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Michael Sonis and Geoffrey J.D. Hewings

Coefficient Change and Innovation Spread in Input-Output Models

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Correspondence address:
Micahel Sonis
Department of Geography
Bar Ilan University
52900, Ramat-Gan
Israel.

E-mail: sonism@mail.biu.ac.il

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Coefficient Change and Innovation Spread in Input-Output Models

Michael Sonis

Correspondence Address: Department of Geography, Bar Ilan University, 52900, Ramat-Gan, Israel and Regional Economics Applications Laboratory, University of Illinois, 607 South Mathews, #220, Urbana, Illinois, 61801-3671 *email*: sonism@mail.biu.ac.il

Geoffrey J. D. Hewings

Regional Economics Applications Laboratory, University of Illinois, 607 South Mathews, #220, Urbana, Illinois, 61801-3671 *email*: hewings@uiuc.edu

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I. Change and Innovation in Input-Output System.

1. Introduction

The most important assumption in the Input-Output theory proposed by its creator Wassily Leontief was the assumption of constancy of direct input coefficients. Leontief constructed the first USA matrix of direct input coefficients, which used for the analysis of the United States economy in 1939. From this time thousands of input-output matrices were assembled for many economies all over the world. Together with the assemblage of input-output matrices the problems of errors and error sensitivity immediately appeared. So, interest in the problem of coefficient change in input-output models is not a recent phenomenon; however, what is most curious about input-output modeling is that analysts, for the most part, have not made the discussion of errors a prominent feature of the presentation of the model, in applications and, especially, in connection with results of impact analyses. However, while the sources of error (in data or in estimation) are often acknowledged, it is rare to find a presentation involving the use of an input-output model in which a statistical confidence interval is assigned to the level of output associated with any given change in final demand.

Attention to change in coefficients in input-output models has been directed to the issue of the effect of error or changes in individual coefficients on the elements of the associated Leontiev inverse matrix (Evans, 1954; Simonovits, 1975; Lahiri and Satchell, 1986). Complementing this approach, there is the issue of coefficient stability and the effect of coefficient change induced by technology, changing markets, structural change and the general effects of economic growth and development. Contributions to this literature include the seminal papers by Sevaldson (1970) and Carter (1970) and the intriguing notions of Tilanus (1966) who, using the annual Dutch input-output tables, suggested a distinction between average and marginal input-output coefficients to parallel the distinctions made in individual consumer consumption theory. Lahiri (1976) approached this problem is a slightly different way, assuming that the choice of input coefficient was a function of the level of demand existing in any given industry. Clearly, Lahiri's ideas provide the entree to the development of a micro-to-macro link in input-output systems in which production choice within the context of an establishment, firm or industry might be modeled in a behavioral setting with the general macroeconomic economy serving to condition choice. In some cases, the choices made at the micro level may, in turn, influence macro-level variables and thus the decision environment faced by other sectors of the economy. In the input-output literature, the models developed by Eliasson (1978) come as close as any to providing this link; the early developments of the transactions value social accounting models (TVSAMs) by Drud, Grais and Pyatt (1985) provided the precursors for extensions towards a more general equilibrium modeling framework. The gradual adoption of computable general equilibrium models, in which the input-output framework is often embedded, has created an even more pressing need for identification of important parameters in the system and an assessment of the role of errors.

For the most part, this work has not been generalizable to all input-output systems; at the regional level, the issue of coefficient change has been more problematical because so many regional and interregional models have been assembled from no survey or partial survey data sources. In this regard, the regional dimension provides the possibility for a new source of error not usually associated with the national level input-output models. The error usually arises in the transfer of the familiar input coefficients into trade coefficients; while Smith and Morrison (1974) evaluated many of the techniques associated with this issue, Stevens and Trainer (1976) suggested that the problem was complicated by the possibilities of differences between the nation and the region in industrial technical structures, a finding confirmed by Israilevich and Hewings (1991).

At the regional level, the debate has been important for focusing attention once again on the *structure* of input-output models and, in particular, on the methods that could be used to ascertain whether two structures were similar. Furthermore, derivative work emanating from this debate has also focused attention on the degree to which notions of importance within the input-output system could be identified. From this work, two complementary approaches to input coefficient change can be identified, namely (1) error analysis and (2) sensitivity analysis. While the two issues will be addressed separately, the distinction is, in many ways, somewhat artificial.

