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Structural Comparison of Input-output tables

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Abstract

Comparison of input-output tables for one country over time, for several countries, or across regions of a given country, have attracted attention, mainly in order to identify patterns of changes of these tables. In this paper the problem of comparisons is envisaged with different approaches, some quantitative and some qualitative. Several methods are applied to a set of 11 tables of regions of Spain.

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Key words

Input-output, regional analysis, qualitative structures, binary matrices

1. Methodological considerations

For comparing input-output tables, two lines of enquiry are possible:

- ✓ the first uses the technical coefficients or the flow tables, as such, and leads to comparisons of quantitative structures, or quantitative comparisons; of particular interest in this area is Soofi (1992) in which structural comparisons are made between Egypt, Morocco and Zambia;

- ✓ the second uses only the binary information related to the existence or non-existence of certain types of relations, and leads therefore to comparisons of qualitative structures, or qualitative comparisons; of particular interest in this area is Holub, Schnabl (1985).

1.1. Quantitative comparisons

Among the different possibilities existing for quantitative comparisons, we have opted for three procedures, the first related to the matrix of direct technical coefficients, and the others to the inverse matrix of direct and indirect coefficients.

(I) Matrix of Similarity

When pairs of I/O tables are compared, it is possible to compute an Index of Similarity (Le Masne):

$$S^{R_1-R_2} = 100\left(1 - 0.5\sum_{i=1}^{n+1}\left|a_i^{R_1} - a_i^{R_2}\right|\right)$$

where R_1 and R_2 correspond to each one of the two tables compared (e.g. each pair of regional tables).

The Le Masne index will be close to 100 in cases of high similarity, and is therefore one of the many statistical distance indicators that can be established for the purpose of studying the similarity between tables.

The Matrix of Similarity (Matrix I) simply reproduces the Indices of Similarity in a set of tables, obviously two by two.

<u>(II) and (III) Matrices of Comparisons of rankings of multipliers</u>

Let us consider the Leontief inverse $(1 - A)^{-1}$. It is possible to sum the rows in order to obtain the output multipliers of uniform expansion of demand for each sector. It is also possible to weight the coefficients of $(1 - A)^{-1}$ according to the share of each sector in final demand and/or to the sectors value added coefficients.

We have opted for the simple un-weighted output multipliers; their ranking provides an ordering of the importance of given sectors to initiate growth processes as a response to final demand. Comparison of the ranks for pairs of countries (regions) provide the elements for filling a matrix of concordance between countries (regions).

In the application to Spain, the three first sectors in the ranking have been compared, and the number of common elements in the two sets has been recorded for each pair of regions (Matrix II).

A similar procedure has been followed starting with the sum of the columns of $(1 - A)^{-1}$ that provides some information as to the overall effect on the rest of the economy of demand growth in a single sector. Comparisons of ranks has allowed to compute a second matrix of concordances (Matrix III).

1.2 Qualitative comparisons

When comparing I/O tables in time or in space, increasing use is made of binary structural qualitative tables in which only the existence (1) or non existence (0) of coefficients or flows, are recorded. In general, some of the relations are excluded from the computations according to criteria that may be related to the size of the coefficient or flow or to the importance of the coefficient in terms of possible impact on outputs.

Like in the previous case of quantitative comparisons, a great number of qualitative comparisons can be made. We have opted for four types of comparisons, the first dealing with total flows, and the other three with layers in the development of indirect effects in the economy.

(IV) Matrix of common large flows

Starting with the input-output flow tables (M) to be compared, first a normalization is made, dividing each element of these tables by the total of flows (i'Mi):

$$N = (i'Mi)^{-1} M$$

This matrix N is then used to compute a binary matrix adopting a threshold criterium.

In the case of the application made for Spain the threshold is related to the simple average of the normalized coefficients 1/*n*, where *n* is the number of non-zero cells in matrix N.

