Various Definitions of Direct and Indirect Requirements in Input-Output Analysis: a Comment and Further Developments

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Abstract

In input-output analysis many definitions of direct and indirect requirements are used. This paper argues that the definition of indirect input requirements used by the U. S. Department of Commerce and suggested by part of the literature (the difference between total and direct requirements) is misleading since total and direct requirements are not homogeneously defined. The paper also identifies the conditions to be met in order to compare performances at system level and the performances at industry levels. Finally it shows that the difference between performances at system level and performances at industry level is explained simply by their different product-mixes.

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In a note which appeared in the Review of Economics and Statistics, Parikh [3] has summarized four main alternative definitions of direct and indirect requirements used in input-output analysis. He has also suggested the use of the following definition given by the U. S. Department of Commerce [5] when there is an interest in estimating the indirect inputs requirements

[1] $\overline{A} = A^a - A$

where

$\overline{A}$ = matrix of indirect inputs requirements

$A^a = [I - A]^{-1}$ = Leontief inverse (total requirements)

A = matrix of intra industry domestic coefficients (direct requirements)

I = Identity matrix

In our opinion, the definition of indirect inputs requirements suggested by Parikh has never been seriously questioned in the input-output literature. This paper argues that the definition [1] is misleading and suggests how it should be amended.

Our argument is based on the fact that A^a and A are not homogeneously defined and, therefore, are not comparable: A^a indicates the *total output* requirements per *unitary* vector of *final demand* while A indicates the *domestic inputs* requirements per *unitary* vector of *total output*. As we are all aware, in input-output analysis, *final* and *total* output may be significantly different due to the weight of the *intermediate inputs*. Therefore, A^a and A do not have the same meaning. For example, the first column a_1^a of matrix A^a shows the vector of total outputs of the different commodities required by the manufacturing at system level of *one unit of the final*

commodity 1. It follows that the vector a^a measures alternatively:

1) *A set of constraints* on the availability of the total outputs that must be fulfilled at system level in order to satisfy an additional unit of final demand of commodity 1, if the commodities indicated by the vector a_1^a are scarse.

2) *The efficacy* of an additional unit of final demand of commodity 1 in activating the production at system level of the commodities indicated by the vector a_1^a, if they are not scarce.

On the other hand, the first column a_1 of matrix A, being computed as the ratio between the value of inputs and the value of the total output of industry 1, measures economic efficiency[1] in the production of *one unit of total output of commodity* 1.

So far, the matrices A and A^a, by capturing two different kinds of performances, *constraints/efficacy at system level* and *economic efficiency at industry level* cannot be compared. This result has general validity. For example, the row vector of *total labour requirements per unitary vector of final demand* $\tilde{l}' = l'*A^a$, often employed in the analysis of the labour productivity at system level[2] and of the theory of value and income distribution in linear production models[3], must not be compared with the row vector of *labour intensities per unit of industry total output* l'. Even the comparison suggested by Gupta-Steedman [3] between system labour productivity changes (measured by the change through time of $\tilde{l}'$)and the labour productivity changes (measured by the change through time of l') has to be considered incorrect.

In our opinion, in order to be comparable, the performances at system and at industry levels have to meet the following conditions:

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[1] Economic efficiency and not technological efficiency. Each input coefficient indicates a partial economic efficiency since it is referred to the use of the specific input considered.

1) Both have to be measured in term of *inputs* or *ouputs.*

2) Both have to indicate *constraints/efficacy* or *efficiency*.

Since in input-output analysis any attempt to find out a notion of performance measuring *constraints/efficay at industry level* is bound to be frustrated[4], we have to concentrate our attention on finding out a notion of performance measuring *efficiency at system level*. This is given by the matrix K

[2] $K = \tilde{A} * \hat{t}^{-1}$

where

$\tilde{A} = A * A^a$ = domestic input requirements at system level.

$t' = i' * A^a =$ row vector of total outputs at system level.

i' = unitary row vector

and where the symbol ^ indicates that the underlying vector has been transformed into a diagonal matrix having the elements of the vector in the main diagonal.

