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LEONTIEF and WALRAS: INPUT-OUTPUT and REALITY

By Ezra Davar

ABSTRACT

Leontief, founder of modern Input-Output analysis, stated (as the Classics and Walras) that the main object of input-output is to describe economic reality as closely as possible. Leontief also claimed that this goal might be reached if there is a reciprocal connection between the theoretical conception of input-output and its empirical treatment. Walras formulated the first mathematical model of general equilibrium theory without an empirical background. On the other hand, Leontief compiled the first empirical input-output, which was a natural expression of real economic life. Walras' model was used as the basis for the theoretical scheme of input-output by Leontief. Naturally, there are some differences between these two systems. Leontief enriched Walras' system in accordance with the changes in real economic life by adding public sector and exports on the demand (consumption) side, and taxation and imports on the supply (input) side. From that moment forward, and during his long life, Leontief applied input-output to different economic topics: dynamic aspect of economic, the choice of technology, trade in the world economy, environmental pollution and so on. At the same time, Leontief's theoretical scheme differs from Walras' system and it does not completely describe reality. For example, Leontief's theoretical scheme describes only the supply side of the economic system. This paper, therefore, discusses the relationship between Walras' theoretical general equilibrium model and Leontief' input-output. In the second section, following the introduction, Walras' original general equilibrium model (relevant part to input-output) will be discussed. Leontief's input-output will presented in the third section. In the fourth section some directions for input-output's perfection and extension in the future, for both theoretical and empirical sides, will be suggested. Finally, the last section will summarize and provide some conclusions.

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Pour observer cette méthode, l'économie politique pure doit emprunter à l'expérience des types d'échange, d'offre, de demande, de marché, de capitaux, de revenus de services producteurs, de produits. De ces types réels, elle doit abstraire, par définition, des types idéaux, et raisonner sur ces derniers,pour ne revenir à la réalite que la science une fois faite et en vue des applications.

Walras (1988(1900))[1]

True advance can be achieved only through an iterative process in which improved theoretical formulation raises new empirical questions and the answers to these questions, in their turn, lead to new theoretical insights.

Leontief (1971)

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Leontief, founder of modern Input-Output analysis, stated (as the Classics and Walras) that the main object of input-output is to describe economic reality as closely as possible. Leontief also claimed that this goal might be reached if there is a reciprocal connection between the theoretical conception of input-output and its empirical treatment. Walras formulated the first mathematical model of general equilibrium theory without an empirical background (Walras, 1954). On the other hand, Leontief compiled the first empirical input-output table, which was a natural expression of real economic life (Leontief, 1936 and 1941). Walras' model was used as the basis for the theoretical scheme of input-output by Leontief. Kuenne writes: "The work of professor W. W. Leontief in building operational interdependence models and that of the Interindustry Division of the Bureau of Labor Statistics in quantifying them, lends Walras' work a more immediate importance (Kuenne, 1954)[2]. Naturally, there are some differences between these two systems. Leontief enriched Walras' system in accordance with the changes in real economic life by adding public sector and exports on the demand (consumption) side, and taxation and imports on the supply (input) side (Leontief, 1986). From that moment forward, and during his long life, Leontief applied input-output to different economic topics: dynamic aspects of the economy (Leontief, 1970a), the choice of technology (Leontief, 1985; Leontief and Duchin, 1986), trade in the world economy (Leontief, 1974), environmental pollution (Leontief, 1970b) and so on. At the same time, Leontief's theoretical scheme differs from Walras' system and it does not completely describe reality. For example, Leontief's theoretical scheme describes only the supply side of the economic system.

This paper, therefore, discusses the relationship between Walras' theoretical general equilibrium model and Leontief's input-output analysis. In the first section, following the introduction, Walras' original general equilibrium model (relevant in part to input-output) will be discussed. Leontief's input-output model will present in the second section. In the third section some directions for input-output's perfection and extension in the future, on both theoretical and empirical sides, will be suggested. Finally, the last section will summarize and provide some conclusions.

