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Utz-Peter Reich

Consistency in Aggregation?
Trying the KLEMS data base

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Keywords: Value added, deflation methods, chaining, additivity, KLEMS data base

Archives: Methods and mathematics, Construction of input-output tables

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Consistency in Aggregation? Trying the KLEMS data base

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Abstract

Value added is a key variable of input-output-tables and national accounts. In order to be used for productivity analysis it must be deflated to yield the "real value" or "volume" of the aggregate. The deflation method that has been standardised in the KLEMS data base produces figures which are inconsistent in aggregation or, - to use another expression, - which are non-additive. Independently of the theoretical position one takes vis-a-vis this fact, the paper studies the question of whether the effect is relevant, statistically speaking. For, if it falls between the margins of error, one may easily ignore it for all practical purposes. The investigation of Danish data, chosen as example, results in mixed findings. For some of the industries inconsistency in aggregation is negligible, for others it comes out at the order of 50 percent of value added. A new deflation method is therefore introduced and tried, which is consistent in aggregation, and has the additional advantage that it clarifies the distinction between the real value and the variation in volume of a value added time series.

1. Introduction

When data of economic growth were compiled on the basis of fixed price indices additivity of the resulting volumes was not a problem. It was taken care of by the method, automatically, and self-understood. It is with the chaining of price and volume indices that the comfortable unity has broken up and a choice between additivity and proper aggregation weights has seemed inevitable, where the choice has been made in favour of the latter. On the basis of Reich (2007), Balk and Reich (2008) have shown, however, that an index exists which combines the two essentials of proper (up to date) weights and additivity, in the abstract. This paper follows suit in asking the question of whether the difference between the traditional and

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the new index is statistically relevant. It does so by studying the problem at a concrete set of data, namely the data furnished by the so-called KLEMS data base.

KLEMS is a new acronym which has made it into the ranks of celebrity in rather little time. It stands for "Das Kapital"[2], Labor, Energy, Materials, and Services, all being studied as factors of production, describing a magnificent data base, containing and managing detailed information on yearly inputs and outputs of some 30 countries over the last three decades on a two digit-industry level within a common accounting framework. The data base is meant to support empirical and theoretical research in the area of economic growth and "the systematic production of high quality statistics on growth and productivity using the methodologies of national accounts and input-output analysis." (Timmer et al. 2008, p. 1)[3] It presents data in nominal terms which are coherent, and indexes in real terms which are not. The paper addresses the second part. The technical term is "consistency in aggregation" or simply "additivity", meaning that the operation of converting figures in nominal terms into figures in real terms is (or in the negative case is not) neutral against the level of aggregation at which it is performed. Opinions are divided about whether this is a serious requirement, or whether its absence is the miss of a quality that is undesirable anyhow. Uncontroversial so far is the view that non-additivity is a mathematically necessary complement going along with the technique of chained indices. The paper investigates both issues. Analyzing the cause of non-additivity of chained indices it finds a way to circumvent it, and shows how the KLEMS data can be transformed to yield accounting figures that are consistent in aggregation.

2. Non-additivity of outputs

The data compiled in national accounts are nominal, by nature, which means that they are expressed in the currency of a specific country at a specific time. The country has a certain territorial extension, and time is measured over a certain interval, usually a year, but both extensions are considered small enough for the model of a "point economy" to hold. A unit of currency is deemed to represent the same economic value wherever and whenever it is spent

[2] Assuming that the famous title is the reason for preferring the German to the English spelling of capital. (One might have argued that letter K is used for capital in economic equations, but in that alphabet M stands for imports and S for saving, and must thus not be used otherwise. The composition of the acronym is not coherent. CLEMS would have been the better choice.

[3] The happy economic paradox deserves to be mentioned that part of the value of the database consists in its being provided for free.

within these limits. This is the precondition on which the construction of economic accounts with resulting balances depends, in order to be meaningful. Such accounting figures can be added and subtracted, because they are expressed in the same homogeneous unit of economic value.

