Structural Decomposition Analyses with Dependent Determinants

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Abstract. Structural decomposition techniques are used to break down the changes in one variable into the changes in its determinants. Typically, these determinants are assumed to be independent. Using the decomposition of value added growth as a prototype example, this paper examines the phenomenon that several of the determinants are not independent. The determinants are termed fully dependent if changes in one determinant cannot occur without corresponding changes in another determinant. In most empirical cases, full dependence exists between groups of determinants, not between separate determinants. It is indicated that dependencies may cause a bias in the results of decomposition analyses. An alternative to overcome this problem is proposed and the findings are illustrated by an empirical study for The Netherlands 1972-1986.

Keywords. Structural decomposition, value added growth, dependent variables.

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1. Introduction

Structural decomposition analyses are nowadays a common descriptive tool in studying changes over time. The central idea is that the change in some variable is decomposed, usually in an additive way, into the changes in its determinants. It thus becomes possible to quantify the underlying sources of the changes. See Rose and Casler (1996) for an overview, recent applications include Oosterhaven and van der Linden (1997), Cabrer *et al.* (1998), Cronin and Gold (1998), Oosterhaven and Hoen (1998), Wier (1998), Albala-Bertrand (1999), Alcala *et al.* (1999), Mukhopadhyay and Chakraborty (1999), Casler (2000), Dietzenbacher (2000), and Milana (2000).

To sketch the idea, consider the standard Leontief model $\mathbf{x} = \mathbf{Ax} + \mathbf{f}$, where $\mathbf{x}$ denotes the vector of sectoral outputs, $\mathbf{A}$ the $n \times n$ matrix of input coefficients and $\mathbf{f}$ the vector of final demands. Its solution is given by $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} = \mathbf{Lf}$, where $\mathbf{L} \equiv (\mathbf{I} - \mathbf{A})^{-1}$ denotes the Leontief inverse. In analyzing the changes in the outputs, the following decomposition may be used[1]

$$\Delta\mathbf{x} = (\Delta\mathbf{L})\mathbf{f}_1 + \mathbf{L}_0(\Delta\mathbf{f}) \tag{1}$$

The first term on the right hand side describes what the changes in the outputs would have been if the input coefficients had changed (inducing a change $\Delta\mathbf{L}$ in the Leontief inverse) but the final demands had remained constant. Similarly, the second term measures the contribution of the final demand changes by expressing the output changes if final demands had changed but technology (reflected by the input matrix $\mathbf{A}$) had been

[1] $\Delta\mathbf{x} = (\Delta\mathbf{L})\mathbf{f}_0 + \mathbf{L}_1(\Delta\mathbf{f})$ is an equivalent expression. It should be noted that the specific form of the decomposition is not unique and the number of equivalent forms increases rapidly when the number of determinants becomes larger. Dietzenbacher and Los (1997, 1998) analyze the sensitivity across decomposition forms.

unchanged. Implicitly, these counterfactual calculations assume that the changes in technology and the changes in final demands can be treated as being independent from each other.

In the example above, the viewpoint of independence between the determinants of the output changes seems reasonable, given the standard assumptions in input-output analysis. In many other cases, however, the assumption of independence between the determinants is incorrect. As an example, consider the vector of sectoral values added $\mathbf{v} = \hat{\mathbf{c}}\mathbf{x} = \hat{\mathbf{c}}\mathbf{L}\mathbf{f}$. Here $\mathbf{c}$ is the vector of value added coefficients (i.e. value added per unit of output) and $\hat{\mathbf{c}}$ is the corresponding diagonal matrix. One of the decomposition forms yields[2]

$$\Delta\mathbf{v} = (\Delta\hat{\mathbf{c}})\mathbf{L}_1\mathbf{f}_1 + \hat{\mathbf{c}}_0(\Delta\mathbf{L})\mathbf{f}_1 + \hat{\mathbf{c}}_0\mathbf{L}_0(\Delta\mathbf{f}) \qquad (2)$$

The first term on the right hand side denotes the change in sectoral values added if only the value added coefficients had changed while the technology and the final demands had remained constant. In the present example, however, it is unlikely that the determinants $\Delta\hat{\mathbf{c}}$ and $\Delta\mathbf{L}$ are independent. Typically, changes in intermediate input coefficients and in value added coefficients affect each other. In some cases they even cannot occur but simultaneously, i.e. they are intrinsically dependent on each other.

In this paper we propose a decomposition form for cases with dependent determinants. In the next section we discuss the extreme case of full dependency. That is, when the sum of the input coefficients a_{ij} and the value added coefficient c_j is fixed. Section 3 analyzes the situation when this sum is allowed to change. Section 4 is devoted to an empirical illustration in which value added changes in The Netherlands between

[2] The other five equivalent forms are $(\Delta\hat{\mathbf{c}})\mathbf{L}_1\mathbf{f}_1 + \hat{\mathbf{c}}_0(\Delta\mathbf{L})\mathbf{f}_0 + \hat{\mathbf{c}}_0\mathbf{L}_1(\Delta\mathbf{f})$, $(\Delta\hat{\mathbf{c}})\mathbf{L}_0\mathbf{f}_1 + \hat{\mathbf{c}}_1(\Delta\mathbf{L})\mathbf{f}_1 + \hat{\mathbf{c}}_0\mathbf{L}_0(\Delta\mathbf{f})$, $(\Delta\hat{\mathbf{c}})\mathbf{L}_0\mathbf{f}_0 + \hat{\mathbf{c}}_1(\Delta\mathbf{L})\mathbf{f}_1 + \hat{\mathbf{c}}_1\mathbf{L}_0(\Delta\mathbf{f})$, $(\Delta\hat{\mathbf{c}})\mathbf{L}_1\mathbf{f}_0 + \hat{\mathbf{c}}_0(\Delta\mathbf{L})\mathbf{f}_0 + \hat{\mathbf{c}}_1\mathbf{L}_1(\Delta\mathbf{f})$, $(\Delta\hat{\mathbf{c}})\mathbf{L}_0\mathbf{f}_0 + \hat{\mathbf{c}}_1(\Delta\mathbf{L})\mathbf{f}_0 + \hat{\mathbf{c}}_1\mathbf{L}_1(\Delta\mathbf{f})$, see Dietzenbacher and Los (1998).

1972 and 1986 are decomposed using both the traditional analysis and the variants proposed in this paper. Section 5 concludes.

2. Full dependency

To describe the case of full dependency we use two examples. *Example 1* refers to the case in which the matrix $\mathbf{A}$ includes import coefficients next to domestic input coefficients. That is, let $\mathbf{Z}$ denote the $n\times n$ matrix of intermediate deliveries, $\mathbf{M}$ the $n\times n$ matrix of imports (all of which are assumed to be competitive) by the production sectors, and $\mathbf{x}$ the vector of domestic outputs. The accounting equations yield $\mathbf{x} = \mathbf{Ze} + \mathbf{f}$ and $\mathbf{x}' = \mathbf{e}'\mathbf{Z} + \mathbf{e}'\mathbf{M} + \mathbf{v}'$, where $\mathbf{e}$ denotes the summation vector, i.e. $\mathbf{e}' = (1,\ldots,1)$.[3] Let $\mathbf{m} = \mathbf{Me}$ denote the vector of total imports, i.e. m_i gives the imports of product i. If we define $\mathbf{A}_d = \mathbf{Z}\hat{\mathbf{x}}^{-1}$ as the matrix of domestic input coefficients and $\mathbf{A}_m = \mathbf{M}\hat{\mathbf{x}}^{-1}$ as the matrix of import coefficients, we can write $\mathbf{x} + \mathbf{m} = (\mathbf{A} \;\; + \mathbf{A}_m)\mathbf{x} + \mathbf{f}$. Using $\mathbf{A} = (\mathbf{A}_d + \mathbf{A}_m)$, representing the technological input coefficients, we have $\mathbf{x} = \mathbf{L}(\mathbf{f}\text{-}\mathbf{m})$ where $(\mathbf{f}\text{-}\mathbf{m})$ denotes the final demands including *net* exports. *Example 2* of full dependency is the application of the decomposition in (2) to a closed economy, i.e. when all imports (and exports) are zero.

