THE END OF TECHNICAL COEFFICIENTS?

An attempt of visual approach to multisectoral dynamics

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1. Introduction

The concept of technical coefficient is one of the fundamentals of the Leontief paradigm which is based essentially on the assumptions that each sector produces one and only one good and the technology used implies fixed coefficients.

These assumptions have created a powerful and reassuring tool for the applied researcher both in the field of data observation (national accounts) and data analysis (Leontief model).

However the presence technologies where input requirements depend on the scale of the sectoral activity is a phenomenon which characterises the process of production. This fact and the increasing prevalence, within the set of goods composing output, of immaterial productions, makes the fixed technical coefficient appear, according the critic analysts, a dated instrument progressively unsuitable to cope with the most relevant and fluid phenomena of the value production of our days.

The difficulty of treatment of the problem, posed in terms of scale dependent input coefficients, in fact, refrains many researchers to try to direct their attention to this possible development. With the result that analysis is kept within the constant coefficient approach which, though limited, is much more submissive. A consolation for applied workers with few data and a lot of ambitions.

2. Dynamics of weak interactions

With the Leontief technical coefficient, technology has had been introduced in macroeconomic modelling, through the introduction of the input requirements. The definition of input requirements, at least in theory, is done through a technological assumption which seem to allow, in principle, their definition in material terms. In practice, however, the reference to an accounting system deriving form SNA, realizes the definition of technical coefficient in terms of an "expenditure" coefficient that practically defines a demand component: the intermediate demand.

The explicitation of this demand component should be relevant also in aggregated macroeconomic models, since intermediate demand usually amounts to a figure of the same relevance of consumption expenditures, and its behaviour does not depend on the same explicative variables. This two considerations should not allow, even in the aggregated income models, the omission of this demand component.

In a multisectoral model it is impossible to omit this component since the accounting identity for which output equals value added no more holds and sectoral output is in principle different from sectoral value added, even in the case that one should adopt a definition of output in terms of "final" products.

Multisectoral interaction, based on the traditional concept of technical coefficient, operates starting from the level of sectoral output through the determination of sectoral input requirements, according fixed technical coefficients. This type of interaction can be defined "weak interaction" since the level of activity doesn't influence the technical coefficient itself.

Weak technical coefficient, i.e. the technical coefficient in the case of weak interaction, is not only an assumption on a demand component of whatever importance, rather it becomes a milestone in the conventional wisdom on multisectoral framework, in particular on multisectoral dynamics.

In this field the restrictions posed by the weak technical coefficient appear more and more severe[1]. The implications of Perron Frobenius theorem on the (weak) Leontief model reflect on two aspects. On the one side all the sectors' outputs grow at the same constant rate, maintaining the same structure. This feature transforms the multisectoral problem into an aggregated one, since, given the feature of global instability, we can concentrate only on those behaviours that do not imply a change in the inner structure of macroeconomic variables, making useless the multisectoral approach.

With weak technical coefficients the only type of dynamic behavior is that represented in Fig. 1. If initial conditions are given according the structure established by the Frobenius eigenvector u_F than sectoral aggregates will stay positive and grow at the rate λ_F^{-1}. Outside this path all trajectories will be attracted by the dominating eigenvalue, λ_D, towards the structures established by its associated eigenvector,u_D, where aggregates become negative after some interactions.

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[1] Given an nxn technical coefficients matrix A and an nxn capital coefficients matrix B, dynamic Leontief model can be written as:

$$x_{t+1} = [I + B^{-1}(I - A)]x_t - B^{-1} f_t$$

Matrix $(I - A)^{-1} B$, by the Perron-Frobenius theorem admits a dominating eigenvalue λ^F with associated eigenvector u^F with all positive elements. The dynamic Leontief model can mantain positive solutions only if initial conditions are given according the proportions determined by u^F. The constant growth factor is given by $1+1/\lambda^F$ and the growth rate path will be unstable since eigenvalue $1+1/\lambda^F$ is the smallest eigenvalue of matrix $[I + B^{-1}(I - A)]$.

On the other hand if initial endowments are not consistent with those associated with the balanced growth path, then these endowments will not adjust towards the required endowments leading to negative values in the solutions.

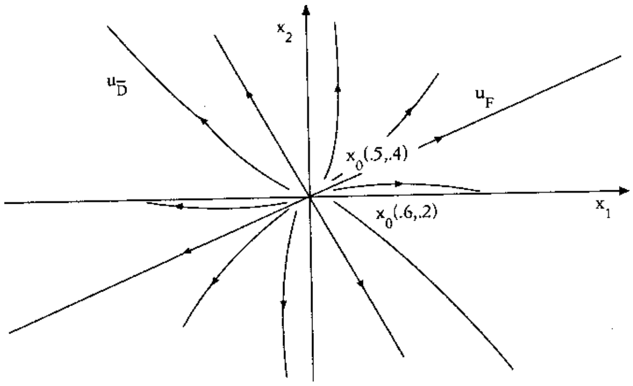

Fig. 1: State space diagram for a two sector model with weak interaction

Weak technical coefficient[2] *a* represents the unit-output input requirement and capital coefficient *b* represents the unit-output fixed capital requirement. The dynamic model is written as:

$$(1) \qquad x_{t+1} = \{1+[1/b](1-a)\}\, x_t - [1/b]\, f_t.$$

Assuming final demand equal to zero the model will admit the origin as stationary state point. Scalar $r=[1/b](1-a)$ represents the balanced growth rate since, starting from non zero initial conditions, x_0, the system will grow by a factor $(1+r)$ in each period. Standard assumptions on coefficients state that the capital coefficient is positive, $b>0$, and technical coefficient is located between zero and one. The system will be then progressively repelled from the origin. This behaviour can be appreciated through the use of the Cobweb diagram. Fig.2 shows the graphical construction of the systems' trajectory $(x_0,x_1,x_2,...)$ starting from initial conditions that are given around the fixed point $x_f = 0$. It is given by the intersection of the function graph *f(x)*, in this case a line with slope *r*, with the bisettrix $x_{t+1}=x_t$.

[2] To avoid notational complications we will refer to the one sector model as long as possible.