2 Error Analysis

Theil's (1957, 1972) pioneering work in entropy decomposition analysis provided a useful way of examining error or change in input structures. He suggested that change could be decomposed into a set of additive components. More recently, Hewings (1984), Hewings and Syversen (1982), Jackson and Hewings (1984) and Jackson, Hewings and Sonis (1990) have explored this technique with reference to data for Washington State. On the other hand, West (1982) has approached error analysis from a relative change perspective, focusing, in particular, on the effects of coefficient error on the multipliers of the associated inverse matrix. Closely allied with this approach is that adopted by Jackson (1986) who developed the notion of a probability density distribution for each coefficient and showed how this "uncertainty" could lead to serious problems in the utilization of the input-output model (Jackson and West, 1989; Wibe, 1982). The relative change approach has also been explored by Xu and Madden (1991).

Lawson (1980) has approached the problem conceptually by considering various forms of error - additive and multiplicative - and the ways in which these might be used in a "rational" approach" to modeling. Closely allied with this line of reasoning would be the work of Stevens and Trainer (1976), Burford and Katz (1981) and Giarratani and Garhart (1991) who have developed some propositions about the major sources of error. The notion of some "rationality" in the error or in the structure of coefficient change of course underlies the widespread application of the RAS or bi-proportional technique in the context of updating (especially at the national level) and estimation (at the regional level, where a national table is often used as a base).. Bacharach's (1970) work revealed a strong link between the RAS technique and the assumptions explicit in linear and nonlinear programming. Matuszewski, Pitts and Sawyer (1964) did in fact propose an LP-RAS technique; in their applications, several coefficients were "blocked out" in the updating algorithm because their true values were either known or could be estimated with what Jensen and West (1980) have referred to as "superior data." To this point, (early 1970s), however, no attempt had been made to assess the degree to which errors in individual coefficients could be ranked or rated in terms of their importance. West (1981) provided some important directions in this regard, suggesting a relationship between coefficient size and the associated multiplier. Several of the techniques and approaches developed for error analysis were subsequently modified to perform sensitivity analysis; these are described in the next section.

3 Sensitivity Analysis

Using a little-known theorem developed by Sherman and Morrison (1950), Bullard and Sebald (1977, 1988) were able to show that, in energy terms, only a very small number of the input coefficients in the US input-output model were analytically important. In applications at the regional level, Hewings (1984) referred to these as *inverse important coefficients*. In a similar fashion, Jensen and West (1980) found that the removal of a large percentage of the entries in an input-output table could be accomplished with little appreciable effect on the results from the use of the model for impact analysis. Subsequently, West (1982) noted that the *size* and *location* of the coefficient within the input-output table provided the major determinant of an individual coefficient's importance. Further work by Morrison and Thumann (1980) and Hewings and Romanos (1981) has extended the sensitivity notions to suggest that the *censal mentality* characterizing the developments of many input-output models (namely, that all entries need to be estimated with the same degree of accuracy) is probably misplaced. This is especially true in the cases in which regional tables are derived from national tables or in the process of updating tables. The results of the sensitivity analysis in combination with statistical estimation techniques suggest that a more "rational" approach to coefficient change could be developed (Jackson and West, 1989).

4 Fields of Influence Approach

A general approach to the problem of coefficient change was proposed by Sonis and Hewings (Sonis and Hewings, 1989, 1991)). one that is based on the notion of a *field of influence* of changes in direct input coefficients. To a large extent, the procedures to be developed are independent of the type of coefficient change; the major objective is the provision of a methodology that is general enough to handle all types of changes - single elements, all elements in a row or column or in all elements of the matrix. The procedure involves the calculation of the ratio of two polynomial functions of changes in contrast to the usual approach which is based on the infinite Taylor series expansion of the Leontief inverse. Moreover, the methodology provides a finite form, one that is eminently capable of realization in the form of a computer algorithm. This *meso-level* economic approach[1] also provides the possibility for uncovering the hierarchical structure of change through the identification of the intensity of influences, an alternative and complementary approach to the *micro-level* structural path analytical methods illustrated by Defourny and Thorbecke (1984).

