Dynamic Extended Input-Output Models

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Introduction

One of the important areas of recent 20 years development, in the field of input-output analysis has been modelling of the linkage between industrial input-output and household activity, especially at regional context. The linkage between household activity and industrial activity are modelled in an input-output analysis by treating household as an ordinary industry, which produces labor and consumes industrial products and included in the transaction matrix. Extended input-output model has been introduced by adding rows and columns to the interindustry flow matrix. A number of different approaches have seen to the extension of input-output models, Some of these work have been based upon the pioneering projects reporting in Mirenyk et. al. (1967), which explored the effects of a rapidly expanding local economy, such as (Batey, Madden.and Weeks 1987, Blackwell 1977, Sadler et. al. 1973, Tiebout 1969). The most interesting of these approaches are those which concentrate on economic-demographic characteristics of the household. The work of (Schunar 1976; Stone 1981; Batey and Madden 1980; 1981; 1983; Van Dijk and Oasterhaven 1986) in particular, has been important in demonstrating the value of input-output analysis as a framework for studying the interrelationships between demographic and economic variables.

The aim of the present paper is a primary preface to the dynamic extended model. first, to examine a sequence of static model elaboration, the sequence can be divided into a set of stages, as starting with the simplest form of static model which household consumption is treated as a component of final demand, and leading eventually to a comprehensive extended model. A large degree of extended model conforms to the principles that apply to a Leontief input-output system, and the only differences concern the presence of positive coefficients in some off diagonal cell of input coefficients matrix (Miller & Blair 1985). Review includes four general types of extended model, and specifically paid more attention to two model and their equations which are going to introduce their dynamic model. Then introduce a set of basic elements and assumptions to dynamic model, and develop two dynamic extended input-output models, the first of which takes account of

different income groups of household, secondly extended model has been developed to take account of household into two separate groups of employed and unemployed workers.

Static Extended Models

Extensions to input-output models have a long history, there has been an increased awareness of the need to improve the specification of household in an input-output model by different scholars. A variety of different approaches developed to household disaggrigation and typically include the study of household income, consumption, income distribution, labor force participation, migration, employment and unemployment, and industrial output. Going back further to the earliest work on the concept by Leontief in 1941. In the simple Leontief input-output model, final demand is exogenous and includes consumption purchases by households, government and export. In the case of households, they earn incomes in payment for their labor inputs in the production process and as consumers they buy goods for final consumption. This simple model, i.e. household-exogenous input-output model, characterized as type I Leontief model (Leontief 1941). It is given by:

$$x = (1-A)^{-1}.\, f \qquad (1)$$

Where x is gross output,

A is technical coefficients,

f is final demand.

In type 1 model the impact of household consuming of industrial output to be assessed, but the effect of a change in industrial output might have upon household income and expenditure, has been ignored. In other words, multipliers ratios of direct and indirect household income changes is specified but induced effects of the presence of household in the economy has not been modeled.

The first development that can be describe as an extension was the closure of the model with respect to the households and involves quite simply the expansion of the simple type I To overcome exogenousity problem, household sector has been transformed from the final demand column and placed inside the technical interrelated table, i.e. make it as one of the endogenous sectors, and more extended models have been provided. With respect to households they are treated endogenously and assumed to behave like other industrial sectors whose outputs are labors and whose inputs are consumption, they were entered into the transaction matrix of input-output model. It is assumed to behave like other industrial sectors with a linear and homogenous consumption function.

So much attempts have been made to disaggregate household into sub-groups. In order to create homogeneous expenditure and earnings categories in the household-endogenous model, a number of studies have adopted a disaggregated household sector. Disaggregation can take a variety of forms under subtitled of type II, III and IV. The model developed by Miyazawa (1976) to disaggregate household into a number of income groups under the assumption of consumption homogeneity, it is called extended input-output model type II. Equations and variables are as follow;

$$\begin{bmatrix} (1-A) & -h_c \\ -h_r & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_H \end{bmatrix} = \begin{bmatrix} d_1 \\ d_h \end{bmatrix} \qquad (2)$$

h_c is a column vector of household consumption,

h_r is a row vector of income from employed coefficients,

X_H is household income, and

d_H is exogenous household income, i.e. income received by resident living in the study area from source outside the area.

and the equations are:

$$(I - A)\,X_1 - h_c(X_H) = d_1$$

$$-h_r(X_1) + X_H = d_H$$

Several criticisms can be made of this form of the extended model. Batey and Madden (1983) suggested that to overcome of the dilemmas household sector should be accompanied by its disaggregation into a number of more homogenous expenditure and earning sub-groups.