The [2] clearly measures *efficiency at system level* since it shows the proportionality between the *domestic inputs* (matrix $\tilde{A}$) and the *total output at system level* (row vector t'). The matrix D

[3] $D = K - A$

correctly defines the magnitude of the difference between the economic efficiency at system and at industry levels.

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[2] See Gupta-Steedman [2], Wolf [6].
[3] See Pasinetti [4].
[4] In fact, the only notion of final ouput available at industry level is that of value added which, as maintained by Arrow [1] (p. 5) is a not observable magnitude. Furthermore, the input-output analysis is based critically on the notion of total output at industry level.

At this point, another important result may be achieved by investigating the relationship between the structure of the economic efficiency at system level and the economic efficiency at industry level. The [2] can be re-written as

$K = A * A^a * \hat{t}^{-1}$

and thus, given

$C = A^a * \hat{t}^{-1}$

[2.1] $K = A * C$

The [2.1] shows that the matrix C is the key factor in transforming the matrix A of industry domestic input coefficients into the matrix K of domestic input coefficients at system level[5]. The matrix C is a normalised Leontief inverse, independent of the structure of the final demand and dependent on the structure of the intra-industry domestic coefficients (matrix A)[6]. Its column vectors measure the relative importance (direct and indirect) of each industry in the manufacturing at system level of any given commodity. In other words, the column vectors of matrix C measure the productive specialisation, that is *the product-mix* used in the manufacturing at system level of any given commodity. Being independent of the structure of the final demand, the matrix C may be used to compare the product-mixes at system level through time and across countries. For example, let

[5] It is easy to show that the matrix *C* has general validity in trasforming any performance at industry level into the corresponding performance at system level. It must be noticed that the matrix C cannot be computed if the inter-industry flows are defined in physical terms. In fact, in this case the row vector t' cannot be computed, being a set of composite commodities.

[6] However it can be easily shown that the same matrix $C^Y = C^X$ is compatible with two different matrices of industry domestic inputs coefficients $Y \neq X$.

$$C^A = \begin{bmatrix} 0.50 & 0.25 & 0.15 \\ 0.20 & 0.30 & 0.30 \\ 0.30 & 0.45 & 0.55 \end{bmatrix} \quad \text{and} \quad C^B = \begin{bmatrix} 0.30 & 0.32 & 0.23 \\ 0.50 & 0.22 & 0.37 \\ 0.20 & 0.45 & 0.40 \end{bmatrix}$$

be the matrices whose column vectors measure the product-mixes used in the manufacturing at system level of cars, plastics and metals in countries A and B, respectively. Limiting our considerations to the manufacturing at system level of cars (first columns of the matrices C^A and C^B), we can notice that the relative importance of the car industry is 50 % in country A and 30% in country B. Furthermore, the information given by the first columns of matrices C^A and C^B could be very powerful in carrying out targeted and selected industrial policy programmes which are aimed to improve the economic efficiency of the manufacturing of cars at system level. In particular, they can be used in ranking ex-ante[7] the industries to which government interventions should be addressed.

Re-writing the [3] as

$$D = A * A^a * \hat{t}^{-1} - A$$

we obtain

[3.1] $D = A * [C - I]$

Given that in conventional input-output models each industry i carries out only the stage of production stage i[8], the identity matrix I can be understood as the matrix of the

[7] However, it must be noticed that the most important industry before government intervention may not be so after government intervention at industry level that modifies the structure of the industry domestic input coefficients. In fact, it can be easily shown that

$$\partial c_{ij(i=1,..,n)} / \partial a_{ji(i=1,..,n)} = 0$$

$$\partial c_{ij(i=1,..,n)} / \partial a_{zi(z,i=1,..,n;z \neq j)} \neq 0$$

where

c_{ij} and and a_{ij} are the generic elements of the matrices *C* and *A*.

product-mix at industry level. Therefore, the [3.1] shows that the difference between the economic efficiencies at system and at industry level are simply explained by the difference in their product-mixes $[C - I]$.

So far, the results of this paper suggest that the works based on the definition given by the U. S. Department of Commerce need to be reconsidered.

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[8] At system level, when commodities are produced by means of other commodities, many production stages are usually involved. For example, the manufacturing of cars at system level involves the production stage performed by the car industry (car production) and those performed by the industries supplying the intermediate inputs to the car industry (plastics production and metal production).