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LEONTIEF AND SAMUELSON ON THE NON-SUBSTITUTION THEOREM
SOME METHODOLOGICAL REMARKS

(Draft – do not quote)

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The non-substitution theorem was first demonstrated by Samuelson and Georgescu-Roegen for a generalized model of Leontief. The theorem raised the question to know if the "deep-seated causes of [economic] development are to be found in the variations of the basic structural relationships themselves_ that is in modification of changes in consumers' tastes_ and changes in the structure of productive processes" (Leontief). Without advanced empirical evidences, this episode directed input-output studies toward general equilibrium theory. This is an anomaly for the philosophy of science: a change in a research program without empirical anomalies but only a certain epistemological and theoretical context.

"The world takes people too much at their words" (Samuelson [1962], p.10)

Before appearing during the 1960's in the quarrels between neoricardian and neoclassical economists, the non-substitution theorem was formulated by Samuelson in 1949[1] for an input-output model of Leontief. The theorem expresses the conditions of validity of Leontief's assumption of constancy of technical coefficients. The non-substitution theorem asserts that in a world with only one primary factor (labor) and without joint production,

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[1] And also, independently, by Georgescu-Roegen [1951].

whatever are the possibilities of substitution between production factors, changes in demand imply no change in technical coefficients, that is to say that once the combination of factors of production is decided it remains unchanged; the dual interpretation of the non-substitution theorem asserts that "under certain specified conditions an economy will have one particular price structure for each admissible value of the profit rate, regardless of the pattern of the final demand" (Salvadori [1987], p.680). As a consequence, according to Samuelson's demonstration the assumption of constancy of technical coefficients is true only under restrictive conditions: the simple model of Leontief (without substitutions between factors of production) is then only a particular case.

Samuelson's interpretation of the non-substitution theorem was first to make a distinction between a general case (with substitutions) and a particular case (the combination of factors of production is given once and fixed outside the marketplace independently of demand[2]) identified by Samuelson as the 'classical model'. The former corresponded to the general conditions treated in the walrasian theory of general equilibrium. Samuelson then sorted out the two cases: the classical model is not only part of general equilibrium theory but a particular case of the latter.

As Samuelson wondered: "Ricardo and Smith would probably have admitted that the relative prices of joint products_ of venison and skin, for example_ would have to be determined by a demand theory and not from labor and land costs alone. One wonders why they did not worry about this "jointness", which every student of Walrasian equilibrium knows to be an intrinsic part of the actual pricing relations among diverse factors and goods" (Samuelson [1959], p.18). The model of Leontief [1949] is then identified as a classical model and, at the same time, considered as a particular case of general equilibrium theory.

Consequently the "classical" case (without substitutions between factors of production and prices are determined by technological data) is not the general case. To paraphrase Samuelson, the classical story is only a part of the general equilibrium theory. In a nutshell the non-substitution then "is here to remind us that it is not easy to escape the imperialism of the general equilibrium theory" (d'Autume [1990], p.246).

This interpretation of the non-substitution theorem (which makes input-output analysis a particular case of general equilibrium theory) was in contradiction with the project of Leontief to build a general theory of economic systems based on production relations independently of demand conditions. Leontief's first answer was to criticize the epistemological model[3] on which the demonstration of the non-substitution theorem lies. In this way Leontief asked the epistemological validity of the theorem and its usefulness for his approach: an empirical analysis of economic systems as multisectors models. This is on this methodological ground that lays Leontief's answer.

Indeed, the non-substitution theorem was presented during the conference organized by the Cowles Commission[4] on linear programming, activity analysis and input-output analysis. This conference gathered together theories and techniques developed during world war two and stimulated by the American administrations and war services (as the US Air Force and the RAND Corporation[5]). All of these approaches used linear mathematical models and alluded to general equilibrium theory and game theory. The developments around the

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[2] In particular, if technical coefficients change this is because of technical change (that is to say a change of technological data).

[3] We call an epistemological model the scientific rules of an explanation. Different epistemological models exist: induction, deduction, abduction etc.

[4] Cowles Commission was created in 1932 by Alfred Cowles to support the *Econometric Society*. First settled in Colorado Springs it moved in Chicago between 1937 and 1954. Marschak and later Koopmans were directof of the Cowles Commission at that time.

[5] The Rand was the think-tank of the USAF.

model of Leontief and especially the demonstrations of the non-substitution theorem were disapproved by Leontief who didn't attend at the conference[6]. This clear disagreement of Leontief is not only theoretical but also methodological.