- **The first** criticism arises from the assumption of a linear and homogeneous consumption function: As households are confined to a single row and column in the model, i.e. one pattern of household consumption is presented, all kinds of household are assumed to have the same wage rate and consumption propensities. This is clearly an unrealistic assumption since any study area can be expected to contain a mixed assembly of households exhibiting widely different consumption patterns (Batey and Madden 1983). Any changes in income and consumption of households as being immediately related to each other, it is clear that decreases in wages to labor do not mean identical decreases in household consumption. As household income falls, or removed completely by redundancy, households do not necessarily spend correspondingly, or disappear altogether from the system and in the reality social security or unemployment benefits take the place of income from employment (Betey & Madden 1981).

- **Secondly**, migrant flows are important elements in the economic system, introducing new consumer into regional economies, or removing existing consumers, so the treatment of migration in an extended input-output model is essential, which is ignored in this model.

- **Thirdly**, propensities consumption are impilicity assumed to apply exclusively to employed households. Unemployed households consumption is treated exogenously as a part of final demand, if it is not considered so it is not influenced by the consumption of employed households.

- **Fourthly**, in this model it is not clear the assumption for the source of newly-employed workers: are they from the local labor or migrants? The impact of their existence before taking up employment has been ignored (Batey P. W. J. 1985, Batey P. W. J.& Weeks M.1988, Batey P. W. J. 1990).

It was an effort to overcome these problems that Mineryk and his colleagues developed a new form of input-output model for their study of the impact of the space program for Boulder, Colorado (Mineryk et el, 1967). To circumvent the problem of linearity of consumption function, Mineryk sub-divided existing workers into a number of income groups, each with different propensity to consume within the local economy, and extended input-output model type III has been provided. Nevertheless, Madden and Batey (1983) believes that Mirenyk et. el. (1967) take a household exogenous input-output model and by interaction develop the induced effects of changes in final demand. Batey, Madden and Weeks (1987) have shown that the Mineryk and Boulder's model as a system of simultaneous equations or an activity-commodity framework. More household disaggregations have been developed on the Mirenyk models, which included workers, who travel between residence and place of work and residence and place of shopping (Madden 1985), and also other study by specifying previous residence of workers (Blackwell 1977) and so on.

A series of approaches have been presented on extended input-output model, which is called type IV by different authors in recent years. The most important characteristics of this type, which has been ignored in other studies, are as follow:

1. The links between economic and demographic models, two links have been specified as the economic-demographic and demographic-economic interfaces. The first of these links represents the effects of that economic factor have on population and the second the effect that demographic factors have on an economy (Batey and Madden 1980).

2. They have identified a particular in-consistency, which arises on the household-endogenous model, concerning unemployment rate (Batey and Madden 1983).

3. They have demonstrated two approaches to the solution of the problem of demographic changes, one based upon an iterative technique and the other using simultaneous method offered by activity analysis, for economic changes, and explored that these two approaches yield identical results (Batey and Madden 1983).

4. They have recognized the importance of modeling the social security payment received by unemployed persons and by old age pensioners (Batey and Madden 1983). Years later Madden and Trigg paid more attention to the migration and unemployment in the extended input-output model and developed last model which was included only one group of migration and unemployment (Madden and Trigg 1990). So they introduced new column for coefficient matrix which is the consumption propensity of unemployed migrants (in most cases to be same as indigenous person) income region and also in two regions formulation. Else where Madden proposed a number of developments to the models of Madden and Trigg(1990) that are intended to remedy that failure. Two levels of unemployment benefits or welfare payments: indigenous and in-migrant were introduced (Madden 1993).

5. They paid more attention to design, construction, application and sensitivity testing of the model, based on the principles of extended input-output analysis, at the metropolitan area level by developing a sub-regional input-output model. For this purpose workforce has been divided into three subgroups, these being employed, short-term unemployed and long-term unemployed or economically inactive workers. This enables them to separate out income received by workers from employment, welfare payments made to the short-term unemployment and those made to the long-term unemployed or economically inactive (Batey, Madden, Scholefield 1993).

6. Madden (1993) introduced that government explicity into the system as a (quasi-) economic sector with different rates of taxation on expenditure, he assumed three different categories of consumers reflecting the interrelationships of different income levels.