1. Walras' Theory of Capital Formation and Credit

Walras considered four separate markets in the capital formation and credit economy: a) services; b) products for consumption; c) new capital goods; and finally d) abstract good (*E*) consisting of *perpetual net income*. The latter is the expression of saving. Here, as in the exchange and production economies, new capital goods and good (*E*) are exchanged according to the rules of free competition and their prices (prices and rate of net income, respectively) are indicated in terms of *numéraire*. In addition, the equilibrium prices of new capital goods and good (*E*) are established by the law of supply and demand: by the relationship between effective demand and effective offers; and by the relationship between the selling prices (demand prices) of products and their costs of production (supply prices). The latter is determined as well as for consumers goods (Davar, 1994). But, Walras' determination of the selling price for capital goods differs from the determination of consumers' goods and selling price for the given concrete capital good (say *k*) is:

$$\Pi^k = p^k / (i+\mu^k+\nu^k) \qquad (1.1)$$

where

Π - is the selling price of capital good;

p - is the gross income of capital good, that is, the price of its service inclusive of both the depreciation charge and the insurance premium;

i - is the rate of net income, is the same for all capital goods;

μ - is the rate of the depreciation charge, will vary with different capital goods;

ν - is the rate of the insurance premium, will also vary with different capital goods.

On the other hand, in order to demand (purchase) capital goods there must be individuals whose incomes exceed their purchase of consumers' goods and services and the aggregate of them is greater then the aggregate of those who are in opposite position, i.e. they consume more than their income. Walras stated that when the positive excess is larger than the amount needed to cover depreciation and insurance, the individual saves. He wrote: "It will be understood that the excess does not constitute true saving unless it is both positive and greater than the sum needed to cover the depreciation and insurance of existing capital goods proper.'(Walras, p.274)

In order to convert this new term, saving, to a term which would be comprehensive, similar to other consumers goods, Walras introduced an abstract (ideal) commodity (*E*) consisting of *perpetual net income* with price $p_e = 1/i$, where i – is the rate of net income. This means that each individual has a certain want for commodity (*E*). Thus, it is demanded (d_e) or offered (o_e), as well as other capital services and a quantity of which is obtained by the condition of maximum satisfaction by its function of utility.

Before describing Walras' general equilibrium system for Capital Formation and Credit, it is necessary to stress that Walras' theory's common approach suggests that the starting model must be a model of individuals' choice with maximization of utility for goods and services. The results of the latter's solution compose basic information for the system of equations. But these models of individuals are not discussed here, because Leontief's approach, unfortunately, does not consider such a problem.

In Capital Formation and Credit, as well as for the Exchange and Production Economies, Walras first formulated a macro model for the general equilibrium state and then described the process of its establishment. The general equilibrium state model of Capital formation and Credit is an extended version of the general

equilibrium state model of the Production Economy. There are additional unknowns for the new capital goods' quantities and prices, and aggregate saving and its price, and an equations system accordingly. Therefore, if we take into account these facts together with the results of the section 1 above, we can formulate Walras' original equation systems describing the general equilibrium state for capital formation and credit, which consists of 8 systems:

1) The system of n equations of the aggregate offer of services for the production of products:

$$O_j^t=F_j^t(p_2, p_3,..., p_m;p_1^t,...,p_t^t,p_1^p,...,p_k^p,p_1^k,...,p_l^k,p_e)=\Sigma o_j^t-d_j^t,(j=1,2,...,t); \quad (1.2a)$$

$$O_j^p=F_j^p(p_2,p_3,...,p_m;p_1^t,...,p_t^t,p_1^p,...,p_k^p,p_1^k,...,p_l^k,p_e)=\Sigma o_j^p-\Sigma d_j^p,(j=1,2,...,k); \quad (1.2b)$$

$$O_j^k=F_j^k(p_2,p_3,...,p_m;p_1^t,...,p_t^t,p_1^p,...,p_k^p,p_1^k,...,p_l^k,p_e)=\Sigma o_j^k-\Sigma d_j^k,(j=1,2,...,l); \quad (1.2c)$$

