### Japan Korea Link Input Output Analysis

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### **1. Introduction**

The two fundamental equations of input output analysis are the material balance equation and price equilibrium equation. The first equation is X=AX+F where A is input output coefficient matrix, X is gross output column vector, and F is final demand column vector. The second equation is P=PA+V where P is price row vector and V denotes a row vector of value added coefficients. Instead of the above two sets of equations, we employ definitive equations of total labour value (total labour requirements) and price equilibrium equations. The total labour value equations are defined as T=TA + L where T is total labour requirements vector (row vector of total labour value), and L is labour coefficient vector (row vector). On the other hand price equilibrium equation can be rewriten as P = PA + L Y where Y is diagonal matrix of value added productivity of direct labour of each sectors. That is,  $v_i = l_i \times y_i$  holds for any sector i.

The objective of this paper is to clarify the relationship between the total labour value(total labour requirements)  $t_j$  and value added productivity of labour  $y_j$  internationally.

In addition, John Roemer's Unequal Exchange Exploitation is tested if the definition can be defined **internationally**, and using the John Roemer's Unequal Exchange Exploitation, the question, "Is Japan exploiting Korea?" will be tested.

#### 2. Data Processing

Source of data are Link Input Output Tables of Japan and Korea published by Institute of Developing Economies, Japan. The year 1975 table is 172 sectors table (528 rows 363 columns for two countries) and year 1985 table is 274 sectors table (**835** rows and 579 columns for two countries). These tables are purchased as magnetic tapes and integrated into 25 sectors tables (53 rows and 52 columns with value

added items and final demand items) by Fortran Programme. Then analyzed according to the following model by Basic Programme.

### 2. Model

Total labour requirements (total labour value) of one unit of commodity j can be defined as .

$$t_j = \sum_i a_{ij} \times t_i + l_j$$
 This can be written in matrix as  $T = TA + L$  This can be solved as  $T = L(I - A)^{-1}$ 

The solved value will be non-negative as long as the economy is reproductive, or it fulfills the Simon Hawking conditions. Because direct labour requirement vector is multiplied be Leontief Inverse Matrix, total labour value can be understood as total labour requirements.

Above most simple definition of total labour requirements(total labour value) are written in matrix as follows: In the following asterisks are suffixed to indicate the understanding that input output coefficients are in monetary terms. However this understanding do not influence the study in any way. That is, all matrix and vectors can be understood both in monetary terms and in physical terms.

Total labour value (requirements) of each sectors considering imported inputs are as follows. In the following,  $M^*$  is import coefficient raw vector.  $t_m^*$  is scalar denoting the total (domestic) labour embodied in one monetary unit of export to purchase same value of import.

$$T^{*} = T^{*}A^{*} + M \cdot t_{m}^{*} + L^{*}$$
(1)

In order to determine the total labour requirements of the two nations simultaneously a new model was developed. This model itself can be used to determine (or replace) the content of foreign produced imported goods and services in link models.

The model to determine the total labour value (total labour requirements) of two countries are as follows.

$$T^{*} = T^{*}A^{*} + M^{*}(t_{m}^{j*}I^{j} + t_{m}^{k*}I^{k}) + L^{*}$$
(2)

Suffix j is for Japan while suffix k is for Korea. Total labour value (requirements) per monetary unit of Japanese import is replaced by total labour value (requirements) of same value of Japanese exports. Suppose  $t_m^{J^*}$  is the total labour value (requirements) of one monetary unit of Japanese import, then it is calculated as the weighted sum of total labour value (requirements) of all industries weighted by the share of that industry's export among total export. That is, total domestic labour requirement of import is calculated as:

$$t_{m}^{j*} = T^{*} \cdot E^{j*}$$
 (3)

Likewise, for the case of Korea, following equation holds.

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$$t_m^{k*} = T^* \cdot E^{k*'}$$
 (4)

 $M^*$  is a row vector of import coefficients where factor 1 to 25 are for Japan and factor 26 to 50 are for Korea.