If we call $\overline{N}$ the qualitative matrix of binary relations of a country (region), then

$$\overline{n}_{ij} = 1 \quad \text{if} \quad n_{ij} \geq \frac{1}{n}$$

$$\overline{n}_{ij} = 0 \quad \text{if} \quad n_{ij} < \frac{1}{n}$$

$$\text{or if} \quad n_{ij} = 0$$

From the different $\overline{N}$ of the country (regions) considered, it is then possible to compute, by pairs of countries, the number of common cells with a value of 1, thus filling a matrix (IV) of common large flows between countries (regions).

(V), (VI) and (VII): Matrices of common flows in layers

Consider the Passinetti matrix of production subsystems X:

$$X = (\mathrm{I} - \mathrm{A})^{-1}\, \hat{y}$$

it is possible to decompose it into production layers (Schnabl, 1994):

$$X = \hat{y} + \mathrm{A}\hat{y} + \mathrm{A}^2\hat{y} + \mathrm{A}^3\hat{y}.....$$

In this case $A\hat{y}$ is the matrix of first step interactions in production flows, connected to a final demand y, $A^2\hat{y}$ is the matrix of second step interactions and $A^3\hat{y}$ the matrix of third step interaction.

For each of these matrices procedures such as those applied to matrix M, can be computed in order to develop the corresponding matrices of common elements in the flows between pair of countries. Thus Matrix V reflects common elements in $A\hat{y}$, Matrix VI in $A^2\hat{y}$ and Matrix VII in $A^3\hat{y}$.

2. Application to a set of Spanish regional tables

In order to apply the different methods of comparison of I/O tables described in point 1 above, a set of comparable 11 input-output tables have been established for Andalusia (1995), Aragon (1992), Asturias (1995), Canarias (1992), Castilla-León (1995), Valencia (1990), Extremadura (1990), Galicia (1990), Madrid (1996), Navarra (1995) and the Basque Region (1995). The common classification has 26 sectors[3].

2.1 Quantitative comparisons

Table 1 portrays the results obtained from the comparison of the Matrix of Similarity (Matrix I), the matrix of comparison of rankings of output multipliers (Matrix II) and the matrix of comparison of rankings of demand impacts (Matrix III).

[3] The comparable tables are part of a project conducted by Ana Mª López at the Institute L. R. Klein of the Universidad Autónoma de Madrid. Prof. López has also performed the computations for the comparisons reported in this paper.

TABLE 1: Quantitative comparisons of the Spanish regions

Similarity Indices of Le-Masne											
MATRIX I	Andalucía	Aragón	Asturias	Canarias	Castilla y León	Com.Valenciana	Extremadura	Galicia	Madrid	Navarra	País Vasco
Andalucía		83.26	78.95	78.65	76.26	78.34	78.78	78.78	80.49	81.94	81.78
Aragón			81.59	79.18	80.22	81.54	81.86	83.39	83.36	82.37	79.80
Asturias				77.65	77.32	78.97	79.30	80.33	78.47	78.23	79.82
Canarias					79.80	76.96	81.81	77.42	78.93	75.81	76.99
Castilla y León						75.45	79.23	77.98	79.24	78.37	75.80
Com.Valenciana							77.00	76.95	79.26	77.20	76.85
Extremadura								81.95	79.92	77.09	76.85
Galicia									78.66	76.64	78.74
Madrid										80.66	80.64
Navarra											81.92
País Vasco											
average	*79.18*	*maximun*	*83.39*	*minimun*	*75.45*						

Comparisons of rankings (output multipliers)											
MATRIX II	Andalucía	Aragón	Asturias	Canarias	Castilla y León	Com.Valenciana	Extremadura	Galicia	Madrid	Navarra	País Vasco
Andalucía		0	0	2	1	0	0	0	0	1	1
Aragón			1	0	2	1	1	2	2	1	0
Asturias				0	1	1	1	1	1	0	0
Canarias					1	0	0	0	0	0	0
Castilla y León						1	1	2	2	1	0
Com.Valenciana							1	1	1	1	0
Extremadura								1	1	0	0
Galicia									3	1	0
Madrid										1	0
Navarra											1
País Vasco											