2) the system of m equations of the aggregate demand of products:

$$D_i = F_i(p_2, p_3,..., p_m;p_1^t,...,p_t^t,p_1^p,...,p_k^p,p_1^k,...,p_l^k,p_e) = \Sigma x_i, \ (i =2,3,...,m);$$

$$D_1 = \Sigma O_j^t p_j^t + \Sigma O_j^p p_j^p + \Sigma O_j^k p_j^k - (\Sigma D_i p_i + E) ; \quad 1.3)$$

3) 1 equation of the aggregate excess of income over consumption:

$$E = D_e p_e = F_e(p_2, p_3,..., p_m;p_1^t,...,p_t^t,p_1^p,...,p_k^p,p_1^k,...,p_l^k,p_e)p_e =$$

$$= F_e(p_2, p_3,..., p_m;p_1^t,...,p_t^t,p_1^p,...,p_k^p,p_1^k,...,p_l^k,i), \quad (1.4)$$

4) the system of n equations of equality between the total quantities of productive services employed (demand) and the total quantities effectively offered for production:

$$\sum a_{ji}^t D_i + \sum a_{jj}^t D_j^k = O_j^t , \qquad (j =1,2,...,t) , \quad (1.5a)$$

$$\sum a_{ji}^p D_i + \sum a_{jj}^p D_j^k = O_j^p , \qquad (j =1,2,...,k) , \quad (1.5b)$$

$$\sum a_{ji}^k D_i + \sum a_{jj}^k D_j^k = O_j^k , \qquad (j =1,2,...,l) , \quad (1.5c)$$

5) the system, consisting of m equations expressing the fact *the selling prices of the products are equal to the cost of the productive services* employed in their production:

$$\sum a_{ji}^t p_j^t + \sum a_{ji}^p p_j^p + \sum a_{ji}^k p_j^k = p_i , \ (i =1,2,...,m) , \quad (1.6)$$

where

$p_1 = 1$;

6) the system of l equations of equality between the selling prices of the new capital goods and their costs of production:

$$\sum a_{jj}^t p_j^t + \sum a_{jj}^p p_j^p + \sum a_{jj}^k p_j^k = P_j , \ (j =1,2,...,l) , \quad (1.7)$$

7) 1 equation of equality between the total of the new capital goods and the aggregate excess of income over consumption in value:

$$\sum D_j^k P_j = E \,, \tag{1.8}$$

8) the system of l equations of the uniformity of the rate of net income for all capital goods proper:

$$P_j = p_j^k / (i+\mu_j+\nu_j) \,, \qquad (j=1,2,...,l) \tag{1.9}$$

These system equations describe, as it was mentioned above, the equilibrium situation, because according to Walras "there will remain $2n+2m+2l+1$ equations to determine exactly $2n+2m+2l+1$ unknowns" (Walras, p.282). This means that the system of equations (1.2)-(1.9) describe the final general equilibrium situation in capital formation. Walras reaches this equilibrium situation in capital formation 'in precisely the same way that we reached equilibrium earlier, first in exchange and then in production.'(Walras, p.282) In this stage, markets of products for consumption and services used for their production were extended by markets of new capital goods, and for the latter, market equilibrium is established as in the previous markets, in other words by the means of *tâtonnement.* In Walras' own words (p.282):

> ... the entrepreneurs manufacturing new capital goods use *tickets* to represent the successive quantities of these *products* which are first determined at random and then increased or decreased according as there is an excess of selling price over cost or vice versa, until selling price and cost become equal.

