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Archives: History of i-o analysis; Methods and mathematics

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ADDITIVE LABOUR VALUES AND PRICES OF PRODUCTION: EVIDENCE FROM THE SUPPLY AND USE TABLES OF THE GERMAN AND GREEK ECONOMY

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JEL classification: B24, C67, D46, D57

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1. INTRODUCTION

In recent years, there have been a growing number of empirical studies in the relationships between labour values, production prices and market prices. The main conclusion of these studies, which are based on Symmetric Input-Output Tables (SIOT) and, therefore, on models of *single* production, is that in actual economies the vectors of labour values and production prices are quite close to that of market prices as this can be judged by alternative measures of deviation.[1] To our knowledge, however, there is *no* relevant study, which is based on the Supply and Use Tables (SUT).

As is well known, the SIOT can be derived from the 'System of National Accounts' (SNA) framework of SUT (see, *e.g.*, United Nations, 1999, chs 2-4), introduced in 1968 (United Nations, 1968, ch.3).[2] Given that in the SUT (SIOT) there are (are no) industries that produce more than one commodity and (neither) commodities that are produced by more than one industry, it follows that the SUT (SIOT) could be considered as the *counterpart* of a joint-product (single-product) system *à la* v. Neumann/Sraffa.[3] Nevertheless, since joint production is the empirically relevant case (see Steedman, 1984; Faber *et al.*, 1998), SUT constitute, doubtless, a *more* realistic 'picture' of the actual economic system than SIOT.

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[1] See Shaikh (1984, 1998), Petrović (1987), Ochoa (1989), Cockshott *et al.* (1995), Cockshott and Cottrell (1997), Chilcote (1997), Tsoulfidis and Maniatis (2002), Zachariah (2006), Tsoulfidis and Mariolis (2007), Tsoulfidis (2008), *inter alia*. A remarkable exception can be found in Steedman and Tomkins (1998), where the price-value deviations are greater than those usually estimated.

[2] For a review of the methods, used to convert the SUT into SIOT, see, *e.g.*, ten Raa and Rueda-Cantuche (2003, pp. 441-447). Amongst the various available methods, the so-called 'Commodity Technology Assumption' is the only one that fulfils a set of important properties of the input-output analysis (see Jansen and ten Raa, 1990). However, the 'Commodity Technology Assumption' is possible to generate economically insignificant results, *i.e.*, *negative* elements in the input-output matrix. For a critical review of the various procedures proposed to overcome this inconsistency, see ten Raa and Rueda-Cantuche (2005).

[3] See, *e.g.*, Flaschel (1980, pp. 120-121), Bidard and Erreygers (1998, pp. 434-436) and Lager (2007). It has to be noted, however, that some of the 'joint' products that appear in the SUT may result from statistical classification and, therefore, they do not correspond with the notion of joint production (see, *e.g.*, Semmler, 1984, pp. 168-169; United Nations, 1999, p. 77).

The purpose of this paper is to estimate, in terms of the usual 'square' linear model of production (for a closed economy with circulating capital and homogeneous labour), the vectors of 'additive labour values' (*i.e.*, direct and indirect labour requirements per unit of net output for each commodity)[4] and actual prices of production associated with the SUT of the German (for the years 2000 and 2005) and Greek (for the years 1995 and 1999) economy, *i.e.*, two European economies that can be expected to possess substantially different production structures. It need hardly be said that in cases of (i) 'rectangular' systems;[5] (ii) heterogeneous labour (see Steedman, 1977, ch. 7 and pp. 178-179; 1985); and (iii) non-competitive imports (see Steedman and Metcalfe, 1981, pp. 140-141; Steedman, 2008, Section 3), any attempt to explore the quantitative relationships between labour values and prices is devoid of economic sense.

The remainder of the paper is organized as follows. Section 2 describes the model.[6] Section 3 presents and critically evaluates the results of the empirical analysis. Section 4 concludes.

2. THE ANALYTIC FRAMEWORK

Assume a closed capitalist economy, which produces n commodities by n linear processes of pure joint production, *i.e.*, a 'square', profitable and productive system,

[4] For this concept, see Steedman (1975, 1976).

[5] It goes without saying that the SUT are not necessarily 'square' (see, *e.g.*, United Nations, 1999, p. 86, §4.41).