Indeed the 1949 conference is influenced by T.C Koopmans' scientific style. Koopmans as a director of research of the Cowles Commission promoted deductive and highly theoretical mathematical economics. He promoted this point of view during the famous controversy of the "measurement without theory"[7]. Leontief then asked the question to know what the methodological value of such demonstrations and developments was[8]. To escape the conclusion of the non-substitution theorem, Leontief attacked the model of explanation on which the demonstrations were grounded: for Leontief theory is not only required to be deductive but first of all to be operational.

This is Leontief's methodological answer that is studied here. This answer raises a first problem as Leontief paradoxically alluded to the same methodological criterion as Samuelson: the requirement of operationality of theory. How can we distinguish between these two criterions: operationality in Leontief's sense and operationality in Samuelson's sense? In other words, is the non-substitution theorem operational in the sense of Leontief and is it in the sense of Samuelson too? Moreover, what is the significance of Leontief's critic on the demonstrations of the non-substitution theorem? To end, what were the consequences of this critic on the research program of Leontief?

After a short account of the simple model of Leontief, the generalized model of Leontief proposed by Samuelson and the demonstration of the non-substitution theorem, an interpretation of the two methodological criteria of operationnality of Samuelson and Leontief is proposed in order to evaluate the significance of the theorem for input-output analysis.

A THEOREM CONCERNING SUBSTITUABILITY IN OPEN LEONTIEF MODELS

The non-substitution theorem was first demonstrated for a generalized open model of Leontief (Samuelson [1951])[9]. We first present the simple open Leontief model and then we expose the generalized model build by Samuelson.

In the open model of Leontief, in contrast to the closed model, final demand from households and State is an exogenous variable. This is an *n* sectors model (*n* homogenous commodities; no joint production). The model determines the equilibrium quantities of inputs and outputs and also the equilibrium prices.

Whether *d* is the column vector of final demands for each industry, A is the matrix of technical coefficients (called structural matrix), X is the vector of outputs, P the (relative) price-vector, R the vector of value-added (wages and profits) and finally, *I* is the identity

[6] See Akhabbar [2005].

[7] Koopmans opened the controversy when he wrote a critical review (1947) of A.Burns and W.C Mitchell's book on economic cycles. Koopmans reproached the two institutionnalists' approach to be blind because of the lack of a theory.

[8] In retrospect, the conference prepared to the modern demonstrations of general equilibrium theory (by Arrow and Debreu in particular).

[9] The non-substitution theorem was first named « substitution theorem ». For an interesting interpretation of this title change see (Pasinetti [1977]).

matrix, one writes the model of Leontief as a double set of equations in vector matrix language:

$$(I\text{-A})X = d \quad \text{and} \quad (I\text{-A})'P = R$$

Technical coefficients are supposed constant: the production function has constant returns to scale and production factors are strictly complementary. As a consequence, if a structural matrix *A* is given, the production function needs no explicit formulation. Labor is a primary factor (that is not produced), this is the $n+1^{th}$ commodity.

To determine the equilibrium quantities, whether the Leontief matrix (I-A) is invertible, then the equilibrium quantity vector is $X^* = (I\text{-A})^{-1}d$. To determine the equilibrium price-vector, two different data are required: technological data (as for the quantity-vector) and, as Leontief didn't build a theory of distribution, a given distribution of value added between wages and profits[10]. For a given wage-profits distribution, the price-vector is determined as follows: $P^* = (I\text{-A})'^{-1}R$.

Relationships between the walrasian general equilibrium theory and the model of Leontief are deeply ambiguous. On the one hand, with constant technical coefficients the model of Leontief has the same structure as the Walras-Cassel model and at once the same production theory: constant returns to scale, general interdependence of productive activities, complementary factors of production.

On the other hand, in contrast to the Walras-Cassel production model, Leontief ignored the separation between inputs ("*services producteurs*") and outputs (the products), with *n* inputs producing *m* products. Leontief had a circular flow vision of the economy, in other words he saw "*the economy as a circular flow*" where goods produce (directly or indirectly) other goods. This circular flow permits the reproduction of the economic system. In retrospect, this circular flow vision brings Leontief closer to Sraffa ([1999], p.111). At that time John Von Neumann (1937) used such a circular flow approach in his own growth model: "*Goods are produced not only from "natural factors of production", but in the first place by each other. Theses processes of production may be circular, i.e good G1 is produced with the aid of good G2, and G2 with the aid of G1.*" (Von Neumann [1945], p.1).

