowing of growth and cycles.

The other point which highlighted the model is that at some conjunctures full employment is impossible even for highly flexible and adaptable labour force. Typically such unemployment occurred at the beginning of transition to new technology when amount of new capital is insufficient for absorbing all the labour freed from old technologies - Fig. 5. In addition it should be noted that the model shows structural changes in labour distribution by industries caused by progress - fig.6 shows total labour, both on new and old technologies, in each of three industries. If we should allow for inertia in the movement of labour, the model could reflect frictional unemployment too.

Explicit showing of old capacities affords to describe the situation when some fixed capitals left unused before scrapping. As a result capacity utilisation varies from 80 to about 100 % - Fig. 4.

Conclusion

The proposed multi-technology approach with explicit new and old technologies have proved that the Input-Output method is the most convenient tool for description of action of 'spontaneous' market forces, rather than a tool for strict 'communist' planning as it was to some extent believed earlier. The approach gives holistic and clear explanation to cycles, unemployment, capacity utilization and other issues.

Further passing to practical application of the multi-technology method to policy making requires shifts of accents in empirical analyses – apart of question how average characteristics of industries are improving from year to year and how R&D could affect them, it is more important to know how in particular new technologies introduced within industries are more progressive. From one hand this is the most natural and self-evident target setting, for progress directly affects just new technologies and new capital embodying it. But from the other hand these specific parameters are concealed beneath average figures of ordinary statistics.

So tackling the task of separate statistical accounting of new and old capital and technologies could provide a break-through in policy modelling.

Appendix: Parameters of the prototype economy

The parameters of the initial old 'o' and final new 'n' technologies are given in parallel, if appropriate.

The economy consists of 3 industries, of which the third one produces only capital goods and is not used as intermediate inputs to any industry. So the last row of the technological matrix is zero:

$$A^{o} = \begin{bmatrix} 0 & 0.46 & 0.55 \\ 0.17 & 0.15 & 0.15 \\ 0 & 0 & 0 \end{bmatrix} \qquad A^{n} = \begin{bmatrix} 0.05 & 0.23 & 0.28 \\ 0.15 & 0.12 & 0.15 \\ 0 & 0 & 0 \end{bmatrix}$$

Total labour force L, is assumed to be constant and equal to 100, **L**=100.

Labour/output ratios:

$$l^{o} = \begin{bmatrix} 0.83 & 0.38 & 0.30 \end{bmatrix} \qquad l^{n} = \begin{bmatrix} 0.48 & 0.11 & 0.12 \end{bmatrix}$$

Capital/output ratios are placed in the diagonal elements of the capital intensity matrix:

$$[\,k^{o}\,] = \begin{bmatrix} 0.50 & & \\ & 0.25 & \\ & & 1.00 \end{bmatrix} \qquad [\,k^{n}\,] = \begin{bmatrix} 0.75 & & \\ & 0.38 & \\ & & 1.75 \end{bmatrix}$$

Capital formation matrix is trivial and shows that capital stocks of all industries are created from output of the third industry only:

$$A^{o}_{k} = \begin{bmatrix} & & \\ & & \\ 1 & 1 & 1 \end{bmatrix} \qquad A^{n}_{k} = \begin{bmatrix} & & \\ & & \\ 1 & 1 & 1 \end{bmatrix}$$

Period of capital functioning is 10 years, $\tau = 10$.

Structure of households' final consumption (it is assumed that the capital good is not directly used for personal consumption) is the following:

$$C = \begin{bmatrix} 0.15 \\ 0.85 \\ 0 \end{bmatrix}$$

Inventory/output ratios matrix (inventories are also required per unit of personal consumption, as shown in A°_{cs} }:

$$A^{o}_{s} = \begin{array}{|c|c|c|}\hline & 0.23 & 0.28 \\ \hline 0.17 & 0.15 & 0.15 \\ \hline & & \\ \hline \end{array} \qquad A^{n}_{s} = \begin{array}{|c|c|c|}\hline 0.02 & 0.09 & 0.11 \\ \hline 0.14 & 0.11 & 0.14 \\ \hline & & \\ \hline \end{array} \qquad A_{cs} = \begin{array}{|c|}\hline 0.08 \\ \hline 0.85 \\ \hline \\ \hline \end{array}$$

Discount coefficient. **d** =10%.

At the initial moment the economy uses only old technology and rests in a steady state with the following values of variables:

$$X^{o}(0) = \begin{array}{|c|}\hline 67.6 \\ 108.2 \\ 6.76 \\ \hline \end{array} \qquad X_{c}(0) = 93.2 \qquad X^{o}_{k}(0) = \begin{array}{|c|}\hline 3.38 \\ \hline 2.70 \\ \hline 0.68 \\ \hline \end{array} \qquad K^{o}(0) = \begin{array}{|c|}\hline 33.8 \\ \hline 27.0 \\ \hline 6.8 \\ \hline \end{array}$$

$$S(0) = \begin{array}{|c|}\hline 33.8 \\ \hline 108.2 \\ \hline \\ \hline \end{array} \qquad X_{\Delta s}(0) = 0.$$

Such economy could increase output only by increasing labour force or shifting to more effective technologies. The second way was considered above.

References

Aulin-Ahmavaara P. (2000), “Dynamic input-output and capital”, *Proceedings of the XIII International Conference on Input-Output Techniques, University of Macerata, Italy, August 21-25th 2000.*

Blanc Mariano Díaz and Ramos Carmen Carvajal (2000), “The foundations of dynamic input-output revisited: does dynamic input-output belong to growth theory?”, G*rupo de investigación en Análisis Input-Output. Departamento de Economía Aplicada. Facultad de Ciencias Económicas y Empresariales*

Idenburg, Annemarth M. and Wilting, Harry C. (2000), “DIMITRI: a Dynamic Input-output Model to study the Impacts of Technology Related Innovations”, *Proceedings of the XIII International Conference on Input-Output Techniques, University of Macerata, Italy, August 21-25th 2000.*

Fougeyrollas, A., LeMouel, P., Zagame, P. (2001), “Endogenous Growth and Environmental Regulation: the Case of the Kyoto Protocol”, *Ecole Centrale de Paris, 19th September 2001, afougey@ecp.fr*

Los, Bart (2000), “Endogenous Growth and Structural Change in a Dynamic Input-Output Model”, *Proceedings of the XIII International Conference on Input-Output Techniques, University of Macerata, Italy, August 21-25th 2000.*

Morishima M. (1964), ”Equilibrium, Stability and Growth, a Multi-sectoral Anlysis”, *London-New York, 1964.*

Ryaboshlyk Volodymyr (2000), “Dvohsektorna Makromonetarna Model Zrostannja”, *The Ukrainian Journal Economist № 6, Kyiv, June 2000.*

Ryaboshlyk Volodymyr (2002), “Combining General Equilibrium and Growth Models: a New Tool for Policymaking”, *Proceedings of the international conference Policy Modeling – EcoMod 2002, Brussels, July 2002. Also available at http://www.ecomod.net/conferences/ecomod2002/papers/ryaboshlyk.pdf*

Ryaboshlyk Volodymyr (2003), “Making New Capital and Innovation Explicit in Growth Modelling”*, The Institute for Prospective Technological Studies, The IPTS Report, vol 78, Oct. 03.*