An important feature in both examples above is that the technological input coefficients and the value added coefficient sum to one in each sector. That is, using $\mathbf{c}' = \mathbf{v}'\hat{\mathbf{x}}^{-1}$,

$$\mathbf{e}' = \mathbf{e}'\mathbf{A} + \mathbf{c}' \qquad (3)$$

[3] Vectors are column vectors by definition, a prime is used to indicate transposition.

The decomposition in (2) yields misleading results. For example, the second term on the right hand side of (2) gives the value added changes that would have occurred if the input coefficients a_{ij} would have changed as they actually have, whereas the value added coefficients and the final demands are fixed. Note, however, that this reflects a nonsensical situation that cannot be given an interpretation. Suppose that $\mathbf{A}_1 < \mathbf{A}_0$,[4] then the value added coefficients $\mathbf{c}'$ cannot remain fixed, given the restriction in (3). In the present setting, changes in the technological input coefficients cannot but affect the value added coefficients. That is, $c_j^1 - c_j^0 = \Sigma_i a_{ij}^0 - \Sigma_i a_{ij}^1$. Consequently, the decomposition in (2) cannot be given any sensible economic interpretation (Wolff, 1985, 1994, points out a similar problem in the context of a decomposition for changes in TFP growth).[5]

In the present case we propose to use the changes in the *mix* of intermediate inputs as a determinant, instead of the changes in the input coefficients. To sketch the idea, suppose that in each sector the value added increases while all intermediate deliveries, i.e. $(z_{ij} + m_{ij})$ for Example 1 and z_{ij} in the case of Example 2, remain the same. As a consequence, the total (gross) outputs increase which in its turn implies a decrease of the input coefficients together with an increase in the value added coefficients. In this particular case, the changes in the value added coefficients have caused the changes in the technological input coefficients. Note, however, that the mix of inputs has not changed. A more appropriate decomposition therefore takes the following form.

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[4] For vectors and matrices we use the following notations. $\mathrm{x} >> \mathrm{y}$ means $x_i > y_i \forall i$, $\mathbf{x} \geq \mathbf{y}$ means $x_i \geq y_i \forall i$, and $\mathbf{x} > \mathbf{y}$ means $\mathbf{x} \geq \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$.

[5] Wolff (1985, 1994) defines the aggregate total factor productivity (TFP) growth rate as ρ = $-[\mathbf{p}' d\mathbf{A} + wd\mathbf{l}' + rd\mathbf{k}']\mathbf{x} / \mathbf{p}'\mathbf{f}$. Here $\mathbf{p}$ is the price vector, $\mathbf{l}$ the vector of labor coefficients showing employment per unit of output, $\mathbf{k}$ the vector of capital stock coefficients, w the uniform wage rate, and r the uniform rate of profit on the capital stock. The sectoral rates of TFP growth are given by π' = $-(\mathbf{p}' d\mathbf{A} + wd\mathbf{l}' + rd\mathbf{k}')\hat{\mathbf{p}}^{-1}$. It immediately follows that ρ = $\pi'\hat{\mathbf{p}}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} / \mathbf{p}'\mathbf{f}$ = $\pi'\hat{\mathbf{p}}(\mathbf{I} - \mathbf{A})^{-1}\hat{\mathbf{p}}^{-1}\hat{\mathbf{p}}\mathbf{f} / \mathbf{p}'\mathbf{f}$ = $\pi'\mathbf{S}\beta$. $\mathbf{S} = \hat{\mathbf{p}}(\mathbf{I} - \mathbf{A})^{-1}\hat{\mathbf{p}}^{-1}$ is the Leontief inverse in value terms and β = $\hat{\mathbf{p}}\mathbf{f} / \mathbf{p}'\mathbf{f}$ is the vector of sectoral shares in total final demand. The decomposition in discrete time yields $\Delta\rho \approx (\Delta\pi')\mathbf{S}\beta + \pi'(\Delta\mathbf{S})\beta + \pi'\mathbf{S}(\Delta\beta)$. The dependence occurs since both $\Delta\mathbf{S}$ and π include the changes $\Delta\mathbf{A}$ in the technical coefficients matrix.

$$
\begin{aligned}
\Delta\mathbf{v} &= \hat{\mathbf{c}}_1\mathbf{L}_1\mathbf{f}_1 - \hat{\mathbf{c}}_0\mathbf{L}_0\mathbf{f}_0 \\
&= \hat{\mathbf{c}}_1\mathbf{L}_1\mathbf{f}_1 - \hat{\mathbf{c}}_0\tilde{\mathbf{L}}_1\mathbf{f}_1 + \hat{\mathbf{c}}_0\tilde{\mathbf{L}}_1\mathbf{f}_1 - \hat{\mathbf{c}}_0\mathbf{L}_0\mathbf{f}_1 + \hat{\mathbf{c}}_0\mathbf{L}_0\mathbf{f}_1 - \hat{\mathbf{c}}_0\mathbf{L}_0\mathbf{f}_0 \\
&= [\hat{\mathbf{c}}_1\mathbf{L}_1 - \hat{\mathbf{c}}_0\tilde{\mathbf{L}}_1]\mathbf{f}_1 + \hat{\mathbf{c}}_0[\tilde{\mathbf{L}}_1 - \mathbf{L}_0]\mathbf{f}_1 + \hat{\mathbf{c}}_0\mathbf{L}_0(\Delta\mathbf{f}) \qquad (4a)
\end{aligned}
$$

where $\tilde{\mathbf{L}}_1 = (\mathbf{I} - \tilde{\mathbf{A}}_1)^{-1}$ and

$$
\tilde{\mathbf{A}}_1 = \mathbf{A}_1\hat{\mathbf{s}}_1^{-1}\hat{\mathbf{s}}_0 \text{ with } \mathbf{s}_i' = \mathbf{e}'\mathbf{A}_i \ (i = 0,1). \qquad (5)
$$

The matrix $\tilde{\mathbf{A}}_1$ has in each column the same distribution of coefficients as $\mathbf{A}_1$ (i.e. $\tilde{a}_{ij}^1 / \tilde{a}_{kj}^1 = a_{ij}^1 / a_{kj}^1$, $\forall$ *i*,*j*,*k*), but has the same column sums as matrix $\mathbf{A}_0$. In other words, matrix $\tilde{\mathbf{A}}_1$ is obtained by multiplying each column of $\mathbf{A}_1$ by a (column-specific) scalar. Note that the same procedure is followed in the RAS approach (see e.g. Stone, 1961, Bacharach, 1970, MacGill, 1977, for recent contributions see van der Linden and Dietzenbacher, 1995, de Mesnard, 1997, Polenske, 1997, Dietzenbacher and Hoen, 1998, Toh, 1998, Gilchrist and St. Louis, 1999, Andreosso-O'Callaghan and Yue, 2000, Jalili, 2000). This approach is used to update input coefficient matrices by sequentially adapting its rows and columns proportionally. The uniform changes in the columns are interpreted so as to reflect the fabrication effects (Stone, 1961), indicating that the proportion of value added in a sector's total purchases has changed (Miller and Blair, 1985). Alternatively, the fabrication effects describe the substitution between total intermediate inputs and value added terms (such as labour and capital).

In the decomposition in (4*a*), the first term gives the fabrication effects, the second term describes the effects due to the change in the mix of intermediate input coefficients (reflecting the substitution among intermediate inputs), and the third term shows how final demand changes affect the sectoral values added.