Thus, the Field of Influence Approach is more general in that it can handle a complete range of changes. In particular, the ability to be able to examine the influence of changes in an arbitrary subset of elements is presented as a major feature of the methodology; it turns out that the familiar RAS or bi-proportional adjustment technique is a special case of coefficient change (see Sonis and Hewings, 1989). In addition, as demonstrated by Sonis and Hewings (1991), the methodology may be extended to issues of decomposition (see Kymm, 1990; Gillen and Guccione, 1990) or the updating of input-output matrices (Snower, 1990 reviews some of the recent work at the national level while Giarratani and Garhart, 1991 provide a similar review at the regional. See also Dietzenbacher, 1990).

The presentation below builds on earlier work (Sonis and Hewings, 1988, 1989, 1990, 1991, 1992) that examined a variety of issues surrounding error and sensitivity analysis, decomposition and inverse important parameter estimation. These ideas are now brought into a general form as a basis for a more complete, general approach. The essential difference between the fields of influence approach and error and sensitivity analysis is that the former are considered as the main vehicle for describing the overall changes in economic relationships between industries created by combinations of changes in technological coefficients. Interpreted in a comparative static framework, it will then be possible to proceed to consideration of evolutionary economic dynamics. This dynamics reflects the innovation diffusion of new technological and administrative changes within Input-Output System.

II. Theoretical Basis for Coefficient Change: A Synopsis

The condensed form of the solution of the coefficient change problem can be presented in the following manner: let $A=\left(a_{ij}\right)$ be an nxn matrix of direct input coefficients; let $E=\left(e_{ij}\right)$ be a matrix of incremental changes in the direct input coefficients; let $B=\left(I-A\right)^{-1}=\left(b_{ij}\right)$, $B\left(E\right)=\left(I-A-E\right)^{-1}$ be the Leontief inverses before and after changes and let *detB*, *detB (E)* be the determinants of the corresponding inverses. Then the following propositions hold (drawing on Sonis and Hewings, 1989, 1991):

Proposition 1. The ratio of determinants of the Leontief inverses before and after changes is the polynomial of the incremental changes, e_{ij}, expressed in the following form:

$$
\begin{aligned}
Q(E) &= \frac{\det B}{\det B(E)} \\
&= 1-\sum_{i_1 j_1} b_{j_1 i_1} e_{i_1 j_1} + \\
&+\sum_{k=2}^{n} (-1)^k \sum_{\substack{i_r < i_{r+1} \\ j_r \neq j_s}} B_{or}\left(j_1, j_2, \ldots j_k ; i_1, i_2, \ldots, i_k\right) e_{i_1 j_1} e_{i_2 j_2} \ldots e_{i_k j_k}
\end{aligned}
\qquad (1)
$$

where:

$$
B_{or}\left(j_1, j_2, \ldots j_k ; i_1, i_2, \ldots, i_k\right)=\begin{vmatrix} b_{j_1 i_1} & b_{j_1 i_2} & \cdots & b_{j_1 i_k} \\ b_{j_2 i_1} & b_{j_2 i_2} & \cdots & b_{j_2 i_k} \\ \vdots & \vdots & \ddots & \vdots \\ b_{j_k i_1} & b_{j_k i_2} & \cdots & b_{j_k i_k} \end{vmatrix}
$$

is a determinant of order k that includes the components of the Leontief inverse B from the ordered set of columns $i_1, i_2, \ldots, i_k$ and rows $j_1, j_2, \ldots, j_k$. Further, in the sum Σ, the products of the changes $e_{i_1 j_1} e_{i_2 j_2} \ldots e_{i_k j_k}$ that differ only by the order of multiplication, are counted only once.

Proposition 2.

This proposition provides a fundamental formula of decomposition of the perturbed Leontief inverse with the help of the matrix fields of influence of changes:

$$B(E) = B + \frac{1}{Q(E)}\left[\sum_{k=1}^{n} \sum_{\substack{i_r \neq i_s \\ j_r < j_{r+1}}} F\left(i_1, i_2, ..., i_k; j_1, j_2, ..., j_k\right) e_{i_1 j_1} ... e_{i_k j_k}\right] \qquad (2)$$

where the matrix field of influence of order *k*, $F\left(i_1, i_2, ..., i_k; j_1, j_2, ..., j_k\right)$, of the incremental changes $e_{i_1 j_1} ... e_{i_k j_k}$ includes the components:

$$f_{ij}\left(i_1, i_2, ..., i_k; j_1, j_2, ..., j_k\right) = (-1)^k \begin{vmatrix} b_{j_1 i_1} & b_{j_2 i_1} & \cdots & b_{j_k i_1} & b_{i i_1} \\ b_{j_1 i_2} & b_{j_2 i_2} & \cdots & b_{j_k i_2} & b_{i i_2} \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ b_{j_1 i_k} & b_{j_2 i_k} & \cdots & b_{j_k i_k} & b_{i i_k} \\ b_{j_1 j} & b_{j_2 j} & \cdots & b_{j_k j} & 0 \end{vmatrix} \qquad i, j = 1, ..., n \qquad (3)$$

