Projecting the Leontief inverse directly by the RAS method

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Abstract

The Leontief inverse is a very useful and powerful tool in input-output analysis. It features in the computation of various kind of multipliers such income and employment multiplier and play an important role in economic impact studies, analysis of structural change and identification of key sectors for development planning. For policy and planning purposes, direct projection of the Leontief inverse become desirable and the RAS method is proposed as a means to do that. This method has some advantages over the conventional method of updating the technical coefficient matrix in order to derive the Leontief inverse. Firstly, it deals with a matrix which more "dense" and hence rounding errors in computation of the R's and S's are reduced. Secondly, the technical coefficient matrix derived from the projected Leontief inverse has a functional form which is more general than the bi-proportional representation. Furthermore, this approach does not require more information than the conventional RAS used to project the technical coefficient matrix.

Key Words

RAS

Biproportional Matrices

Input Output Tables

Decomposition

Projecting the Leontief inverse directly by the RAS method

1. Introduction

The RAS method is most commonly applied to update the technical coefficient matrix in input-output analysis. More recently, Snower(1990) has suggested including other information in the refinement and application of the RAS method. Nonetheless, the basic RAS method is popularly used in balancing a matrix depicting flows (such as labor mobility, business transaction and trade) among destinations and origins when only the marginal totals and a previous completed matrix is available. In using the input-output model for analysis and forecasting, the Leontief inverse is a matrix that analysts are familiar and it is used extensively in their work.

Symbolically, the equation relating the output of industries and the final demand is given by:

$$X = (I - A)^{-1}F \qquad (1)$$

where X is the vector of output, F is the vector of final demand, A is the technical coefficient matrix, and $(I-A)^{-1}$ is the Leontief inverse matrix.

The elements of the Leontief inverse are coefficients which indicate the impact of a unit change in the exogenous final demand on the output of the industry. The importance of each element in analysis is that it has captured both direct as well as indirect effects arising from the interdependence of sectors or industries in the production of goods and services to meet the final demand. Thus not surprisingly, there are a panoply of impact multipliers (such income and employment multipliers, forward and backward linkages) which attempts to measure and rank the sectoral contributions in terms of value-added, job creations according, foreign exchange earnings and other economic criteria.

As a forecasting tool, equation (1) is an appealing and easy to use formula. When the vector of final demand (F) in a future period is ascertained, the output of each sector to satisfy the final demand can be estimated by simply pre-multiplying F by the Leontief inverse. The reliability of the forecast will depend both on the accuracy of the forecasted final demand vector and that of the Leontief inverse. The latter is in turn dependent on the currency and accuracy of the technical coefficient matrix A.

For projecting the Leontief inverse, the usual practice were to update the matrix A by the RAS method, before deriving the updated Leontief inverse. In this paper, we consider the merit of updating the Leontief inverse directly using the RAS method.

2. The RAS Method

For the ease of exposition, we shall consider an input output table compile for an economy partitioned into three sectors. Let A and B denote the initial (existing) and the final (updated) technical coefficient matrices respectively. The RAS method postulates that B is bi-proportionally related to A :

$$B = RAS \qquad (2)$$

where R and S are diagonal matrices

and that $\sum_j b_{ij}Q_j = U_i$ and $\sum_i b_{ij}Q_j = V_j$.

Writing the marginal constraints in the extended form for the case of a three sectors economy:

$$b_{11}Q_1+b_{12}Q_2+b_{13}Q_3 = U_1 = r_1a_{11}s_1Q_1 + r_1a_{12}s_2Q_2 + r_1a_{13}s_3Q_3 \Rightarrow \mathbf{r_1 = U_1/\sum_j a_{1j}s_jQ_j}$$

$$b_{21}Q_1+b_{22}Q_2+b_{23}Q_3 = U_2 = r_2a_{21}s_1Q_1 + r_2a_{22}s_2Q_2 + r_2a_{23}s_3Q_3 \Rightarrow \mathbf{r_2 = U_2/\sum_j a_{2j}s_jQ_j}$$