In order to solve the equation, two matrix,  $I^{j}$  and  $I^{k}$  are introduced.

The factor of  $I^{j}$  is a diagonal matrix that elements relate to Japan are 1, while all other factor is 0.

$$I_{ij}^{j=1}$$
 for  $i=j$  and for  $1 \le i \le 25$ )  $I_{ij}^{j=0}$  (for  $i \ne j$  and for  $1 \le i \le 25$  else  $26 \le i \le 50$ 

The factors of  $I^{k}$  are such that the diagonal element related to Korea are 1 and all the rest are 0.

$$I_{ij}^{k} = 1 \text{ for } i = j \text{ and for } 26 \le i \le 50 \quad I_{ij}^{k} = 0 \text{ for } i \neq j \text{ and for } 26 \le i \le 50 \text{ else } 1 \le i \le 25$$

$$I_{ij}^{j} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & \ddots \\ \ddots & \ddots & 1 & \dots \\ \ddots & \ddots & 0 & \ddots \\ \ddots & \ddots & 0 & \ddots \\ \ddots & \ddots & 0 & 0 \\ 0 & \dots & 0 & 0 \end{pmatrix} \qquad I_{k}^{k} = \begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \ddots & \ddots & 1 & \dots \\ \ddots & \ddots & 1 & 0 \\ 0 & \dots & 0 & 1 \end{pmatrix} \tag{5}$$

Rewriting the above equation by replacing  $T_d^* = L^* (I - A^*)^{-1}$ 

$$T^{*} = T_{d}^{*} + M^{*} (t_{m}^{j*} I^{j} + t_{m}^{k*} I^{k}) (I - A^{*})^{-1}$$
(7)

By inserting this equation (7) to equations (3) and (4) and arranging gives the following equations.

$$t_{m}^{j*} = T_{d}^{*} E^{j*'} + M^{*} t_{m}^{j*} I^{j} (I - A^{*})^{-1} E^{j*'} + M^{*} t_{m}^{k*} I^{k} (I - A^{*})^{-1} E^{j*'}$$
(8)

$$t_{m}^{k*} = T_{d}^{*} E^{k*'} + M^{*} t_{m}^{j*} I^{j} (I - A^{*})^{-1} E^{k*'} + M^{*} t_{m}^{k*} I^{k} (I - A^{*})^{-1} E^{k*'}$$
(9)

Noticing that both  $t_m^{j*} t_m^{k*}$  are scalars, rearranging gives

$$(1 - M^* I^J (I - A^*)^{-1} E^{j*'}) t_m^{j*} - (M^* I^k (I - A^*)^{-1} E^{j*'}) t_m^{k*} = T_d^* E^{j*'}$$
(10)

$$-M^{*}I^{j}(I-A^{*})^{-1}E^{k*'}t_{m}^{j*} + (1-M^{*}I^{k}(I-A^{*})^{-1}E^{k*'})t_{m}^{k*} = T_{d}^{*}E^{k*'}$$
(11)

Above two equations can be solved by recognizing them as two simultaneous equations by Cramer's solution:

$$t_{m}^{j*} = \frac{\begin{vmatrix} T_{d}^{*}E^{j*'} & -M^{*}I^{k}(I-A^{*})^{-1}E^{j*'} \\ T_{d}^{*}E^{k*'} & 1-M^{*}I^{k}(I-A^{*})^{-1}E^{k*'} \\ T_{d}^{*}E^{k*'} & 1-M^{*}I^{k}(I-A^{*})^{-1}E^{j*'} \\ \frac{1-M^{*}I^{j}(I-A^{*})^{-1}E^{j*'} & T_{d}^{*}E^{j*'} \\ -M^{*}I^{j}(I-A^{*})^{-1}E^{k*'} & T_{d}^{*}E^{k*'} \\ -M^{*}I^{j}(I-A^{*})^{-1}E^{k*'} & T_{d}^{*}E^{k*'} \\ \end{vmatrix}$$
(12)