[6] See Appendix 1 for the available input-output data as well as the construction of relevant variables.

and in which commodity prices deviate from the prices of production. Homogeneous labour is the only primary input and there is only circulating capital, whilst labour is not an input to the household sector. Moreover, the net product is distributed to profits and wages that are paid at the beginning of the common production period and there are no savings out of this income.[7] Finally, the givens in our analysis are (i) the vector of market prices; (ii) the technical conditions of production, that is, the triplet $\{\mathbf{B}, \mathbf{A}, \mathbf{a}\}$, where $\mathbf{B}$ represents the $n \times n$ Make matrix, $\mathbf{A}$ the $n \times n$ Use matrix (both $\mathbf{B}$ and $\mathbf{A}$ are expressed in physical terms), and $\mathbf{a}^{\mathrm{T}}$ the $1 \times n$ vector of employment levels process by process ('T' is the sign for transpose); and (iii) the real wage rate, which is represented by the $n \times 1$ vector $\mathbf{d}$.

On the basis of these assumptions, the vector of additive labour values, $\mathbf{v}$, the total 'surplus value', S, and the vector of production prices, $\mathbf{p}$, related to the processes *actually* used in the economy under consideration, may be estimated from the following relations

$$\mathbf{v}^{\mathrm{T}}\mathbf{B} = \mathbf{v}^{\mathrm{T}}\mathbf{A} + \mathbf{a}^{\mathrm{T}} \tag{1}$$

$$S \equiv \mathbf{v}^{\mathrm{T}}\mathbf{u} \tag{2}$$

$$\mathbf{p}^{\mathrm{T}}\mathbf{B} = (1+r)(\mathbf{p}^{\mathrm{T}}\mathbf{A} + w\mathbf{a}^{\mathrm{T}})$$

or

$$\mathbf{p}^{\mathrm{T}}\mathbf{B} = (1+r)\mathbf{p}^{\mathrm{T}}\mathbf{C} \tag{3}$$

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[7] We hypothesize that wages are paid *ante factum* (for the general case, see Steedman, 1977, pp. 103-105) and that there are no savings out of this income in order to follow most of the empirical studies on this topic (see footnote 1).

where $\mathbf{u} \equiv [\mathbf{B} - \mathbf{C}]\mathbf{e}$ represents the 'surplus product', $\mathbf{e}$ ($\equiv [1,1,...,1]^{\mathrm{T}}$) the summation vector, $\mathbf{C}$ ($\equiv \mathbf{A} + \mathbf{d}\mathbf{a}^{\mathrm{T}}$) the 'augmented' Use matrix, w ($= \mathbf{p}^{\mathrm{T}}\mathbf{d}$) the money wage rate in terms of production prices, and r the uniform rate of profits. Provided that $[\mathbf{B} - \mathbf{A}]$ and $\mathbf{B}$ are non-singular, (1), (2) and (3) entail that

$$\mathbf{v}^{\mathrm{T}} = \mathbf{a}^{\mathrm{T}}[\mathbf{B} - \mathbf{A}]^{-1} \tag{4}$$

$$S = (1 - \mathbf{v}^{\mathrm{T}}\mathbf{d})\mathbf{a}^{\mathrm{T}}\mathbf{e} \tag{5}$$

$$\mathbf{p}^{\mathrm{T}} = (1 + r)\mathbf{p}^{\mathrm{T}}\mathbf{D} \tag{6}$$

where $\mathbf{D} \equiv \mathbf{C}\mathbf{B}^{-1}$. Relations (4), (5) and (6) imply that (i) $\mathbf{v}$ is uniquely determined; (ii) S is positive iff the unit 'value of labour power', $\mathbf{v}^{\mathrm{T}}\mathbf{d}$, is less than 1; and (iii) $(1 + r)^{-1}$ is an eigenvalue of the matrix $\mathbf{D}$ and $\mathbf{p}^{\mathrm{T}}$ is the corresponding left-hand side eigenvector. Nevertheless, *nothing* guarantees the existence of a (semi-) positive solution for ($\mathbf{v}, r, \mathbf{p}$).[8]

Finally, it should be stressed that any 'complication' related to (1)-(2) and/or (3) (*i.e.*, inconsistency, non-unique solution for $\mathbf{v}$, non-unique economically significant solution for $\mathbf{v}$ and/or $(r, \mathbf{p})$, co-existence of positive (non-positive) 'surplus value' with non-positive (positive) profits) does not constitute, as is well known, any problem for the v. Neumann/Sraffa-based analysis;[9] it indicates rather a inner limit of the 'labour theory of value'.

[8] See Sraffa (1960, §§ 69-72), Schefold (1971, pp. 25-26 and 31-34; 1978), Steedman (1977, chs 10-12), Filippini and Filippini (1982), Fujimoto and Krause (1988) and Hosoda (1993).

[9] See Steedman (1977, chs 12-13; 1992), Kurz and Salvadori (1995, ch. 8) and Bidard (1997).