This means that the l equilibrium prices of these new capital goods are determined by the comparison between two different prices: cost of production (designated by P (see system (1.7)) and selling prices (designated by Π). The latter did not appear in the equilibrium situation as P substituted them in the system (1.9). This is similar for m equilibrium prices of commodities for consumption (there are two different prices: cost of production (designated by p and determined by system (1.6)) and selling prices (designated by π, which do not appear in system (1.3), since they were substituted by p). Also, for n equilibrium quantities of services (there are two different quantities: offered (designated by O, see system (1.2)) and demand (designated by D^h, which did not appear in equilibrium situation, since they were substituted by O, in system (1.5)). And finally, for the aggregate demand of new capital goods (designated by D) which did not appear in equilibrium situation, since it was substituted by the aggregate excess of income over consumption (designated by E) in system (1.8).

This means that in order to establish the equilibrium by *tâtonnement* it is necessary to formulate the working system by transforming systems (1.3), (1.5), (1.8) and (1.9) respectively. The working (starting) model in the capital formation has the following form:

1) mutual dependence between effective offer total quantities of services and their offered prices p:

$$O_j^t=F_j^t(\pi_2, \pi_3,...,\pi_m;p_1^t,...,p_t^t,p_1^p,...,p_k^p,p_1^k,...,p_l^k,p_e)=\Sigma o_j^t-d_j^t,(j=1,2,...,t); \tag{1.10a}$$

$$O_j^p=F_j^p(\pi_2,\pi_3,...,\pi_m;p_1^t,...,p_t^t,p_1^p,...,p_k^p,p_1^k,...,p_l^k,p_e)=\Sigma o_j^p-\Sigma d_j^p,(j=1,2,...,k); \tag{1.10b}$$

$$O_j^k=F_j^k(\pi_2,\pi_3,...,\pi_m;p_1^t,...,p_t^t,p_1^p,...,p_k^p,p_1^k,...,p_l^k,p_e)=\Sigma o_j^k-\Sigma d_j^k,(j=1,2,...,l); \quad (1.10c)$$

2) mutual dependence between effective aggregate demand quantities of products and their selling prices π:

$$D_i = F_i(\pi_2, \pi_3,..., \pi_m;p_1^t,...,p_t^t,p_1^p,...,p_k^p,p_1^k,...,p_l^k,p_e) = \Sigma x_i, \ (i=2,3,...,m);$$

$$D_1 = \Sigma D_j^{th} p_j^t + \Sigma D_j^{ph} p_j^p + \Sigma D_j^{kh} p_j^k - (\Sigma D_i p_i + E) ; \quad (1.11)$$

3) mutual dependence between the aggregate excess of income over consumption and rate of net income i:

$$E = D_e p_e = F_e(\pi_2, \pi_3,..., \pi_m;p_1^t,...,p_t^t,p_1^p,...,p_k^p,p_1^k,...,p_l^k,p_e)p_e =$$

$$= F_e(p_2, p_3,..., p_m;p_1^t,...,p_t^t,p_1^p,...,p_k^p,p_1^k,...,p_l^k,i), \quad (1.12)$$

4) the total demand quantities of productive services are obtained by means of total demand quantities of products (included new capital goods):

$$\sum a_{ji}^t D_i + \sum a_{jj}^t D_j^k = D_j^{th} , \qquad (j=1,2,...,t) , \quad (1.13a)$$

$$\sum a_{ji}^p D_i + \sum a_{jj}^p D_j^k = D_j^{ph} , \qquad (j=1,2,...,k) , \quad (1.13b)$$

$$\sum a_{ji}^k D_i + \sum a_{jj}^k D_j^k = D_j^{kh} , \qquad (j=1,2,...,l) , \quad (1.13c)$$

5) cost of production for products are obtained by the offered prices of services

$$\sum a_{ji}^t p_j^t + \sum a_{ji}^p p_j^p + \sum a_{ji}^k p_j^k = p_i , \ (i=1,2,...,m) , \quad (1.14)$$

6) cost of production of new capital goods are obtained by offered prices of services:

$$\sum a_{jj}^t p_j^t + \sum a_{jj}^p p_j^p + \sum a_{jj}^k p_j^k = P_j , \ (j=1,2,...,l) , \quad (1.15)$$

