“DYNAMIC EQUILIBRIUM IN INPUT-OUTPUT MODELS: THEORY AND EMPIRICAL APPLICATIONS”

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Abstract

The paper reviews different alternative formulations of dynamic input-output establishing in each case whether they are equilibrium or non equilibrium - unstable – models.

A new formulation introducing production delays is presented with an example of an empirical application.

Introduction

Microeconomics studies the behaviour of elementary economic units.

Macroeconomics deals with the behaviour of large economic aggregates like national economies, considered as units.

Input-output analysis does not deal with units whether large or elementary: it is devoted to enlighten the relationship between units – more or less aggregate -.

Therefore IO is the proper tool both to explore the nature of economic structure and the conditions and development of general equilibrium processes.

The static input-output approach provides interesting information – sometimes highly counterintuitive – obtained from a rigorous mathematical treatment of past data. In spite of its rigorous conceptual foundation it often falls short of producing forecasts comparable by experimentation to the data collected from reality at later stages.

Usually the blame for these discrepancies is put on the many practical difficulties that occur when gathering the data necessary to determine the coefficients, the time it takes to collect them and the degree of aggregation chosen.

On the theoretical front, the static approach, by definition, does not provide information about how the structure works over time and does not bring any light on the ways or even whether the economy will reach an equilibrium status following impacts coming from economic policy actions.

This type of information requests the use of dynamic models. In this paper we review some dynamic models and their quality as equilibrium models and posit a new theoretical one that describes properly the equilibrium processes.

We also make reference to a practical application of the latter.

Analysis of the equilibrium quality of dynamic input-output models.

1. Models with capital accumulation. Leontief's dynamic model.

The well known balance equation of this model is:

$$x(t) = A\,x(t) + Y(t) + B\,x'(t) \qquad \text{(E.1)}$$

where x is a vector of sector output over time.

A is a matrix of current coefficients.

Y is a vector of final demands over time.

B is a matrix of capital coefficients.

(E.1) also can be written:

$$x'(t) = B^{-1}[\, I - A\,]\,x(t) - B^{-1}\,Y(t) \qquad \text{(E.2)}$$

Calling $M = B^{-1}[\, I - A\,]$ and $N = -B^{-1}$, (E.2) becomes:

$$x'(t) = M\,x(t) + N\,Y(t) \qquad \text{(E.3)}$$

The solution of this set of differential equation is trivial:

$$x(t) = e^{Mt}x(0) + \int_0^t e^{M(t-\tau)}\,N\,Y(\tau)\,d\tau \qquad \text{(E.4)}$$

This equation allows to describe the behaviour over time of the outputs of each sector following the shape of the final demands and given the momentum of the economy at time 0 of the model (represented by x(0)).

When Y(t) = K, that is when final demands are constant, (E.4) becomes

$$x(t) = [\, I - A\,]^{-1}K + R(T) \qquad \text{(E.5)}$$

Where $R(t) = e^{Mt}[\, x(0) - [I - A]^{-1}K\,]$ (E.6)

The situation (E.5) of Leontief's dynamic model without growth (final constant demands) coincides with that of its static model only if the term R(t) = 0.

This requests $e^{Mt} \longrightarrow 0$ when $t \longrightarrow \infty$ or, what is the same, that the eigenvalues of matrix M have negative real parts.

We have elsewhere proved (1) that if the coefficients of the capital matrix are positive, the previous condition never happens and therefore it can be said that the Leontief's dynamic model is always unstable. It describes the reality as if equilibrium could never be reached.

(1) See demonstration in "The foundations of dynamic input-output revisited". M. Blanc – Carmen Ramos www.uniovi.es/iude

2. Models without capital accumulation.

A.C. Chiang (2) has suggested a different type of dynamic input-output model as follows:

First he defines dynamic output as x(t). Then dynamic total demand as Ax(t)+Y(t)

Assuming that they are not equal, he assumes further that change x'(t) in output happens to adjust the discrepancies between supply and demand and therefore that:

$$x'(t) = Ax(t) + Y(t) - x(t) \qquad \text{(E.6)}$$

or

$$x'(t) = [A - I]\, x(t) + Y(t) \qquad \text{(E.7)}$$

Chiang does not elaborate any further neither provides the solution of (E.7), but his model deserves some attention:

- First it is noteworthy that Chiang's model (E.7) is identical to Leontief's dynamic model (E.2) with the coefficient matrix B equal - I. This would imply that if Chiang's model makes sense, matrix B cannot be a capital coefficient matrix and that jeopardizes the value of Leontief's dynamic.

- Second M = [A – I] which is stable if Hawkins-Simon conditions hold. Therefore Chiang's model is a stable equilibrium model.