Comparisons of rankings (demand impacts)											
MATRIX III	Andalucía	Aragón	Asturias	Canarias	Castilla y León	Com.Valenciana	Extremadura	Galicia	Madrid	Navarra	País Vasco
Andalucía		2	1	1	1	1	1	2	1	2	1
Aragón			1	1	2	2	1	2	2	2	1
Asturias				1	1	1	1	1	2	1	2
Canarias					2	2	1	1	1	0	2
Castilla y León						3	0	1	2	1	1
Com.Valenciana							0	1	2	1	1
Extremadura								2	1	1	2
Galicia									1	1	1
Madrid										2	2
Navarra											1
País Vasco											

Inspection of Table 1 shows that the Le Masne index is in general quite high (75% average), with one of the regions (Aragon) showing systematically a smaller distance from the others: the structure of Aragon is probably more representative of an “average” Spanish table.

Inspection of matrices II and III shows that there is in general more common rankings for the sum of the columns of the inverse, than for the sum of rows. Once again, Aragon confirms its “central” role.

2.2 Qualitative comparisons

Table 2 portrays the matrices for agreements or the existence of flows in the matrix $\overline{N}$ and in the different layers previously described.

TABLE 2: Qualitative comparisons of the Spanish regions

Agreements on flows on matrix N (above average)											
MATRIX IV	Andalucía	Aragón	Asturias	Canarias	Castilla y León	Com.Valenciana	Extremadura	Galicia	Madrid	Navarra	País Vasco
Andalucía		69	75	71	68	60	65	70	65	58	66
Aragón			71	62	72	73	62	81	77	76	74
Asturias				69	71	60	67	72	70	64	69
Canarias					65	56	68	63	63	51	60
Castilla y León						62	64	74	63	67	66
Com.Valenciana							57	64	65	59	62
Extremadura								63	55	51	56
Galicia									65	66	69
Madrid										60	76
Navarra											77
País Vasco											
average	*65.89*	*maximun*	*81.00*	*minimun*	*51.00*						

Agreements on flows on matrix Ay (above average)											
MATRIX V	Andalucía	Aragón	Asturias	Canarias	Castilla y León	Com.Valenciana	Extremadura	Galicia	Madrid	Navarra	País Vasco
Andalucía		74	76	74	55	63	63	66	75	56	74
Aragón			69	65	54	70	62	72	74	64	77
Asturias				69	60	60	63	69	75	62	80
Canarias					50	65	66	61	72	50	72
Castilla y León						55	49	50	64	47	62
Com.Valenciana							56	64	71	52	67
Extremadura								56	57	45	60
Galicia									64	52	70
Madrid										59	82
Navarra											70
País Vasco											
average	*63.80*	*maximun*	*82.00*	*minimum*	*45.00*						

Agreements on flows on matrix A^2y (above average)											
MATRIX VI	Andalucía	Aragón	Asturias	Canarias	Castilla y León	Com.Valenciana	Extremadura	Galicia	Madrid	Navarra	País Vasco
Andalucía		31	31	33	27	31	26	25	34	27	31
Aragón			29	26	25	32	24	29	29	25	29
Asturias				29	34	30	22	28	33	27	33
Canarias					29	29	25	24	34	23	31
Castilla y León						30	20	23	32	26	30
Com.Valenciana							26	27	34	25	30
Extremadura								26	24	19	23
Galicia									26	23	30
Madrid										24	30
Navarra											28
País Vasco											
average	*27.84*	*maximun*	*34.00*	*minimum*	*19.00*						

Agreements on flows on matrix A^3y (above average)											
MATRIX VII	Andalucía	Aragón	Asturias	Canarias	Castilla y León	Com.Valenciana	Extremadura	Galicia	Madrid	Navarra	País Vasco
Andalucía		19	19	20	15	18	17	18	23	16	20
Aragón			16	14	14	16	14	17	18	15	16
Asturias				17	14	15	15	19	19	16	18
Canarias					13	16	14	14	19	15	15
Castilla y León						16	12	13	16	14	15
Com.Valenciana							14	14	19	16	14
Extremadura								15	15	13	14
Galicia									16	13	18
Madrid										16	19
Navarra											14
País Vasco											
Average	*16.00*	*maximun*	*23.00*	*minimum*	*12.00*						