3. RESULTS AND THEIR EVALUATION

The application of the previous analysis to the SUT of the German (for the years 2000 and 2005) and Greek (for the years 1995 and 1999) economy gives the following results:[10]

(i). The matrices $[\mathbf{B}-\mathbf{A}]$ and $\mathbf{B}$ are non-singular. Consequently, $\mathbf{v}$ can be uniquely estimated from (4), and $\mathbf{p}$ can be estimated from (6).

(ii). The matrices $[\mathbf{B}-\mathbf{A}]^{-1}$ contain negative elements. Consequently, the systems under consideration are not 'all-productive' and, therefore, they do not have the properties of a single-product system (Schefold, 1971, pp. 34-35; 1978; see also Kurz and Salvadori, 1995, pp. 238-240).[11]

(iii). The vectors of labour values of the German and the Greek economy for the year 1995 are positive. However, the vector of labour values of the Greek economy for the year 1999 contains five negative elements, which correspond with the 'primary products' of the following industries: 01 (Agriculture, hunting and related service activities); 11 (Extraction of crude petroleum and natural gas; service activities incidental to oil and gas extraction excluding surveying); 23 (Manufacture of coke,

[10] *Mathematica 5.0* is used in the calculations. The analytical results are available on request from the authors.

[11] A commodity is said to be 'separately producible' in system $\{\mathbf{B},\mathbf{A}\}$ if it is possible to produce a net output consisting of a unit of that commodity alone with a nonnegative intensity vector. A system of production is called 'all-productive' if all commodities are separately producible in it. Thus, if $\{\mathbf{B},\mathbf{A}\}$ is 'all-productive', then $[\mathbf{B}-\mathbf{A}]^{-1} \geq \mathbf{0}$ (*ibid*.).

refined petroleum products and nuclear fuels); 61 (Water transport); and 67 (Activities auxiliary to financial intermediation).[12]

(iv). The 'surplus values' are positive (German economy: $\mathbf{v}^{\mathrm{T}}\mathbf{d} \simeq 0.48$ (2000), 0.45 (2005), Greek economy: $\mathbf{v}^{\mathrm{T}}\mathbf{d} \simeq 0.28$ (1995), 0.16 (1999)).

(v). The systems of actual prices of production of the German economy have a unique, positive solution for $(\mathbf{p}, r)$, and $(1+r)^{-1}$ are the dominant eigenvalues of the matrices $\mathbf{D}$. The system of actual prices of production of the Greek economy for the year 1995 (for the year 1999) has 20 (16) positive, 4 (8) negative and 34 (34) complex conjugate solutions for r, and only economically insignificant solutions for $\mathbf{p}$. Consequently, in the case of the Greek economy, positive 'surplus value' co-exists with economically insignificant $(\mathbf{p}, r)$.[13]

(vi). In the German economy, the deviations of actual prices of production from additive labour values are in the area of 15%, whilst the deviations of market prices from additive labour values or actual prices of production are in the area of 57% (as these can be judged from the 'd distance').[14, 15] Finally, in the Greek economy for the

[12] The additive labour values of the German and Greek economy are reported in the Appendix 2, Tables 2.1-2 and 2.3-4, respectively.

[13] The eigenvalues of systems (6) are reported in the Appendix 3.

[14] Consider the deviation of $\mathbf{x}^{\mathrm{T}}$ ($\geq \mathbf{0}^{\mathrm{T}}$) from $\mathbf{y}^{\mathrm{T}}$ ($\geq \mathbf{0}^{\mathrm{T}}$). The '$d$ distance', which has been proposed by Steedman and Tomkins (1998, pp. 381-382), constitutes a *numéraire*-free measure of deviation and is defined as

$$d \equiv \sqrt{2(1-\cos\theta)}$$

where θ is the angle between $\mathbf{x}^{\mathrm{T}}\hat{\mathbf{y}}^{-1}$ and $\mathbf{e}$ ($\hat{\mathbf{y}}$ denotes the diagonal matrix formed from the elements of $\mathbf{y}$).

year 1995, the deviation of market prices from additive labour values is almost 87% (see Table 1). Thus, we conclude that the price-additive labour value deviations are, by and large, considerably greater than those estimated on the basis of SIOT.[16]

Table 1. Deviations of prices from additive labour values; German and Greek economy

d distance (%)	Germany 2000	Germany 2005	Greece 1995	Greece 1999
Actual prices of production vs. additive labour values	14.1	16.1	–	–
Market prices vs. additive labour values	56.8	56.3	87.0	–
Market prices vs. actual prices of production	57.7	57.5	–	–

The next issue that comes up is whether the systems under consideration are characterized by $\mathbf{E}(r) \equiv [\mathbf{B} - \mathbf{A}(1+r)]^{-1} > \mathbf{0}$ for some $r > 0$. As is well known, $\mathbf{E}(r) > \mathbf{0}$ is a sufficient condition for the existence of an *interval* of r, in which a joint production system retains all the essential properties of indecomposable single-

[15] It may be noted that for $s_w = 0.1$ ($= 0.2$), where s_w represents the fraction of wages saved (see Appendix 1), the deviation of actual prices of production from additive labour values in the German economy for the year 2000 is almost 16.0% (18.1 %), whilst the deviation of market prices from actual prices of production is almost 58.0% (58.4%).