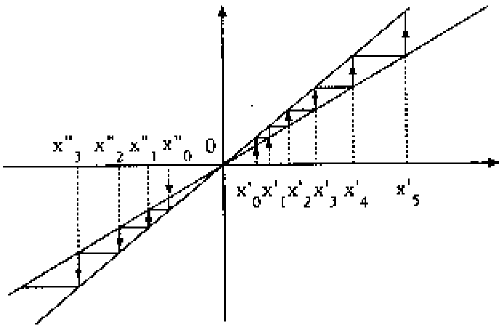

Fig. 1: State space diagram for a two sector model with weak interaction

This is the reason why the most relevant results in the field of "weak" analysis have been obtained within the model without investment - fixed capital reproduction - where starting from the technical coefficient matrix, the static multiplier analysis, linkage analysis and key sectors analysis have been developed.

3. Technical coefficient disintegration

If we want to model a more advanced specification of the unit-output input requirement than we should make the weak technical coefficient react to the level of activity according some reasonable assumptions. We will, then, get a multisectoral framework where technical coefficients, no more fixed, are suitable for quantifying the implications of the differentials in sectoral returns to scale and the swings in the value content of immaterial inputs. In the simplest case of dependency, the linear dependency, we could can write:

$$m/x = a + a'x$$

With respect to the traditional case of *weak interaction*, in this case a further interaction is superimposed which operates on input requirements starting from the scale of sectoral output. This type of interaction that will be called *strong interaction* implies the activation of two causal links: the first between sectoral output and, given a unit (-output) input requirement, corresponding inputs required, the other originating from sectoral ouput for the determination of unit(-output) input requirement.

If we confine to the case of positive linear strong interaction, output can be realised as long as required inputs do not exceed output itself. However it can be expected that approaching a predetermined output level, x_b, that we define as sustainable output or steady state output it will be more and more difficult to obtain output given the increasing rate of input requirements. The output levels which can be mantained are those for which the input demands, m, is not greater than output x

$$m/x = a+a'x \leq 1$$

which means

$$x \leq (1-a)/a'.$$

If we put

$$x_b = (1-a)/a'$$

we can write

$$x \leq x_b.$$

The new Leontief model with strong interaction i.e. the scale dependent input requirements (SDIR) model will be given by:

(2) $$x_{t+1} = [1+(1-a)/b]x_t - (a'/b)x_t^2 - 1/b\, f_t.$$

Cobweb diagram in fig.3 shows the existence of a second non zero fixed corresponding to the intersection of parabola $y = [1+(1-a)/b]z-(a'/b)z^2$ with the line $y=x$; where $y=x_{t+1}$ and $z=x_t$.

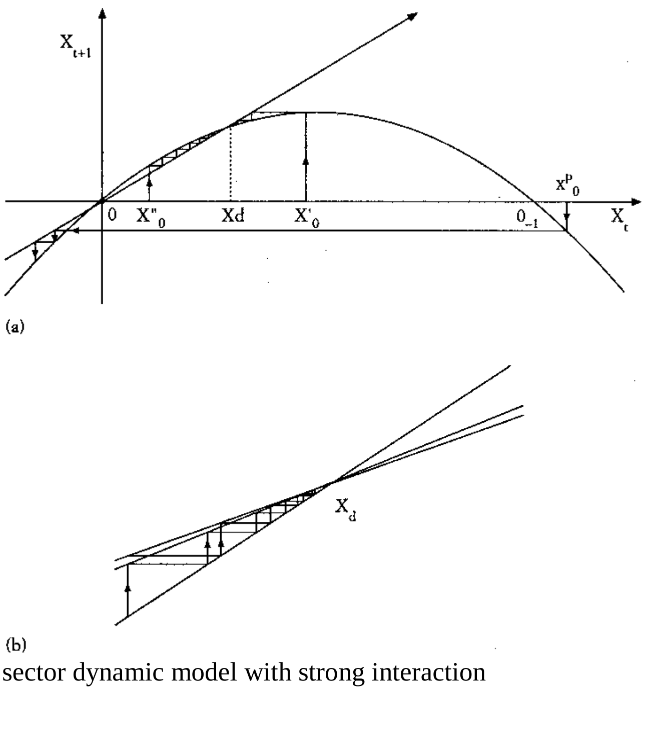

Fig.3: Cobweb for the one sector dynamic model with strong interaction

From Fig.3a we note that the trajectory converges towards x_{d2}, which represents the level of sustainable output, x_b. We can linearize around the fixed point. Fig.3b shows a magnification of

Fig.3a around the fixed point. The Cobweb diagram obatined thruogh the use of parabola is practically the same as that obtained using the tangent line. In both cases trajectories move towards the fixed point x_d, which for this reason is defined as attractive, as long as the slope of the line $f(x_t)=(a'/b)x_t+[1+(1-a)/b]$ is less than unit.

Moreover we are able to determine the basin of attraction of the fixed point x_d. This region, $D(x_d)$, designates the initial conditions for which trajectories are attracted by the fixed point. In our case, see fig.3(a), this region is given by the interval limited by the repelling fixed point $x_{d1}=0$ and the intersection of the parabola with the horizontal ax. In fact we note that an initial condition given outside this interval will generate a divergent trajectory towards - ∞.

We then attain the general conclusion that the SDIR model has a the fixed point which is attractive if $|f'(x_d)|<1$, and repelling if se $|f'(x_d)|>1$; it possesses an asymptotical output structure x_d towards which it tends or from which is repelled.

In less formal terms we have come to the conclusion that if we renounce to the concept of (fixed) technical coefficients we do not change a simple detail in a model specification. We demolish a consolidated conventional wisdom on multisectoral dynamics according which the stability of a dynamical system is established once and for all according its structural coefficients.

Stability and instability can coexist in the same framework, so that a new problem arises that of determining the borders across which our trajectories will be no more pulled towards a reasonable objective but pushed to explosion.

Within the model the unit-output input coefficient breaks into several components disintegrating the identificabily of the technical coefficient.

4. Dynamics of strong interactions

Even if our analysis will finally lead us to resort to numerical simulation and avoid algebra, we need now to carry on a bit more with formal analysis, to identify the dynamic patterns can be further found in a framework of strong interaction, other than the balanced growth path. We take into consideration the case where $|f'(x_d)|=1$. [In this case it may happen that the system is attracted by some equilibrium configuration other than a fixed point or infinity.]