When the territorial boundary is transgressed this measurement breaks down, obviously, because the currency is not current outside. A similar limit holds for the interval in time, which is less obvious because change is not discrete here, but it needs not much reflection to realize that a euro of year 2008 is not of the same value as a euro of 1975, and the two cannot be added directly be employed within one and the same account. A technique must be applied to make values of the two years comparable. It is called technique of deflation and proceeds as follows:

Consider an elementary nominal aggregate of transactions at the lowest level of aggregation v_t for year *t* (e.g. pharmaceuticals at the two digit level). Price statistics furnishes a corresponding price index p_t describing the movement of the currency's purchasing power in respect to this aggregate. A *volume index* q_t is defined from these data by means of

(1) $$q_t = \frac{1}{p_t}\frac{v_t}{v_0} ,$$

where v_0 is the nominal value of the aggregate at some reference year 0, chosen arbitrarily. The *volume index* of a higher aggregate Q_t at time *t* is then defined as

(2) $$Q_t = \prod_{s=1}^{t} \frac{\sum p_{s-1} q_t}{\sum p_{s-1} q_{s-1}} ,$$

a multiplicative chain of yearly Laspeyres-indices[4]. The *volume* $V_0\, Q_t$ of the high aggregate is given by the product of the volume index Q_t and the nominal value of the aggregate in the reference year V_0 with

(3) $$V_0 = \sum v_0 ,$$

[4] Some countries prefer the Fisher index, an option we do not discuss at this point.

the high aggregate being additively composed from the low ones[5]. As a result the *volume* $V_0 Q_t$ of a nominal aggregate V_t may be expressed as

(4) $$V_0 Q_t = V_0 \times \prod_{s=1}^{t} \frac{\sum p_{s-1} q_s}{\sum p_{s-1} q_{s-1}} \ .$$

In words, one accounts for the price change of the high aggregate by re-valuing the nominal values v_t of each elementary aggregate at previous year prices, summing them up, and multiplying the resulting ratios.

Table 1 applies the method to the output[6] of the first some 30 industries of the Danish economy over the period 1970 to 2005[7]. The first column contains the nominal values of industry output in year 1970 (in Danish Kroner of 1970). The second and third column show the volume index for years 1970 and 2005, respectively, according to equations (1) and (2) with reference year 1995. In column (4) the volume for year 2005 is derived following equation (4). The figures of column (1) are divided by those of column (2) and multiplied by those of column (3).

Column (5) demonstrates the resulting inconsistency in aggregation by summing the volumes of the sub-aggregates. For example, direct deflation of the aggregate AtB, comprising (A1) agriculture, (A2) hunting and forestry, and (B) fishing, is 24,780 million 1970Kr., while the volume calculated by summing the deflated components comes out at 25.145 million 1970Kr.. The balance given in the last column, is - 365 million 1970Kr.. which is a small amountin this case, not worth mentioning, perhaps But in other industries the discrepancy reaches a dimension that cannot be ignored, see aggregate C, mining and quarrying, as an extreme case. And each aggregation level produces its own results. Class 23t25, chemical, rubber, plastics, fuel, for example, shows a volume of 22,035 million 1970Kr. if you aggregate classes (23) +

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[5] More precisely, we would write $V_0 = \sum_{i=1}^{n} v_0^i$, where $i = 1,...,n$ enumerates the elementary product groups, but we suppress the product group index in order to simplify notation.

[6] „gross output" in KLEMS terminology. It is a deplorable accident, and unnecessary potential source of confusion, especially for non-English speakers that the enormous effort that went into harmonizing English terminology in the international SNA1993, and the ensuing European ESA1995 has not been honored by producers of the KLEMS data base and the proper terms adopted. The paper keeps strict conformity with that standard.

[7] The Danish economy has been chosen, because its statistical office has produced detailed data for the long period.

(24) + (25), - not shown in the table, - and a volume of 22,840 million 1970Kr. if you aggregate (23) + (244) + (24x) + (27) as done in Table 1.

Table 1 Aggregative inconsistency of volume growth of selected Danish industry output between 1970 and 2005, official method