III. Particular Cases of Coefficient Changes and their Fields of Influence:

3.1. Changes in one element and direct (first order) fields of influence

The specific applications begin with the initial scheme of Sherman and Morrison (1950); assume the change occurs only in one place, (i_1, j_1), i.e.,

$$e_{ij} = \begin{cases} e & i = i_1,\ j = j_1 \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

For each component $b_{ij}(e)$ of the Leontief inverse *B(E)*, the following Sherman-Morrison formula holds:

$$b_{ij}(e) = b_{ij} + \frac{b_{i i_1} b_{j_1 j} e}{1 - b_{j_1 i_1} e} \qquad (5)$$

where:

$$\frac{\det B}{\det B(e)} = 1 - b_{j_1 i_1} e \qquad (6)$$

These two formulae serve as a basis for the definition of the direct (first order) field of influence of change, which is the matrix $F(i_1; j_1)$ with the components

$$f_{ij}\left(i_1; j_1\right) = b_{i i_1} b_{j_1 j} \qquad (7)$$

Obviously, this matrix can be presented as a multiplication of the i_1^{th} column of the Leontief inverse *B* on its j_1^{th} row; thus the following matrix structural equation holds:

$$F(i_1; j_1) = \left(f_{ij}\left(i_1; j_1\right)\right) = \begin{pmatrix} b_{1 i_1} \\ b_{2 i_1} \\ \vdots \\ b_{n i_1} \end{pmatrix} \left(b_{j_1 1}\ b_{j_1 2}\ ...\ b_{j_1 n}\right) \qquad (8)$$

Thus, for the construction of the field of influence $F(i_1; j_1)$ associated with the change in the location (i_1, j_1) on the Leontief inverse, one should go to the symmetric location (j_1, i_1) and multiply the corresponding i_1-column on j_1-row of the Leontief inverse.

Another important presentation of the direct field of influence is:

$$F(i_1; j_1) = BP_{i_1 j_1} B \tag{9}$$

where the matrix $P_{i_1 j_1}$, that has a unit value at the location (i_1, j_1) of the intersection of the i_1^{th} row and j_1^{th} column and zeros elsewhere. The condition (9) can be checked by direct matrix multiplication.

In matrix notation, equation (5) can be interpreted to mean that the new Leontief inverse *B*(*e*) is equal to the sum of two matrices – the original Leontief Inverse, *B,* and the field of influence, $F(i_1; j_1)$, multiplied on the rational fraction function of *e*:

$$B(e) = B + \frac{e}{1-b_{j_1 i_1} e} F(i_1; j_1) = B + \frac{1}{1-b_{j_1 i_1} e} BE_{i_1 j_1} B \tag{10}$$

where the matrix $E_{i_1 j_1} = eP_{i_1 j_1}$ has a component, *e,* on the intersection (i_1, j_1) of the i_1^{th} row and j_1^{th} column and zeros elsewhere.

It is important to stress that the field of influence does not depend on the size, *e,* of change; it depends on the *location* of change in the matrix of direct inputs. The structure of the field of influence determines the distribution of impacts of change in the intra/inter sectoral economic dependencies on the "surface" of the Leontief Inverse.