7) the total value of new capital goods is obtained by summation of demand value of all new capital good proper:

$$\sum D_j^k \Pi_j = D , \quad (1.16)$$

8) the system of l equations of the uniformity of the rate of net income for all capital goods proper:

$$\Pi_j = p_j^k / (i+\mu_j+\nu_j) , \qquad (j=1,2,...,l) \quad (1.17)$$

The number of equations in systems (1.10)-(1.17) is the same as in the previous systems (1.2)-(1.9) and it equals $2n+2m+2l+1$, but the number of unknowns is enlarged and it equals $3n+3m+3l+2$. This means that there are $n+m+l+1$ additional unknowns: 1) m selling prices of consumers goods (π); 2) n demand quantities of

services (D^h); 3) l selling prices of new capital goods (Π); and 4) one aggregate demand quantities of new capital goods (D). Therefore, in order to begin the process of adjustment (*tâtonnement*) a certain number of unknowns, namely ($n+m+l+1$), have to be given, as in the cases of exchange economy and production economy.

Walras realized his approach for the capital formation and credit assumed that: 'Let us suppose that we come to a market, where a certain price of net income $p'_e = 1/i'$ plus l quantities of new capital goods D'_k, $D'_{k'}$, $D'_{k''}$,... to be manufactured, plus n prices of services, plus m quantities of final products to be manufactured, are all determined in random (Walras, p.284)'. By substituting these given magnitudes in systems (1.10)-(1.17) Walras obtained four couples of magnitudes (values): 1) n quantities of services offered O (1.10) and their demanded quantities D^h (1.13); 2) m-1 selling prices of products π (1.11) and their cost of production p (1.14); 3) l selling prices of new capital goods Π (1.17) and their cost of production P (1.15); 4) total saving E (1.12) and total demand value of new capital goods (investment) D (1.16). If there is equality, perchance, between different magnitudes for all four couples then there is equilibrium. Generally, however, that is not fulfilled.

Therefore, Walras' used *tâtonnement* to establish equilibrium similar to exchange and production economies. Namely, in addition to iterative processes for the previous economies, in Capital formation and credit there are two additional iterative processes. The first is the establishment of equilibrium in the market for new capital goods by means of their selling prices and cost of production. If the selling price of new capital goods is greater than their cost of production, the quantity produced will increase and their selling price will fall; if their selling price is lower than their cost of production, the quantity produced will diminish and their selling price will rise. In equilibrium their selling price and their cost of production are equal. The second is the establishment of equilibrium in the market for an abstract good *net income*. If the total saving (total value of *net income*) is less than the total value of new capital goods, then the rate of income has to rise, and in opposite case has to fall. The equilibrium rate of income is established when the total saving is equal the total value of new capital goods (for more detail of this processes see Davar 1997).

Nevertheless, it is necessary to stress that Walras formulated the law for the capital formation and credit, similar to that for exchange and production economies, which not only indicates the ways of establishment of equilibrium, but also states the equilibrium's state. In other words, equality between total savings and total new capital goods is established by a comparison between them, i.e., independently, but not by means of the relationship between total income and total consumption. This means that in Walras' case, the budget constraint' law, for the Capital formation is also a by-product of his law as well as that for the production economy (Davar, 1994). Here, the total used income is determined by the sum of its components: consumption and new capital goods (investment). In addition, equality between total saving and total new capital goods guarantees equality between using and producing total incomes. So, in Walras' theory, equilibrium between total saving and total investment is primary, the budget constraint law is secondary, and not the opposite, as is presented in the literature since Walras' time. It is necessary to stress that here we do not discuss problems of existence, uniqueness and variation of prices (re-establishment of equilibrium state) since they are irrelevant for the goal of the paper (Davar, 1994, 1997, 1998)..

2. Leontief's Input-Output Analysis

Leontief started his first book about input-output analysis with the following words: 'THIS MODEST VOLUME describes an attempt to apply the economic theory of general equilibrium – or better, general interdependence – to an empirical study of interrelations among the different parts of a national economy as revealed through covariations of prices, outputs, investments, and incomes.' (Leontief, (1941), 1960, p.3) This means, that Leontief tried to apply neo-classical (Walras') general equilibrium to practical economic life. He first compiled empirical input-output table and then drew its theoretical scheme.