This idea of circularity was also available in activity analysis. To end, modern versions of general equilibrium theory, like models of Arrow-Debreu and McKenzie, didn't maintained Walras' distinction between inputs and products. As a conclusion the idea of circularity isn't incompatible with general equilibrium theory. However Leontief [1928] and others considered the circular flows approach as an alternative to the *homo oeconomicus*-based theory of Walras.

To end, the question of the determination of prices calls for a distinction between two different cases: (1) the model of Leontief with prices equal to costs (production prices), corresponds to Walras' assumption of perfect competition with zero-profits. In this case the model of Leontief is a simplified representation of the walrasian general equilibrium; (2) the model of Leontief with a surplus (positive profits) is closer to the classical model. Both examine the condition of reproduction of the economy, which is a characteristic of the reproduction viewpoint in economics (opposed to the scarcity viewpoint)[11].

While Leontief argued that his model was a simplified general equilibrium model this interpretation is correct only for the first editions of Walras' *Elements of pure political economy* and ignores the developments of general equilibrium theory of production after Pareto and Barone's critical analysis of the constancy of technical coefficients. From the point

[10] It is not supposed to be a unique rate of profit.
[11] See Gilibert [1998].

of view of walrasian economics, the non-substitution theorem is the missing link between the case without substitutions between factors of production (Walras-Cassel-Leontief models and classical models) and the case with substitutions (Walras-Pareto models): the theorem connects input-output analysis (the model of Leontief) and general equilibrium theory.

Focused on the role of demand, the non-substitution theorem makes a distinction between the case without substitutions and the case with substitutions. Indeed, when one determines the equilibrium quantity-vector $X^* = (I-A)^{-1}d$, it is supposed that a change in final demand *d* implies no change in technical coefficients *A*, that is to say that the combination of factors of production remains constant when demand changes. This is this last assumption that is questioned by Samuelson's demonstration of the non-substitution theorem.

Samuelson demonstrated the non-substitution theorem at the linear programming conference organized by the *Cowles Commission* in Chicago in 1949[12]. According to Samuelson, the non-substitution theorem reconciles the model of Leontief (constant technical coefficients and the value added distribution is given) with the theory of general equilibrium (variable technical coefficients and the theory of marginal productivity rules income distribution). For Samuelson, "actually all his [Leontief's] theory in its present form is compatible with the more general case of substitutability**"** (Samuelson [1951], p.142-143). In other words, the situation described by the model of Leontief isn't contradictory with and not exclusive of the theory of general equilibrium: it is a particular case of the walrasian theory of general equilibrium. Indeed, Samuelson shows that under certain circumstances "all desirable substitutions have already been made by the competitive market, and no variation in the composition of final output or in the total quantity of labor will give rise to price change or substitution" (*Ibid.* p.143); otherwise, whether the factors of production are substitutable, a change in final demand, relative prices or in available labor, substitutions between factors occur and technical coefficients change. In a nutshell the theorem raises the question of the validity of a price-quantity equilibrium independent of consumers' tastes and of the structure of demand.

In order to generalize Leontief's model[13], Samuelson took the same set of assumptions except the constancy of technical coefficients[14]: the generalized production function allows substitutions among factors of production. This is a *generalized model of Leontief*.

Let X_i be the total output of industry *i*, for all *i* = 1 … *n*

x_{ij} output of the *i-th* industry used by industry *j*

a_{ij} is the quantity of input *i* used to produce one unity of output *j*

and C_i the final demand ("final output") of the *i-th* industry.

Quantity equilibrium is defined for each industry by: $X_i = \sum_{j=1}^{n} x_{ij} + C_i$.

Labor is the *n+1*th commodity and is the sole primary factor. The final demand for labor is assumed equal to zero ($C_{n+1} = 0$), and : $X_{n+1} = 0 + \sum_{j=1}^{n} x_{n+1\,j}$.

[12] At the conference, Koopmans, Arrow and Georgescu-Roegen proposed three others demonstrations of the non-substitution theorem.

[13] That is to say to allow substitutions between factors.

[14] It must be underlined that, moreover, Samuelson changed the vocabulary and then the viewpoint of the analysis: he spoke of 'primary factors', 'isoquants' etc.