It should be mentioned that the matrix $\tilde{\mathbf{A}}_1$ may be considered as a benchmark which can be chosen in many ways. In our case, the benchmark implies that a change in the value added coefficient c_j affects all input coefficients a_{ij} proportionally. In general, this seems an attractive benchmark, but whenever additional information is available an alternative benchmark may be more appropriate. For example, one might know that a certain increase in c_j caused a reduction in a single coefficient a_{kj}, because new capital goods installed in sector j substitute inputs from sector k. Then, the matrix $\tilde{\mathbf{A}}_1$, and thus $\tilde{\mathbf{L}}_1$ in equation (4*a*), can be specified in such a way that all changes in c_j are reflected in an opposite change in a_{kj}^1. The only requirement is that the column sums of $\tilde{\mathbf{A}}_1$ must equal the column sums of $\mathbf{A}_0$. In any case, the second term of equation (4*a*) should be interpreted as the effect due to changes in the intermediate input structure different from the changes as reflected by the benchmark.

In Dietzenbacher and Los (1998), it was argued that the number of mutually equivalent decomposition forms is n!, when there are n determinants. In the present case, we apply the principle of nested or hierarchical decompositions (see e.g. Sonis and Hewings, 1990, Oosterhaven and van der Linden, 1997, Oosterhaven and Hoen, 1998). That is, in the first step the dependent determinants are taken together as if they were a single determinant. Writing $\gamma = \hat{\mathbf{c}}\mathbf{L}$ implies that the number of determinants is now n-1, so that the number of different decompositions is reduced to (n-1)!. In the second step $\Delta\gamma$ is decomposed in two ways. That is,

$$\begin{aligned}\Delta\gamma = \hat{\mathbf{c}}_1\mathbf{L}_1 - \hat{\mathbf{c}}_0\mathbf{L}_0 \quad &= (\hat{\mathbf{c}}_1\mathbf{L}_1 - \hat{\mathbf{c}}_0\tilde{\mathbf{L}}_1) + (\hat{\mathbf{c}}_0\tilde{\mathbf{L}}_1 - \hat{\mathbf{c}}_0\mathbf{L}_0)\\ &= (\hat{\mathbf{c}}_1\mathbf{L}_1 - \hat{\mathbf{c}}_1\tilde{\mathbf{L}}_0) + (\hat{\mathbf{c}}_1\tilde{\mathbf{L}}_0 - \hat{\mathbf{c}}_0\mathbf{L}_0)\end{aligned}$$

with $\tilde{\mathbf{L}}_0 = (\mathbf{I} - \tilde{\mathbf{A}}_0)^{-1}$ and $\tilde{\mathbf{A}}_0 = \mathbf{A}_0\hat{\mathbf{s}}_0^{-1}\hat{\mathbf{s}}_1$. As a consequence, the total number of different, but equivalent, decomposition forms equals 2(n-1)!. The three forms that, in the present example, are equivalent to (4*a*) are as follows.

$$\Delta\mathbf{v} = (\hat{\mathbf{c}}_1\mathbf{L}_1 - \hat{\mathbf{c}}_0\tilde{\mathbf{L}}_1)\mathbf{f}_0 + \hat{\mathbf{c}}_0(\tilde{\mathbf{L}}_1 - \mathbf{L}_0)\mathbf{f}_0 + \hat{\mathbf{c}}_1\mathbf{L}_1(\Delta\mathbf{f}) \quad (4b)$$

$$= (\hat{\mathbf{c}}_1\tilde{\mathbf{L}}_0 - \hat{\mathbf{c}}_0\mathbf{L}_0)\mathbf{f}_0 + \hat{\mathbf{c}}_1(\mathbf{L}_1 - \tilde{\mathbf{L}}_0)\mathbf{f}_0 + \hat{\mathbf{c}}_1\mathbf{L}_1(\Delta\mathbf{f}) \quad (4c)$$

$$= (\hat{\mathbf{c}}_1\tilde{\mathbf{L}}_0 - \hat{\mathbf{c}}_0\mathbf{L}_0)\mathbf{f}_1 + \hat{\mathbf{c}}_1(\mathbf{L}_1 - \tilde{\mathbf{L}}_0)\mathbf{f}_1 + \hat{\mathbf{c}}_0\mathbf{L}_0(\Delta\mathbf{f}) \quad (4d)$$

The first term in each equation gives the fabrication effects, i.e. the effects due to a change in the value added coefficients combined with a corresponding change in the column sums of the input coefficients matrix. For the first terms in (4*a*) and (4*b*), the fabrication effects are based on the input mix of period 1, while they are based on the input mix of period 0 for the first terms in (4*c*) and (4*d*).

It is interesting to note that the traditional form as in (2) has a tendency to bias the effects of changes in the value added coefficients, when compared to equations (4). Suppose, for example, that $\Delta\hat{\mathbf{c}} = \hat{\mathbf{c}}_1 - \hat{\mathbf{c}}_0 >> 0$, then it follows from (3) that $\mathbf{s}_1 << \mathbf{s}_0$. Consequently, $\tilde{\mathbf{A}}_1 = \mathbf{A}_1\hat{\mathbf{s}}_1^{-1}\hat{\mathbf{s}}_0 > \mathbf{A}_1$ and using the power series expression for the Leontief inverse it follows that $\tilde{\mathbf{L}}_1 >> \mathbf{L}_1$ (where it is assumed that $\mathbf{A}_1$ is irreducible). Next compare the effects due to changes in the value added coefficients in (2), i.e. $(\Delta\hat{\mathbf{c}})\mathbf{L}_1\mathbf{f}_1$, with the fabrication effects caused by changes in value added coefficients as in (4*a*), i.e. $(\hat{\mathbf{c}}_1\mathbf{L}_1 - \hat{\mathbf{c}}_0\tilde{\mathbf{L}}_1)\mathbf{f}_1$. The difference is given by $(\hat{\mathbf{c}}_1\mathbf{L}_1\mathbf{f}_1 - \hat{\mathbf{c}}_0\mathbf{L}_1\mathbf{f}_1) - (\hat{\mathbf{c}}_1\mathbf{L}_1\mathbf{f}_1 - \hat{\mathbf{c}}_0\tilde{\mathbf{L}}_1\mathbf{f}_1)$ = $\hat{\mathbf{c}}_0(\tilde{\mathbf{L}}_1 - \mathbf{L}_1)\mathbf{f}_1 >> 0$ (provided $\mathbf{c}_0 >> 0$ or $\mathbf{f}_1 >> 0$). The effects as measured in (2), i.e. $(\Delta\hat{\mathbf{c}})\mathbf{L}_1\mathbf{f}_1$, are all positive and are all larger than the effects as measured in (4*a*). A comparison of (4*b*)-(4*d*) with their corresponding forms similar to (2) gives the same result.

Intuitively speaking, this result is very plausible. If $\mathbf{c}_1 >> \mathbf{c}_0$, it must be the case that in each column of the matrix $\mathbf{A}_1$ at least some element must decrease since its column sums must decrease. This latter aspect is completely neglected in the decomposition in (2), yielding an overestimation of the effects of changing value added coefficients. For the effects of the changes in the intermediate input coefficients the same result holds, with a negative sign. That is, the effects of changing input coefficients as measured by the second term in (2) are negative, while the effects of changes in the intermediate input mix in (4*a*) are less negative (or maybe even positive). It should be stressed that these results for the comparison between the effects in (2) and in (4*a*) are based on the case where each value added coefficient increases. In the general case where some value added coefficients increase while some others decrease, simple comparative results cannot be obtained without further assumptions on the structure of production.