Resorting to parameter r, that will be defined as potential growth rate, and sustainable output, x_b, eq.(2) can be rewritten as

(3) $$x_{t+1}-x_t = r\,[1 - (x_t/x_b)]x_t.$$

In the case of weak interaction the system's growth rate would be determined by r, which represents the share of resources not used as materials per unit output,$(1-a)$ and capital coefficient b. When output is greater than x_b inputs will become greater than the corresponding output. So when $x_t>x_b$ capital stock (or inventories) will begin to decrease to allow for production and this will imply a negative growth rate $(x_{t+1}-x_t)<0$. When output is less than x_b, the accumulation process can take place since htere are resources left, once that inputs have been used in the producing process. When $x_t<x_b$ then $(x_{t+1}-x_t)>0$. If the output level is low compared to x_b, which means that resources are abundant after having employed the materials then the actual growth rate should be near the potential growth rate r. However when output grows the actual output growth rate decreases and becomes equal to one when $x_t=x_b$.

The equilibrium output for model (3) can be determined putting $x_{t+1} = x_t = x_d$:

$$x_d = (1+r)x_d - (r/x_b)x^2{}_f$$

getting the values

$$x_{d1}=0 \quad \text{e} \quad x_{d2}=x_b.$$

We can choose the dimension of our units, defining a unit as x_b so that $x_{d2}=1$. Through this procedure we standardise the model so that output and final demand that up to now has been assumed equal to zero, can be expressed as a proportion of sustainable output.

The Leontief Scale dependent input coefficients (SDIR) model becomes:

(4) $$x_{t+1}=(1+r)\,x_t - rx_t^2.$$

We can easily show that $x_d=0$ e $x_d=1$ are fixed points, and that

$$f(x) = (1+r)x - rx^2 \qquad f'(x) = 1+r-2rx.$$

The fixed point in the origin, the *Leontief fixed point*, $x_{d1}=0$, will always be repelling since for $0\leq a<1$ and $b>0$, $f'(0)=1+r>0$. The other fixed point $x_{d2}=1$ will be attractive if $||f'(1)|=|1-r|<1$ that is $0<r<2$. Se $b=(1-a)/2$, the balanced growth rate r becomes equal to 2, and the method based on the derivative becomes inconclusive since, as we said, $f'(1)=-1$. For establishing the stability of x_d we need to define an iterated map. The resulting dynamic model $x_{t+2}=g(x_t)=f(f(x_t))$ also admits x_d a fixed point. After some calculus we get

$$g'(x_d)=1$$

$$g''(x_d)=0$$

$$g'''(x_d)=-2f'''(x_d)-3(f''(x_d))2.$$

If $g'(x_d)=1$, $g''(x_d)=0$ e $g'''(x_d)<0$ the Cobweb diagram is such that the curve switches form upper concave to lower concave in the fixed point so that x_d results attractive. If $r>2$ the fixed point $x_d=1$ is repelling, since $|f'(1)| = |1+r2r| >1$. If initial endowment x_0 is chosen in the proximity of the fixed point $x_d=1$, then x_t gets away to reach a 2-cycle, as shown in Fig. 4(a). The fixed point $x_d=0$ is repelling since $f'(0)=1+r>1$. If we construct a diagram as in Fig. 4(b) starting from low initial conditions, for example $x(0)=0.1$, trajectory x_t will be repelled from the origin $x_d=0$ and attracted by the same 2-cycle.

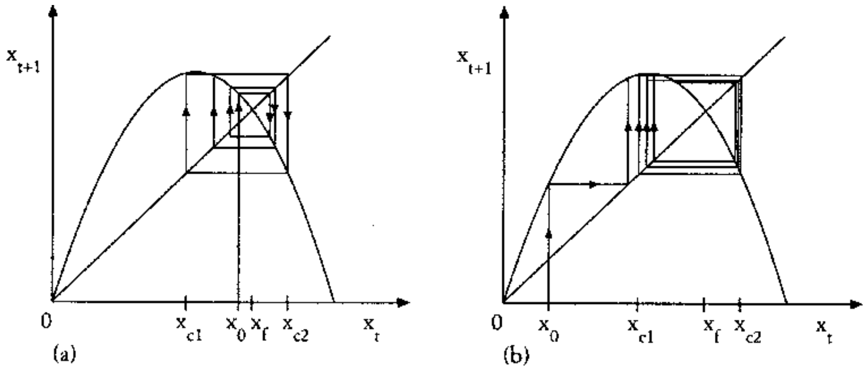

Fig. 4: Repelling fixed points

To show algebraically that a 2-cycle is attractive we need to refer to the iterated map:

$$x_{t+2}=f(f(x_t))=f^{(2)}(x_t).$$

If for the iterated map, $f^{(2)}(x_t)$, both x_{c1} and x_{c2} are attractive fixed points, then these points will constitute an attracting 2-cycle for $f(x(_t))$. We will now determine for which values of r our system exhibits a 2- cycle. Fixed points that satisfy $x_d=f(f(x_d))$ ove $f(x_d)=(1+r)\,x_d-rx_d^2$ are:

$$x_c = \frac{2 \pm r\sqrt{r^2-4}}{2r}$$

Since $f'(x)=(1+r)-2rx$ then

$$f'(x_{c1})f'(x_{c2}) = [1+r-2r\frac{2+r+\sqrt{r^2-4}}{2r}][1+r-2r\frac{2+r-\sqrt{r^2-4}}{2r}] = 1-r^2+4 = 5-r^2$$

If

$$-1<5-r^2<1$$

that is

$$2<r<\sqrt{6}=2.449$$

the *2*-cycle is attractive. When *r* increases the stable cycle doubles its period and this process is known as "period doubling". When *r* increases further and becomes greater than a threeshold level then the "chaotic regime" is activated which is characterised by sensitivity to initial conditions and aperiodic trajectories within limited intervals. Sensitivity to initial conditions means that to trajectories having very similar initial conditions x_0 e $(x_0+\varepsilon)$ for ε positive, after some iterations, will exhibit dynamic behaviours rather dissimilar.