$$b_{31}Q_1+b_{32}Q_2+b_{33}Q_3 = U_3 = r_3a_{31}s_1Q_1 + r_3a_{32}s_2Q_2 + r_3a_{33}s_3Q_3 \Rightarrow \mathbf{r_3 = U_3/\sum_j a_{3j}s_jQ_j}$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\text{Column totals} = \quad V_1 \qquad V_2 \qquad V_3$$

$$\textbf{Value of s} = \frac{\mathbf{V_1}}{\sum_i \mathbf{r_i a_{i1} Q_1}} \quad \frac{\mathbf{V_2}}{\sum_i \mathbf{r_i a_{i2} Q_2}} \quad \frac{\mathbf{V_3}}{\sum_i \mathbf{r_i a_{i3} Q_3}}$$

The substitution factor for ith industry is:

$$r_i = U_i/\sum_j a_{ij}s_jQ_j \qquad (3)$$

and the fabrication factor for the jth industry is:

$$s_j = V_j/\sum_i r_ia_{ij}Q_j \qquad (4)$$

Thus an iterative procedure which is convergent (Bacharach, 1970), can be started by estimating the substitution factors (r's) by letting fabrication factor equals unity for each sector. With the computed value of the r_i in (3) they are used to compute the fabrication factors in (4). The procedure is repeated until the values of computed factors do not differ from one iteration to another.

The RAS Identity

In matrix notation the RAS identity can be written as:

$$B = RAS \equiv R^*AS^* + (R - R^*)A\,S^* + R^*A(S-S^*) + (R-R^*)A(S-S^*) \qquad (5)$$

where R* and S* are some arbitrary diagonal matrices.

The significance of (5) can be better appreciated when the identity is written for the (i-j)th element of B:

$$b_{ij} = r_i a_{ij} s_j = r_i^* a_{ij} s_j^* + (r_i - r_i^*)\, a_{ij} s_j^* + r_i^*\, a_{ij}(s_j - s_j^*) + (r_i - r_i^*)\, a_{ij}(s_j - s_j^*) \qquad (6)$$

In particular when $r_i^* = s_j^* = 1$ (indicating basis of no change),

$$b_{ij} = r_i a_{ij} s_j = a_{ij} + (r_i - 1)\, a_{ij} + a_{ij}(s_j - 1) + (r_i - 1)\, a_{ij}(s_j - 1)$$

$$b_{ij} - a_{ij} = (r_i - 1)\, a_{ij} + a_{ij}(s_j - 1) + (r_i - 1)\, a_{ij}(s_j - 1) \qquad (7)$$

The change in the technical coefficient is partitioned into three components: the first is due to row effect (substitution), the second is due to column effect (fabrication), and the third is the interaction between the row and column.

If the interaction term in (7) is ignored, then the resulting equation is equivalent to that proposed by Friedlander (1967)

$$b_{ij} - a_{ij} = (r_i - 1)\, a_{ij} + a_{ij}(s_j - 1)$$

$$= \mu_i a_{ij} + \lambda_j a_{ij} \qquad (8)$$

Using the set of marginal constraints, the factors μ's and λ's can be solved from the simultaneous equation system(Henry, 1973) :

$$U^o - U = \text{diag}(U).\mu + A.\text{diag}(Q^o).\lambda$$

$$V^o - V = \text{diag}(Q^o).A'.\mu + \text{diag}(V).\lambda \qquad (9)$$

where U^o is a vector whose ith element is the intermediate sale of the ith sector during the base year, i.e. $U^o_i = \sum_j a_{ij} Q^o_j$ and V^o is the vector of intermediate purchase during the base year. A diagonal matrix whose diagonal elements are from a vector X is denoted as diag(X).

The Friedlander method can thus be considered as a linear approximation of the RAS method. One drawback of the Friedlander method is that it does not preserve sign in the final matrix, i.e. positive entries in the initial matrix A can turn negative in the final matrix B. One suggestion to deal with this negativity problem is to add to every (i-j)th element of the final matrix the term $\mu_i a_{ij} \lambda_j$. This tantamount to recognise the presence of an interaction term as indicated in equation (7). The resulting adjusted matrix is one approximating the RAS solution.