As a final remark, it seems at first sight that there is a connection to the strand of literature that takes so-called interaction effects into consideration. For example, including interaction effects into the decomposition of $\Delta\gamma$ yields

$$\begin{aligned}\Delta\gamma &= \hat{\mathbf{c}}_1\mathbf{L}_1 - \hat{\mathbf{c}}_0\mathbf{L}_0 = (\Delta\hat{\mathbf{c}})\mathbf{L}_1 + \hat{\mathbf{c}}_1(\Delta\mathbf{L}) - (\Delta\hat{\mathbf{c}})(\Delta\mathbf{L}) \\ &= (\Delta\hat{\mathbf{c}})\mathbf{L}_0 + \hat{\mathbf{c}}_0(\Delta\mathbf{L}) + (\Delta\hat{\mathbf{c}})(\Delta\mathbf{L})\end{aligned}$$

The third term on the right hand side of both expressions measures the interaction effects, which reflect the interaction of changes in $\hat{\mathbf{c}}$ and changes in $\mathbf{L}$. It seems as if this exactly captures what we are trying to measure. Closer inspection, however, shows that the dependence covered by the interaction terms is entirely different from the dependence that is the point of focus in this paper.

The dependence considered by us is theoretical in its nature and stems from the underlying input-output model. In particular the adding-up constraint in (2) yields that there is full dependence between $\Delta\mathbf{c}'$ and $\mathbf{e}'(\Delta\mathbf{A})$. The interaction terms, however, reflect an “empirical dependence”. That is, they indicate the extent to which the numerical changes in the two determinants have the same sign or have opposite signs.[6]

[6] Note that this empirical dependence becomes rather complex for more elaborate decompositions since the number of interaction terms involved, increases rapidly when the number of determinants increases.

This empirical dependence is present in decomposition studies, no matter whether the determinants are theoretically dependent or independent. This may be illustrated by using the decomposition in (1). From a theoretical viewpoint, the determinants $\Delta\mathbf{L}$ (caused by $\Delta\mathbf{A}$) and $\Delta\mathbf{f}$ are independent, given the context of the input-output model again. Yet the decomposition based on the use of interaction terms yields

$$\begin{aligned}\Delta\mathbf{x} &= (\Delta\mathbf{L})\mathbf{f}_1 + \mathbf{L}_1(\Delta\mathbf{f}) - (\Delta\mathbf{L})(\Delta\mathbf{f}) \\ &= (\Delta\mathbf{L})\mathbf{f}_0 + \mathbf{L}_0(\Delta\mathbf{f}) + (\Delta\mathbf{L})(\Delta\mathbf{f}).\end{aligned}$$

The interaction terms provide information on the empirical relation between the $\Delta\mathbf{L}$ and $\Delta\mathbf{f}$ terms. Of course, if many empirical studies (across countries and over time, using the same decomposition) showed similar results for the interaction terms, it would seem reasonable to check the underlying theoretical model. So, empirical dependence may indicate theoretical relations that have been hidden hitherto. The present paper, however, considers only adding-up constraints which are well-known but which have never been taken into account in decomposition analyses.

3. Dependency in the general case

This section discusses the case of determinants that are dependent, but not fully dependent. To this end, we adapt the two examples of the previous section. In contrast to Example 1 in Section 2, *Example 3* assumes additionally that there is also a vector of non-competitive imports, which are treated as primary factors. *Example 4* adapts Example 2 of the closed economy in the previous section by assuming that imports are introduced as part of the primary costs (and at the same time exports as part of the final demand). For both cases, there is now also a vector of sectoral non-competitive imports next to the vector of sectoral values added. Focussing on Example 3, the vector $\mathbf{d}'$ is defined as the row vector of non-competitive import coefficients, i.e. measured per unit of output, and equation (3) now changes into

$$\mathbf{e}' = \mathbf{e}'\mathbf{A} + \mathbf{c}' + \mathbf{d}' \qquad (6)$$

Note that the column sums of the intermediate input coefficients plus the non-competitive import coefficients on the one hand, and the value added coefficients on the other hand, are fully dependent.

Applying the decomposition form (2) to the present case means that implicitly two assumptions are made. The first term in (2) describes the effects of changes in the value added coefficients under the assumption that these changes are counterbalanced by equal changes (but with the opposite sign) in the non-competitive import coefficients. The second term in (2) gives the effects of changes in the intermediate input coefficients under the assumption that the total increase (decrease) in a sector's inputs is outweighed by a equal decrease (increase) of this sector's non-competitive imports. Both assumptions imply in this example that non-competitive imports are implicitly viewed as balancing items, or residuals.[7]

The results in the previous section suggest an alternative approach. That is, when value added coefficients change it is assumed that the column sums of the technological input matrix **A** plus the row vector of import coefficients change in the opposite way, creating fabrication effects. Note that the structure of the coefficients within a column remains the same. It thus is explicitly assumed that when value added coefficient c_j is increased by α%, each input coefficient a_{ij} as well as the import coefficient d_j is

[7] In the literature, typically these implicit consequences are not even mentioned. In a decomposition analysis of labour productivity growth, Dietzenbacher, Hoen and Los (2000) explicitly point out the problem of dependency and the option to use imports as a residual term.

decreased by the same percentage (viz. $\alpha c_j / 1 - c_j$). Value added terms (such as labour and capital) substitute intermediate inputs and imports, whose mix remains unchanged. In the same fashion, we assume that changes in the non-competitive import coefficients also induce the column sums of **A** to change. So under this assumption, using less non-competitive imports implies that more of each intermediate input is required. This causes a second fabrication effect. The changes in the technological input coefficients are again changes in the mix of intermediate inputs. Using $\gamma = \hat{\mathbf{c}}\mathbf{L}$ we arrive at the following expression.

$$\begin{aligned} \Delta\gamma &= \hat{\mathbf{c}}_1\mathbf{L}_1 - \hat{\mathbf{c}}_0\mathbf{L}_0 \\ &= (\hat{\mathbf{c}}_1\mathbf{L}_1 - \hat{\mathbf{c}}_0\tilde{\mathbf{L}}_1) + (\hat{\mathbf{c}}_0\tilde{\mathbf{L}}_1 - \hat{\mathbf{c}}_0\breve{\mathbf{L}}_1) + (\hat{\mathbf{c}}_0\breve{\mathbf{L}}_1 - \hat{\mathbf{c}}_0\mathbf{L}_0) \end{aligned} \tag{7a}$$

with $\tilde{\mathbf{L}}_1 = (\mathbf{I} - \tilde{\mathbf{A}}_1)^{-1}$ and $\breve{\mathbf{L}}_1 = (\mathbf{I} - \breve{\mathbf{A}}_1)^{-1}$, where

$$\tilde{\mathbf{A}}_1 = \mathbf{A}_1(\hat{\mathbf{s}}_1 + \hat{\mathbf{d}}_1)^{-1}(\hat{\mathbf{s}}_0 + \hat{\mathbf{d}}_0) \tag{8}$$

$$\breve{\mathbf{A}}_1 = \mathbf{A}_1\hat{\mathbf{s}}_1^{-1}\hat{\mathbf{s}}_0 \tag{9}$$

The first term on the right hand side of (7*a*) expresses the change in γ caused by a change in the value added coefficients, under the assumption that these changes simultaneously led to proportional adaptations of the column sums of **A** and the non-competitive import coefficients **d**. The second term indicates the effects due to changes in the mix of domestically produced intermediate inputs and non-competitive imports, under the assumption that the value added coefficients and the composition of domestically produced intermediate inputs had remained unchanged. The third term on the right hand side of (7*a*) shows how the changes in the composition of domestically produced intermediate inputs would have affected γ under the assumption that value added coefficients and non-competitive import coefficients had been constant. Note that the column sums of $\breve{\mathbf{A}}_1$ are equal to the column sums of $\mathbf{A}_0$, i.e. $\mathbf{e}'\breve{\mathbf{A}}_1 = \mathbf{s}_0'$.