We can follow the first phases of the process with reference to Fig.5(a)-(d). IN Fig. 5(a) we have represented the iterated map $f^{(2)}(x_t)$ together with the original function $f(x_t)$. Technology is defined by technical coefficient $a=0.2$, and by capital coefficient $b=0.57$. The potential growth rate will be equal to *1.4* and fixed point x_d, is unique and attractive. As shown in this figure $f(x_d)$ and $f^{(2)}(x_d)$ coincide and $f'(x_d)$ and $f'^{(2)}(x_d)$ are less than unit.

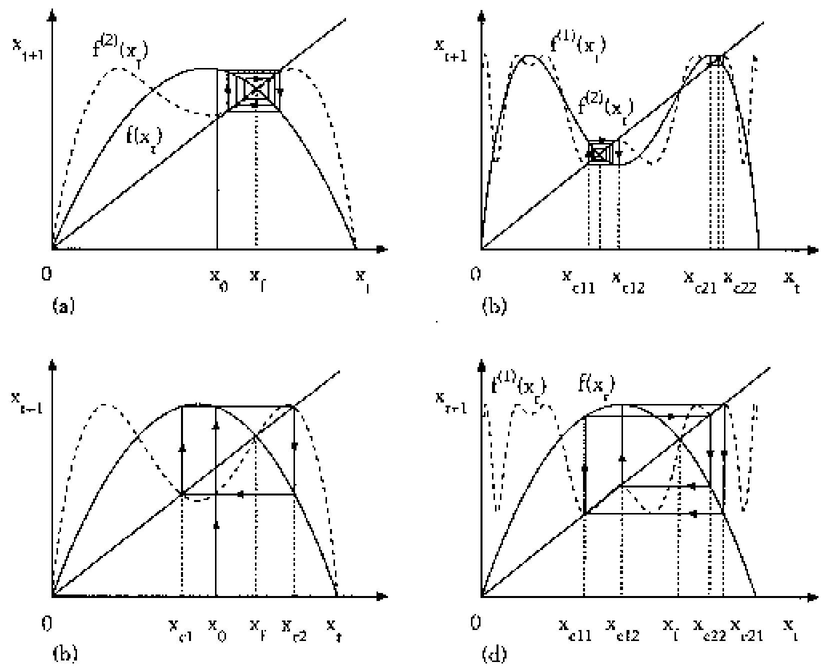

Fig. 5: (a) Attractive fixed point. (b) 2-cycle emergence. (c) (d) 4-cycle emergence.

In Fig.5(b) the formation of an attractive 2-cycle is shown. Capital coefficient *b* has fallen from *0.57* to *0.33* so that the potential growth rate *r* has increased to *2.42*.

Fixed point x_d has now become repelling, as it emerges from the slope of the original function evaluated in the fixed point $|f'(x_d)| > 1$. A 2-cycle has generated given by the two points on the iterated map, x_{c1} e x_{c2}. This cycle is attractive, since the slope of the iterated map, $f_{(xt)}$, evaluated in the two points, is less than unit, $|f'^{(2)}(x_{c1})| < 1$ and $|f'^{(2)}(x_{c2})| < 1$.

Fig.5(c) shows the emergence of an attractive 4-cycle. The capital coefficient has decreased to *0.32* and *r* has increased to *2.5*. The two points x_{c1} and x_{c2} become repelling since the derivative of the iterated map, $f^{(2)}(x_t)$, evaluated in the two points x_{c1} and x_{c2}, becomes greater than unit. While on the iterated map $f^{(4)}(x_t)$ four fixed points appear x_{c11}, x_{c12}, x_{c21}, x_{c22} for which $|f'^{(4)}(x_{cij})| < 1$ with $i,j = 1,2$. Further decreases in coefficient *b* lead to the chaotic regime. Fig.6(a) shows an example of aperiodic trajectory for x_t, $0<t<100$.

We can show in a diagram on the plane *(b,a)* all the possible equilibrium configurations and the critical values that determine the transition from one to another. Fig. 6(a) shows for each couple technical coefficient and capital coefficient the type of equilibrium configuration.

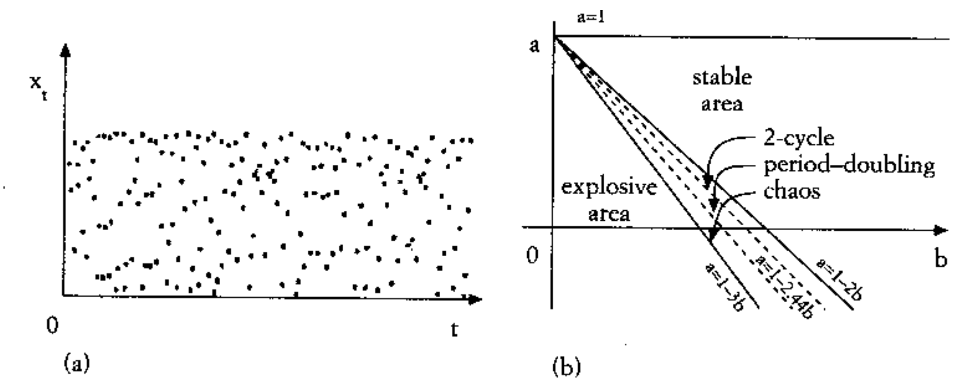

Fig. 6: (a) Chaotic trajectory. (b) Critical values for coefficients a and b.

In this framework the introduction of time-invariant final demand, $f_t = f^*$, will transform model (4) as follows:

(5) $$x_{t+1} = (1+r)x_t - rx^2_t - (1/b)f^*$$

where variables are measured with respect to x_b.

Final demand operates in the sense of modifying the intercept of the parabola. The parabola in fact slides down until the two points coincide. A further increase in final demand generates the disappearance of the two fixed points.