Here we do not describe in detail the empirical input-output table of Leontief, since it is well known and familiar. We will just stress some of its attributes that are required for the following discussion. First, consumption (final uses) is divided into several categories: private and public (government) consumption, investment, changes in the stocks and export, and value added is divided into the several items: wages, profits and other value added, taxes, subsidies and imports for production. Second, the data, in general, were in monetary terms, that means prices and quantities were not separated. Before describing Leontief's theoretical input-output model it is necessary to stress that from the very beginning Leontief gave importance and significance to presenting prices and quantities separately (Leontief, 1960, and 1974). Therefore, Leontief presented two different versions of input-output systems. First, in general, one where quantities (physical) and prices (absolute-money) are separated (Leontief 1965 & 1974), and second, according to empirical input-output, where quantities are money terms and prices are in relative terms (Leontief, 1960 (1941) &1986). In order to differentiate between these two systems, in the following the first (physical quantities and money prices) will be presented in the bold characters and the second (monetary quantities and relative prices) in ordinary characters.

Leontief's open input-output systems might be presented as the following (Leontief, 1965 & 1974):

a) the quantitative demand model

$$\boldsymbol{x} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\,\mathbf{y} \tag{2.1}$$

b) the money (absolute) prices model

$$\boldsymbol{p}' = (\boldsymbol{I} - \boldsymbol{A}')^{-1}\,\boldsymbol{v}' \tag{2.2}$$

where

$\boldsymbol{x}$ – a column vector ($n\times1$) of total outputs in physical terms;

$\boldsymbol{A}$ – a matrix ($n\times n$) of direct input coefficients in physical terms;

$\boldsymbol{y}$ – a column vector ($n\times1$) of final uses (private and public consumption, investment, export) in physical terms;

$\boldsymbol{p}$ – a row vector ($1\times n$) of money (absolute) prices in money/physical terms;

$\boldsymbol{v}$ – a row vector ($1\times n$) of a value added (imports for production, value of labor and capital, taxes and subsidies) in money/physical terms;

I – a unit matrix ($n\times n$).

In such an approach there is a tacit assumption that there is an equilibrium situation. What this means is that demand equals supply for goods and factor services and cost of production (supply price) equals demand price for goods. Also, it is assumed that that there exist required quantities of factor services. In addition, it is necessary to

stress that value added is composed by the value of factor items (Leontief, 1986; Davar, 1994).

Leontief's empirical input-output table, as we have mentioned above, is expressed in monetary terms. Therefore, for the theoretical description of the empirical input-output, the above model was used changing only its terminology:

a) the quantitative demand model

$$x = (I - A)^{-1} \text{ y} \tag{2.3}$$

b) the money (absolute) prices model

$$p' = (I - A')^{-1} v' \tag{2.4}$$

where

x – a column vector (n×1) of total outputs in monetary terms;
A – a matrix (n×n) of direct input coefficients in monetary terms;
y – a column vector (n×1) of final uses in monetary terms;
p – a row vector (1×n) of relative prices in money terms;
v – a row vector (1×n) of a value added in money terms.

Before going on to discuss these models, it is necessary to point out that the term 'relative price' has specific meaning, namely the prices of goods in equilibrium is equal to one. Let us do not discuss here the problems connected with such an interpretation of relative prices (see Davar, 1994). However, Ghosh (1959) suggested that the supply model can be described by means of a total output of goods that may be obtained for a given value added. The latter approach has been wide discussed in the input-output literature (Augustinovics, 1970; Dietzenbacher, 1997; Oosterhaven, 1988, & 1996). At the same time, it has to be stressed that Leontief, to our best knowledge, never and nowhere related to Ghoshian type models. Meanwhile, this is the time to decide what is the genuine sense of Ghoshian type models.