Description	Code	(1) 1970 nominal mill. Kr.	(2) 1970 index 1995	(3) 2005 index =100	(4) 2005 volume mill. Kr.	(5) sum subagg. mill. Kr.	(6) balance mill. Kr.
AGRIC., HUNT., FOR. AND FISHG.	AtB	14,015	60.1	106.2	24,780	25,145	-365
AGRIC., HUNTING AND FORESTRY	A	12,989	58.2	109.7	24.468	24,375	93
Agriculture	1	12,631	58.1	108.7	23,636		
Forestry	2	358	63.8	131.7	739		
FISHING	B	1,026	89.0	66.8	770		
MINING AND QUARRYING	C	495	8.2	178.2	10,805	29,557	-18,752
M. A. Q. OF ENERGY PROD. MATRL.	10t12	3	0.0	202.7	29,97		
M. AND Q. EXCEPT E. P. M.	13t14	492	108.4	79.3	360		
FOOD , BEVERAGES, TOBACCO	15t16	22,369	61.1	104.1	38,076	38,085	-9
Food and beverages	15	21,792	60.6	104.4	37,441		
Tobacco	16	640	94.3	94.8	644		
TEXTILE , LEATHER AND FOOTW.	17t19	4,718	111.1	65.5	2,783	2,877	-94
Textiles and textile	17t18	4,188	109.6	72.0	2,749	2,774	-25
Textiles	17	2,125	88.5	79.0	1,897		
Wearing App., Dress., Dying Of Fur	18	2,063	142.6	60.6	877		
Leather, leather and foot wear	19	530	123.7	23.9	103		
WOOD AND OF WOOD AND CORK	20	1,719	68.9	113.9	2,842		
PULP, PAPER, PRINTG., PUBLG.	21t22	5,597	80.3	102.8	7,164	7,237	-73
Pulp, paper and paper	21	1,560	61.2	89.9	2,293		
Printing, publishing and reproduction	22	4,037	87.5	107.1	4,942	4,944	-2
Publishing	221	2,466	98.8	114.2	2,850		
Printing and reproduction	22x	1,571	73.7	98.3	2,095		
CHEMICAL, RUBBER, PLAST., FUEL	23t25	6,537	51.6	147.3	18,655	22,840	-4,185
Coke, refined petroleum, nucl. Fuel	23	1,718	95.6	101.5	1,825		
Chemicals and chemical	24	3,195	36.3	176.7	15,562	16,367	-805
Pharmaceuticals	244	486	12.5	282.1	10,958		
Chemicals excl. Pharmaceuticals	24x	2,709	53.7	107.2	5,409		
Rubber and plastics	25	1,624	42.7	122.1	4,648		

Source: KLEMS 2008 and own calculations

Statistical policies vis-à-vis this state of affairs vary. There are three options, essentially. One may show the inconsistency in aggregation, and not bother; one may bother and avoid to show it; and one may eliminate it by distributing it over the tables. Not un-typically, perhaps, the first policy has been adopted by the Dutch, the second one by the Germans, and the third one by the French.

The theoretical literature is divided, too. In a survey carried out for the European Community Al et al. (1986) summarize:

"Where the deflation of transaction totals in the national accounts and input-output tables is concerned, the chain-approach would appear to be clearly preferable (to the fixed base-approach, UPR) since users of such information are primarily interested in the way in which the changes in the aggregates came about. It is clearly only in a minority of cases that these data are used to make a comparison between two moments separated by a long time interval. However, attention should also be given to a practical disadvantage of the chain approach. It is not possible with this system to produce additively consistent tables in deflated values. The tables produced by multiplying the bases basket by the volume chain index for an observation moment separated from the base moment by more than one period will exhibit aggregation discrepancies." (p. 358) They conclude that deflated data of this type are useful for short term business cycle analysis, and less so for long term growth analysis.

When after a political turmoil in their Senate the United States decided to switch from fixed base price indices to chained indices, Ehemann et al. (2002) defended the resulting inconsistency in aggregation by two arguments:

"Interestingly, additivity was not mentioned as a desirable property of the (earlier fixed-weighted, UPR) estimates." (p. 37)

"The goal of additivity in an index that also meets other requirements of national income accounting may not only be unattainable, but undesirable." (p. 40)

Nevertheless they also admit:

"Despite considerable efforts to educate users …, BEA's 1996 adoption of chained price and quantity measures initially resulted in some criticism because of the loss of additivity." (p. 37)

Whelan (2002) takes an intermediary position:

"A crucial feature of this chain aggregation methodology … is that the real aggregate of X and Y will generally not be the arithmetic sum of the real series for X and Y… As a result, mistaken calculations based on real NIPA data have become common in the work of academic, policy, and business economists." (p. 219)

By way of example, Whelan investigates U.S. GDP growth with, and without, computer output. He demonstrates the fallacies of naïve compilation, writing his paper as "A guide to U.S. chain aggregated data" and suggesting some alternative ways of manipulating these data.

In summary, the three authors agree that the non-additivity feature is a mathematical necessity common to all chain indices. They diverge in the evaluation of this characteristic. As to the latter, some further insight may be gained by turning from the output to the input side of the accounts.

3. Non-additivity of inputs

Working with the textbook identity of Y = C +I + G + X - M, Whelan (2002) follows the expenditure approach to compiling GDP, which is natural, as it represents the commonly used des-aggregation of this figure. In contrast, the KLEMS databank, pursuing the goal of growth and productivity measurement, necessarily turns to the product approach. The product approach aggravates the issue of consistency in aggregation by demanding not only that aggregates are added up across industries, but that a balance be defined which closes the accounts for every industry. And more than that, this balance represents the final goal of the accounting procedure. In business accounts it reveals the profit gained in the operation, in national accounts it describes the value added to the value of products consumed in the operation of an industry. If these balances are out of order the whole point of establishing the accounts goes astray.