Production of each good depends on a homogenous production function $F_i(.)$ of first degree (constant returns to scale). There is no joint production, this is a one-commodity-per-industry case :

$$X_i = F_i(x_{1i}, x_{2i}, ..., x_{n+1i})$$

Hence, technical coefficients are not constant and we write the simple relation :

$$F_i\left(a_{1i}, a_{2i}, ..., a_{ni}\right) = 1$$

The equilibrium is found for each industry *k*, given the final demand for other industries, in such a way that the final demand for that industry *k* (C_k) is maximized (given technical constraints and the available quantity of labor (efficiency criteria)).

Each industry *k*, is ruled by following the optimization program:

$$\begin{cases} Max\, C_k = X_k - \sum_{j=1}^{n} {}_{k \neq j}\, x_{kj} \\ X_k = F_k(x_{1k}, x_{2k}, ..., x_{n+1\kappa}) \\ F_i(x_{1i}, x_{2i}, ..., x_{n+1i}) - \sum_{j=1}^{n} x_{ij} = C_i \quad \forall i = 1, ..., n \;\; et \;\; i \neq k \\ X_{n+1} = 0 + \sum_{j=1}^{n} x_{n+1\,j} \end{cases}$$

Solution of the program is found using the Langrange's multipliers. One shows easily that technical coefficients are independent of the C_i (final demands), of the prices and of X_{n+1} (the available quantity of labor).

The non-substitution theorem is powerful because it makes an initially postulated assumption (the constancy of technical coefficients) the consequence of a set of reduced assumptions. Indeed, while Leontief assumed the constancy of technical coefficients, Samuelson showed that in a generalized model of Leontief the constancy is true under precise conditions (only one primary factor, no joint production etc.). In other words, in the generalized model of Leontief, the constancy is the result of a deduction from fundamental assumptions. From a logical point of view, the status of the theoretical proposition concerning the constancy of technical coefficients changes in the generalized model: this is no more an initial assumption but the result of a deduction (a theorem).

However, to make such a move, Samuelson's production function is no more synthesized by a set of technical coefficients (like in the simple model of Leontief) and postulates the existence of a general derivable production function.

We show in what follows that Samuelson's demonstration isn't only the result of a theoretical strategy (to show that the model of Leontief is a particular case of the general equilibrium theory). Samuelson's demonstration follows methodological rules he defined as operational methodology.

DEDUCTIVE METHODOLOGY AND OPERATIONNALISM

Samuelson's non-substitution theorem is a strategic piece of the hegemonic ambitions of the general equilibrium theory expressed during the conference organized in 1949 by T.C Koopmans. The conference was a symposium on the "a fundamental problem of normative economics: the best allocation of limited means toward desired ends" (Koopmans [1951], p.1). The conference had indeed two dimensions: an exposure of the development of planning techniques during world war two and the improvement of general equilibrium theory. The former interest reminds us that general equilibrium theory was a favourable argument for the possibility of a socialist calculation (view supported by Oskar Lange[15]). The latter was focused on normative implications of general equilibrium theory. Indeed, the normative economics Koopmans dealt with is the one concerned with the results of a competitive general equilibrium and the optimum of Pareto. As a consequence most of the normative propositions enounced were part of general equilibrium theory.

In fact the conference was a place to make a synthesis of the different approaches of general equilibrium theory, to introduce new mathematical tools, and to get closer to theory of game: this conference was a crucial step toward a new statement of general equilibrium theory[16].

This restatement was influenced by Koopmans' deductive methodology he put under practice during the conference; also he was leading the "measurement without theory" against the NBER. The controversy opposed theorists for whom theory guides measurement, like Koopmans, Arrow, Samuelson, Leontief, Hurwicz or Klein, to the empiricists like Mitchell and Vining (NBER). However theorists didn't developed only one methodology but different way to make theories, sometimes opposed one to another.

Samuelson's methodology is famous. The methodological criterion he chose to change the scientific vocabulary of economics was an operational one he defined in 1938. However, when using the operational criterion Samuelson wasn't alone to do so. This term 'operational' was common in Cambridge and especially in Harvard. Indeed, while empirical and experimental studies had been developing in social sciences since the end of the 19th century, there were very different ways to articulate theory and observations, even under the influence of the Vienna circle: the operational methodology is one of them.