A simple extended input-output model type VI, which has been formulated by Batey, Madden and Scholefield (1993), is given by:

$$\begin{bmatrix} X_1 \\ X_h \\ s \cdot u \end{bmatrix} = \begin{bmatrix} I-A & -h_c^e & -h_c^u \\ -h^a & 1 & 0 \\ -s.l & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} d_1 \\ d_h \\ s \cdot p \end{bmatrix} \qquad (3)$$

Where;

X_1 is industrial gross output,

X_h is total income to employed worker,

h_c^e is the consumption propensity vector of employed workers,

h_c^u is the consumption propensity vector of unemployed workers,

h^a is income coefficient vector of employer workers,

l is the vector of labour demand coefficients,

P is total number of workers,

s is welfare benefit payable to one unemployed worker,

u is the number of unemployed workers.

The equations here are:

$$(I-A)X_1 - h_c^e \cdot X_h - h_c^u \cdot s \cdot u = d_1$$

$$-h^a X_1 + X_h = d_h$$

$$l \cdot X_1 + u = p$$

In type IV model and its equations employment and unemployment groups can be divided into the subgroups with different propensities to consume, or migration and indigenous, or welfare benefits, for simplicity they have been considered as a group. Different multiplier which can be combined to form income multipliers, production multipliers and employment multipliers represent the effect of explicity modeled demographic-economic interaction (Batey and Madden 1983).

Batey and Rose (1990) have presented a critical survey of researches on extended input-output models, they mentioned extended input-output models have analyzed the effects of output and employment changes as an exogenous increase in regional planning. But they have failed in the regional economy and other elements in a regional system: population, transportation, energy, environment, etc. Further problem which, static extended input-output model is dealing with referred to its assumptions that we hope can solve few of them in dynamic extended input-output model.

Dynamic Extended Input-Output Model

In this section first approach on dynamic extended model which has been formulated by Batey and Madden (1983) introduced. Secondly, the assumptions which have been recognized, are required for dynamic purpose are discussed, and thirdly three dynamic extended input-output models are presented.

Batey and Madden developed a (quasi-) dynamic extended input-output model by the form of:

$$(I - A - B)X_{it} + B\,X_{it-1} - H_c\,X_{ht} = d_t$$

Where,

B is a matrix of capital coefficients, representing stocks of industries used per unit of output of industry,

t is time superscript,

H_c is the matrix of consumption vector, and

X_h is the subset of household turnover activities.

They have mentioned that a new B matrix is introduced into the system, but no more discussion has been build on it, moreover they explained problems are dealing with dynamic model at regional level such as: availability of data on capital stock measures, relationships of capacity to output, the definition of the capacity and so on. They have emphasized that changes in demographic variables will have to be obtained by estimation. This is an introduction to the dynamic extended input-output model.

Assumptions on Dynamic Extended Model

Even though many authors have attempted too much to overcome the dilemmas which are dealing with the static extended input-output models. Some problems have been remained in the static extended model which we believe that can be solved by dynamizing extended input-output model are as follow:

- In the static extended input-output model is assumed implicity that households consume all of their income, or if assumed that household (or worker in some models), consumes a proportion (average or marginal propensity to consume) of their income. The rest of what they earn has not been modeled. And it is satisfied for industries. In the other words can be mentioned that in static extended input-output model saving has been ignored. Although Stone and Weale (1986) described a model with saving and investment but they are considered as exogenous variables. In the model which we are going to introduce saving is modeled as an endogenous variable.

- Each industry sells all of their products and nothing remained. As is clear producer needs to have raw materials and intermediate goods at least few months before it would be used in the production process. Of course all of industry's production are not being sold immediately, so some of the finished goods even

intermediate goods, raw materials have been remained for next period of production process. If one says that the stock of industries are included into the capital formation. Other question is in mind: changes in stock is endogenous or exogenous? If it is exogenous so should be as a part of final demand whilst naturally is depends on the output level, so could be endogenous.

- In the static extended input-output investment is exogenous and is a part of final demand. Which is called capital formation. In the economic point of view investment is a function of the output, so could be introduce as an endogenous in the dynamic extended input-output model.

To overcome above dilemma dynamic extended models have been provided which in the next section dynamic models corresponding two types of static extended model will be discussed.