As could be expected, the average of agreements on large flows (above average) between regions, decreases from matrix IV (65.89) to matrix V (63.80), to matrix VI (27.84) and finally to matrix VII (18.00) as the number of relations decline with the successive layers (there are more total relations than relations on a first step response to demand, and more in this first step that in the second step, etc.).

Once again, inspection of matrix IV confirms the surprising central position of Aragon, already established in the quantitative comparisons, but when the layers are analysed this situation changes, with the Basque Region occupying a more central position in layers 1 and 2 and Madrid in layers 2 and 3.

3. Further comparative analysis

In order to combine the quantitative and qualitative approaches, we have further transformed the seven comparative matrices into seven binary matrices, with (1) when the observed relation between two regions is above the average.

Thus, if we consider Table 1, for Matrix I the index of similarity of Le Masne is 83.26 between Andalusia and Aragon, while the average for this matrix is an index of 79.18. Thus, in the binary version of Matrix I, 1 will appear in the cell linking these two regions.

TABLE 3: Binary matrices of relations between regions

Similarity Indices of Le-Masne											
MATRIX I	Andalucía	Aragón	Asturias	Canarias	Castilla y León	Com.Valenciana	Extremadura	Galicia	Madrid	Navarra	País Vasco
Andalucía		1	0	0	0	0	0	0	1	1	1
Aragón			1	1	1	1	1	1	1	1	1
Asturias				0	0	0	1	1	0	0	1
Canarias					1	0	1	0	0	0	0
Castilla y León						0	1	0	1	0	0
Com.Valenciana							0	0	1	0	0
Extremadura								1	1	0	0
Galicia									0	0	0
Madrid										1	1
Navarra											1
País Vasco											

Comparisons of rankings (output multipliers)											
MATRIX II	Andalucía	Aragón	Asturias	Canarias	Castilla y León	Com.Valenciana	Extremadura	Galicia	Madrid	Navarra	País Vasco
Andalucía		0	0	1	1	0	0	0	0	1	1
Aragón			1	0	1	1	1	1	1	1	0
Asturias				0	1	1	1	1	1	0	0
Canarias					1	0	0	0	0	0	0
Castilla y León						1	1	1	1	1	0
Com.Valenciana							1	1	1	1	0
Extremadura								1	1	0	0
Galicia									1	1	0
Madrid										1	0
Navarra											1
País Vasco											

Comparisons of rankings (demand impacts)											
MATRIX III	Andalucía	Aragón	Asturias	Canarias	Castilla y León	Com.Valenciana	Extremadura	Galicia	Madrid	Navarra	País Vasco
Andalucía		1	0	0	0	0	0	1	0	1	0
Aragón			0	0	1	1	0	1	1	1	0
Asturias				0	0	0	0	0	1	0	1
Canarias					1	1	0	0	0	0	1
Castilla y León						1	0	0	1	0	0
Com.Valenciana							0	0	1	0	0
Extremadura								1	0	0	1
Galicia									0	0	0
Madrid										1	1
Navarra											0
País Vasco											

Agreements on flows on matrix N (above average)											
MATRIX IV	Andalucía	Aragón	Asturias	Canarias	Castilla y León	Com.Valenciana	Extremadura	Galicia	Madrid	Navarra	País Vasco
Andalucía		1	1	1	1	0	0	1	0	0	1
Aragón			1	0	1	1	0	1	1	1	1
Asturias				1	1	0	1	1	1	0	1
Canarias					0	0	1	0	0	0	0
Castilla y León						0	0	1	0	1	1
Com.Valenciana							0	0	0	0	0
Extremadura								0	0	0	0
Galicia									0	1	1
Madrid										0	1
Navarra											1
País Vasco											