3. Applying the RAS procedure to the Leontief Inverse

Let M denotes the initial Leontief inverse matrix and N denotes the final (updated) Leontief inverse obtained by using the RAS procedure. So

$$M = (I - A)^{-1} \quad \text{and} \quad N = HMK = (I - B)^{-1} \qquad (10)$$

For the final year, the following must hold:

$$(I - B)^{-1}F = NF = (HMK)F = Q \qquad (11)$$

where F and Q are respectively the final demand vector and gross output vector known for the final year. To avoid confusion, the letters H and K are used in place of R and S to denote the diagonal matrices of substitution and fabrication factors. Writing the system in full for the case of a three sectors economy, we have:

$$n_{11}F_1+n_{12}F_2+n_{13}F_3 = Q_1 = h_1m_{11}k_1F_1+h_1m_{12}k_2F_2+h_1m_{13}k_3F_3 \Rightarrow \mathbf{h_1 = Q_1/\sum m_{1j}k_jF_j}$$

$$n_{21}F_1+n_{22}F_2+n_{23}F_3 = Q_2 = h_2m_{21}k_1F_1+h_2m_{22}k_2F_2+h_2m_{23}k_3F_3 \Rightarrow \mathbf{h_2 = Q_2/\sum m_{2j}k_jF_j}$$

$$n_{31}F_1+n_{32}F_2+n_{33}F_3 = Q_3 = h_3m_{31}k_1F_1+h_3m_{32}k_2F_2+h_3m_{33}k_3F_3 \Rightarrow \mathbf{h_3 = Q_3/\sum m_{3j}k_jF_j}$$

The solution for the substitution factors, **h** are obtained assuming the fabrication factors (**k**) are known. Thus

$$h_i = Q_i/\sum_j m_{ij}k_jF_j \quad ; \quad i = 1,2,...,n \qquad (12)$$

The price framework corresponding to the Leontief system is:

$$P = (I - B')^{-1}W = N'W = (HMK)'W \qquad (13)$$

where P is the vector of prices for industrial products, B' is the transpose of the technical coefficient matrix in the target year, and W is a vector of industrial values added per unit of output. Writing the system for the case of a three sector economy, we have:

$n_{11}W_1+n_{21}W_2+n_{31}W_3 = P_1 = h_1m_{11}k_1W_1+h_2m_{21}k_2W_2+h_3m_{31}k_3W_3 \Rightarrow \mathbf{k_1 = P_1/\sum m_{i1}h_iW_i}$

$n_{12}W_1+n_{22}W_2+n_{32}W_3 = P_2 = h_1m_{12}k_2W_1+h_2m_{22}k_2W_2+h_3m_{32}k_2W_3 \Rightarrow \mathbf{k_2 = P_2/\sum m_{i2}h_iW_i}$

$n_{13}W_1+n_{23}W_2+n_{33}W_3 = P_3 = h_1m_{13}k_3W_1+h_2m_{23}k_3W_2+h_3m_{33}k_3W_3 \Rightarrow \mathbf{k_3 = P_3/\sum m_{i3}h_iW_i}$

The fabrication factor for the jth industry can be computed as:

$$k_j = P_j/\sum_i m_{ij}h_iW_i \quad ; \quad j = 1,2,...,n \qquad (14)$$

Once again, an iterative procedure can be set up using equations (12) and (14) to obtain estimates of the substitution and fabrication factors.

<u>The Implied Relationship between Matrix A and Matrix B</u>

The projected Leontief inverse matrix can be used to solve for the implied technical coefficient matrix. We have

$$(I - B)^{-1} = H(I - A)^{-1}K$$

$$= [K^{-1}(I - A)H^{-1}]^{-1}$$

Hence ,

$$I - B = K^{-1}(I - A)H^{-1}$$

$$= K^{-1}H^{-1} - K^{-1}AH^{-1}$$

and $$B = I - K^{-1}H^{-1} + K^{-1}AH^{-1} \qquad (15)$$

or $$B = A + (I - A) + K^{-1} (I-A)H^{-1} \qquad (16)$$

A few properties of the matrix B can be noticed from equation (15). Firstly, functional relationship between B and A is slightly more general than simply bi-proportional.