Table 2 takes industry 23t25, chemical, rubber, plastics and fuel, as an example. It compiles the volume of output, of intermediate consumption[8], and of gross value added in year 2005 on the basis of year 1970. The first column does so for the whole industry, columns (2) to (5) do the same for each sub-industry. We find heavy non-additivity when aggregating across

[8] „intermediate inputs" in KLEMS terminology, a strange compound because you can have either intermediate consumption as opposed to final consumption, or secondary inputs as opposed to primary inputs.

industries. When we deflate output of the aggregate industry directly we arrive at 18,655 mill. 1970Kr., while when we first deflate the sub-industries, and add up afterwards, the joint output comes out at 22,837 mill. 1970Kr, as shown in table 1, already. For intermediate consumption, the discrepancy is less significant, 11,387 as against 13,273 million 1970Kr. It is largest, naturally, for the balance of output and intermediate consumption, namely gross value added, where direct deflation yields 6,822 million 1970Kr., while adding the deflated sub-industries yields a volume of 9,076 million 1970Kr. Which of the two figures is the true value added to be used for analysis of growth and productivity?

Table 2 Aggregative inconsistency in determining value added of the industry "Chemical, rubber, plastics and fuel"

	(1)	(2)	(3)	(4)	(5)
Industry code*)	23t25	23	244	24x	25
Output 1970 (mill. 1970Kr.)	6,537	1,718	486	2,709	1,624
Volume index 1970 (1995=100)	51.6	95.6	12.5	53.7	42.7
Volume index 2005 (1995=100)	147.3	101.5	282.1	107.2	122.1
Volume 2005 (mill. 1970Kr.)	18,655	1,825	10,958	5,410	4,644
Sum of sub-industries (mill. 1970Kr.)	22,837				
Intermediate cons. 1970 (mill. 1970Kr.)	4,544	1,585	269	1,721	969
Volume index 1970 (1995=100)	57.5	90.7	16.0	56.3	39.4
Volume index 2005 (1995=100)	144.1	103.8	290.7	113.9	126.2
Volume 2005 (mill. 1970Kr.)	11,387	1,814	4,874	3,483	3,103
Sum of sub-industries (mill. 1970Kr.)	13,273				
Gross value added 1970 (mill. 1970Kr.)	1,993	133	217	988	655
Volume index 1970 (1995=100)	44.0	261.2	9.5	62.6	52.0
Volume index 2005 (1995=100)	150.5	22.0	269.6	93.7	115.6
Volume 2005 (mill. 1970Kr.)	6,822	11	6,128	1,480	1,457
Sum of sub-industries (mill. 1970Kr.)	9,076				

*) see table 1 for description

Source: KLEMS 2008 and own calculations

Studying these figures, one can hardly agree with the conclusion that consistency in aggregation is undesirable. On the contrary, it seems to be an essential element of any compilation method that is intended to produce meaningful balances in what can justly be called an "economic account".

Whelan (2002) explains the cause of the inconsistency very well. Straight addition of volume series of sub-aggregates fails to account for the effect of relative price shifts between them. They are added carrying the weights of the base period, neglecting that these weights change during the period of observation, and therefore - so his conclusion, - the method of straight addition is incorrect. But abstaining from such addition, as he advises, is not an effective remedy. For imagine two countries using different levels of aggregation due to different coverage in primary data, which level is one to trust? A "value added" that is not additive is a contradiction in terms. There is no other remedy than to go one step further, and to design a method that incorporates consistency in aggregation intrinsically, thus breaking the hitherto believed law that chain indices are inconsistent in aggregation by mathematical necessity, and presenting other options. This is the task of the next sections.

4. Additivity in the short run

Point of departure is the apparent mathematical link between index weights that are up to date, on the one hand, and inconsistency in aggregation, on the other. To demonstrate it consider two low aggregates A and B that are to be aggregated to a high aggregate AB. Following definitions (1) to (4) we have

(5) $$q^A{}_t = \frac{1}{p^A{}_t}\frac{v^A{}_t}{v^A{}_0}\ , \qquad q^B{}_t = \frac{1}{p^B{}_t}\frac{v^B{}_t}{v^B{}_0}$$

(6) $$Q^A{}_t = \prod_{s=1}^{t}\frac{\sum p^A{}_{s-1}q^A{}_t}{\sum p^A{}_{s-1}q^A{}_{s-1}}\ , \qquad Q^B{}_t = \prod_{s=1}^{t}\frac{\sum p^B{}_{s-1}q^B{}_t}{\sum p^B{}_{s-1}q^B{}_{s-1}}$$