In economics, H. Schultz (1938) was the first to allude to an *operational* methodology. According to Schultz, his demand estimation method was "an illustration of what … Bridgman … has called 'the operational procedure' for determining the meaning of a concept [in this case the meaning of demand]" (*in* Cohen [1995], p.59). Indeed, the physicist P. Bridgman (Harvard) developed in his *Logic of modern physics* (1927) the philosophical doctrine he called operationalism (also called operationism). This doctrine states that the meaning of a scientific concept is synonymous with the set of operations entering its definition. In other words, the meaning of a concept is given by an explicit definition of this concept stating experimental or mental operations of measure. This is the experimental version (and not *mentalist*) of Bridgman's

[15] Leontief was interested in this dimension of general equilibrium theory (see Leontief's correspondance with Oskar Lange ([HUG, 4517.5] « 1947-1950 »)).

[16] See Weintraub [1983].

operationalism that was successfully exported in social sciences, psychology and philosophy[17].

The particularity of operationism is that it proposes a strong relationship (synonymy) between theoretical terms and observational terms (direct observations or sense data), without distinction between induction and deduction. Nevertheless in contrast to induction, operationism considers theory as necessary *and* requires theoretical terms to be operational, that is to say to be empirically meaningful (while induction is dubious about theory).

In philosophy, Carnap (in Cambridge *Mass*. at that time) showed in 1936-1937 that operationism is the strong version of verificationism and at once that this methodological program is impracticable! For the philosophy of science, Carnap's article ended the operational program (but Hempel, Popper and Goodman continued to criticize and to discuss operationism during the forties and the fifties) and ended also the verificationist program. This led to the creation of what Carnap called logical empiricism[18].

In Cambridge again, first Samuelson (in 1938), a student of Leontief at Harvard and then working at the MIT, and later Leontief (in 1949), at Harvard, will allude to the same operational criterion.

SAMUELSON'S OPERATIONAL THEOREMS

Samuelson first studied the neoclassical theory of demand and utility and raised the question: "Does not the whole utility analysis become meaningless in the operational sense of modern science?" (Samuelson [1938], p.344). The process underlying the *Foundations of Economic Analysis* [1947] was a translation of economic theory into an operational language of science.

For Samuelon "a meaningless theory according to this criterion is one which has no empirical implications by which it could be refuted under ideal empirical conditions" (*Ibid*.). The definition Samuelson gave of his operational criteria was stable in time and he didn't remove it: "My approach looked backward in summarizing "economically" (in the Mach-Vienna Circle sense) the "meaningful" (testable and, in principle, refutable) core of constrained-budget demand theory." (Samuelson [1998], p.1380). The project of Samuelson was to eliminate from "the theory of consumer behaviour any '*vestigial traces of the utility concept*', consequently stripping consumption theory '*to its bare implications for empirical realism*'" (Samuelson (1938) *in* Cohen [1995], p.65-66). According to Samuelson this is that method which permits to enounce *operational theorems*.

It is important to underline that the definition Samuelson gave of the operational meaning of a theory isn't the same than neither Bridgman's definition nor Carnap's version of operationalism. In contrast to Carnap and Bridgman's approaches, the methodology Samuelson adopted is not a criterion of meaning but a criterion of falsification. Blaug [1980] argued that Samuelson's operationalism was a mixture of empiricism and falsificationism[19]. Whether we admit Blaug's position, Cohen [1995] rejected it. This latter underlines that the book of Popper, *Die Logik der Firschung* published in 1934, was translated in English late in the fifties (1959) and he supposes

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[17] Behaviourism is particularly influenced by operationism.

[18] We consider in what follows that Carnap's version of operationism is the general one.

[19] Karl Popper is considered as the father of falsificationism. This methodology doesn't permit to find the empirical meaning of a concept but only to say if it is a scientific one and if it is not yet falsified.

that Samuelson didn't read it. However, Popper's ideas were largely transmitted and known in the English literature of philosophy of science early in the thirties. Moreover, Samuelson's methodology is clearly not an operational one but a version of falsificationism: according to such a methodology, theory doesn't need direct correspondences with data but only indirect empirical implications.

To bring empirical implications to the theory of consumer, Samuelson chose to start his analyses with the data consumers reveal in the marketplace. The utility function is then the one maximized which corresponds to the revealed preferences. This function is a transitive and complete ordinal utility function, and satisfies the weak axiom of revealed preference. Ordinal utility is then purged from introspection and get an operational meaning, according to Samuelson: the function is defined "as that which is maximized by means of *revealing preferences* in the market place" (Cohen [1995], p.66).