Using the expression for $\Delta\gamma$, the decomposition for $\Delta\mathbf{v}$ becomes

$$\Delta\mathbf{v} = (\hat{\mathbf{c}}_1\mathbf{L}_1 - \hat{\mathbf{c}}_0\tilde{\mathbf{L}}_1)\mathbf{f}_1 + \hat{\mathbf{c}}_0(\tilde{\mathbf{L}}_1 - \breve{\mathbf{L}}_1)\mathbf{f}_1 + \hat{\mathbf{c}}_0(\breve{\mathbf{L}}_1 - \mathbf{L}_0)\mathbf{f}_1 + \hat{\mathbf{c}}_0\mathbf{L}_0(\Delta\mathbf{f}) \qquad (10)$$

It should be emphasized again that the decomposition is not unique. In the previous section, $\Delta\gamma$ could be decomposed in two different ways, which led to four alternative decompositions for $\Delta\mathbf{v}$, i.e. equations (4*a*)-(4*d*). In the present case, $\Delta\gamma$ consists of three determinants, suggesting 3! = 6 equivalent decomposition forms. However, in order to ensure that the changes in the value added coefficients are simultaneously divided between the domestically produced intermediate inputs and the non-competitive imports, changes in the value added coefficients must be considered "before" the changes in the imports.[8] This requirement rules out three (of the six) decomposition forms. Next to (7*a*) we have

$$\Delta\gamma = (\hat{\mathbf{c}}_1\mathbf{L}_1 - \hat{\mathbf{c}}_0\tilde{\mathbf{L}}_1) + (\hat{\mathbf{c}}_0\breve{\mathbf{L}}_0 - \hat{\mathbf{c}}_0\mathbf{L}_0) + (\hat{\mathbf{c}}_0\tilde{\mathbf{L}}_1 - \hat{\mathbf{c}}_0\breve{\mathbf{L}}_0) \qquad (7b)$$

with $\breve{\mathbf{A}}_0 = \mathbf{A}_0\hat{\mathbf{s}}_0^{-1}\hat{\mathbf{s}}_1(\hat{\mathbf{s}}_1 + \hat{\mathbf{d}}_1)^{-1}(\hat{\mathbf{s}}_0 + \hat{\mathbf{d}}_0)$,

$$\Delta\gamma = (\hat{\mathbf{c}}_1\tilde{\mathbf{L}}_0 - \hat{\mathbf{c}}_0\breve{\mathbf{L}}_0) + (\hat{\mathbf{c}}_0\breve{\mathbf{L}}_0 - \hat{\mathbf{c}}_0\mathbf{L}_0) + (\hat{\mathbf{c}}_1\mathbf{L}_1 - \hat{\mathbf{c}}_1\tilde{\mathbf{L}}_0) \qquad (7c)$$

with $\tilde{\mathbf{A}}_0 = \mathbf{A}_0\hat{\mathbf{s}}_0^{-1}\hat{\mathbf{s}}_1$.

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[8] In deriving (7*a*), it was assumed that the value added coefficients are changed first, followed by changes in the non-competitive imports coefficients and finally by changes in the intermediate input coefficients. The mutually equivalent forms are obtained from the other five possible orderings of this sequence of changes. However, if non-competitive imports coefficients are changed before the value added coefficients are changed, it is no longer possible that value added terms substitute intermediate inputs and non-competitive imports alike, i.e. leaving their mix invariant.

The right hand sides of equations (7*b*)-(7*c*) are structured in the same way as (7*a*) is. That is, the first term reflects the fabrication effects due to changes in the value added coefficients, the second term the fabrication effects due to changing non-competitive import coefficients, and the third term gives effects due to the changes in the input mix. The final structure includes also the $\Delta\mathbf{f}$ terms and is obtained as a nested decomposition again. Since $\mathbf{v} = \gamma\mathbf{f}$, we have two equivalent forms for the decomposition of $\Delta\mathbf{v}$ into $\Delta\gamma$ and $\Delta\mathbf{f}$ terms. Next, $\Delta\gamma$ can be decomposed into its determinants according to the three equivalent forms (7*a*)-(7*c*). As a consequence, for the decomposition of $\Delta\mathbf{v}$ there are now six different, but equivalent, forms.

4. An illustration for the Netherlands, 1972-1986

The previous sections indicated that the common structural decomposition framework for value added growth (equation (2)) may yield biased results for the relative importance of the three sources which are usually distinguished: changes in value added per unit of gross output, changes in intermediate input coefficients and changes in final demand. Nevertheless, the proposed alternatives (equations (4) and (10)) would merely provide an academic improvement if the empirical size of the bias would appear to be negligible. To give an illustration of the practical consequences of sticking to the usual framework, the three decomposition methods (2), (4) and (10) will be compared for a decomposition of real value added changes in The Netherlands between 1972 and 1986.

The data are taken from OECD (1995). Six tables are used: the domestic transactions tables (files NLDIOK72 and NLDIOK86), the imported transactions tables (NLMIOK72 and NLMIOK86) and the total transactions tables (NLTIOK72 and NLTIOK86). The elements of all these tables are expressed in constant, 1980 prices and contain transactions between 33 industries.[9] Each table also contains an additional row and column with (relatively small) “statistical discrepancies”. Since this section does not aim at deeper insights into the causes of Dutch value added growth, the values in these rows and columns are treated in a rather rough way. The rows were added to the value added rows and the columns were added to the final demand columns.[10]

For our calculations, the imports transactions table provides a square matrix of competitive imports (matrix $\mathbf{M}$ in Example 1). The total transactions table provides a row vector of non-competitive imports. On the basis of this data set, we are able to empirically analyze the following three cases.

Mixed imports case. All available information from the database is used as it is. This case exactly reflects the situation sketched in Example 3 in Section 3, where intermediate deliveries include competitive imports while non-competitive imports are included as a separate row.

For many published input-output tables, however, these detailed data are not available and the distinction between competitive and non-competitive imports cannot be made. That is, imports are either included as a row vector or are included as a column of negative exports. We have tried to simulate also these situations by adapting (or actually abusing) our original database.[11]

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9 See Appendix A for the sector classification. The sectors “non-ferrous metals” and “radio, TV & communication equipment” were deleted from the tables because they did not use any inputs nor produced any output (the OECD apparently included them in order to maintain a uniform sector classification for several countries).

10 Another (similar) adjustment was unavoidable for the 1986 table, since its rows and columns do not add up to exactly identical values for a number of sectors. In this case, the difference was compensated for in the final demand column.

11 It should be borne in mind that the purpose of this empirical study is to get some insight into the bias of the results on the basis of actual data. Whereas the outcomes for the mixed imports case are also indicative for the changes in the Netherlands, the results for the next two cases are not. They should be viewed as the findings for a hypothetical country, say the Neverlands.

Competitive imports case. For this case, the existing row of non-competitive imports was redistributed proportionally over the matrix of competitive imports (and the final demands were adapted correspondingly). This case then exactly corresponds to Example 1 in Section 2 for full dependency, where only competitive imports were included.

Non-competitive imports case. This case is obtained by aggregating the existing matrix of competitive imports into a single row and adding this to the existing row of non-competitive imports. It reflects the situation described by Example 4 in Section 3.

The results for the value added decompositions for the aggregate Dutch economy are documented in Table 1. The coefficients of variation are calculated on the basis of the equivalent decomposition formulae hinted at in the discussions of equations (2), (4) and (10).[12]

TABLE 1 ABOUT HERE

The results for the mixed imports case show that the differences between the approaches - equations (2) and (10) - are very significant. The negative contribution of the change in value added coefficients according to the proposed methodology is only ten percent of the corresponding value according to the traditional method. With regard to changes in intermediate input coefficients, both the size and the sign of the estimated contributions are different, leaving the contribution of changed final demand vectors almost identical.[13]

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[12] Note that the diagonal matrix $\hat{\mathbf{c}}$ was replaced by the vector $\mathbf{c}'$ to investigate the contributions of the determinants for the aggregate economy using equations (2), (4) and (10).

[13] For ease of comparison, only the sums of the second and third term in (10) are reported in Table 1, as $\Delta\mathbf{L}$. On average, the contribution of changing non-competitive import coefficients was slightly negative (-1.5%), while the contribution of changing domestic intermediate input structures accounted for only 0.3% of the total effect.