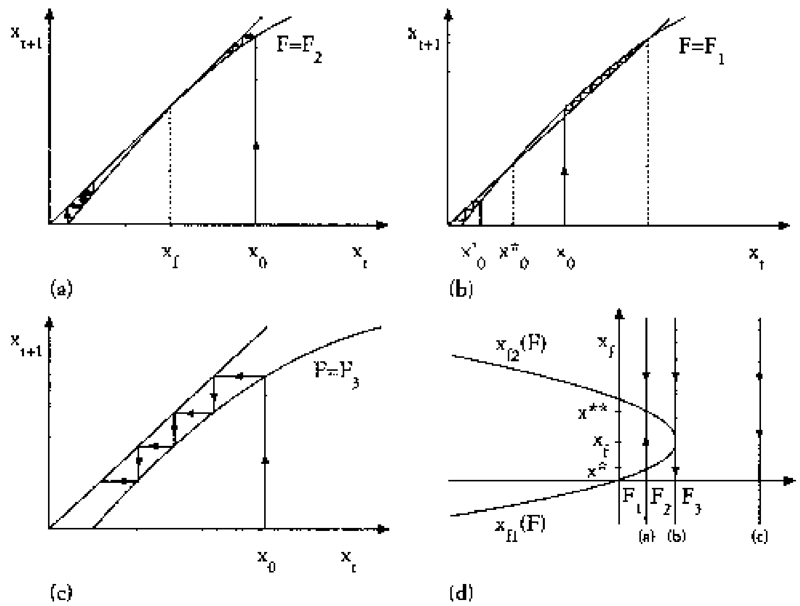

Fig. 7: Effects of final demand on the output trajectory

5. Focus, saddle and node as alternatives to the balanced growth path

We will now try to generalise the one sector model to the n sector case. The unit intermediate requirements of goods of the ith type per unit of output of the jth sector can be written as

$$m_{ij}/x_j = a_{ij} + a'_{ij}x_j$$

Intermediate requirements in matrix notation will be given by:

$$m_t = Ax_t + C\tilde{x}_t x_t$$

where matrix C=[a'$_{ij}$] and matrix $\tilde{x}_t$ is a diagonal matrix whose diagonal elements are given by the elements of vector x_t. The n sector model will be given by:

(6)
$$x_{t+1} = [I + B^{-1}(I - A)]x_t - B^{-1}C\tilde{x}_t x_t - B^{-1}f_t$$

As in the one-sector case, we may refer to a linearization of system (6) around the fixed point. After having calculated the fixed point, we obtain the matrix of first derivatives, the Jacobian matrix

$$J(x_d) = \begin{bmatrix} f^{11}(x_{1d}, x_{2d}) & f^{12}(x_{1d} x_{2d}) \\ f^{21}(x_{1d}, x_{2d}) & f^{22}(x_{1d}, x_{2d}) \end{bmatrix}$$

$$= [I + B^{-1}(I - A)] - 2B^{-1}C\tilde{x}_{dt}$$

In a convenient vicinity of fixed point x_d containing the fixed point, i.e. within its basin of attraction, the trajectories of dynamical system (6) are those exhibited by linear model:

(7) $$x_{t+1} = [(I + B^{-1}(I - A)) - 2B^{-1}C\tilde{x}_d]x_t$$

where vector x_t is now measure with respect to x_d. This means that the growth paths will be measured with reference to an axes system conveniently oriented and centred on the fixed point.

For a discussion of the possible shapes of the growth paths of a strong interaction dynamic system it is convenient to assume that the model has been transformed into its canonical form.

The state space diagram of the actual model can differ since non singular transformation U distorts the diagram. However transformation U doesn't change the character and the properties of these diagrams. The possible growth paths depend on the characteristics of the fixed point. The fixed point can in fact be a node, a focus, a saddle or a centre.

If the fixed point is a node the growth path doesn't exhibit fluctuations. Matrix J has two real and distinct eigenvalues, λ_1 e λ_2, and is written as:

$$J = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

The two eigenvalues can be less than unit in modulus, $|\lambda 1| < |\lambda 2| < 1$, then the fixed point is a stable tangent node. For $x_{1,t=0} = x_{2,t=0} = 0$ we get the fixed point. For $x_{1,t=0} = 0$ e $x_{2,t=0} \neq 0$ we get the x_2 ax (fixed point excluded) and we note that while t gets to infinity x_2 tends to 0. Similarly for $x_{2,t=0} = 0$ e $x_{1,t=0} \neq 0$ we get the x_1 ax (with exclusion of the fixed point) and for t that tends to infinity x_1 tends to 0. In general the motion along a whatever path consists in the asymptotical approach of the origin as can be seen in Fig. 8(a). In the case where $|\lambda_2| < |\lambda_1| < 1$ then the growth paths will become tangent to the x_2 ax. If on the contrary the two eigenvalues are greater than one in modulus the fixed point is an unstable tangent node similar to the case in Fig. 8(a) but with the growth paths in reversed orientation.

If matrix J has a repeated eigenvalue λ but is not diagonal, which happens when the number of repetitions of the real eigenvalue is less than the dimensions of the state space then it will be written as:

$$J = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$$

In this case if the eigenvalue is less than one, $\lambda<1$, than the fixed point is said stable improper node since x_1 and x_2 move towards zero when t tends to infinity, as shown in Fig. 8(d). If

$\lambda > 1$ then the fixed point is said unstable improper node. The growth rates of the model in the space state are similar to those shown in Fig.8(d) but with reversed arrows.