[16] Compare with the findings from the empirical studies mentioned in footnote 1. For example, the market price-labour value deviation estimated on the basis of the 19×19 SIOT of the Greek economy for the year 1995 is almost 23.6% (Tsoulfidis and Mariolis, 2007, p. 428, Table 1). Since the *theoretically* maximum value of $\cos\theta$ equals $1/\sqrt{n}$, the theoretically maximum value of the 'd distance', D, equals $\sqrt{2[1-(1/\sqrt{n})]}$. Thus, the normalized 'd distance', defined as d/D, is almost $23.6/124.2 \simeq 19\%$ and, therefore, considerably lower than the normalized 'd distance' estimated from the relevant 58×58 SUT, which is almost $87.0/131.8 \simeq 66\%$.

product systems (Schefold, 1971, p. 35). The investigation can be based on the following theorem (Bidard, 1996, p. 328): Consider the eigensystems associated with the pair $\{\mathbf{B}, \mathbf{A}\}$, namely

$$\lambda \mathbf{B}\mathbf{x} = \mathbf{A}\mathbf{x} \tag{7}$$

$$\lambda \mathbf{y}^{\mathrm{T}}\mathbf{B} = \mathbf{y}^{\mathrm{T}}\mathbf{A} \tag{8}$$

There exists $r > -1$ such that $\mathbf{E}(r) > \mathbf{0}$ iff there exist $(\lambda, \mathbf{x}, \mathbf{y}) > \mathbf{0}$, where $\mathbf{x}$ is determined up to a factor.

The estimation of the characteristic values and vectors that correspond with the pairs $\{\mathbf{B}, \mathbf{A}\}$ of the Greek and German economy gives the following results: The eigensystems of the German economy for the years 2000 and 2005 have 21 positive (and simple) and 17 positive (and simple) eigenvalues, respectively, whilst the eigensystems of the Greek economy for the years 1995 and 1999 have 18 positive (and simple) and 15 positive (and simple) eigenvalues, respectively.[17] However, there are no positive eigenvectors and, therefore, the said condition is not satisfied.[18]

4. CONCLUDING REMARKS

The exploration of the relationships between additive labour values and actual prices using a usual linear model of joint production and data from the Supply and Use tables of the German and the Greek economy, gave the following results: (i). In the

[17] See Appendix 4.

[18] It is important to note that this attribute of the considered systems is independent of the composition and the level of the real wage rate and, therefore, does not rely on our hypothesis that there are no savings out of wages.

German economy (for the years 2000 and 2005), positive additive labour values and positive actual prices of production co-exist with positive 'surplus value'. (ii). In the Greek economy (for the years 1995 and 1999), economically insignificant additive labour values and/or actual prices of production co-exist with positive 'surplus value'. (iii). The deviations of market prices from additive labour values and actual prices of production are in the range of 56%-87% (as this can be judged from the 'd distance'). (iv). A sufficient condition for the existence of an interval of the uniform rate of profits, in which the systems under consideration retain all the essential properties of indecomposable single-product systems, is violated.

Since in the real world joint production constitutes the rule, these findings would seem to be of some importance and, especially, cast doubts on the logic of the 'empirical labour theory of value' (Stigler, 1958, p. 361). Nevertheless, future research efforts should use input-output data from various countries, concretize the model by including the presence of fixed capital and the degree of its utilization, depreciation, turnover times, taxes and subsidies, and explore the relationships between prices and hypothetical changes in income distribution.

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APPENDIX 1: A NOTE ON THE DATA

The SUT of the German (for the years 1995 and 1997 through 2005) and Greek (for the years 1995 through 1999) economy are available via the Eurostat website (http://ec.europa.eu/eurostat). The corresponding levels of sectoral employment of the German economy are included in the SUT, whilst those of Greek economy are provided by the National Statistical Service of Greece.