These two types of models of input-output are similar to Walras' model (see above) and even worse, instead of Leontief' empirical input-output are more widely applied and closer to reality. First, final uses are expressed as one whole component and the possibility is lost to catch a variety of different categories, which is characterized by its own specific moments. The same is true for a value added. Labor is also presented empirically as a single row, and not as various professions similar to reality, but also the row of labor was joined with other items of factors. Leontief was the first understanding this flaw and attempted to correct it (Leontief & Duchin, 1986). Second, the relationship between quantity and price for both goods and factors services, has stayed vague, and consequently the same is true for the relationship between theoretical model and empirical analysis.

One of the main flaws of Walras' general equilibrium theory is its static character. Walras' formulated a very progressive theory of capital formation and credit (see previous section), but the latter is static. Therefore, Leontief and other authors suggested a dynamic input-output model. There are a lot of various version of the dynamic input-output model, but we will discuss here Leontief's version because it is characterized by common attributes of all them. Leontief's formulated his dynamic input-output model as the followings (Leontief, 1970):

$$x_t - A_t x_t - B_{t+1}(x_{t+1} - x_t) = c_t \tag{2.5}$$

where

x_t – the column (n×1) vector of sectoral outputs produced in year t;
c_t – the corresponding column vector of deliveries of final demand excepted the new capital stocks of fixed and working capital (inventories) in year t;
A_t – the square (n×n) of technical (input) coefficients in year t;
B_t – the corresponding square matrix of capital coefficients in year t;

It is necessary to point out that Leontief's dynamic model of Input-output (2.5) is formulated in monetary (nominal) terms. Therefore, its corresponding price model describes relative prices (Brody, 1970; Ciaschini, 1993). However, this model is far from reality: 'A time-phased vector of final demand – premultiplied by a given dynamic inverse – may arithmetically yield negative total direct and indirect output requirements for some goods in some periods of time. If so, at least some of balance equations in system (3) do not represent the real world. As everyone who has dealt with this kind of system knows, the problem arises because eq. (3) assumes full capacity utilization in all sectors all the time (Leontief, 1970, p.22).'

In addition, this dynamic model does not describe the real process of saving and investment, as it existed in Walras' static model (vide supra). This is because, prices are relative and such categories as income and interest rate and insurance are absent. As Kuenne writes: 'Although the Leontief matrix may be an historical derivative of the Walrasian general equilibrium system, its right to a position as an operational logical derivative mat be questioned. Disaggregation and interdependence are necessary but not sufficient for such a claim. Unfortunately, each economist must judge on a largely intuitive basis the relative importance of such adjustment present in the Walrasian and absent in the Leontief framework, and therefore the range of applicability of the latter; (Kuenne, 1954)'. So, on the one hand Walras' Capital formation and credit describes reality very closely, but it is static. On the other hand Leontief's dynamic input-output model is dynamic, but it is far from reality. The future task to combine them.

3. Leontief's Input-Output Analysis in the Future

In order for Leontief's Input-Output analysis to become a genuinely useful tool for economic analysis and planning in the future, several decisive actions are required. First, its *theoretical basis must be reformulated in accordance with Walras' theoretical treatment and real economic life*, namely, adding and enlarging the demand part (final use), and consideration of price discrimination for goods and primary factors, especially for labor.

Second, empirical input-output *must be extended to accord with reality*. This means that primary factors and categories of final uses have to be presented in a highly disaggregated form, in order to attend to underemployment problem, as it should be dealt with. As regards fixed capital, its presentation must also be divided by levels, in order to formulate and use a truthful dynamic input-output model. And, exporting countries must also present exports in final uses in order to measure the influence of exchange rates.

Third, *real integration of money and interest rate* in both theoretical and empirical input-output must be done.

Finally, a genuine association between Walras' theoretical and Leontief's empirical input-output must be achieved. *Input-output with mixed measurement* for goods and for primary factors, in our opinion, might be one of solutions for this serious problem[3].