The final matrix B is a sum of a diagonal matrix ($I-K^{-1}H^{-1}$) and a bi-proportional term. Secondly, diagonal elements of B can possibly be negative[1]. This is a disadvantage as the elements of the technical coefficients are non-negative. Thirdly, zero elements of the matrix A will remain as zero entries in the matrix B. The first two properties are contrary to those of the RAS method applied directly on the A matrix.

4. Modified HM*K Method

Instead of applying RAS directly to the Leontief inverse matrix, let's consider the projection of the sum of the power series of the technical coefficient matrix by the RAS method:

$$[B + B^2 + B^3 + ...] = H. [A + A^2 + A^3 + ...].K$$

which is equivalent to:

$$(I-B)^{-1} - I = H. [(I-A)^{-1} - I].K$$

or $$N - I = H.[M-I].K = H.M^*.K \qquad (17)$$

In other words, the RAS method is applied to the matrix (M^*) formed by the Leontief Inverse minus an identity matrix. The factors, h and k can be obtained from the an iterative procedure as illustrated in the case of a three sector economy below:

$$U_1 = Q_1\text{-}F_1 = h_1m^*_{11}k_1F_1+h_1m^*_{12}k_2F_2+h_1m^*_{13}k_3F_3 \Rightarrow \mathbf{h_1 = U_1/\sum m^*_{1j}k_jF_j}$$

$$U_2 = Q_2\text{-}F_2 = h_2m^*_{21}k_1F_1+h_2m^*_{22}k_2F_2 +h_2m^*_{23}k_3F_3 \Rightarrow \mathbf{h_2 = U_2/\sum m^*_{2j}k_jF_j}$$

$$U_3 = Q_3\text{-}F_3 = h_3m^*_{31}k_1F_1+h_3m^*_{32}k_2F_2+h_3m^*_{33}k_3F_3 \Rightarrow \mathbf{h_3 = U_3/\sum m^*_{3j}k_jF_j}$$

The solution for the substitution factors, **h** are obtained assuming the fabrication factors (**k**) are known. Thus

$$h_i = U_i/\sum_j m^*_{ij}k_jF_j \quad ; \quad i = 1,2,...,n \qquad (18)$$

From the price framework corresponding to the Leontief system in the three sectors economy:

$$P_1\text{-}W_1 = h_1m^*_{11}k_1W_1+h_2m^*_{21}k_2W_2+h_3m^*_{31}k_3W_3 \Rightarrow \mathbf{k_1 = (P_1\text{-}W_1) /\sum m^*_{i1}h_iW_i}$$

$$P_2\text{-}W_2 = h_1m^*_{12}k_2W_1+h_2m^*_{22}k_2W_2+h_3m^*_{32}k_2W_3 \Rightarrow \mathbf{k_2 = (P_2\text{-}W_2) /\sum m^*_{i2}h_iW_i}$$

[1] Consider the first diagonal element of B, $b_{11} = 1 - (1-a_{11})/r_1s_1$. It is possible that $(1-a_{11}) > r_1s_1$, and so b_{11} will be negative.

$P_3\text{-}W_3 = h_1m^*_{13}k_3W_1+h_2m^*_{23}k_3W_2+h_3m^*_{33}k_3W_3 \Rightarrow \mathbf{k_3 = (P_3\text{-}W_3) /\sum m^*_{i3}h_iW_i}$

The fabrication factor for the jth industry can be computed as:

$$k_j = (P_j\text{-}W_j) /\textstyle\sum_i m^*_{ij}h_iW_i \quad ; \quad j = 1,2,...,n \qquad (19)$$

What is the implied relationship between B and A in the HM*K model?