(7) $$V_0^A = \sum v_0^A\ , \qquad V_0^B = \sum v_0^B$$

(8) $$V_0^A Q_t^A = V_0^A \times \prod_{s=1}^{t} \frac{\sum p_{s-1}^A q_s^A}{\sum p_{s-1}^A q_{s-1}^A} \ , \qquad V_0^B Q_t^B = V_0^B \times \prod_{s=1}^{t} \frac{\sum p_{s-1}^B q_s^B}{\sum p_{s-1}^B q_{s-1}^B}$$

We also have the simple aggregation in nominal values of

(9) $$V_t^{AB} = V_t^A + V_t^B \quad .$$

Applying the same definitions to the high aggregate AB yields a volume of

(10) $$V_0^{AB} Q_t^{AB} = (V_0^A + V_0^B) \times \prod_{s=1}^{t} \frac{\sum p_{s-1}^A q_s^A + \sum p_{s-1}^B q_s^B}{\sum p_{s-1}^A q_{s-1}^A + \sum p_{s-1}^B q_{s-1}^B} \quad .$$

Equation (10) describes the procedure of adding the values in current prices and those in prices of the previous year first, and then forming the volume index. Aggregation precedes deflation. In contrast, if we want to deflate first and then aggregate we must add the two equations 8 which yields

(11) $$V_0^{AB} Q_t^{AB} = V_0^A \times \prod_{s=1}^{t} \frac{\sum p_{s-1}^A q_s^A}{\sum p_{s-1}^A q_{s-1}^A} + V_0^B \times \prod_{s=1}^{t} \frac{\sum p_{s-1}^B q_s^B}{\sum p_{s-1}^B q_{s-1}^B}$$

which is clearly different from equation 10. The two operations of deflation and aggregation are not interchangeable. The method is inconsistent in aggregation, by mathematical necessity, which opinion is state of the art, at present. The question now is not whether chaining is right or wrong as compared to a fixed base index, but whether equations (1) to (4) represent the only way in which chaining may be performed. If this particular form of chaining is not additive, there are others, perhaps, that are.

As has been said above inconsistency in aggregation is a problem less for the short run than for the long run. In fact, if you consider two adjacent periods only the present index is perfectly consistent. We begin, therefore, our search for a new index at this point. For two adjacent periods *t-1* and *t* equations (8) simplify to

(12) $$V_{t-1}^A Q_t^A = V_{t-1}^A \times \frac{\sum p_{t-1}^A q_t^A}{\sum p_{t-1}^A q_{t-1}^A} = \sum p_{t-1}^A q_t^A$$

and

(13) $$V_{t-1}^{B}Q_{t}^{B} = V_{t-1}{}^{B} \times \frac{\sum p_{t-1}^{B}q_{t}^{B}}{\sum p_{t-1}^{B}q_{t-1}^{B}} = \sum p_{t-1}^{B}q_{t}^{B} \quad .$$

Equation (10) also simplifies to

(14) $$V_{t-1}^{AB}Q_{t}^{AB} = (V_{t-1}^{A} + V_{t-1}^{B}) \times \frac{\sum p_{t-1}^{A}q_{t}^{A} + \sum p_{t-1}^{B}q_{t}^{B}}{\sum p_{t-1}^{A}q_{t-1}^{A} + \sum p_{t-1}^{B}q_{t-1}^{B}} = \sum p_{t-1}^{A}q_{t}^{A} + \sum p_{t-1}^{B}q_{t}^{B} \quad .$$

The sum of expressions (12) and (13) equals expression (14). The standard index is fully consistent in aggregation for the very short run between two adjacent periods. Concerning economic policy it follows that the method is unproblematic when used for business cycle analysis, as already remarked by Al et al. (1986). There is one caveat, however. Consistency in aggregation between two adjacent periods requires that a Laspeyres or a Paasche index be used (Balk and Reich 2008), or their arithmetic average. Any other index, the Fisher index in particular, does not yield this result.

5. Additivity in the long run

Comparing equations (14) and (10) it appears that the difficult element in the latter is the Greek letter Π, which stands for the operation of product multiplication. It says that the growth factors (one plus the growth rate) of each year are multiplied by each other in series to yield the volume index Q_t with respect to year 0. The idea seems not far fetched to try the other possible operation, namely to replace the letter denoting multiplication by the letter Σ for summation, so that instead of multiplying the growth factors we sum the growth differences, using the expression

(15) $$\sum_{s=1}^{t}\sum p_{s-1}(q_{s} - q_{s-1}) = \sum_{s=1}^{t}\sum p_{s-1}\Delta q_{s} \ .$$

This is not a combination of sums and products which can never be additive, but a double sum which can. The inner sum runs over the different commodities which index is not shown, and the outer sum runs over the interval (0,t) of years under consideration.