Through to the year 1999, the Supply Tables describe 59 products and 59 industries, whilst the Use Tables describe 59 products and 60 industries. The German SUT for the years from 2000 onwards describe 59 products and 59 industries. These tables are revised and they are not comparable with those of preceding years. The products are classified according to CPA (Classification of Products by Activity), whilst industries are classified according to NACE (General Industrial Classification of Economic Activities within the European Communities). Given that technical change over time could be expected to be rather 'slow', we have chosen to apply our analysis to the tables of the German economy for the years 2000 and 2005, and of the Greek economy for the years 1995 and 1999. Namely, if we leave apart the non-symmetric tables of the German economy (before the year 2000), we maximized the chronological distance amongst the SUT of each country.

In the SUT of both countries, all elements associated with the product and industry 12 (Mining of uranium and thorium ores) equal zero and, therefore, we remove them from our analysis. In the case of the German economy, all the elements associated with the product and industry 13 (Mining of metal ores) of the Make matrices (*i.e.*, the part of the Supply Tables that describes domestic production) equal zero and, therefore, we remove them from our analysis, whilst there are elements associated with the product 13 in the Use matrices (*i.e.*, the part of the Use Tables that describes intermediate consumption) that are positive. In order to derive symmetric matrices, we aggregate the product 13 of the Use matrices with the product 27 (Basic metals). This choice is based on the fact that product 13 is mainly used by the industry 27 (Manufacture of basic metals). Thus, we derive Make and Use matrices of dimensions 57×57 for the German economy. In the case of the Greek economy, the Use matrices include an additional, *fictitious* industry named 'Financial Intermediation Services Indirectly Measured (FISIM)'. In order to derive symmetric matrices, we apply the aggregation that United Nations (1999, p.135, §5.76) recommend for this case. Namely, we aggregate the fictitious industry with the industry 65 (Financial intermediation, except insurance and pension funding). Thus, we derive Make and Use matrices of dimensions 58×58 for the Greek economy.

In the Supply Tables, goods and services are measured at ‘basic prices’, whilst in the Use Tables all intermediate costs are measured in ‘purchasers’ prices’. The derivation of the SUT at basic prices is based on the method proposed by United Nations (1999, ch. 3 and pp. 228-229). All the SUT used in our analysis are in current prices. The market prices of all products are taken to be equal to one; that is to say, the physical unit of measurement of each product is that unit which is worth of a monetary unit (see, *e.g.*, Miller and Blair, 1985, p. 356).

Wage differentials are used to homogenize the sectoral employment (see, *e.g.*, Sraffa, 1960, §10; Kurz and Salvadori, 1995, pp. 322-325), *i.e.*, the jth element of the vector of employment levels process by process, $\mathbf{a} \equiv [\mathrm{a}_j]$, is determined as follows: $\mathrm{a}_j = l_j (w_j^m / w_{\min}^m)$, where l_j and w_j^m are total employment and money wage rate, in terms of market prices, of the jth sector, respectively, and $w_{\min}^m$ is the minimum sectoral money wage rate in terms of market prices. Alternatively, the homogenization of employment could be achieved, *for example*, through the economy’s average wage; in fact, the empirical results are robust to alternative normalizations with respect to homogenization of labour inputs. Furthermore, by assuming that workers do not save and that their consumption has the same composition as the vector of private households consumption expenditures, $\mathbf{c}$, directly available in the SUT, the vector of the real wage rate, $\mathbf{d}$, is determined as follows: $\mathbf{d} = (w^m_{\min} / \mathbf{e}^{\mathrm{T}}\mathbf{c})\mathbf{c}$, where $\mathbf{e}^{\mathrm{T}} \equiv [1,1,...,1]$ represents the vector of market prices (see also, *e.g.*, Okishio and Nakatani, 1985, pp. 66-67). It goes without saying that the empirical results (on the deviations of actual prices of production from labour values and market prices) are robust to the assumption that a certain relatively small fraction of wages, s_w, is saved; in this case the vector of the real wage would be equal to $[(1 - s_w)w^m_{\min} / \mathbf{e}^{\mathrm{T}}\mathbf{c}]\mathbf{c}$. Finally, it should be noted that, in the available SUT, we do not have data on fixed capital stocks. As a result, our investigation is based on a model with circulating capital.