Using the notations defined above:

$$N - I = HM^*K = H.[M - I].K$$

and so

$$N = I + H.[M - I].K \qquad (20)$$

$$(I - B) = [I + H.(M - I).K]^{-1}$$

rearranging,

$$B = I - [I + H.(M - I).K]^{-1}$$

$$= H. [KH + (M\text{-}I)^{-1}]^{-1} .K$$

$$= H. [KH - I + A^{-1}]^{-1}.K \qquad (21)$$

$$= H. [A + A\{(I\text{-}KH)^{-1} - A\}^{-1}A].K \qquad (22)$$

$$= H.Z.K \qquad (23)$$

where $Z = [A + A\{(I\text{-}KH)^{-1} - A\}^{-1}A]$

Equation (23) [2] indicates that the final matrix is bi-proportionally related to Z. In the absence of substitution and fabrication effects, H=K=I, then obviously B = A in equation (21). The properties of the B from the HM*K method will be identical to that of the standard RAS method if the matrix $\{(I\text{-}KH)^{-1} - A\}^{-1}$ in equation (22) is always non-negative[3]. However, this is not necessary so and hence elements of B can be possibly be negative. In the empirical exercise reported in section 5, there four out of

[2] The derivation of the equations prior to equation (23) are obtained by repeated use of the matrix relationship : $[A\text{-}BD^{-1}C]^{-1} = A^{-1} + A^{-1}BGCA^{-1}$; where $G = [D\text{-}CA^{-1}B]^{-1}$.

[3] The matrix of technical coefficient A is a non-negative matrix. But the matrix $\{(I\text{-}KH)^{-1} - A\}$ is not a dominant diagonal matrix, hence the product of its inverse and matrix A gives a matrix which may have negative elements. One sufficient condition (but not necessary) for $\{(I\text{-}KH)^{-1} - A\}$ to be a dominant diagonal matrix is $h_ik_i < 1$ for every i. A discussion on the properties of dominant diagonal matrices can be found in Heal, Hughes and Tarling(1974).

a hundred elements which are negative. However, all the negative elements are very small and they correspond to the zero elements in the initial matrix A.

Furthermore, using the RAS identity, equation (20) can be written as:

$$N - M = (R-I)M^* + M^*(S-I) + (R-I)M^*(S-I) \quad (24)$$

which indicates that the change in the Leontief Inverse can be decomposed into three components: the row effect, column effect and the interaction effect.

As shown in Toh(1998), the estimated substitution and fabrication factors can be interpreted as instrumental variable estimates. The asymptotic variances of the IV estimators are :

$$AVar(h_i) = \sigma^2.\Sigma_j F_j^2/[\Sigma_j k_j^* m^*_{ij} F_j]^2 \quad (25)$$

and

$$AVar(k_j) = \sigma^2.\ \Sigma_i W_i^2\ /[\Sigma_i h_i^* m^*_{ij}\ W_i]^2 \quad (26)$$

where h_i^* and k_j^* are values of h_i and k_j at the end of the convergent iterative procedure, and σ^2 is the variance of the error term[4].

The availability of the variances of h_i and k_j allows the construction of confidence interval for the projected multiplier coefficients based on the HM*K method. Denoting the predicted technical coefficient as n_{ij}^f, then

$$n_{ij}^f = h_i.m^*_{ij}.k_j$$

The asymptotic variance of b_{ij}^f can be approximated[5] by:

$$AVar(n_{ij}^f) = (m^*_{ij}.k_j)^2.AVar(h_i) + (h_i.m^*_{ij})^2.AVar(k_j) \quad (27)$$

The forecast of the output of the ith industry, given the vector of final demands (F*) is

$$Q_i^f = \Sigma_j\ n_{ij}F^*_j \quad (28)$$

and the associated approximate variance of this forecast will be[6]:

$$Var(Q_i^f) = \Sigma_j\ F^{*2}_j.AVar(n_{ij}) \quad (29)$$

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[4] The underlying statistical model is: $n_{ij} = h_i m^*_{ij} k_j + e_{ij}$; where e_{ij} is the error term possessing the usual properties that $E(e_{ij})=0$ and $Var(e_{ij})=s^2$ for all i and j.