This is clearly the same principle which is employed in the theory of revealed preference and the non-substitution theorem: the explanation of empirical data (actual technical coefficients / consumed quantities) is expressed as an optimization program of a derivable mathematical function (final net output / utility function) under constraints (the production function and the quantities equilibrium / budgetary constraint). To solve the program the method of Lagrangean multipliers is used.

On the one hand, the non-substitution theorem shows that a production function with substitution exists which induces under certain circumstances constant technical coefficients (one *observes* constant technical coefficients) and which induces under other circumstances variable technical coefficients (one *observes* variation of technical coefficients).

On the other hand, in the same way as the production function obtains an operational meaning by its references to variations of measurable technical coefficients, the utility function gains empirical significance once leaving mysterious allusions to introspection[20]. These functions are operational by means of indirect references to observation: this is the deductive method and not the canonical operational method of Carnap and Bridgman. Indeed, Carnap's operational methodology requires direct correspondences between concepts and observation: to gain an operational meaning the mathematical function must be defined by synonymous measure operations…

As a consequence, Samuelson's method lies on two stages: (1) enunciation of minimal ('economical') assumptions under which rest deduction and calculation; (2) expression of the explanation of the problem as an optimization program.

Ultimately then, the non-substitution theorem is operational according to Samuelson's definition of the operational meaning of a theory, but this definition is completely different from the one Carnap proposed. On the other hand, Samuelson's criterion is falsificationist (close to Popper) and corresponds to the one proposed by Koopmans at the *Cowles Commission*: the empirical content (and not operational meaning) of a hypothetic-deductive theory comes from the singular statements

[20] Samuelson's methodology is understandable as a methodology concerning the validity of assumptions. Here, he replaced an unoperational assumption (a function of utility based on introspection) by an operational assumption (a utility function based on market data).

(observable) deduced from the general assumptions[21]. This deductive method is very different from the canonical operational methodology which is close to Leontief's own methodology.

THE OPERATIONAL SCIENCE OF LEONTIEF AND THE DEMONSTRATIONS OF THE THEOREM

While Leontief and Samuelson alluded to the same operational methodology, their definitions are different. On the one hand, Samuelson proposed a falsificationist criterion and dealt with deductive approach: theory is indirectly confirmed or infirmed. On the other hand, Leontief rejected Samuelson (and Koopmans') deductive methodology: each scientific term has to be directly empirically founded. In fact, whether Samuelson's operationism is close to Popper's falsificationism, Leontief defined a methodology close to Carnap's definition of operationism.

Since the 1920's Leontief had been looking for a methodology which would bring together the "nomological order" (general laws) and the "idiographic order" (facts) (Leontief [1928]). Leontief's methodological doctrine requires theoretical terms to be directly observable. Moreover this doctrine is compatible with classical economics and its research of *objective* forces. The requirement of directly observable terms implies that Leontief's criterion is a meaning methodological criterion. Leontief called it 'operational meaning' [1949] [1952] [1958].

As a witness of soviet debates about the existence of empirical laws underlying political economy, Leontief clearly chose against those who proposed substitution of a science of history and social essences for political economy. This view implies that political economy is a science which studies empirical phenomenon (like cycles, economic dynamic etc.). This science is ruled by an empirical methodology and based on a theoretical framework: theory is a necessary preliminary to empirical analysis (Leontief [1927]). However, theory is strictly controlled by empirical rules: this is what Leontief called "the methodology of theory" (Leontief [1928]). Without such rules, theorizing leads to the making of empty theoretical boxes.

Leontief's criticism of the subjective economic theory, based on individual behaviour, and especially the theory of consumer, was related to his view of economics as an empirical science. As a consequence, objective theory is preferred and focused on the directly observable factors like the flows of commodities[22]. As a consequence, the methodological stakes of the non-substitution theorem is now clear: if the theorem is operational then Samuelson is right and economic theory is incomplete without individual data of consumers' tastes, which is the point of view of *colesmen* like Arrow, Koopmans, Hurwicz, Klein etc.

Paradoxically Leontief [1949] alluded to Samuelson's operational criterion to challenge the stakes of the non-substitution theorem. In fact, the paradox is solved once one admits that Leontief and Samuelson's definitions of operationism are different. Samuelson's methodology is a version of the deductive approach of the Cowles Commission while Leontief rejects what he called the "deductive shool" ([1952]).