For the dynamic purpose, some assumptions are required for model which we are going to introduced in this part. It is assumed that households do not spend their incomes completely, they save a part of their income as saving for the future and invest it as different kinds of investment have been clarified such as; investment into the bank with the certain profit or buying shares or bonds to take part in industries activities. So their income includes wages or salary from the work and profit from investment, both of them have been modelled. Their income wages are inside the transaction matrix and their profit could be inside the intersectoral capital coefficients matrix which define in the dynamic input-output model. It should be mentioned that transaction matrix and capital coefficients matrix have same dimension by this definition. Besides, households capital formation has been excluded from the final demand and in dynamic extended model one column of the final demand should be transferred into the intersectoral capital coefficient matrix. Two dynamic extended models developed and their equations have been formulated. First, dynamic extended model which is corresponding into the type II is as follow:

$$\begin{bmatrix}(1-A) & -h_c\\ -h_r & 1\end{bmatrix}\begin{bmatrix}X_1\\ X_H\end{bmatrix}-\begin{bmatrix}B & h_s\\ h_p & b\end{bmatrix}\begin{bmatrix}\Delta X_t\\ \Delta X_H\end{bmatrix}=\begin{bmatrix}d_1\\ d_H\end{bmatrix} \qquad (4)$$

where,

B is inter-sectoral capital coefficients matrix in the conventional dynamic input-output model,

h_s is column of saving output ratio (propensity to saving) or $(1-h_c)$,

h_p is row of profit to output ratio (profit on saving),

b is a scalar of intrahousehold coefficients (could be zero),

ΔX_t is vector of output growth,

ΔX_H is scalar of income growth,

d_1 is a vector of final demand,

d_H is exogenous income received by workers.

Equations here are:

$$(I-A)X_1 - h_c X_H - B\Delta X_1 - h_s \Delta X_H = d_1$$
$$-h_r X_1 + X_H - h_p \Delta X_1 - b\Delta X_h = d_H$$

First of the above equation is the usual Leontief formulation with $h_c X_H$ household consumption, $B\Delta X_1$ the sock of industries, and $h_s \Delta X_H$ household saving. Second equation is a set levels of income, $h_r X_1$ wage payment to labor, $h_p \Delta X_1$ profit received by workers on saving. More disaggregation can be provided on the household saving such as; indigenous household saving, in-migrant household saving and unemployment household saving. Now, second dynamic extended model for static extended input-output type IV with two household groups i.e. employment and unemployment groups provided, which may be interesting. The formulation are here:

$$\begin{bmatrix} I-A & -h_c^e & -h_c^u \\ -h^a & 1 & 0 \\ -sl & 0 & 1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_h \\ su \end{bmatrix}-\begin{bmatrix} B & h_s^e & h_s^u \\ 0 & r\,h_s^e & 0 \\ 0 & 0 & r\,h_s^u \end{bmatrix}\begin{bmatrix} \Delta X_1 \\ \Delta X_h \\ \Delta s.u=0 \end{bmatrix}=\begin{bmatrix} d_1 \\ d_h \\ s.p \end{bmatrix} \qquad (5)$$

where.

h_s^e is saving output ratio for employed household or propensity to saving for employed workers,

h_s^u is saving output ratio for unemployment household or propensity to saving for unemployed workers,

r is interest rate on saving account or profit rate on investment,

$\Delta s.u$ is equal to zero because the amount of benefit is legislated by the policy makers and in short term is not being changed.

Equations are here:

$$(I-A)X_1-h_c^e\cdot X_H-h_c^u\cdot s\cdot u-B\Delta X_1-h_s^e\Delta X_H-h_s^u\Delta s.u=d_1$$

$$-h^a X_1+X_H-r\,h_s^e\Delta X_H=d_h$$

$$l\cdot X_1+u=p$$

in the above equations should define some variables:

$h_c^e X_H$ is employed household consumption,

$h_c^u.s.u$ is unemployed household consumption,

$B\Delta X_1$ is investment by industries,

$h_s^e\Delta X_H$ is employment household saving,

$h_s^e\Delta s.u$ is unemployment household saving,

$r\,h_s^e\Delta X_H$ is income receipt from saving or investment of employed household.

Model (5) can be more extended by diaggregating employed households into the income groups, and unemployed workers into the short time and long term unemployed workers. In an experimental works data avaiblitlity is the main point to construct the model. The next step is application of the model in an experimental works.

Conclusion

Static extended input-output model is notably broader than in conventional input-output models, and typically includes the study of household income and consumption and the interactions among various institutions: income distribution, migration, labor force participation, employment and unemployment, and industrial output. Dynamic extended input-output model is also notably broader than in conventional dynamic input-output models, which includes the study of household saving, profit and industrial investment.

Extended input-output has many benefits, one of the principle benefit of extended input-output models lies in the calculation of impact multipliers and forecasting industries investments. A wide variety of multipliers may be derived, which are analogous to income, employment, unemployment, migration. Regarding to the dynamic extended model, it would be interesting to pay attention to the variety of multipliers same as static model in the further research.

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