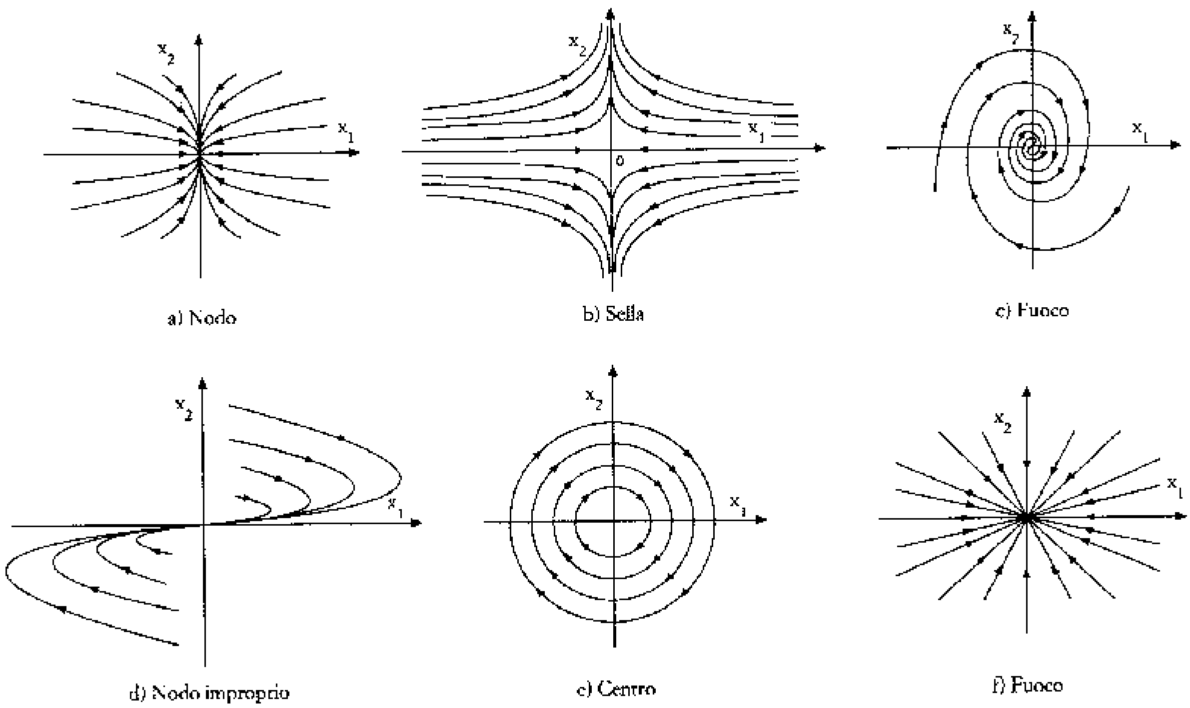

Fig. 8: Growth paths for output

When the fixed point is a focus than the growth paths move with oscillations towards (or from) the fixed point. Matrix J has complex eigenvalues and is written as:

$$J = \begin{bmatrix} \alpha & \mu \\ \mu & \mu \end{bmatrix}$$

where α represents the real part and μ the imaginary part.

If the real part α is les than one, the fixed point is said stable focus and the paths in the state space appear as shown in Fig. 8(c). If the real part is greater than unit the fixed point is said an unstable focus and the paths are similar to those shown in Fig. 8(c) with the reversed arrows.

A special case is detected when matrix J has a repeated eigenvalue. It can be written as:

$$J = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

and we will have that $x_2/x_1 = (x_{2,t=0}/x_{1,t=0})$ or $x_2 = (x_{2,t=0}/x_{1,t=0})x_1$. This means that the growth paths will be lines to the fixed point generated according the ratio between initial endowments *(x2,t=0/x1,t=0)*.

If the eigenvalue is lower than unit the points will move towards the fixed point as in Fig.8(f). If its greater than one they will be divergent and the fixed point will be an unstable focus.

When the fixed point is a saddle the paths will move firstly towards the fixed point ut in its vicinity will begin to diverge. This will happen when the eigenvalues will be the one greater than unit and the other less than unit, $\lambda_1<1<\lambda_2$. For $x_{1,t=0}=0$ *e* $x_{2,t=0}=0$ we get the x_2 ax (with exclusion of the fixed point) while x_2 will tend to infinity as time passes on. The motion along the positive semiax x_1 points towards the fixed point and the motion along the positive semiax x_2 diverges from the fixed point as can be seen in fig. Fig.8(b). In the case where $\lambda_2<1<\lambda_1$ then the orientation of the paths in the same figures would be reversed.

If the fixed point is a centre the growth paths will nor approach nor get away from the fixed point. In this case matrix J has complex and conjugated eigenvalues with real part equal to zero generating paths similar to circles centred on the fixed point. If the imaginary part is greater than zero, $\mu>0$, the direction of the motion is clockwise while if it is less than zero, $\mu<0$ the motion direction is anticlockwise as shown in Fig. 8(a).

6. Visual multisectoral analysis

However the variety multisectoral dynamic paths shown in Fig. 8 doesn't exhaust the dynamic possibilities of a strong interaction multisectoral model. It refers only to those cases where a fixed point exists and we have been able to detect it, so that the its linearization has been made possible. The emergence of cycles and turbulence cannot be studied in this way. We need to actually compute the trajectories of the system to see how it behaves when the fixed point from attracting becomes repelling. A useful tool is to compute the basin of attraction of the attracting fixed point, region $D(x_d)$ in chapter 3, and to see how it varies under different values of the parameters.

For each initial endowment (condition) we will determine the number of iterations needed to reach the fixed point or infinity. We will then represent o+n the *x,y* plane those initial conditions that approach the fixed point (or infinity) within the same number of iterations with dots of same colour. Lets calculate a basin of attraction for a strong interaction model where the parameters are given by:

A = [a_{11}=0.00612	a_{12}= 0.00010	a_{21}= 0.01605	a_{22}=0.00566]
C = [c_{11}=0.20890	c_{12}= 0.03900	c_{21}= 0.04030	c_{22}=0.04000]
B = [b_{11}=0.10000	b_{12}= 2.00000	b_{21}= 0.20000	b_{22}=0.04000]
f = [f_1=0.500000	f_2= 0.80000]		

and we let the capital coefficient b_{21} vary.

a) Existence of an attracting focus

The morphology of the basin of attraction in the case of existence of an attracting fixed point is shown in Fig.9(a); this happens when parameter b_{21} is put equal to 3. The basin of attraction of fixed point in F, which, as we will see, is an attracting focus, is given by the almost triangular region in the centre of the figure. Outside this region the system cannot survive, trajectories are explosive, but with different speeds which are denoted, in the same figure, by different colours.

In the lower border of this region one can detect a repelling 2cycle C. All the points of the border represent the extension to two sectors of the Leontief repelling fixed point discussed in chapter 4 for the one sector case.

In fact a trajectory starting precisely on the border with neither converge or diverge but remain confined to the border.