Here

$$\Delta = \begin{vmatrix} 1 - M^* I^j (I - A^*)^{-1} E^{j*'} & -M^* I^k (I - A^*)^{-1} E^{j*'} \\ -M^* I^j (I - A^*)^{-1} E^{k*'} & 1 - M^* I^k (I - A^*)^{-1} E^{k*'} \end{vmatrix}$$
(14)

By returning these results to the above equation gives the total labour requirements (value) of two nations simultaneously.

$$T^{*} = T_{d}^{*} + M^{*} (t_{m}^{j*} I^{j} + t_{m}^{k*} I^{k}) (I - A^{*})^{-1}$$
(15)

Model including capital cost indicated lower goodness of fit, and so hereafter we will neglect capital cost.

# 4-2 Regression of inverse of value added productivity regressed by total labour

### requirements(value)

Inverse of value added productivity, 1/Y, (amount of direct labour per nominal added value) regressed by total labour requirements (total labour value) show the following results.

	Dependent variable	Constant	Explanatory Variable T	<i>R</i> <sup>2</sup>	Number of data
975 Japan & Korea	1/Y	-0.0358 (0.244)	1.206 (10.72)	0.71	49
975 Japan	1/Y	-0.0943 (2.40)	1.878 (10.09)	0.82	25
975 Korea	1/Y	-0.159 (0.78)	1.449 (4.89)	0.52	24
985 Japan & Korea	1/Y	-0.014 (0.442)	1.223 (12.85)	0.78	49
985 Japan	1/Y	-0.0297 (2.88)	1.693 (14.3)	0.9	25
985 Korea	1/Y	-0.0499 (1.157)	1.459 (64.55)	0.65	24

The value in () are t values. The regression coefficient of explanatory variable T are stable around 1.2 to 1.9. This results are also obtained by input output analysis of US, Japan, and Korea for 1960 to 1985 that this coefficient is between 1.3 to 2.0. Since Y = V/L where L is direct labour coefficient, above results can be interpreted that value added productivity are linearly correlated to the ratio of direct labour requirement to total labour value (total labour requirements). Such relation that inverse of value added productivity can be explained by total labour value(requirement) are verified not only within a nation but also internationally.

# 4-3 Regression analysis of Value Added Productivity Regressed by Total Labour Productivity

Value Added Productivity of Direct Labour is regressed by Total Labour Productivity. Sectors such as Oil Products and Electricity, Water Supply and Gas (either (regional) monopoly or oligopoly) industries digress from the trend. Such results are obtained by national analysis, and such nature are verified internationally through link models as well.

	Dependent	Constant term	Explanatory	$R^2$	Number of data
	Variable		Variable 1/T		
975 Japan and	Y	-1.75	1.79	0.26	49 Kr no
Korea		(0.244)	(10.72)		unclassified sector
975 Japan &	Y	-0.734	1.275	0.76	47 Jp Kr oil
Korea		(0.255)	(11.91)		products deleted
985 Japan &	Y	-18.34	2.75	0.23	49 Kr unclassified
Korea		(0.39)	(3.72)		no data
855 Japan and	Y	-4.63	1.53	0.53	48 Jp oil products
Korea		(0.35)	(7.29)		deleted
985 Japan and	Y	-2.194	1.176	0.87	45 Jp Kr
Korea		(0.558)	(16.63)		Electricity+Water
					deleted