The advantages of the proposed approach over the traditional methodology are most prominently reflected in the results for the competitive imports case. In the aggregate, in the absence of other primary inputs than value added components, the change in final demand (the third term) must always equal the change in value added, since $v_i = \mathbf{e}'\mathbf{v}_i = \mathbf{c}_i'\mathbf{L}_i\mathbf{f}_i = \mathbf{e}'(\mathbf{I}-\mathbf{A}_i)\mathbf{L}_i\mathbf{f}_i = \mathbf{e}'\mathbf{f}_i = f_i$, with i=0,1. Using the traditional methodology (2), this equivalence is not found in two of the six equivalent formulae.[14] In these formulae, the $\Delta\mathbf{f}$ terms read $\mathbf{c}_1'\mathbf{L}_0\Delta\mathbf{f}$ and $\mathbf{c}_0'\mathbf{L}_1\Delta\mathbf{f}$, respectively, whereas the other four formulae contain as corresponding $\Delta\mathbf{f}$ terms either $\mathbf{c}_0'\mathbf{L}_0\Delta\mathbf{f}$ or $\mathbf{c}_1'\mathbf{L}_1\Delta\mathbf{f}$. Substituting $\mathbf{c}_0' = \mathbf{e}'(\mathbf{I}-\mathbf{A}_0)$ in $\mathbf{c}_0'\mathbf{L}_0\Delta\mathbf{f}$ for example yields

$$\mathbf{c}_0'\mathbf{L}_0\Delta\mathbf{f} = \mathbf{e}'(\mathbf{I}-\mathbf{A}_0)\mathbf{L}_0\Delta\mathbf{f} = \mathbf{e}'\Delta\mathbf{f} = \Delta f = \Delta v \qquad (11)$$

Hence, the aggregate $\Delta\mathbf{f}$ effect is exactly equal to the change in the total value added. This equality clearly does not hold when **c** and **L** have different time indices, which is the case for two of the six formulae under the traditional methodology (2). This explains why the reported average of the $\Delta\mathbf{f}$ effects differs from 100%. Considering the third term in equations (4*a*)-(4*d*), it follows that the proposed decompositions all yield contributions of final demand change exactly equal to 100%.

Further, the traditional approach yields nonzero contributions for the first two terms. Such values do not have a sensible interpretation, since a reduction of value added due to changed value added coefficients cannot take place without a reduction of aggregate final demand by an equal amount. The equations (4*a*)-(4*d*) do not suffer from such implausible outcomes. For the aggregate economy, i.e. replacing $\hat{\mathbf{c}}$ by $\mathbf{c}'$, we find for the first term in (4*a*) that $\mathbf{c}_1'\mathbf{L}_1\mathbf{f}_1 - \mathbf{c}_0'\tilde{\mathbf{L}}_1\mathbf{f}_1 = \mathbf{e}'\mathbf{f}_1 - \mathbf{c}_0'\tilde{\mathbf{L}}_1\mathbf{f}_1$. Now note that $\mathbf{c}_0' = \mathbf{e}'(\mathbf{I}-\mathbf{A}_0) = \mathbf{e}'(\mathbf{I}-\tilde{\mathbf{A}}_1)$ so that also $\mathbf{c}_0'\tilde{\mathbf{L}}_1\mathbf{f}_1 = \mathbf{e}'\mathbf{f}_1$. Hence, the first term in (4*a*) equals zero for the aggregate

[14] See footnote 2 for the equivalent forms.

economy. Equation (11) shows that the third term in (4*a*) equals Δv. Consequently, also the second term in (4*a*) must be equal to zero. Similar results can be obtained for equations (4*b*)-(4*d*). The outcomes in Table 1 confirm this finding that the contributions of value added coefficient changes and of changes in the input structure are zero indeed.[15]

The non-competitive imports case yields the smallest (though still substantial) differences between the traditional decomposition framework (2) and the alternative decomposition of equation (10). This is due to the larger 'cushion' of non-competitive imports which, as was argued already, is assumed to be a balancing item in the traditional approach.

The decomposition results for the aggregate Dutch economy show that explicitly considering dependencies yields largely different conclusions with respect to the relative importance of the determinants of value added growth. Table 2 confirms that the empirical difference between the traditional methodology and the proposed approach may be very substantial at the sector level as well.

TABLE 2 ABOUT HERE

For reasons of space, only the decomposition results for the first determinant (changes in value added coefficients) are presented.[16] Again, the averages taken over the equivalent formulae are presented, together with some information on the spread around these values (the coefficient of variation, the maximum and the minimum). In general, the main

[15] The theoretical findings above are based on the assumption that all column sums of **A** are positive for both periods. The positive coefficients of variation reported in Table 1 are caused by the column sums of **A** for industry 33. For 1972, it was positive, whereas it was zero for 1986. This implied that the column of $\mathbf{A}_{1986}$ could not be 'blown up' to the size of $\mathbf{A}_{1972}$, and that the aforementioned sufficient condition for the first and second term to be zero was not fulfilled. To obtain the inverse of the diagonal matrices **s** of column sums of **A** in the presence of industries without intermediate inputs, the corresponding elements of **s** were set equal to 10^{-10}. Consequently the average $\Delta\mathbf{c}$ and $\Delta\mathbf{L}$-effects are not exactly equal to zero. Their closeness to zero causes the relatively large size of the coefficient of variation.

[16] Table 2 contains the results for the mixed imports case. The tables in Appendix B present the results for the two alternative cases. In general, the results are similar to those reported here.

conclusions from the analysis of the aggregate economy appear to be valid also for the analysis at the sector level. For the sectors for which the traditional approach yields extremely negative contributions (for example, sectors 3, 6, 13, 14 and 23), the results obtained using the new decompositions are (often substantially) closer to zero. The main exception is sector 33 ("other producers"), but this is an 'unusual' sector in the sense that it neither used any intermediate inputs in 1986 nor produced any intermediate inputs in both years. Hence, not too much attention should be paid to the results for this particular sector (see also footnote 11). The sign of the negative contribution is reversed for four sectors if the proposed method is adopted, that is for sectors 2, 10, 12 and 16. Sector 17 is the only case for which the negative contribution of value added coefficients change is more negative when the new decomposition is applied than it is when the traditional methodology is used. For the sectors with a positive contribution according to the traditional decompositions, the results are more diversified. Sign reversals do not occur, but in four sectors the contribution appears to be lower, whereas for the remaining nine sectors a stronger positive contribution is found when the proposed method is used. All in all, the contribution of changes in the value added coefficients moves in the same direction for 26 out of 33 sectors. That is, negative contributions become less negative or positive and positive contributions increase if the proposed methodology is used instead of the traditional methodology. In general, the variation around the average for equivalent formulae is approximately equal for both methods. The main exceptions are sectors 4 and 6, the results of which obtained with the proposed approach once more confirm the sensitivity to different but equivalent weights emphasized in Dietzenbacher and Los (1997, 1998).

5. Conclusions and discussion

This paper shows that traditional structural decomposition analyses may involve conceptual problems if two or more of the specified determinants of change are theoretically dependent. The most well-known decomposition analyses for which such dependencies cause trouble are those in which value added change is attributed to changes in value added coefficients, changes in the matrix of input coefficients and changes in the final demand vector. It is demonstrated that these decompositions either yield results which are impossible to interpret correctly, or involve implicit and hardly defendable assumptions with regard to one or more 'balancing' or 'residual' terms (often non-competitive import coefficients).

An alternative approach is proposed, which does not suffer from these drawbacks. Some investigations which use input-output tables for the Dutch economy in 1972 and 1986 show that the results obtained with the new decomposition method may differ to a substantial extent from the ones obtained with the traditional, deficient approach.