APPENDIX 2: ADDITIVE LABOUR VALUES (ALV) OF THE GERMAN AND GREEK ECONOMY

Table 2.1. ALV; Germany, 2000

CPA	ALV	CPA	ALV
01	46.7	40	63.4
02	57.0	41	34.0
05	39.7	45	64.6
10	172.5	50	71.7
11	38.1	51	62.5
14	55.5	52	66.6
15	58.3	55	61.3
16	42.7	60	75.9
17	68.6	61	38.4
18	66.5	62	50.2
19	68.8	63	63.7
20	63.2	64	47.2
21	63.4	65	64.3
22	55.7	66	61.4
23	37.6	67	43.6
24	63.7	70	17.4
25	68.5	71	10.7
26	65.9	72	64.9
27 $\oplus$ 13	67.4	73	72.6
28	71.2	74	50.8
29	73.8	75	82.6
30	59.5	80	94.6
31	73.3	85	68.9
32	66.4	90	47.1
33	73.0	91	95.6
34	74.5	92	49.1
35	78.0	93	26.4
36	70.6	95	110.8
37	56.5		

Table 2.2. ALV; Germany, 2005

CPA	ALV	CPA	ALV
01	52.3	40	52.0
02	39.4	41	33.1
05	37.0	45	63.6
10	160.1	50	68.4
11	47.9	51	64.2
14	63.4	52	68.3
15	60.2	55	64.4
16	49.1	60	77.2
17	67.1	61	36.8
18	65.1	62	55.3
19	62.9	63	61.0
20	59.6	64	42.9
21	61.8	65	56.1
22	55.1	66	57.9
23	45.3	67	40.2
24	60.3	70	15.1
25	64.9	71	11.4
26	68.2	72	74.5
27 $\oplus$ 13	61.5	73	80.4
28	69.6	74	54.2
29	71.8	75	84.4
30	63.4	80	96.5
31	77.4	85	68.9
32	66.1	90	46.9
33	68.4	91	96.6
34	71.3	92	52.0
35	74.4	93	26.6
36	68.1	95	116.4
37	59.1		

Table 2.3. ALV; Greece, 1995

CPA	ALV	CPA	ALV
01	148.8	37	238.9
02	1244.4	40	482.9
05	356.8	41	674.3
10	903.5	45	460.2
11	195.1	50	301.1
13	967.3	51	413.8
14	482.3	52	220.3
15	339.3	55	261.2
16	290.5	60	488.0
17	505.5	61	486.2
18	479.0	62	731.7
19	482.5	63	845.6
20	523.4	64	541.6
21	575.8	65	1844.8
22	652.6	66	678.9
23	280.5	67	297.4
24	614.5	70	39.5
25	605.4	71	194.2
26	646.0	72	594.5
27	546.1	73	693.2
28	623.6	74	385.7
29	682.9	75	1027.8
30	580.9	80	994.6
31	620.2	85	538.4
32	586.4	90	742.4
33	666.6	91	899.5
34	585.2	92	557.9
35	1009.9	93	231.5
36	486.2	95	1308.3

Table 2.4. ALV; Greece, 1999

CPA	ALV	CPA	ALV
01	-27.3	37	136.0
02	1651.7	40	32.8
05	31.0	41	895.2
10	637.9	45	313.7
11	-7852.7	50	291.9
13	814.8	51	329.4
14	169.9	52	163.0
15	201.1	55	169.9
16	240.6	60	173.8
17	441.3	61	-270.9
18	428.3	62	397.1
19	431.9	63	618.5
20	467.2	64	318.5
21	485.2	65	1459.5
22	585.9	66	525.1
23	-5857.4	67	-14.6
24	451.9	70	29.7
25	501.7	71	145.2
26	277.0	72	688.1
27	372.2	73	677.8
28	499.8	74	399.5
29	631.5	75	971.3
30	666.3	80	1044.3
31	491.1	85	542.2
32	494.8	90	573.8
33	596.0	91	844.3
34	450.3	92	493.9
35	979.6	93	188.5
36	468.8	95	1280.6

APPENDIX 3: EIGENVALUES OF THE SYSTEMS OF ACTUAL PRICES OF PRODUCTION OF THE GERMAN AND GREEK ECONOMY

Table 3.1. Eigenvalues of $\mathbf{D}$*; Germany, 2000*

1	0.740	22	0.091
2	0.358	23	$-0.050 \pm 0.069i$
3	0.316	24	0.083
4	$0.297 \pm 0.026i$	25	$0.076 \pm 0.014i$
5	0.288	26	-0.072
6	0.256	27	0.069
7	0.246	28	$0.058 \pm 0.028i$
8	$0.241 \pm 0.006i$	29	0.061
9	$0.216 \pm 0.015i$	30	0.059
10	0.207	31	$0.032 \pm 0.045i$
11	0.195	32	$-0.032 \pm 0.039i$
12	$0.172 \pm 0.071i$	33	0.034
13	0.185	34	0.032
14	$0.183 \pm 0.007i$	35	0.019
15	$0.173 \pm 0.042i$	36	$0.016 \pm 0.003i$
16	0.158	37	$0.001 \pm 0.015i$
17	0.145	38	$-0.003 \pm 0.006i$
18	$0.136 \pm 0.007i$	39	0.002
19	$0.119 \pm 0.005i$	40	0.0001
20	$0.098 \pm 0.033i$		
21	0.096		