[5] The covariance between h_i and k_j is assumed to be negligible in the formulae.

[6] The final demand vector, F* is assumed to be known with no error.

4. Empirical Illustration

As an empirical illustration of the computation suggested in the previous sections, the data from three available input-output tables for the Singapore economy were used. The tables compiled at five years interval are for the years 1978, 1983 and 1988. In the following we present our computation based on the industries being aggregated into 10 sectors according to the standard classification in the national account.

In Table 1, the usual statistical measures of accuracy were computed to evaluate the performance of the RAS method applied to the matrix A directly and the HM*K method in predicting the technical coefficient matrix A and the Leontief inverse matrix[7]. For the projection of the matrix A for the year 1983, the HM*K method is able to provide a higher level of accuracy than that obtained by the usual RAS method. However, in projection the matrix A for 1988, the usual RAS method appears to be the better method according to the statistical measures of accuracy present. In the case of projection of the Leontief inverse, what is said of the projection of matrix A applies. However, the margin of superiority is not a substantial one. Both methods seem equally good in updating the technical coefficient matrix and the Leontief inverse.

Table 1 : Comparing Prediction by the Alternative RAS Method

		RAS	***RAS***	***HM*K[+]***	***HM*K[+]***
		1978 - 83[a]	***1983 - 88[b]***	***1978 - 83[a]***	***1983 - 88[b]***
Matrix A	***R-SQ***	0.8709	0.9966	0.9997	0.8370
	Chi-Square	0.9724	0.4712	0.0183	1.2728
	MSE[*]	0.0003	0.0003	0.0069	0.6142
	MAE[+]	0.0094	0.0103	0.0006	0.0081
Leontief inverse	***R-SQ***	0.9484	0.9992	0.9999	0.9969
	Chi-Square	0.4924	0.1280	0.0033	0.5062
	MSE	0.0001	0.0001	0.0030	0.4438
	MAE	0.0048	0.0052	0.0007	0.0086

Notes:
a The initial matrix is for 1978 and the final matrix is for 1983;
b The initial matrix is for 1983 and the final matrix is for 1988;
+ HM*K : Applying RAS to the matrix $[(I-A)^{-1} - I]$

[7] The predictions of A and the Leontief Inverse using RAS directly on the Leontief inverse were done. However, the diagonal elements of the implied B matrix were substantially negative for two sectors. Hence the results are not reported together with those from the HM*K method.

R-SQ : Square of the correlation coefficient between the actual and predicted;
Chi-Square = $\sum(\text{observed- expected})^2/\text{expected}$
MSE = Mean Square Error; MAE = Mean Absolute Error

Table 2 : Distribution of Absolute Forecast Error

Method of Projection	**RAS Method Applied to Matrix A**		**HM*K Method RAS Applied to the Matrix $M^* = (I-A)^{-1} - I$**	
Projecting Matrix A	**A 78-83**	**A 83-88**	**A 78-83**	**A 83-88**
	A	***B***	***C***	***D***
L.T. 0.0005	11	11	54	16
0.0005 -- 0.0010	9	9	27	9
0.0010 -- 0.0050	39	36	19	39
0.0050 -- 0.0100	15	14	0	13
0.0100 -- 0.0200	12	12	0	12
0.0200 -- 0.0300	7	10	0	4
0.0300 -- 0.0400	1	2	0	3
0.0400 -- 0.0500	1	0	0	2
G.E. 0.0500	5	6	0	2
Total	100	100	100	100
Projecting Matrix $(I-A)^{-1}$	**$(I-A)^{-1}$ 78-83**	**$(I-A)^{-1}$ 83-88**	**$(I-A)^{-1}$ 78-83**	**$(I-A)^{-1}$ 83-88**
	A1	***B1***	***C1***	***D1***
L.T. 0.0005	21	18	51	13
0.0005 -- 0.0010	13	13	24	12
0.0010 -- 0.0050	36	38	25	36
0.0050 -- 0.0100	14	13	0	17
0.0100 -- 0.0200	15	14	0	11
0.0200 -- 0.0300	0	3	0	2
0.0300 -- 0.0400	0	0	0	5
0.0400 -- 0.0500	0	0	0	1
G.E. 0.0500	1	1	0	3
Total	100	100	100	100