[21] For Koopmans, empirical content is provided by econometric tests and estimations.

[22] However, while the relationships between an objectivist philosophy and operationalism are complex, both are rooted in philosophical materialism.

Leontief's rejection of the deductive method is due to his rejection of the testability methodology: assumptions are indirectly confirmed by means of empirical tests. As a consequence, testability implies indirect relations between theoretical statements and observational statements. Econometrics belongs of course to tests methodology. Against the deductive school, Leontief chose a radical solution: to theoretical operations it must directly correspond observable operations in the economic system; that is to say that each theoretical term is explicitly defined in terms of real operations. In a radical expression, this operationism is clearly related to Carnap's model of explanation.

As a consequence the hypothetical general derivable production function postulated by Samuelson has no empirical (or operational) meaning. At the opposite, only technical coefficients, measured and directly related to facts, are operational. According to the Leontief criterion of operationality this is enough to reject the non-substitution theorem. However, whether Samuelson's demonstration is not operational according to Leontief, other demonstrations may be acceptable? Indeed, two others demonstrations of Koopmans and Arrow kept the expression of technical coefficients unchanged and don't use mathematical derivable production functions.

THE DEMONSTRATIONS AND THE IMPRACTICABILITY OF OPERATIONISM

Koopmans and Arrow didn't use the same mathematical language than Samuelson. There language is identical to Leontief's matrix language: they use matrix language and geometrical interpretation of vectors. This is the language of linear programming. Koopmans [1951] demonstrated the theorem for a 3 industries case and Arrow generalized to *n* industries (Arrow [1951]).

While Samuelson's demonstration of the non-substitution theorem isn't operational because of the use made of a mathematical production function[23], examination of the operationality of Arrow and Koopmans' demonstrations leads to the limits of Leontief's methodology.

Whether the mathematical language employed by Koopmans and Arrow is close to Leontief's one, they use a maximum principle (like Samuelson). This maximum principle isn't the one of rational behavior theory and then doesn't make the demonstration a subjectivist theory (that is to say an explanation of economic phenomenon based on individual behavior). However, one supposes that the marketplace "just as the ideal competitive system acts so as to get as much outputs with as little inputs as is possible" (Samuelson [1959], p.31). Operationality of such a maximum principle is questionable (but at the same time questions the significance of the operational methodology).

Applying a strict operational criterion (like the one defined by Carnap), we find no correspondence in the real economic systems with the maximum principle: it is impossible to define explicitly a principle of this kind[24]. Moreover it is not possible to consider that the maximum principle is tautological (that is to say neither right nor wrong). As a consequence the maximum principle would not be acceptable. However it is impossible to reject it according to the operational criterion otherwise most of theoretical terms would be rejected too. Moreover, this principle is here a minimal

[23] Of the kind $y = f(X)$.

[24] In fact it is impossible to find operational meaning to most of usual theoretical terms, like for instance, dispositional terms ("breakable", "constant" etc.)

maximum one: even the classical school admits that the techniques chosen in the system is the one which cost is the lower; in fact any production function implicitly corresponds to a maximizing principle[25], even Leontief's production function. It is then difficult to reject this principle only because it is now explicitly stated.

To end, the logical form of Samuelson, Koopmans and Arrow's theoretical proposition, that is to say a theorem, is by definition meaningless as it deals with hypothetic situations (if… then…). As far as science deals with actual worlds only, possible words are excluded. As a consequence, Leontief was very sceptical about the value of theorems in economics and considered that despite the matrix algebra used by Koopmans, the distance between Samuelson and Koopmans is very short except that "the axiomatic assumptions of the linear approach are typically more restrictive than those conventionally employed by neoclassical theory […] and thus [Koopmans] has been able to arrive at more specific implications. The question naturally arises what gain there is in this for immediate or possibly for later explanation of observed facts." (Leontief [1958], p.104).

To conclude, in its strict expression, Leontief's operational criterion of meaning attenuates the non-substitution theorem validity. However, such a criterion leads to a rejection of usual theoretical terms as the maximum principle. It seems that whether one doesn't introduce new theoretical statement in the model of Leontief, it is difficult not to examine the non-substitution theorem.

ESCAPING THE NON-SUBSTITUTION THEOREM?

In 1949 Samuelson, Koopmans, Georgescu-Roegen and Arrow presented their demonstration of the non-substitution theorem during the linear programming conference. The demonstrations were published in 1951 in the 13th monograph of the Cowles Commission (Koopmans, [1951]).