Table 3.2. Eigenvalues of $\mathbf{D}$*; Germany, 2005*

1	0.727	22	0.097
2	0.383	23	$0.094 \pm 0.018i$
3	0.316	24	0.095
4	$0.297 \pm 0.006i$	25	0.080
5	0.290	26	0.069
6	0.257	27	0.066
7	0.244	28	0.059
8	$0.230 \pm 0.004i$	29	-0.059
9	$0.225 \pm 0.012i$	30	$0.021 \pm 0.044i$
10	0.221	31	$0.046 \pm 0.010i$
11	$0.182 \pm 0.071i$	32	-0.044
12	0.190	33	0.043
13	$0.183 \pm 0.044i$	34	$0.026 \pm 0.008i$
14	0.186	35	$-0.004 \pm 0.024i$
15	0.169	36	-0.020
16	$0.154 \pm 0.009i$	37	0.019
17	$0.141 \pm 0.025i$	38	0.015
18	0.141	39	$0.002 \pm 0.012i$
19	$0.122 \pm 0.010i$	40	0.0003
20	$0.114 \pm 0.035i$	41	0.0001
21	$-0.034 \pm 0.091i$		

Table 3.3. Eigenvalues of $\mathbf{D}$ *; Greece, 1995*

1	0.711	22	$0.009 \pm 0.051\,i$
2	0.540	23	$-0.036 \pm 0.032\,i$
3	0.370	24	0.047
4	0.350	25	$-0.041 \pm 0.017\,i$
5	0.281	26	0.041
6	0.257	27	$0.011 \pm 0.023\,i$
7	0.196	28	0.024
8	$0.191 \pm 0.016\,i$	29	$0.018 \pm 0.005\,i$
9	$0.132 \pm 0.131\,i$	30	$-0.014 \pm 0.006\,i$
10	0.162	31	$-0.006 \pm 0.012\,i$
11	$0.136 \pm 0.009\,i$	32	0.011
12	0.126	33	-0.008
13	$-0.077 \pm 0.091\,i$	34	0.008
14	0.100	35	0.007
15	$0.029 \pm 0.073\,i$	36	$-0.002 \pm 0.002\,i$
16	$0.066 \pm 0.041\,i$	37	0.002
17	$0.070 \pm 0.016\,i$	38	-0.001
18	0.071	39	0.0003
19	-0.067	40	$1.061\text{x}10^{-7}$
20	$-0.013 \pm 0.062\,i$	41	$-3.418\text{x}10^{-19}$
21	$0.056 \pm 0.002\,i$		

Table 3.4. Eigenvalues of $\mathbf{D}$ *; Greece, 1999*

1	$0.461 \pm 0.511\,i$	22	$-0.009 \pm 0.044\,i$
2	0.645	23	-0.041
3	0.403	24	$0.002 \pm 0.039\,i$
4	0.344	25	$-0.031 \pm 0.019\,i$
5	0.256	26	$0.029 \pm 0.022\,i$
6	$-0.209 \pm 0.140\,i$	27	0.034
7	$0.200 \pm 0.125\,i$	28	-0.022
8	$0.189 \pm 0.035\,i$	29	$-0.005 \pm 0.016\,i$
9	0.185	30	$0.015 \pm 0.005\,i$
10	0.165	31	$0.003 \pm 0.011\,i$
11	0.153	32	-0.011
12	0.121	33	-0.010
13	0.110	34	0.008
14	$0.107 \pm 0.025\,i$	35	$-0.002 \pm 0.002\,i$
15	0.104	36	0.002
16	$0.079 \pm 0.019\,i$	37	-0.001
17	$0.056 \pm 0.029\,i$	38	0.001
18	-0.059	39	-0.0002
19	$0.056 \pm 0.012\,i$	40	$-1.018\text{x}10^{-7}$
20	0.050	41	$1.411\text{x}10^{-18}$
21	$0.019 \pm 0.043\,i$		