The black belts around the attracting fixed point F indicate subregions, within the basin, where the trajectory converges at the same velocity. Fig.9(b) shows more clearly further details. The subregion in white within the basin designates trajectories that converge within less of 20 iterations and some trajectories have been actually drawn on it. Trajectories initiating in the vicinity of point C are repelled. If they start within the border, first adjust proportions towards those of a common guideline and then reach the focus by means of a common path ***g***. Those initiating outside the border are repelled to infinity.

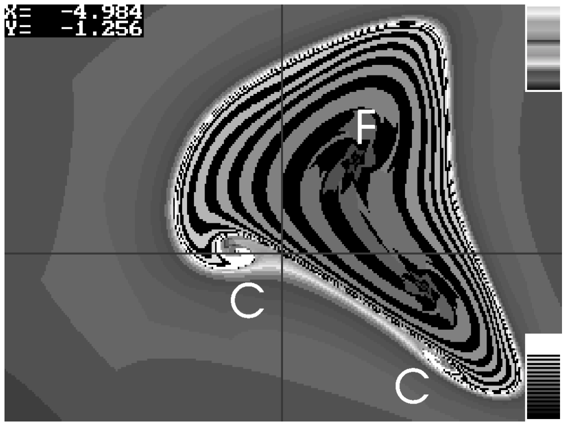

Fig.9(a)

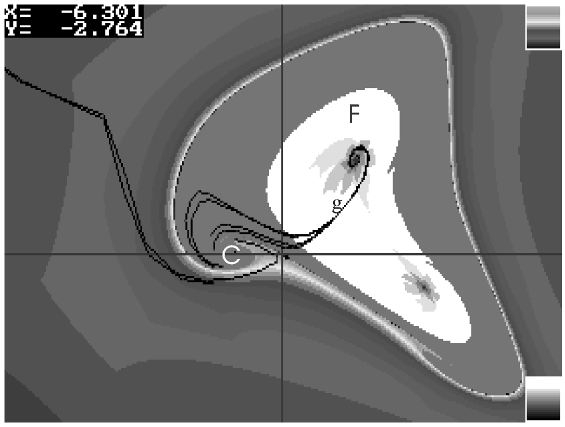

Fig.9(b)

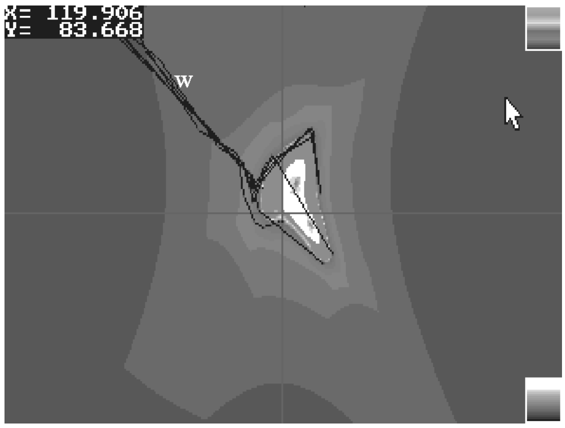

Fig.9(c)

The wider picture in Fig 9(c) shows, however, that all trajectories initiating near and outside the border of the basin of attraction assume the typical Leontief divergent pattern attracted by a north - east path ***w*** to explosion.

b) Disappearance of the attracting focus

b) If parameter b_{21} is equal to 0,78 the fixed point vanishes i.e. the multisectoral economy is no more able to warrant a fixed sustainable output vector. However the trajectory within the basin, *g*, doesn't explode. It is now attracted towards an oscillating sustainable output path, which remains confined within the basin as in fig 10(a). Starting outside the trajectory diverges along the path *w*.

For further decrease of the parameter the basin of attraction breaks generating small regions of instability inside the non explosive area.

In Fig. 10(b) we see a trajectory starting from one of these "holes" within the basin that diverges. Fig 10(c), on the contrary, shows the same basin with a trajectory that remains confined within the basin.

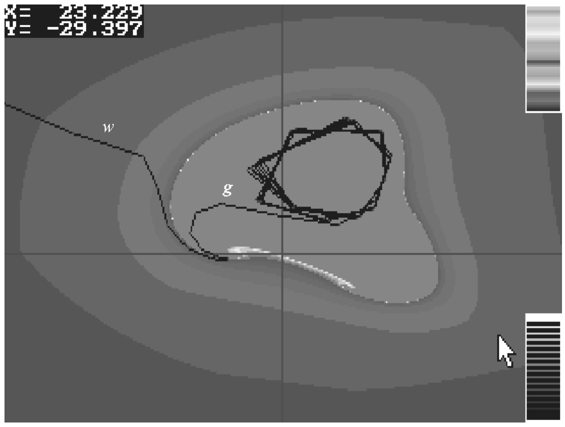

Fig.10(a)

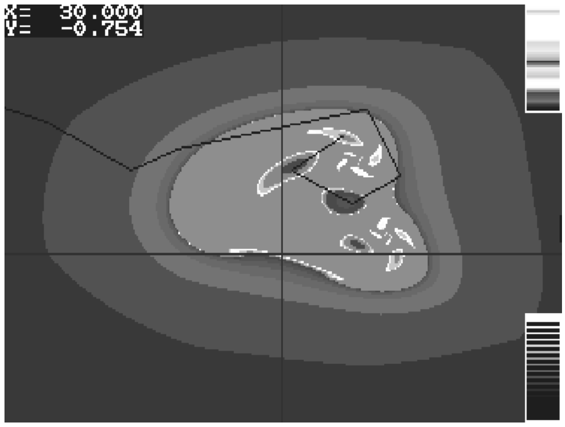

Fig.10(b)

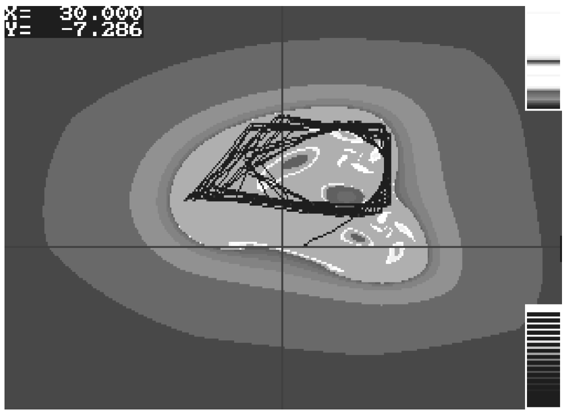

Fig.10(c)

c) Dissolution of the basin of attraction

Finally for further decrease of the parameter b_{21} the system reaches the chaotic regime. In this case also the basin of attraction becomes chaotic as can be appreciated from Fig. 10(a). Here starting from the chaotic area of the basin we will get chaotic trajectories. The "holes" of instability still persist within the chaotic basin.