3.1.1. Tolerance intervals of change

Equation (5) provides a solution for the following basic question (Bullard and Sebald, 1977): assuming that each element of the direct input coefficient matrix *A* is permitted to vary anywhere within a specified tolerance interval, what would be the generated tolerance interval for each element of the Leontief inverse? Bullard and Sebald (1977, 1988) proposed the following solution; consider the change ε in an element $a_{i_1 j_1}$ from *A* within the tolerance interval

$$\alpha \le e \le \beta \tag{11}$$

Then, according to (5), the change in each element b_{ij} from the Leontief inverse *B* will be:

$$\delta_{ij}(e) = \frac{b_{ii_1} b_{j_1 j} e}{1 - b_{j_1 i_1} e} \tag{12}$$

Obviously,

$$\frac{\partial \delta_{ij}(e)}{\partial e} = \frac{b_{ii_1} b_{j_1 j}}{(1 - b_{j_1 i_1} e)^2} \tag{13}$$

Assuming that all the elements of B are positive it follows that $\frac{\partial \delta_{ij}(e)}{\partial \varepsilon} > 0$ and the function $\delta_{ij}(e)$ increases monotonically. This means that:

$$\frac{b_{ii_1} b_{j_1 j} \alpha}{1 - b_{j_1 i_1} \alpha} \leq \delta_{ij}(e) \leq \frac{b_{ii_1} b_{j_1 j} \beta}{1 - b_{j_1 i_1} \beta} \tag{14}$$

and this provides the tolerance intervals for the changes in the components of the Leontief inverse caused by individual change in the (i_i, j_1) cell of the matrix A.

3. 2. Change in one column.

Consider the changes $\begin{pmatrix} e_{1j_1} \\ e_{2j_1} \\ \vdots \\ e_{nj_1} \end{pmatrix}$ which occur in only one j_1^{th} column of the matrix of direct inputs A. Then the matrix of increments $E = (e_{ij})$ will have the components

$$e_{ij} = \begin{cases} e_{i,j_1} = e_i & i = 1, 2, ..., n,\ j = j_1 \\ 0 & j \neq j_1 \end{cases} \tag{15}$$

In this case the Propositions 1 and 2 may be written in the form given by Sherman and Morrisson (1949):

$$Q(E) = 1 - \sum_{s=1}^{n} b_{j_1 s} e_s;\ b_{ij}(E) = b_{ij} + \frac{b_{ij_1} \sum_{s=1}^{n} b_{sj} e_s}{1 - \sum_{s=1}^{n} b_{j_1 i} e_s} \tag{16}$$

or the Leontief inverse can be written in the matrix form:

$$B(E) = B + \frac{\sum_{s=1}^{n} F(s; j_1) e_s}{1 - \sum_{s=1}^{n} b_{j_1 s} e_s} \tag{17}$$

IV. Links with Diffusion of Technological Innovation

One of the major criticisms directed at the use of input-output models has been its inability to handle technological change in coefficients induced by new innovations. In this chapter, some preliminary steps will be taken to link some of the ideas of error and sensitivity analysis with work that has primarily focused on individual innovations and their diffusion within an economy. This chapter provides some *ex ante* forecasts of structural change derived from a general equilibrium forecasting model in which the input-output structure assumes a prominent role. From these forecasts, the challenge presented would be the reverse of the normal innovation analysis - namely, to infer the nature, direction and causality chains of the innovation processes that generate the forecast set of input-output structures.

Instead of tackling some of the difficult empirical issues, some theoretical analysis will be next presented

IV.1. Schumpeterian wave in a leading sector.

Consider a social accounting system, presented in the form of a matrix of direct inputs, $A=\left(a_{ij}\right)$. Introducing the value added of the j_1 sector:

$$a_{n+1,j_1}\left(t\right)=1-\sum_{i=1}^{n}a_{ij_1}\left(t\right) \qquad (18)$$

From (18), column j_1 can be considered as a frequency vector for the use of inputs from all other sectors; technological change will then be portrayed as a *direct inputs adjustment process.* This adjustment process may be considered as a competition for shares of the direct inputs; by placing the analysis with a relative share competitive environment, significant benefits for modeling and interpretation arise. The analysis of general adjustment dynamics and its asymptotic behavior (if time $t\to\infty$) recently

studied in detail in (Sonis, Dridi, Hewings, 2006). In this chapter we consider the simple adjustment dynamics converging to the attracting equilibrium in only one leading sector j_1 associated with the Schumpeterian wave of diffusion of technological innovation (see Sonis, 1983; 1986; 1991; 2002). The corresponding matrix of direct inputs $A\left(t\right)$ has a form:

$$A\left(t\right)=\begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1j_1}\left(t\right) & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2j_1}\left(t\right) & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ldots & \vdots \\ \vdots & \vdots & \ldots & \vdots & \ldots & \vdots \\ \vdots & \vdots & \ldots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nj_1}\left(t\right) & \ldots & a_{nn} \end{bmatrix} \qquad (19)$$