Conclusions

In this paper the relationship between Walras' theoretical general equilibrium theory and Leontief's input-output analysis was discussed. It was shown that Walras' theory served as the background of Leontief's approach. At the same time, it was also shown that there are several attributes characteristic to each approach, both differently and specifically. On the one hand, Walras' approach clearly discussed problems of prices (absolute-money) and quantities, progressive theory of serving, income and investment, income and interest rates, and integration of money in the general equilibrium framework. On the other hand, Leontief's input-output analysis based on an empirical database describing the reality very closely, including government (public) sector and international trade (import and export of goods and services). Therefore, in the conclusion of the paper, some suggestions on the basis of combination of positive attributes of Walras' theory and Leontief's analysis are offered to improve the applicability of input-output system for practical goals.

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[1] Jaffe (1954, p.71) translated this thus: "Following this same procedure, the pure theory of economics ought to take over from experience certain type concepts, like those of exchange, supply, demand, market capital, income, productive services and products. From these real-type concepts the pure science of economics should then abstract and define ideal-type concept in terms of which it carries on its reasoning. The return to reality should not take place until the science is completed and then only with a view to practical applications."

[2] We cannot agree with such a statements as "We may conclude that, setting aside purely formal similarities, the analysis of Leontief and Walras have little in common." (Kurz, H. D. & N. Salvadori, 2000, p. 174). First, as the authors observe "Leontief at times has himself expressed the opinion that his analysis and that Walras are compatible with one another." Second, the authors had numerous arguments that show the opposite. The authors suggest two arguments. The first – method – they state that Leontief '… insisted that the investigation should focus on 'directly observable basic structural relationships' … and not, like Walras's general equilibrium theory, on utility, demand functions etc., that is, things are not directly observable.' (p. 174) There are some points to stress. First, in our opinion, this is opposite to that which Leontief claimed repeatedly, that is a reciprocal connection between empirical and theoretical treatments (see our quotation). Second, the fact that until today such categories as utility, demand functions are not observable is one of basic flaws of Leontief's approach and that of general modern economic theory. Finally, there must be a comparison only between relevant parts of two theories because Walras' approach is much more comprehensive than Leontief's (viding infra). The second argument of the authors is the content of the theory. There are many problematic statements, but let us discuss this one which characterized the common style of the authors. They write: "… in Isnard as well as Leontief, the parameters that determine relative prices are technological and institutional data, whereas in Walras's case of the pure exchange economy the 'effective demands' are ultimately rooted in the agent's utility maximizing disposition. There is a real and close similarity between those of Leontief and Isnrd, whereas is only a questionable one between those of Leontief and Walras." (p.174) This statement is "correct", but the authors compare here two different approaches, Leontief's production economy and Walras' exchange economy. Not only is this incorrect but also there is an illusion that the readers (majority of them), who are not familiar Walras original work that Walras discussed only an exchange economy. It is necessary to point out that the authors in the notes write: 17. Hence, the appropriate point of reference would be Walras's developed theory including the production of consumption goods and the reproduction of capital goods proper (and money economy – E.D.). For the comparison of that theory with the 'classical' theory, see Kurz & Salvadory (1995, pp.23-6) Why did the authors not compare Walras' Production or Capital economies with Leonief's one? And what the reader might find in the source with the authors recommend: "It is beyond the scope of this book to discuss in detail the merits and demerits of Walras's general equilibrium analysis …" (Kurz & Salvadory, 1995, p. 25). Finally, such an eminent author of Walras' theory as Jaffé writes "I believe I have evidence enough to indicate that Isnard was not merely "a precursor of Leon Walras," but actually a direct progenitor of the Walrasian general equilibrium model.(Jaffé, p.56) and 'The resemblance between Isnard and Walras are so close both in the terminology and in the analytical employment of the concept that …' (p. 76). So, if 'There is a real and close similarity between the contribution of Leontief and Isnard, …' then the same is true between Leontief and Walras and not contrary as the authors have claim (vide supra).

[3] See our paper at this Conference "Input-Output in Mixed Measurement".