***Note**: <A> Predicting 1983 I-O coefficients based on applying RAS to 1978 coefficients.*
<B> Predicting 1988 I-O coefficients based on applying RAS to 1983 coefficients.
*<C> Predicting 1983 I-O coefficients based on applying HM*K to 1978 inverse.*
*<D> Predicting 1988 I-O coefficients based on applying HM*K to 1983 inverse.*
Columns A1, B1, C1 and D1 are the corresponding distributions for projecting the multipliers, i.e. the elements of the Leontief inverse.

The forecast errors were subjected to further analysis in Table 2. A frequency distribution of the absolute forecast error for each of the cases is shown in Table 2. In congruence with the conclusion reached earlier, the application of the RAS method to the Leontief inverse (HM*K method) does not seem to provide more accurate projection of the technical coefficients or the multipliers than applying the RAS method directly on the coefficient matrix A. However, even if the HM*K method cannot claim superiority, its loss in projection accuracy appear marginal and can be outweighed by the convenience gained in computation of forecasts and the associated statistical confidence intervals of the forecasts.

In Table 3, the output multipliers for each of the sectors are presented. The output multiplier for the jth sector is given by the sum of the elements in the jth column of the Leontief inverse. It has the simple interpretation that for a unit increase in the final demand for the product of the jth sector, the output multiplier measures the additional output (from all sectors) in the economy. The output multiplier has been used extensively by development economists in measuring the backward linkages of sectors to help identification of key sectors targeted for preferential assistance[8]. Algebraically, the output multiplier for the jth sector is:

$$\phi_j = \sum_i m_{ij} \quad ; \qquad \text{where } m_{ij} \text{ are the (i-j)th element of the Leontief inverse.}$$

Column 2 of Table 3 presents the output multipliers based on the projected 1988 Leontief inverse matrix. With the exception of the agriculture/forestry sector, the absolute percentage error is less than one per cent for all sectors. The corresponding standard errors of the forecasts are shown in column 5. All sectors have the actual values of the output multipliers contained within the 95% confidence intervals shown in columns (6) and (7).

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[8] For more detail discussion and other references, see chapter 11 of Bulmer -Thomas(1982).

Table 3: PROJECTED OUTPUT MULTIPLIERS

	ACTUAL 1988	*HM*K 1988*	*Forecast Error*	*Absolute Percentage Error*	*Standard Error of Forecast*	*Lower Bound of 95% C.I.*	*Upper Bound of 95% C.I.*
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*
AGR/FOREST	1.4334	1.4381	-0.0047	1.24	0.0217	1.3722	1.4591
MANUFACT-URING	1.2565	1.2566	-0.0001	0.06	0.0152	1.2268	1.2878
UTILITY	1.3144	1.3891	-0.0748	0.53	0.0205	1.2804	1.3623
CONSTRUCT	1.3838	1.4106	-0.0268	0.73	0.0257	1.3224	1.4250
COMMERCE	1.5369	1.5456	-0.0088	0.28	0.0115	1.5181	1.5642
TRANSPORT	1.2665	1.2661	0.0004	0.18	0.0119	1.2403	1.2881
COMMS	1.1833	1.1872	-0.0039	0.14	0.0089	1.1671	1.2027
FINANCE	1.3288	1.3307	-0.0019	0.13	0.0096	1.3114	1.3496
BIZ SERVICE	1.4039	1.4131	-0.0092	0.40	0.0127	1.3842	1.4349
OTHER SERVICES	1.3088	1.3067	0.0021	0.24	0.0127	1.2802	1.3310

Notes:
Column (1) is the actual 1988 output multipliers
*Column (2) is the projected output multipliers using the HM*K method;*
Column (3) is the forecast error = column(1)-column(2)
Columns (6) and (7) are respectively the lower and upper bounds of the 95% confidence intervals of the forecast.