The Schumpeterian S-shaped wave can be presented with the help of the difference equations for captive Logistic growth probabilistic chain of the set $a_{ij_1}\left(t\right)$ of technological coefficients in sector j_1 :

$$a_{ij_1}\left(t+1\right)=N_0\frac{\left(a_{ij_1}\left(t\right)-N_i\right)e^{u_i}}{\sum_{s=1}^{n+1}\left(a_{sj_1}\left(t\right)-N_s\right)e^{u_s}}+N_i$$
$$N_0+\sum_{s=1}^{n+1}N_s=1;\ 0\le N_0,N_s\le 1 \qquad (20)$$

where u_i, $i = 1, 2, ..., n+1$ are the temporal marginal utilities of inputs from sectors *i*, and $N_1, N_2, ..., N_{n+1}$, $0 \le N_s \le 1$ are the widths of the minimal use of inputs from all sectors. In essence, the N_s may be considered as the width of the *technological niches* for each input in sector j_1 .

$$N_0 = 1 - \sum_{s=1}^{n+1} N_s \qquad (21)$$

is the total width of the possible changes in inputs outside of all technological niches.

Without loss of generality, assume that:

$$u_1 > u_2 > ... > u_{n+1} \qquad (22)$$

The solution of the system of difference equations (20) has the following form (see Sonis, 1983, 1986):

$$a_{ij_1}(t) = N_0 \frac{\left(a_{ij_1}(0) - N_i\right) e^{u_i t}}{\sum_{s=1}^{n+1} \left(a_{sj_1}(0) - N_s\right) e^{u_s t}} + N_i \qquad (23)$$

The fixed point of the adjustment dynamics (20) is

$$r = \begin{pmatrix} N_0 + N_1 \\ N_2 \\ \vdots \\ N_{n+1} \end{pmatrix} \qquad (24)$$

It is possible to prove that this vector is the attractor of the dynamics (23) (see Sonis, 1983, 1986) and therefore:

$$\begin{aligned} &\lim_{t \to +\infty} a_{1j_1}(t) = N_1 + N_0 \\ &\lim_{t \to +\infty} a_{ij_1}(t) = N_i \qquad i = 2, 3, ..., n+1 \end{aligned} \qquad (25)$$

from which the stabilization stage A_∞ of the Schumpeterian cycle within sector j_1 may be derived:

$$A_\infty = \begin{bmatrix} a_{11} & a_{12} & \ldots & N_0 + N_1 & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & N_2 & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ldots & \vdots \\ \vdots & \vdots & \ldots & \vdots & \ldots & \vdots \\ \vdots & \vdots & \ldots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \ldots & N_n & \ldots & a_{nn} \end{bmatrix} \qquad (26)$$

The qualitative picture of the redistribution of inputs within sector j_1 can be completed as follows: the share $a_{1j_1}(t)$ of the input with the most efficient use (i.e., with the maximal temporal marginal utility) will monotonically increase from its niche width of N_1 to a new level $N_1 + N_0$. The share $a_{n+1j_1}(t)$ of the most inefficient input monotonically decreases to the level of its preservation niche N_{n+1} . Because of inequality $N_i \le a_{ij_1}(t) \le N_0 + N_i$ the technological coefficient $a_{ij_1}(t)$ in sector j_1 always include the part N_i , which is interpreted as a captivity of the direct input from the sector *i*. This captivity could be zero, indicating that the input has been replaced entirely or the source of inputs may move to another region. The dynamics of shares of other inputs is wave-like; they monotonically increase to their maximal level and, after that, decrease to the preservation niche levels, N_i (see figure 1).