- Third, its solution is equal to Leontief's static. Therefore it could be considered the dynamic generalization of the latter, should it not be true that Chiang's model presents a mathematical flaw: the dimensions of the left hand side of this equation (E.6) are different than those of the right hand side.

On the left $x'(t) = \frac{d[x(t)]}{dt}$ has dimensions unit of output per t^2.

(since x(t) is a flow whose dimensions are unit of output per time).

On the right, the dimensions are only u/t, unless the coefficients A, which have dimensions units of i per unit of j change to $\frac{ui}{uj.t}$

But this would completely alter the meaning of the coefficients and the interpretation of the model.

A solution is requested, and this leads us to posit a final approach, which can be a good base for improved empirical research.

(2) "Fundamental methods of Mathematical Economics" Alpha C. Chiang. Mc.Graw-Hill, Inc. p. 619.

3. A new proposal.

Starting with the dynamic definitions of

- Output: x(t)
- Demand: Ax(t) + Y(t)

We define a new variable, capital **X**(t), as

$$\mathbf{X}(t) = \int_0^t [Ax(t) + Y(t) - x(t)]\, dt$$

which represents the accumulation over time of the difference (excess or default) of demand over production. That variable represents the amount of unfilled orders of each sector, that is its backlog.

By definition

$$\mathbf{X}'(t) = Ax(t) + Y(t) - x(t) \qquad \text{(E.8)}$$

But if t_i is the time it takes to produce one unit of product i, the portion of unit produced per unit of time will be $1/t_i$.

The amount of units of i produced per unit of time corresponding backlog $\mathbf{X_i}(t)$ will be :

$$x_i(t) = \frac{1}{t_i} \cdot \mathbf{X_i}(t)$$

If we call T a matrix of production times defined as $T = \begin{bmatrix} t_1 & 0 \ldots 0 \\ 0 & t_2 \ldots 0 \\ 0 \ldots\ldots & t_n \end{bmatrix}$

we will have: $T^{-1} = \begin{bmatrix} \frac{1}{t_1} & 0 & 0 \ldots\ldots 0 \\ 0 & \frac{1}{t_2} & 0 \ldots\ldots 0 \\ 0 & 0 & \frac{1}{t_3} \ldots\ldots 0 \\ \ldots\ldots & & \\ 0 & 0 \ldots\ldots\ldots\ldots & \frac{1}{t_n} \end{bmatrix}$

We can write then:

$x(t) = T^{-1} \mathbf{X}(t)$ (E.9)

and

$\mathbf{X'}(t) = [A - I] T^{-1} \mathbf{X}(t) + Y(t)$ (E.10)

It is important to remind that (E.10) allows to know the evolution of backlogs, not the outputs but the latter, are obtained easily through (E.9).

If the make Y(t) = K , the solution of (E10) becomes:

$\mathbf{X}(t) = T [I - A]^{-1} K + R_2 (t)$ (E11)

With $R_2 (t) = e^{[A - I]T} [\mathbf{X}(0) - T [I - A]^{-1} K]$

(E11) describes the evolution ever time of the backlogs of all the industrial sectors of the economy.

The outputs follow the equation:

$x(t) = [I - A]^{-1} k + e^{[I- A] T} [x(0) - [I - A]^{-1} K]$ (E12)

$e^{[I- A] T}$ tends to zero if the Hawkins-Simons conditions hold and (E12) is a generalization of Leontief's static model.

<u>A simulation model based on this new development</u>

(E10) can be computed numerically by iterating the calculation of the following equation:

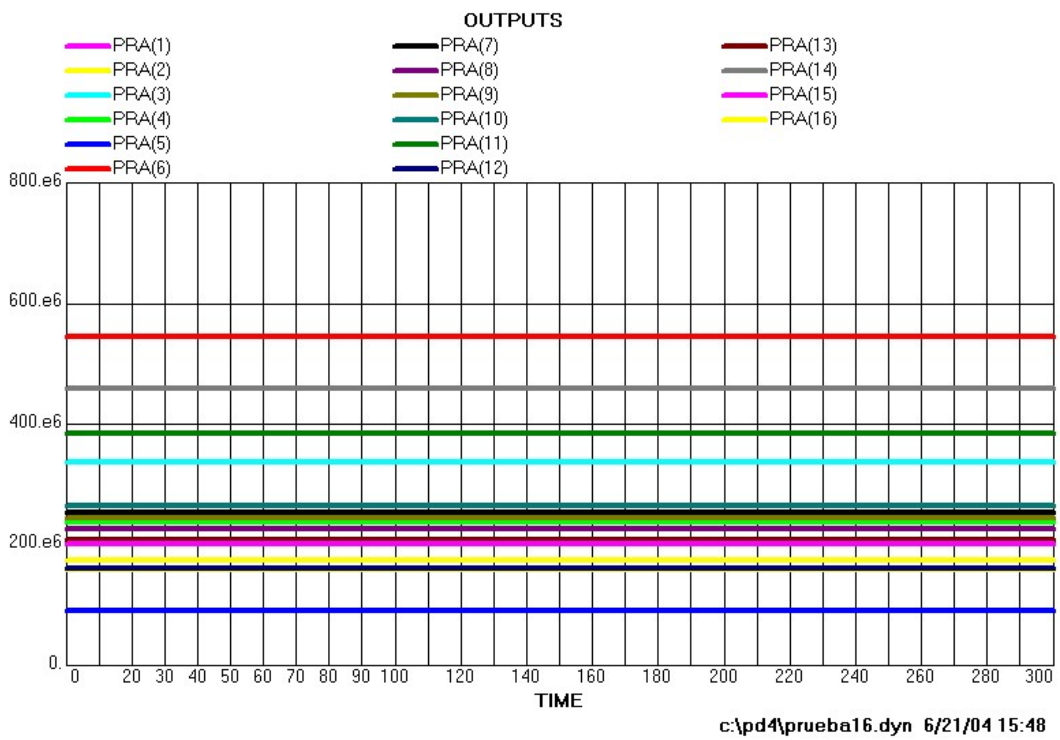
$$\mathbf{X}(t + dt) = \mathbf{X}(t) + dt \left[[A - I] T^{-1} \mathbf{X}(t) + Y(t) \right],$$

provided that we give a starting value to x(t), a value to dt and a series of successive values to Y(t).

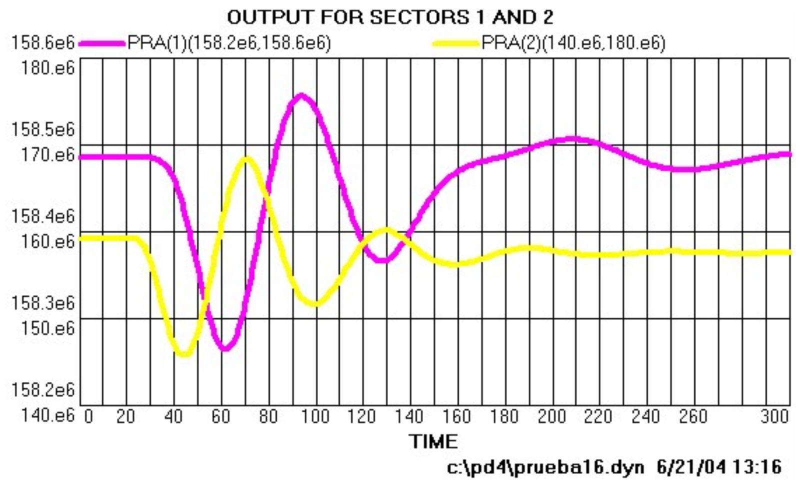
We have applied this method to the regional table for 1995 of Asturias, a region of the north of Spain. The table we have taken is the version with sixteen sectors. The model uses dt equal one week and a matrix T of production maturities of each of the sixteen sectors.

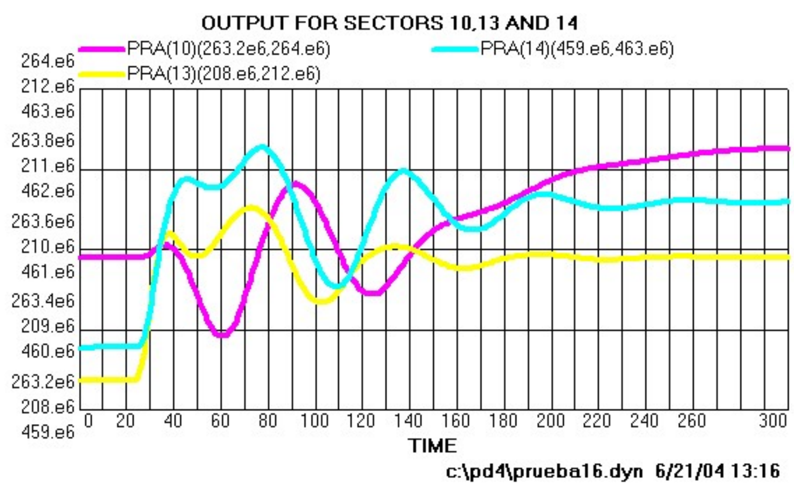
We present here two simple applications:

1. Starting from any value of the initial backlogs and using the real values of final demands for the year, outputs evolve until they stabilize at the value corresponding to Leontief's static model.



2. Simulation of an impact of increase of demand for electricity of 10% that produces a wave effect in the outputs of the sixteen sectors over time.





Thus the propagation of effects over time though the economy can be studied and traced as if we were doing a laboratory experiment.