Given the results reported in this paper, the question arises whether the traditional approach may also yield biased results for analyses in which changes in variables other than value added are decomposed. In our view, the answer to this question is affirmative. Dependency problems emerge whenever changes in variables for which (firms in) sectors have to pay are considered. That is, dependency occurs when a change in a single input coefficient (the one central to such a study) causes changes in at least one other input coefficient, due to the adding-up constraint. Such problems do not only emerge if the variable to be decomposed relates to one or more primary input categories (such as labor, imports, or value added), but also if it is closely tied to one or more sectoral outputs (the use of coal, for example). We will not present an exhaustive list of types of structural decomposition analyses in which the approach proposed in this paper might yield better indicators of the relative importance of the specified determinants. Instead, we will briefly discuss two issues which are often studied with the aid of structural decomposition analysis for which our alternative methodology might be relevant. These issues are changes in labor requirements and changes in energy use.

In our view, traditional decompositions of changes in labor requirements (see e.g. Forsell, 1990) are also vulnerable to dependency problems. In such decompositions, changes in labor requirements are generally attributed to changes in sectoral labor coefficients, changes in the production structure and changes in the level and composition of final demand. If labor requirements are expressed in value terms (wages and salaries), the similarity to the analysis in this paper is evident. Wages are costs incurred to use a primary input. A change in sectoral wage coefficients cannot but cause a change of equal size with opposite sign in the sum of input coefficients for intermediate inputs and other primary inputs. In this case, we would propose to decompose the change in wages into (i) the effects of changing wage coefficients under the assumption of an unchanged mix of other input coefficients, (ii) the effects of a change in the ratio between the aggregates of the other primary input coefficients on the one hand and the intermediate input coefficients on the other, (iii) the effects of a changed intermediate input composition and (iv) the effects of changes in the final demand vector.

Even if labor requirements are expressed in physical terms, such as jobs or man-years, dependency problems are very likely to occur. Only in the case in which changes in physical labor coefficients are exactly offset by equal opposite changes in the remuneration of labor, the other input coefficients can remain unchanged. In principle, one could apply the same alternative framework as sketched above for the decomposition in value terms. It might be, however, that theories and empirical evidence on the formation of wage rates, profit rates and prices in the presence of input substitution and/or labor-saving innovations offer insights which would suggest a further modification.

Dependency problems do not only emerge in analyses in which changes in the use of primary inputs are decomposed, but also if changes in the output of one or more sectors are studied in the traditional way. This can be illustrated by referring to structural decomposition analyses of changes in energy use (see e.g. Lin and Polenske, 1995, Mukhopadhyay and Chakraborty, 1999, and Jacobsen, 2000). Most of these studies make use of so-called "hybrid" input-output tables, in which the output of energy-producing sectors is expressed in physical terms and the output of non-energy sectors in money terms. In general, changes in the use of energy of some sort (say, coal) are decomposed into (i) changes in the sectoral coal input coefficients, (ii) changes in the production structure, and (iii) changes in the level and composition of final demand. Now, a decrease in the physical use of coal per dollar of output of a non-energy sector will lead to either higher input coefficients for at least one non-coal input or a higher value added coefficient, unless the price of coal changes exactly inversely. The traditional decomposition methodology implicitly assumes that value added coefficients are used as balancing terms. Our alternative approach yields a decomposition that is similar to the one for changes in values added or changes in labor requirements.

The elaborate analysis of value added change in the previous sections and the necessarily superficial discussion of structural decomposition analyses with regard to other variables in this section, indicate that there is ample room for application of the methodology proposed in this paper to a wide range of issues. Future studies should answer the question whether empirical differences between the outcomes of the traditional decompositions and decompositions purposefully avoiding dependent determinants are as large as they are in the value added case presented in this paper.

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Appendix A: Industry classification for the Dutch input-output tables

1	Agriculture, forestry & fishing	18	Other transport
2	Mining & quarrying	19	Motor vehicles
3	Food, beverages & tobacco	20	Aircraft
4	Textiles, apparel & leather	21	Professional goods
5	Wood products & furniture	22	Other manufacturing
6	Paper, paper products & printing	23	Electricity, gas & water
7	Industrial chemicals	24	Construction
8	Drugs & medicines	25	Wholesale & retail trade
9	Petroleum & coal products	26	Restaurants & hotels
10	Rubber & plastic products	27	Transport & storage
11	Non-metallic mineral products	28	Communication
12	Iron & steel	29	Finance & insurance
13	Metal products	30	Real estate & business services
14	Non-electrical machinery	31	Community, social & personal services
15	Office & computing machinery	32	Producers of government services
16	Electrical apparatus, nec	33	Other producers
17	Shipbuilding & repairing		

Appendix B: Results for the alternative cases

In Table 2 (in the main text), the decomposition results by sector are documented for the mixed imports case in which the original OECD (1995) classification of imports into competitive and non-competitive imports is maintained. Tables B.1 and B.2 present corresponding results for the other two cases on imports discussed in the main text.

TABLE B.1 ABOUT HERE

TABLE B.2 ABOUT HERE

Table 1: Decomposition results for the aggregate Dutch economy, 1972-1986*

Nature of imports	Methodology	Δc		ΔL		Δf	
Mixed	Traditional	-19.9	(0.19)	16.6	(0.18)	103.3	(0.04)
	Proposed	-2.0	(0.19)	-1.2	(0.26)	103.2	(0.01)
Competitive	Traditional	-19.9	(0.19)	19.8	(0.17)	100.1	(0.04)
	Proposed	0.0	(1.00)	0.0	(1.00)	100.0	(0.00)
Non-Competitive	Traditional	-20.0	(0.21)	6.8	(0.20)	113.2	(0.04)
	Proposed	-10.8	(0.23)	-2.5	(0.35)	113.3	(0.03)

* Percentage of total value added change, averaged over equivalent decompositions (coefficient of variation between brackets).

Table 2: Decomposition results by sector, mixed imports case.

	Effect of value added coefficient change as % of total							
	Traditional Methodology				Proposed Methodology			
industry	avg	cv	max	min	avg	cv	max	min
1	16.2	0.22	20.1	12.4	34.0	0.20	42.2	26.7
2	-19.2	0.31	-12.6	-26.8	61.9	0.21	75.1	48.8
3	-733.1	0.20	-584.4	-883.5	-659.6	0.20	-530.6	-788.8
4	14.4	0.14	16.9	12.2	0.9	3.37	4.3	-3.4
5	133.4	0.07	149.6	118.8	52.7	0.10	60.4	48.6
6	-160.3	0.17	-132.7	-187.4	-13.7	0.71	1.1	-22.6
7	-31.1	0.34	-19.7	-43.0	-12.1	0.48	-6.2	-18.2
8	28.2	0.30	37.9	19.0	30.0	0.26	39.1	20.5
9	230.0	0.20	280.0	180.0	208.4	0.21	260.0	162.4
10	-14.4	0.30	-9.6	-20.0	9.8	0.20	12.1	7.2
11	36.8	0.12	41.1	32.5	57.6	0.13	69.7	48.7
12	-45.4	0.23	-32.0	-60.9	20.7	0.09	24.6	18.8
13	-96.7	0.16	-80.0	-113.2	-40.7	0.14	-31.7	-47.7
14	-99.1	0.21	-77.3	-121.5	-52.8	0.24	-39.9	-65.8
15	53.2	0.43	81.7	27.1	57.1	0.36	82.6	29.6
16	-0.8	0.31	-0.5	-1.2	6.3	0.23	8.2	4.1
17	-51.1	0.13	-43.8	-58.0	-58.4	0.09	-51.7	-65.5
18	97.5	0.03	102.0	91.4	82.6	0.06	91.5	76.7
19	-68.5	0.31	-45.5	-92.9	-50.1	0.31	-33.1	-66.8
20	-53.9	0.29	-36.6	-72.9	-35.3	0.26	-26.1	-44.6
21	-52.8	0.27	-37.6	-68.6	-27.9	0.29	-19.1	-36.6
22	7.5	0.48	14.1	2.8	27.6	0.48	53.3	14.5
23	-125.5	0.21	-96.1	-156.9	-93.8	0.18	-72.0	-114.1
24	-47.6	0.08	-43.4	-51.8	-25.9	0.04	-24.2	-27.2
25	-44.7	0.17	-36.7	-52.7	-32.6	0.17	-26.3	-38.8
26	-8.7	0.18	-7.1	-10.3	-3.6	0.35	-1.7	-5.2
27	1.6	0.22	1.9	1.2	18.3	0.19	23.8	14.2
28	1.9	0.36	2.7	1.1	12.3	0.21	15.4	8.9
29	2.9	0.29	4.0	2.0	18.6	0.21	23.7	12.9
30	0.5	0.35	0.8	0.3	5.8	0.24	7.6	3.7
31	-18.8	0.22	-14.6	-23.0	-13.4	0.24	-9.6	-16.9
32	0.0	0.00	0.0	0.0	0.0	0.00	0.0	0.0
33	-560.0	0.01	-552.8	-567.1	-560.0	0.01	-552.8	-567.1
aggregate	-19.9	0.19	-15.9	-24.0	-2.0	0.19	-1.6	-2.5