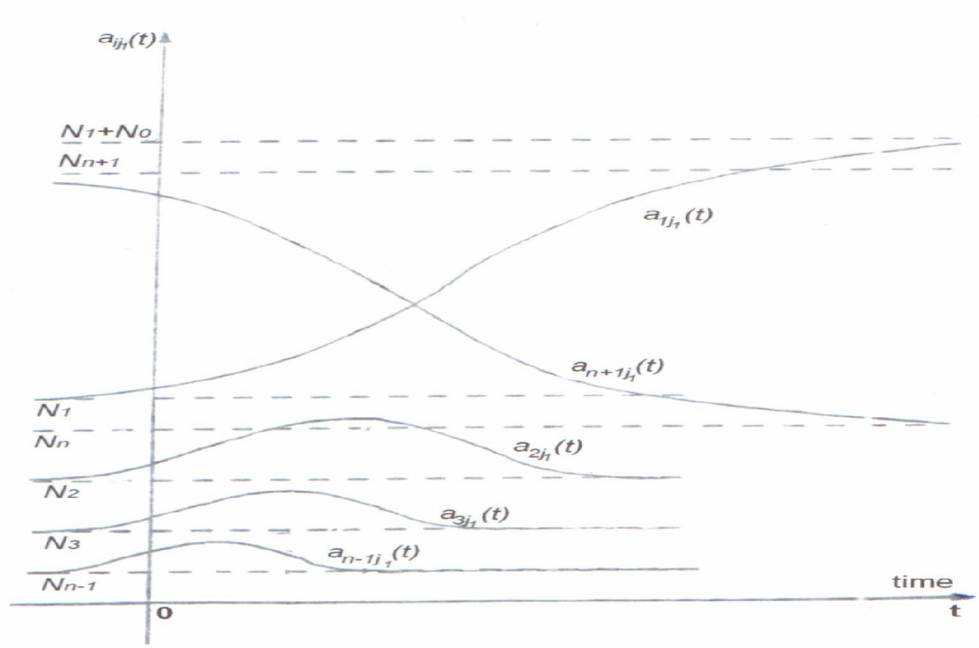

Figure 1. Schumpeterian wave in leading sector.

VI.2. Dynamics of Leontief inverse.

Now we describe the input-output dynamics of Leontief inverse $B(t)=\left(I-A(t)\right)^{-1}$ presenting the economic action of Schumpeterian wave approaching the stabilization attracting state (24). Introduce the matrix of changes

$$E(t)=\left(e_{ij}(t)\right)=A(t)-A(0) \qquad (27)$$

or, in the coordinate form:

$$e_{ij}(t)=\begin{cases} e_{i,j_1}(t)=a_{i,j_1}(t)-a_{i,j_1}(0) & i=1,2,...,n,\ j=j_1 \\ 0 & j\neq j_1 \end{cases} \qquad (28)$$

The transformation of the economy from the initial state at time *0* can be presented through the use of formulae (16-17):

$$Q\left(E(t)\right)=1-\sum_{i=1}^{n} b_{j_1 i}(0)e_{ij_1}(t)=1-\sum_{i=1}^{n} b_{j_1 i}(0)a_{ij_1}(t)+\sum_{i=1}^{n} b_{j_1 i}(0)a_{ij_1}(0)=$$
$$=1+\sum_{i=1}^{n} b_{j_1 i}(0)a_{ij_1}(0)-\sum_{i=1}^{n} b_{j_1 i}(0)\left[N_i+N_0\frac{\left(a_{ij_1}(0)-N_i\right)e^{u_i t}}{\sum_{s=1}^{n+1}\left(a_{sj_1}(0)-N_s\right)e^{u_s t}}\right] \qquad (29)$$

and

$$B\left((t)\right)=B(0)+\frac{1}{Q\left(E(t)\right)}\left\{\sum_{s=1}^{n}F\left(j_s \quad s\right)\left(a_{sj_1}(t)-a_{sj_1}(0)\right)\right\} \qquad (30)$$

or

$$b_{ij}\left(E(t)\right)=b_{ij}(0)+\frac{b_{ij_1}(0)\sum_{s=1}^{n}b_{sj}(0)\left(N_s+N_0\frac{\left(a_{sj_1}(0)-N_s\right)e^{u_s t}}{\sum_{r=1}^{n+1}\left(a_{rj_1}(0)-N_r\right)e^{u_r t}}-a_{sj_1}(0)\right)}{Q\left(E(t)\right)} \qquad (31)$$

The difference equation (31) presents the diffusion of simple Schumpeterian wave through the fields of influence of the changes in the case that Schumpeterian wave appears in some leading sector. The analogous but much more complicated formulae can be presented if different Schumpeterian waves will appear in different sectors of input-output economy. These formulae can be used for forecasting of structural changes within input-output economic dynamics if Schumpeterian waves have various diffusion forms (Sonis, 2003).

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