At that time Leontief was creating at Harvard University the Harvard Economic Research Project (HERP)[26], and was occupied with the dynamic input-output model and the first Conference on Input-Output Analysis held in 1951 in Dreibergen. Leontief didn't attend at the conference and disapproved the development around Koopmans' activity analysis. The answer of Leontief to these theoretical developments is the collective works of the HERP published in 1953: *The Studies in the Structure of the American Economy* [1953].

Theses works are not only theoretical and empirical answers to the *Cowles Commission*' but also methodological. The introduction of the *Studies* expressed the position of Leontief in the measurement without theory controversy and he noted that "the theory is required to be operational" (Leontief [1953], p.3). His methodological answer was criticized at that time as intolerant[27] as it rejected radical empiricism (the NBER), deductive methodology (Koopmans, Arrow, Samuelson) and econometrics (Klein, Haavelmo).

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[25] Samuelson noted once : « As I shall discuss in connection with the role of maximum principles in natural science, the pump-line trajectory of a falling apple and the elliptical orbit of a wandering planet may be capable of being described by the optimizing solution for a specifiable programming problem. But no one will be tempted to fall into a reverse version of the Pathetic Fallacy and attribute to the apple or the planet freedom of choice and consciously deliberative minimizing." (Samuelson, 1972, p.250).

[26] Created in 1948 thanks to funds from the US army and the Ford Foundation.

[27] See Arrow [1953], Klein [1953] and Hurwicz [1955].

However, Leontief's methodological "intolerance" is balanced in the *input-output* studies: without alluding to the non-substitution theorem, changes in technical coefficients are conceived as the consequence of technical change **and** changes in demand and tastes of consumers. "The […] more deep-seated causes of development are to be found in the variations of the basic structural relationship themselves; that is in modification of the consumers' tastes, and changes in the structure of productive processes" (Leontief [1953], p.12).

Nevertheless, Leontief underlined ironically that "no other field of economic inquiry can suffer so much from theoretical over-simplification as the study of household behaviour" (Leontief [1953], p.15); according to him, this problem "cannot be easily approached 'from below' via economic psychology or quantitative sociology for the simple reason that neither of these disciplines does yet actually exist" (*Ibid.*).

As a matter of facts, the *Studies* represent a turn in input-output analysis[28] as it considered implicitly two versions of input-output economics: the simple model and the generalized model. One finds in the *Studies* a measure of the effects of variation of demand on technical coefficients. These *input-output* works are followed by Anne Carter's works on structural change [1970] but also by empirical studies like Arrow and Hoffenberg [1959]. This new research direction brought together the simple and the generalized model of Leontief without however giving new theoretical insights to understand the role of demand.

As a consequence, after the demonstration of the non-substitution theorem new questions arose in input-output economics. This episode didn't, however, permit to overtake the distinction between a classical case (the simple model of Leontief without substitutions) and a general case (the walrasian theory of economic general equilibrium). A clear consequence of that confrontation of input-output analysis and linear programming is the development of applied general equilibrium models outside input-output economics. For instance, based on the algorithm Dantzig developed to solve maximum problems arising in generalized model of Leontief, Scarf developed mathematical algorithms for general equilibrium models; Shoven and Whalley noticed about applied general equilibrium model they exposed: "The models reported here extend Wassily Leontief's work on empirical Walrasian models based on fixed input-output coefficients by incorporating substitution effects in both production and demand and by including more than one consumer" ([1984], p.1008). Application of general equilibrium theory was fed by theoretical works on the generalized model of Leontief and also by empirical works on input-output tables and social accounting matrices.

FINAL REMARKS

The non-substitution conducts to a paradoxical conclusion: the model of Leontief is a classical model (ricardian model would say Samuelson) and at once a model of general equilibrium as a particular case of it! Then the question is no more to know whether the model of Leontief is classical or not (it is) but if such a classical model is a particular case of general equilibrium or not (which is another discussion).

Strangely enough, the non-substitution theorem didn't give the signal for the development of empirical analysis of structural change within a classical framework while the 1949 demonstrations opened the road for the application of general equilibrium theory. This

[28] A first turn was the collaboration with the Bureau of Labor Statistics which led to abandon the closed model. A second turn is the creation of the HERP.

episode explains why the classical theoretical framework proposed by Leontief gave rise to the empirical application of walrasian models.

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