Agreements on flows on matrix Ay (above average)											
MATRIX V	Andalucía	Aragón	Asturias	Canarias	Castilla y León	Com.Valenciana	Extremadura	Galicia	Madrid	Navarra	País Vasco
Andalucía		1	1	1	0	0	0	1	1	0	1
Aragón			1	1	0	1	0	1	1	1	1
Asturias				1	0	0	0	1	1	0	1
Canarias					0	1	1	0	1	0	1
Castilla y León						0	0	0	1	0	0
Com.Valenciana							0	1	1	0	1
Extremadura								0	0	0	0
Galicia									1	0	1
Madrid										0	1
Navarra											1
País Vasco											

Agreements on flows on matrix A^2y (above average)											
MATRIX VI	Andalucía	Aragón	Asturias	Canarias	Castilla y León	Com.Valenciana	Extremadura	Galicia	Madrid	Navarra	País Vasco
Andalucía		1	1	1	0	1	0	0	1	0	1
Aragón			1	0	0	1	0	1	1	0	1
Asturias				1	1	1	0	1	1	0	1
Canarias					1	1	0	0	1	0	1
Castilla y León						1	0	0	1	0	1
Com.Valenciana							0	0	1	0	1
Extremadura								0	0	0	0
Galicia									0	0	1
Madrid										0	1
Navarra											1
País Vasco											

Agreements on flows on matrix A^3y (above average)											
MATRIX VII	Andalucía	Aragón	Asturias	Canarias	Castilla y León	Com.Valenciana	Extremadura	Galicia	Madrid	Navarra	País Vasco
Andalucía		1	1	1	0	1	1	1	1	1	1
Aragón			1	0	0	1	0	1	1	0	1
Asturias				1	0	0	0	1	1	1	1
Canarias					0	1	0	0	1	0	0
Castilla y León						1	0	0	1	0	0
Com.Valenciana							0	0	1	1	0
Extremadura								0	0	0	0
Galicia									1	0	1
Madrid										1	1
Navarra											0
País Vasco											

Table 3 portrays the results thus obtained. Again if we refer to Matrix I, it is easy to verify the central position of Aragon as all the relations of this region with the rest of the regions shows a value of 1. The central role of Madrid and the Basque region in the output layers can also be verified by inspection of the binary matrices V, VI and VII.

In order to complete these comparisons, a final matrix has been computed by simply adding the seven binary matrices. This new matrix, shown on table 4, will reach a maximum of 7 for a cell when, according to the seven approaches adopted, two regions are always rather similar, in structural terms, both quantitatively and qualitatively.

TABLE 4: Total agreements between regions according to seven quantitative and qualitative criteria

Total Agreements											
	Andalucía	Aragón	Asturias	Canarias	Castilla y León	Com.Valenciana	Extremadura	Galicia	Madrid	Navarra	País Vasco
Andalucía		6	4	5	2	2	1	4	4	4	6
Aragón			6	2	4	7	2	7	7	5	5
Asturias				4	3	2	3	6	6	1	6
Canarias					4	4	3	0	3	0	3
Castilla y León						4	2	2	6	2	2
Com.Valenciana							1	2	6	2	2
Extremadura								3	2	0	1
Galicia									3	2	4
Madrid										4	6
Navarra											5
País Vasco											

Inspection of Table 4 definitely confirms the central role of the Aragon table, that is specially close to Valencia, Galicia, Madrid and also to Andalusia and Asturias. Furthermore, Asturias is close to the Basque Region, Galicia and Madrid; and, Madrid and the Basque Country, two of the more advanced industrialized regions of Spain, are also structurally rather close from each other.

4. Final considerations

This paper has presented a number of initial methodologies and results of a larger project on the patterns of regional structural change in Spain. These results point to some aggregated characteristics of the Spanish regional economy and are encouraging for more in depth studies of the relation between quantitative and qualitative analysis.

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