But in its simplicity, this is only half of the solution. For expression 15 is additive over products, but it is not additive over time, because the currency in which prices p_t are expressed is not constant in its value over time, but varies with monetary policy and inflation. In order to correct for the variation imposed in this way on the measurement unit of the accounts we introduce the concept of “real price” (in analogy to the concept of “real wage”), which is the nominal price divided by the general price level. We define the *real price* $\tilde{p}_t$ (as opposed to nominal price p_t) by

(16) $$\tilde{p}_t = \frac{p_t}{\Lambda_t}$$

and the corresponding *real value* (as opposed to nominal value v_t) by

(17) $$\tilde{v}_t = \frac{v_t}{\Lambda_t}$$

where Λ_t is the *general price level* at time *t*. The question of how to measure the general price level is a discussion of its own, too involved to be taken up in this paper. For our purpose it is sufficient to postulate

(18) $$\Lambda_t = \prod_{s=1}^{t} \frac{\sum q^C{}_t \, p_t}{\sum q^C{}_t \, p_{t-1}} \quad .$$

The vector q_t^C stands for the commodity basket employed for measuring the general price level, commonly either the consumer price index, or the implicit GDP deflator, where the latter would be preferable in the context of a national accounting system.

The additive aggregate volume growth of period *t* in respect to period *0* is then given by

(19) $$V_0 \Delta Q_t = \sum_{s=1}^{t} \sum \tilde{p}_{s-1} \Delta q_s \; .$$

Table 3 applies this formula to the industry "Chemical, rubber, plastics and fuel." The years 1970 - 1975 are calculated consecutively, the rest of the series till 2005 is summarized in the last column. Calculation begins by transforming all nominal values into real values, measured in the currency of one single year 1970. Contrary to what ideally ought to be used for measuring the general price level, the price index of value added total of all industries has been used in this compilation, because it is simple to retain from the KLEMS tables and close to the implicit GDP deflator. The index has grown by a factor 6.18 over the whole period. By means of the index nominal output is transformed into real output, technically speaking, lines (3), (7) and (11) are divided by line (2) and multiplied by 20.1, thus being normalised to year 1970.

Volume indices of output, intermediate consumption and gross value added are taken from the KLEMS database. The corresponding growth in volume is found by multiplying real values by the growth rate deduced from the indices. The output volume growth between years 1970 and 1971 is thus given by 6,537$*$(53.4/51.6 - 1) = 232 million 1970Kr., and similarly for intermediate consumption and gross value added. This calculation is fully identical with the present standard. What is new is not compilation of the growth rates, but their chaining (equation 19 as opposed to equation 8).

The test of consistency in aggregation is provided by the last line of table 3, which compares the balance of output and intermediate consumption to value added. The discrepancy is much smaller than in table 2, but it is not zero. For year 1973, for example, direct deflation of value added yields a growth of 88 million 1970Kr., while indirect deflation yields 167. Over the whole period the difference is 4394 as against 4869 million 1970Kr., a discrepancy of 10 percent. The reason is not hard to guess. Any numerical exercise comes to the point where rounding errors play a role. In fact, if you do the above calculation independently on a desk computer you find that 6537 (53.4/51.6 - 1) is actually equal to 190, but the internal digits held in the compilation programme yield the other figure.

Inconvenient as they are, rounding errors actually furnish a further argument for the requirement of an additive deflation method, because only if the method provides figures that are additive in theory, can rounding errors be detected and controlled in practice.

Table 3 Additive volume growth of the industry "Chemical, rubber, plastics and fuel" (code 23t25)

1	Year	1970	1971	1972	1973	1974	1975	2005/1970
2	General price index (1995=100)	20.1	21.6	23.6	26.6	30.2	34.6	6.18
	Output							
3	Nominal (current mill. Kr.)	6,537	7,125	7,555	9,206	14,208	13,554	
4	Real (mill. 1970Kr.)	6,537	6,636	6,438	6,960	9,445	7,878	
5	Volume index (1995=100)	51.6	53.4	58.3	62.4	59.5	55.0	
6	Volume growth (mill. 1970Kr.)	--	232	604	447	-324	-706	12,292
	Intermediate consumption							
7	Nominal (current mill. Kr.)	4,544	5,061	5,169	6,287	11,054	9,834	
8	Real (mill. 1970Kr.)	4,544	4,714	4,405	4,753	7,348	5,716	
9	Volume index (1995=100)	57.5	58.3	59.8	63.7	63.5	55.7	
10	Volume growth (mill. 1970Kr.)	--	65	124	280	-9	-911	7,425
	Value added							
11	Nominal (current mill. Kr.)	1,993	2,064	2,386	2,919	3,154	3,720	
12	Real (mill. 1970Kr.)	1,993	1,922	2,033	2,207	2,097	2,162	
13	Volume index (1995=100)	44.0	47.1	58.4	60.9	51.6	55.8	
14	Volume growth (mill. 1970Kr.)	--	144	459	88	-336	169	4,394
15	Output - intermediate consumpt.	--	167	481	167	-315	205	4868