Table B.1: Decomposition results by sector, competitive imports case.

	Effect of value added coefficient change as % of total							
	Traditional Methodology				Proposed Methodology			
industry	avg	cv	max	min	avg	cv	max	min
1	16.2	0.22	20.1	12.4	36.0	0.19	44.1	28.2
2	-19.1	0.31	-12.6	-26.8	68.8	0.20	83.3	54.7
3	-732.6	0.19	-584.4	-883.5	-650.7	0.19	-527.1	-774.4
4	14.4	0.14	16.8	12.2	-3.2	1.18	1.4	-8.4
5	133.4	0.07	150.1	118.3	48.9	0.11	54.5	43.4
6	-160.0	0.16	-132.7	-187.4	16.8	0.87	37.5	0.2
7	-31.0	0.34	-19.7	-43.0	-6.6	0.75	-1.6	-11.6
8	28.2	0.30	37.9	19.0	29.6	0.26	39.3	20.7
9	229.3	0.19	273.8	186.1	204.0	0.19	245.8	162.5
10	-14.4	0.30	-9.6	-20.0	19.5	0.19	24.4	15.0
11	36.8	0.11	41.0	32.5	62.1	0.14	74.2	51.4
12	-45.0	0.17	-37.1	-52.7	41.5	0.17	50.6	33.3
13	-96.4	0.15	-80.0	-113.2	-29.0	0.17	-23.2	-35.6
14	-98.9	0.20	-78.3	-119.5	-43.6	0.24	-31.4	-57.2
15	52.6	0.43	81.7	27.1	54.6	0.34	83.2	30.7
16	-0.8	0.30	-0.5	-1.2	7.7	0.21	10.1	5.6
17	-51.1	0.13	-43.8	-58.0	-59.3	0.09	-52.1	-65.6
18	97.1	0.07	107.8	83.1	80.9	0.10	91.5	72.0
19	-68.4	0.31	-45.5	-92.9	-45.1	0.31	-30.2	-60.5
20	-54.0	0.30	-36.6	-72.9	-33.3	0.29	-23.2	-43.6
21	-52.7	0.26	-37.6	-68.6	-26.0	0.28	-18.1	-34.1
22	7.5	0.48	13.9	2.7	35.9	0.44	60.1	16.9
23	-125.4	0.20	-96.1	-156.9	-88.7	0.17	-70.3	-108.6
24	-47.6	0.08	-43.4	-51.8	-24.5	0.05	-23.1	-26.2
25	-44.7	0.17	-36.9	-52.5	-31.0	0.18	-25.0	-37.2
26	-8.7	0.17	-7.1	-10.3	-2.9	0.47	-1.3	-4.9
27	1.6	0.22	1.9	1.2	19.4	0.20	24.8	14.6
28	1.9	0.36	2.7	1.1	12.6	0.20	16.0	9.6
29	2.9	0.28	4.0	2.0	18.7	0.21	24.5	13.7
30	0.5	0.35	0.8	0.3	5.9	0.25	8.1	4.0
31	-18.8	0.22	-14.6	-22.9	-12.9	0.24	-9.3	-16.7
32	0.0	0.21	0.0	0.0	0.0	0.21	0.0	0.0
33	-560.0	0.01	-552.8	-567.1	-560.0	0.01	-552.8	-567.1
aggregate	-19.9	0.19	-15.9	-24.0	0.0	1.00	0.00	0.00

Table B.2: Decomposition results by sector, non-competitive imports case

	Effect of value added coefficient change as % of total							
	Traditional Methodology				Proposed Methodology			
industry	avg	cv	max	min	avg	cv	max	min
1	16.3	0.25	20.7	12.0	26.5	0.24	33.1	20.1
2	-19.5	0.32	-12.6	-26.8	4.8	0.36	6.2	1.8
3	-735.7	0.21	-567.0	-911.6	-698.3	0.22	-544.0	-853.9
4	14.4	0.11	16.4	12.5	11.8	0.13	13.3	9.9
5	133.8	0.08	153.6	117.1	106.1	0.07	117.1	97.5
6	-161.8	0.22	-117.4	-213.4	-82.9	0.25	-59.7	-105.2
7	-31.3	0.37	-19.7	-43.0	-27.1	0.38	-16.7	-37.4
8	28.6	0.34	38.5	18.9	30.1	0.33	40.1	20.1
9	232.4	0.25	315.3	159.1	219.1	0.27	298.3	154.4
10	-14.6	0.33	-9.6	-20.0	-5.9	0.44	-3.2	-8.7
11	36.9	0.14	43.4	31.0	48.1	0.15	58.7	40.2
12	-45.2	0.20	-35.0	-56.4	-31.2	0.20	-24.8	-37.7
13	-97.3	0.20	-75.3	-122.3	-65.3	0.21	-51.7	-80.5
14	-99.1	0.22	-75.8	-123.1	-84.0	0.22	-64.3	-106.5
15	54.4	0.50	81.7	27.1	53.8	0.50	81.3	26.4
16	-0.8	0.34	-0.5	-1.2	1.1	0.15	1.3	0.8
17	-51.2	0.13	-43.8	-58.1	-55.5	0.09	-49.8	-61.0
18	98.0	0.01	99.0	97.4	93.2	0.02	95.1	91.5
19	-69.3	0.35	-44.3	-94.7	-66.9	0.36	-42.9	-91.1
20	-54.9	0.34	-35.1	-75.2	-54.2	0.35	-35.0	-74.1
21	-53.4	0.32	-34.3	-73.9	-44.2	0.36	-28.0	-60.6
22	7.2	0.29	10.0	4.7	11.9	0.30	17.7	8.0
23	-125.7	0.21	-96.1	-156.9	-101.0	0.19	-77.1	-123.7
24	-47.7	0.08	-43.4	-51.8	-29.2	0.04	-27.6	-30.6
25	-44.7	0.18	-36.4	-53.3	-36.2	0.17	-29.6	-42.8
26	-8.7	0.18	-7.0	-10.3	-4.5	0.23	-2.9	-5.9
27	1.6	0.22	1.9	1.2	16.8	0.19	21.2	13.2
28	1.9	0.36	2.7	1.1	10.5	0.24	13.6	7.0
29	2.9	0.29	4.0	2.0	15.3	0.26	20.5	9.4
30	0.5	0.35	0.8	0.3	4.5	0.28	6.1	2.6
31	-18.8	0.22	-14.5	-23.1	-14.3	0.23	-10.7	-17.8
32	0.0	0.00	0.0	0.0	0.0	0.00	0.0	0.0
33	-560.0	0.01	-552.8	-567.1	-560.0	0.01	-552.8	-567.1
aggregate	-20.0	0.21	-15.5	-24.8	-10.8	0.23	-8.0	-14.2