In Table 4, using the RAS identity in equation (5), the output multipliers projected by the HM*K method are decomposed into the three components. In the bi-proportional model, the multipliers can only be solved in relative terms. To overcome this, we follow Van der Linden and Dietzenbacher (1997) in asserting that the sum of all substitution effects equals zero. This implies that the restriction $\sum\varphi_i h_i = 1$, where $\varphi_i=U_i/\sum_j U_j$. Three sectors, agriculture/forestry, utility and construction have positive row effects, and the largest of which is recorded for the construction sector. Only two sectors, transportation and communications have positive figures for both the column and row effects. With the exception of the transportation and communication sector, the interaction effects are negative across all sectors.

Table 4: DECOMPOSITION OF OUTPUT MULTIPLIERS, 1988

	ACTUAL 1988	HM*K 1988	BASE 1983	Row Effect	Column Effect	Interaction	Forecast Error
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
AGR/FOREST	1.4334	1.4381	1.4682	0.0042	-0.0563	-0.0005	-0.0047
MANUACT-URING	1.2565	1.2566	1.1928	-0.0008	0.0656	-0.0003	-0.0001
UTILITY	1.3144	1.3891	1.5439	0.0148	-0.2311	-0.0063	-0.0748
CONSTRUCT	1.3838	1.4106	1.4475	0.0191	-0.0892	-0.0038	-0.0268
COMMERCE	1.5369	1.5456	1.5224	-0.0402	0.0639	-0.0049	-0.0088
TRANSPORT	1.2665	1.2661	1.3406	-0.0132	-0.0657	0.0025	0.0004
COMMS	1.1833	1.1872	1.2087	-0.0117	-0.0128	0.0007	-0.0039
FINANCE	1.3288	1.3307	1.3271	-0.0205	0.0255	-0.0016	-0.0019
BIZ SERVICE	1.4039	1.4131	1.3280	-0.0192	0.1070	-0.0063	-0.0092
OTHER SERVICES	1.3088	1.3067	1.3032	-0.0038	0.0063	-0.0001	0.0021

Notes:
Column (1) is the actual 1988 output multipliers
*Column (2) is the projected output multipliers using the HM*K method;*
Column (3) is the actual 1983 output multipliers;
Columns (4), (5) and (6) are the components computed according to the RAS identity;
Column (7) is the forecast error = column (1) - column (2)

5. Conclusion

In this paper we consider the projection of the Leontief inverse matrix by using the RAS method on a initial Leontief inverse matrix. The varied uses of the Leontief inverse provide ample justification for the elements of the Leontief inverse matrix to be projected directly rather than as a by-product of projecting the technical coefficient matrix. In applying the RAS method directly to the Leontief Inverse matrix (HMK method) , we encounter the problem of having diagonal elements of the projected inverse being less than unity which implies the associated technical coefficient matrix will have negative diagonal elements. A modified method (HM*K method) which involves applying RAS to the Leontief inverse minus an identity matrix, corrected the major problem of the HMK method. The HM*K still possess drawback of giving negative elements in the implied technical coefficient matrix but is unlikely to be serious and prevalent empirically. There are several advantages of the HM*K method. It begins with an initial matrix which is more *dense*, i.e. there less elements with zero value, which enables higher computational accuracy. Moreover, it yields direct estimates of the Leontief inverse which is in turn used frequently for projecting output and other economic multipliers. Furthermore, the computation of asymptotic standard errors of the forecasts are more easily effected than in the standard RAS method.

In the empirical exercise, the conventional RAS method is compared with the HM*K method. The HM*K method cannot claim superiority in forecast performance. However, the projections by both method are similar. Perhaps more empirical work will have to be done to provide a more definitive answer.

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