# 4-4 J.E.Roemer's Unequal Exchange Exploitation,

J.E.Roemer's definition of Unequal Exchange Exploited is, in Roemer's words, a sector is defined as Unequal Exchange Exploited when no matter how it spends its sector's income on goods it cannot purchase goods embodying as much labour as it supplied. A sector is defined as Unequal Exchange Exploiting if no matter how it chooses its income (value added generated in its sector) on goods it always commands more labour than it supplied. The remaining sectors are called Unequal Exchange Neutral. Sector j is UE Exploited when

$$l_j > v_j \cdot t_{\max}^*$$
  $t_{\max}^* < \frac{l_j}{v_i} = \frac{1}{y_i}$ 

Sector J is UE Exploiter when

$$l_j < v_j \cdot t^*$$
)<sub>min</sub>  $t^*_{min} > \frac{l_j}{v_i} = \frac{1}{y_i}$ 

Then, most industries fall into the category of neutral. In order to avoid this problem, neutral industries are classified into industries facing favourable exchange and those facing unfavourable exchange. Industries facing favourable exchange are those that the purchased it's own commodity embodies greater amount of labour than that sector actually supplied. Sector j faces favourable exchange when

$$t_j > \frac{t_j}{v_i} = \frac{1}{y_i}$$
 holds. Unfavourable is the case when  $t_j < \frac{t_j}{v_i} = \frac{1}{y_i}$ 

holds. Results follows.

	1975			
	Japan	Korea	Japan	Korea
.Iron & Steel				Ν
Nonferrous Metal				
.Metal Products				
. General Machinery				
.Electric Machinery				
Automobiles				
Other transportation Machinery				
Precision Machinery				
Construction				
0 Food Processing				
1 Textile Yarn				
2 Textile Apparel				
3 Ceramics				
4 Wood processing				
5 Paper Printing				
6 Chemical & Rubber				
7 Other Manufacturing				
8 Agri-forestry, fishery				
9 Mining				
0 Oil Refinery & Coal products				
1Electricity water Gas supply				
2 Commerce				
3 Transportation, Communicat'n Storage				

4 Finance Insurance real estate		
5 Unclassified	n.a.	n.a.

N stands for UE Neutral. F stands for favourable exchange. U stands for unfavourable exchange.

# Conclusion

1. Roemer's definition of UE Exploitation can be defined internationally. Hypothesis that Japan is exploiting Korea was not verified, however some industries realizing high value added per labour are classified as exploiting industries in both countries.

2. Value added per labour and total labour productivity are correlated. Scatter diagramme shows the structure of the economy. For both Korea and Japan, oil products (refinery) and gas, electricity, water supply digress from the trend.

3. Inverse of value added productivity of labour are significantly correlated to total Labour value (requirements). The regression coefficients are stable between 1.2 to 1.9. and these results support the results obtained by national time-series analysis. That is, inverse of value added productivity of labour can be explained by total labour requirements and the coefficients vary at the different stage (phase) of development or different phase of trade cycle.

# **References**

OKISHIO, N., "Marx's Economics" (in Japanese) Chikuma Shobo, 1977.

OKISHIO, N., 'Major Theorem of Labour Theory of Value' (in Japanese), "Kobe University Economics Research Annuals", No.36, 1989.

NAKAJIMA, A., 'Total Labour Inputs, Value Added per Labour and Economic Development' "Kyoto University Economic Review (in Japanese)" Vol.150, No. 4, 1992.

NAKAJIMA, A., 'Value Added per Direct Labour and Total Labour Inputs: Process of Economic Development and Total Labour Values,' "Kyoto University Faculty of Economics Working Paper" No. 31, 1995.

NAKAJIMA, A., Value Added per Direct Labour and Total Labour Inputs in the Process of Economic Development' in "Churalongkorn Journal of Economics", Bangkok, Thailand, forthcoming . ROEMER, J., "A General Theory of Exploitation and Class", Harvard University Press, 1982. ROEMER, J., 'Unequal Exchange, Labor migration and International Capital Flows: A theoretical

Synthesis' in Desai, P., ed. "Marxism, Central Planning and the Soviet Economy", pp34-60,1987. TenRAA, "Linear Analysis of Competitive Economies" LSE Handbooks in Economics, Harvester Wheatsheaf, 1995.