Source: KLEMS 2008 and own calculations

Comparing the results of the two methods depicted in tables 2 and 3, we find an output growth of 12,292 million 1970Kr. for the additive index, as against 18,665 - 6,537 = 12,128 million 1970Kr. for the multiplicative index. The corresponding pairs of numbers are 7,425 as against 11,387 - 4,544 = 6,843 million 1970Kr for intermediate consumption and 4,394 as against 6,822 -1,993 = 4,829 million 1970Kr. for gross value added. It seems that the rule proposed by Whelan not to deflate at the low level and add, but to first aggregate and then deflate comes close to employing an additive index.

6. Interpretation and comparison

Formulas are needed in order to communicate compilation procedures. They are, however, of little use, if there is no corresponding interpretation in concept. The search for consistency in aggregation is not only a matter of practical convenience. It also sheds light on some conceptual matters of the dealing with varying values and prices in economic statistics, which have always been there, but never quite addressed and cleared up.

The first of these issues of interpretation concerns equation 1. The question is about the precise meaning of the variable q_t. In many papers and textbooks of economic statistics q_t is called and treated like a quantity, pounds of butter, for example. This goes together with the microeconomic view that the value of an economic transaction is given by the quantity of a homogeneous good and the price at which it is bought, which is unique because the good is homogeneous. The reality of macroeconomic statistics differs from this model. The nominal value addressed in equation 1 is not homogenous, but an aggregate of many similar, but inhomogeneous, goods ("milk products"). The price index attached to it is also an average of many prices collected for different types of outlets, locations and goods. The ratio of the two numbers does not yield a homogeneous quantity. It is an expression of value corrected for the change in purchasing power of the domestic currency within a certain class of many goods or services. It is called volume to mark the distinction from quantity[9]. and it measures quality as well.

The second issue of interpretation concerns the placement of the volume concept in time. It is common to treat a price index in equation 1 as if it were a price, but this is inadequate. A price index is not an average of prices, because that does not exist, but of price variation in time[10], it is not a state variable but a process variable, it does not describe a location in the space of prices, but a movement or speed (percent/year). Best example is the overall price index used to measure inflation. It is expressed as a rate per year where the resulting level is arbitrary and irrelevant. The same holds for the complementary variable volume. The recognition that volume is a variable of movement (growth) and not of the state of an economy is expressed in table 3 by not adding the growth in volume to the nominal base year value. The addition would be incoherent, because the base year value includes all movements in prices which have taken place up to that year, while the subsequent movement in volume abstracts from them.

The third and last issue of interpretation is the most interesting one, perhaps. The distinction between absolute (nominal) and relative (real) prices is well known theoretically, but in

[9] For a detailed explanation of the difference between volume and quantity see SNA1993 par. 16.12.
[10] You cannot average the price of a pound of butter with the price of a litre of milk, you can average their rates of change.

practice it is rarely recognized, not even in price statistics itself. The distinction between the "real" economy and its nominal counterpart reflects the insight that the nominal price of a good is determined by two forces, supply and demand ruling on the market of the specific good, on the one side, and the purchasing power of the money offered in exchange for it, on the other. As the latter is measured by some average of all prices, we may identify the real price movement of a good with its price movement relative to the movement of the average general price level (equation 16). In this way we neatly separate the real economy from the monetary one, decomposing a nominal value change not into two but into three distinct components, the volume change, the relative or real price change, and the monetary change of the general price level,

(20) $$v_t = (\Lambda_t \times \tilde{p}_t \times q_t) v_0$$

The three-partition is used in the SNA in order to distinguish real income from the volume of product and real from neutral holding gains. The idea may be extended to other parts of the system as well, and is naturally built in within an additive volume index.