For a further decrease in b_{21} the basin of attraction completely vanishes. All initial conditions generate explosive trajectories, which however have not the same velocity of explosion since more dense areas generate trajectories that remain in the vicinity of the origin for more iteration than those starting from less dense (darker) areas, see Fig. 10(b)(c).

One could also see this sequence in the reverse way i.e. for increasing values of the parameter. Starting from Fig. 11(c) back wards we get a sort of "genesis of stability" starting from instability through chaos to stability. From this figure in fact we get information on the location of initial conditions which generate less unstable trajectories. This information is valuable both in designing and in evaluating changes in the parameters in view of reinforcing the "stability" of the system.

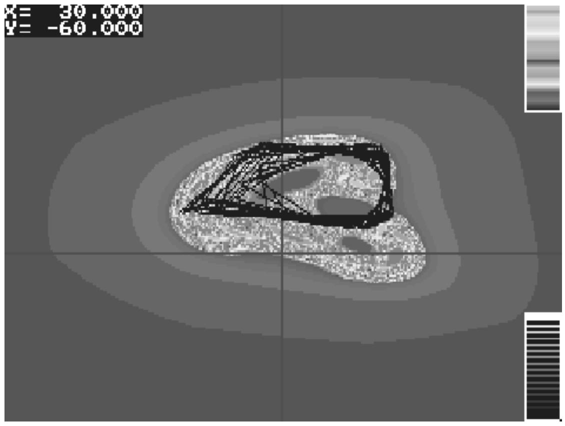

Fig.11(a)

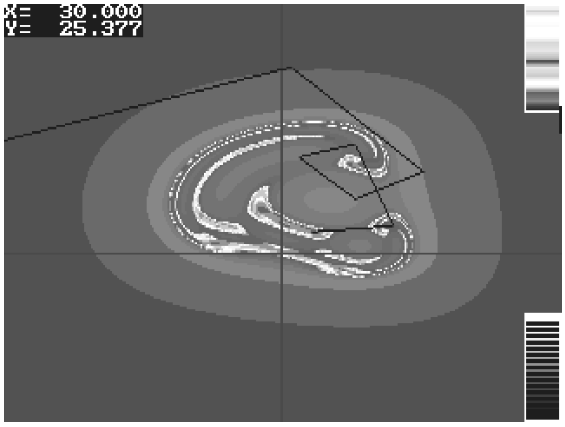

Fig.11(b)

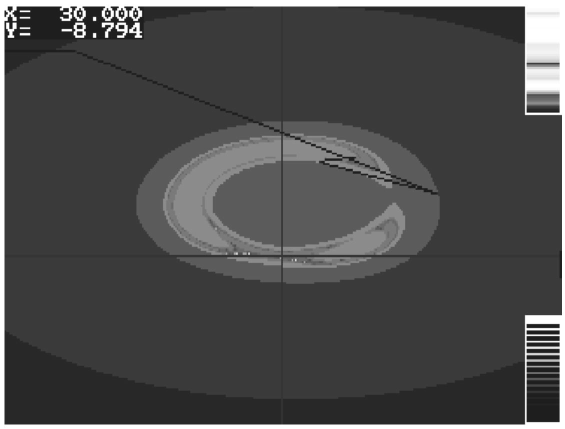

Fig.11(c)

7. Conclusions

Conventional wisdom in multisectoral analysis is that dynamics generating inside multisectoral models with fixed technical coefficients and capital reproduction is characterized by a high degree of instability. The introduction of strong interaction seems to put into crisis the concept of technical coefficient, that of the balanced growth path and also the method of obtaining insights on the dynamic behaviours exclusively through the mathematical analysis of the parametric structure of the model.

If technical coefficient is not fixed it could be worthwhile to model the industrial absortions directly as demands by each sector. These industrial demands will contribute with their parameters to the reduced (recursive) form of the model without a specific functional meaning of each parameter. Alternatively we could still model the technical coefficient, seen as the intermediate demand per unit output, which reacts to the level of output of the same sector or of other relevant sectors, in a way that keeps the functional meaning of each parameter.

Under the point of view of mathematical analysis, through the idea of linearization around the fixed point, it is possible to discover some typical dynamic behaviours which update the traditional idea of balanced growth path. The existence of a sustainable output vector, which can be seen as a node, a focus, a saddle or a centre, adds dynamical patterns within the analysis of multisectoral dynamics. This investigation, however, is possible only when the fixed point is known. So that we need, preliminarily, a method for determining the fixed point which usually is not based on mathematical analysis but on numerical methods.

Moreover we need to resort to numerical analysis as exclusive tool in the case where the fixed point dos not exist. We can in fact find further dynamic patterns in connection with the fact that the attracting fixed point can transform into an n-cycle and from there into a chaotic attractor. Through the period doubling onset to chaos, the problematics of facing "turbulence" is introduced in multisectoral dynamics.

In the context of strong interaction, then, numerical simulation plays both the role of an applied and a qualitative tool. It can reveal situations that cannot be detected on the basis of the analysis of the parameters of the model and which became apparent when analysing and comparing the shapes of the basins of attractions and their changes.

This analytical procedure seems to anticipate a sort of "visual" approach in the discussion of the numerical results for multisectoral analysis; an idea that could not appear so eccentric once that data evidence becomes reliable and multisectoral analysis not confined to the unnecessary long run.

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THE END OF TECHNICAL COEFFICIENTS?

An attempt of visual approach to multisectoral dynamics

Maurizio Ciaschini

1. Introduction

2. Dynamics of weak interactions

3. Technical coefficient disintegration

4. Dynamics of strong interactions

5. Focus, saddle and node as alternatives to the balanced growth path

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