APPENDIX 4: EIGENVALUES OF THE PAIRS $\{\mathbf{B},\mathbf{A}\}$ OF THE GERMAN AND GREEK ECONOMY

Table 4.1. Eigenvalues of $\{\mathbf{B},\mathbf{A}\}$ *; Germany, 2000*

1	0.488	22	0.089
2	0.359	23	$-0.045 \pm 0.071i$
3	0.314	24	$0.076 \pm 0.005i$
4	$0.295 \pm 0.024i$	25	-0.071
5	0.286	26	$0.058 \pm 0.039i$
6	$0.246 \pm 0.009i$	27	0.068
7	0.242	28	0.064
8	0.238	29	0.059
9	$0.215 \pm 0.025i$	30	0.053
10	0.208	31	$-0.034 \pm 0.022i$
11	$0.201 \pm 0.010i$	32	0.035
12	0.183	33	$0.027 \pm 0.008i$
13	$0.181 \pm 0.014i$	34	0.024
14	$0.167 \pm 0.046i$	35	0.017
15	0.147	36	0.015
16	$0.144 \pm 0.014i$	37	$-0.003 \pm 0.012i$
17	$0.139 \pm 0.007i$	38	$0.004 \pm 0.009i$
18	0.132	39	0.0001
19	0.112	40	0
20	$0.104 \pm 0.028i$		
21	$0.092 \pm 0.008i$		

Table 4.2. Eigenvalues of $\{\mathbf{B},\mathbf{A}\}$ *; Germany, 2005*

1	0.502	22	$0.095 \pm 0.001i$
2	0.383	23	$0.085 \pm 0.026i$
3	0.318	24	0.069
4	$0.296 \pm 0.007i$	25	$0.064 \pm 0.001i$
5	$0.263 \pm 0.025i$	26	-0.057
6	0.244	27	0.051
7	$0.230 \pm 0.008i$	28	$0.047 \pm 0.019i$
8	$0.224 \pm 0.010i$	29	-0.047
9	0.220	30	0.047
10	$0.197 \pm 0.057i$	31	0.025
11	$0.195 \pm 0.030i$	32	$0.019 \pm 0.010i$
12	0.186	33	-0.021
13	0.176	34	0.020
14	0.168	35	$0.003 \pm 0.015i$
15	$0.153 \pm 0.006i$	36	0.015
16	0.138	37	$-0.004 \pm 0.008i$
17	$0.122 \pm 0.033i$	38	0.0001
18	$0.115 \pm 0.020i$	39	0
19	$0.116 \pm 0.010i$		
20	0.111		
21	$-0.031 \pm 0.090i$		

Table 4.3. Eigenvalues of $\{\mathbf{B}, \mathbf{A}\}$ *; Greece, 1995*

1	0.689	22	0.044
2	0.420	23	0.043
3	0.363	24	$0.031 \pm 0.027\,i$
4	0.348	25	$-0.037 \pm 0.019\,i$
5	0.282	26	$0.006 \pm 0.040\,i$
6	0.250	27	-0.039
7	0.199	28	$-0.016 \pm 0.023\,i$
8	0.194	29	0.025
9	$0.144 \pm 0.108\,i$	30	$0.022 \pm 0.008\,i$
10	$0.161 \pm 0.010\,i$	31	$-0.003 \pm 0.018\,i$
11	$0.129 \pm 0.009\,i$	32	-0.014
12	0.128	33	0.008
13	$-0.076 \pm 0.093\,i$	34	-0.007
14	0.115	35	0.006
15	$0.029 \pm 0.073\,i$	36	$0.003 \pm 0.001\,i$
16	$0.066 \pm 0.036\,i$	37	-0.003
17	-0.069	38	0.001
18	$0.066 \pm 0.003\,i$	39	$-0.0001 \pm 0.0005\,i$
19	$-0.010 \pm 0.061\,i$	40	0
20	0.060	41	0
21	0.051	42	0

Table 4.4. Eigenvalues of $\{\mathbf{B}, \mathbf{A}\}$ *; Greece, 1999*

1	$0.397 \pm 0.515\,i$	22	$-0.008 \pm 0.041\,i$
2	0.648	23	-0.042
3	0.400	24	$0.023 \pm 0.027\,i$
4	0.337	25	$0.033 \pm 0.007\,i$
5	0.255	26	0.028
6	$-0.208 \pm 0.140\,i$	27	$-0.021 \pm 0.018\,i$
7	$0.207 \pm 0.047\,i$	28	$-0.006 \pm 0.022\,i$
8	0.185	29	-0.019
9	$0.181 \pm 0.026\,i$	30	$-0.013 \pm 0.010\,i$
10	$0.147 \pm 0.013\,i$	31	$0.015 \pm 0.005\,i$
11	0.126	32	$-0.003 \pm 0.013\,i$
12	0.121	33	0.008
13	0.112	34	-0.007
14	0.101	35	0.003
15	0.081	36	$0.001 \pm 0.001\,i$
16	$0.067 \pm 0.033\,i$	37	-0.001
17	0.067	38	-0.0001
18	$0.034 \pm 0.049\,i$	39	0
19	-0.060	40	0
20	$0.046 \pm 0.021\,i$	41	0
21	0.050		