Table 4 Growth of the industry "Real estate, renting and business activities" (million 1970Kroner)

Industry code	K	70	71	72	73	741t4	745t8
Output 1970	16885	10155	485	349	576	3996	1324
Output 2005	66098	30167	2627	8327	1520	13721	9736
Additive volume growth	44970	12908	3042	11678	1014	9081	7246
Multiplicative volume growth	39725	11062	3791	23132	1136	8444	5979
Intermediate consumption 1970	5316	2247	351	126	223	1645	724
Intermediate consumption 2005	26803	9063	1615	4574	837	6914	3800
Additive volume growth	23709	7473	1438	5013	640	5881	3265
Multiplicative volume growth	25173	8227	1584	6010	648	5851	3408
Energy							
Intermediate consumption 1970	186	61	17	7	14	48	38
Intermediate consumption 2005	547	79	13	55	33	202	164
Additive volume growth	50	-30	2	34	-1	28	17
Multiplicative volume growth	-49	-27	-13	8	-6	-8	-6
Materials							
Intermediate consumption 1970	1929	967	27	33	63	615	224
Intermediate consumption 2005	7582	4999	81	323	208	1433	538
Additive volume growth	9191	6871	73	555	181	1082	429
Multiplicative volume growth	7350	3187	81	781	237	1051	554
Services							
Intermediate consumption 1970	3201	1219	307	86	146	982	461
Intermediate consumption 2005	18674	3985	1521	4196	596	5279	3097
Additive volume growth	17493	3580	1386	4450	467	4778	2831
Multiplicative volume growth	19059	3722	1591	5768	491	5249	2980
Value added 1970	11569	7908	134	223	353	2351	600
Value added 2005	39296	21104	1012	3753	683	6806	5937
Additive volume growth	21198	5363	1605	6685	306	3263	3977
Multiplicative volume growth	17174	4788	1032	16578	317	2981	2534
Sum of sub-industries	28230						

K	REAL ESTATE, RENTING AND BUSINESS ACTIVITIES
70	Real estate activities
71	Renting of machinery and equipment

72	Computer and related activities
73	Research and development
741t4	Legal, technical and advertising
745t8	Other business activities, n.e.c

Source: KLEMS 2008 and own calculations

For concluding our analysis, table 4 presents a comparison between the traditional multiplicative chained index and the new additive one, where industry K, "Real estate, renting and business activities" has been chosen as example, contributing a major portion to the economic growth of the period. Its real output (output in constant money units) grew from 16,885 to 66,098 million 1970Kr. Of this growth 44 970 million 1970Kr. are due to growth in volume (meaning more production), when measured by an additive index, and 39,725 1970Kr. when measured by the traditional multiplicative index, a significant difference.

Looking at the lower level of aggregation we find that the spread of two indices is of similar magnitude except for the computer industry. Here the enormous growth in output from 349 to 8,327 million 1970Kr. in 35 years has been generated by an increase in production of 11,678 million 1970Kr. according to the additive index and of 23,132 Kr. according to the multiplicative index. The difference between the two figures is due to the different treatment of price change. The multiplicative index attaches the volume growth between time 1 and time t to the price of time 0 (see equation 4), while the additive method recognises the fact that within the time interval real prices have fallen so that later volume growth has less economic weight in the time series (equation 19).

The growth in volume of 11,678 million 1970Kr. seems paradox, as it overshoots the growth in real value actually transacted (8,327 - 345 = 7,982 million Kr.), but the account is closed by the movement of real prices which has been negative (7,982 - 11,678 = - 3,400 million 1970Kr.), thus transferring a significant part of the value added growth to the industry's customers through the price mechanism. The separation of growth in a volume and a price component is analytical, not real, and separate integration of these movements leads to virtual points, not actual displacement in the corresponding mathematical space.

Further down in table 4 advantage is taken of the possibility to separate classes of intermediate consumption within the KLEMS database. The two volume indices yield similar results for intermediate consumption total. The general tendency of the two indices to move in parallel is corroborated by the break-up into the three categories of energy, materials and services. But for the growth of gross value added the differences are more pronounced as is to be expected for a balance of two larger aggregates. At industry 72, computer and related industries, the additive decomposition shows a gross value added growth in volume of 6,685 million 1970 Kr., which includes the relative price decrease of the products over the studied period, while excluding it in the multiplicative index yields a growth of 16,758 million 1975Kr.

Inconsistency in aggregation of the multiplicative index is strikingly demonstrated, once again, by summing volume growth of the sub-industries, which yields 28,283 million 1970Kr. as opposed to a figure of 17,174 1970million Kr. found when deflating the high aggregate directly. The contradiction deserves to be taken seriously and addressed. Value added tells the income has been earned in operating an industry and that may be distributed to labour and capital. It does not seem reasonable to look at an industry like "Computer and related activities" (code 72) individually, deduce labour and capital income, and to forget that part of this income disappears when the industry is aggregated with its neighbours. Consistency in aggregation is a condition a priori, - not of price statistics, perhaps, - but of economic accounts.

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