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FROM ECONOMIC STRUCTURE TO POWER STRUCTURE : A STRUCTURAL ANALYSIS[1]

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Abstract

Power is absent in many economic studies, though everyone knows that social and economic agents often look to increase their own influence on the behaviour of others. In this paper, a power structure is associated to an economic structure, the power resulting from the influence transmitted and received by the agents. It is assumed that the agents have an initial measurable opinion that will change over time as a result of influence transmitted through the power structure induced by the economic structure. A power structure implies the existence of one or more equilibrium opinions, which constitute the unanimous compromise between the totality or a part of the agents. A classification of power structure would appear to be linked to the nature of the equilibrium opinion. The stationary case is when a compromise is reached immediately, as in terror equilibrium or price equilibrium in oligopoly. In most cases, a compromise requires several stages of negotiation. Due to the nature of the power structure, a unanimous compromise would take into consideration all initial opinions, while in a dictatorial structure, only those of a dominating agent subset would be considered. These results illustrate the logic of economic globalisation. Moreover, if agents do not manifest resistance to the influence of others, they are considered as versatile; thus several compromises are possible with the cyclical adherence of subgroups of agents.

[1] We would like to thank Mrs Phyllis Smith (*Institut Supérieur des Langues Vivantes*, University of Liege) for her English language assistance in the preparation of this manuscript.

Social power is the foothold of the Marxist economists, but, during the last century, other non-Marxist economists also considered power as fundamental to explain economics. We cite, first, the Austrian school, with E. Von Boehm Bawerk(1914), F. Von Wieser (1926) who introduced the psychological dimension in economics, and second, J. Schumpeter (1942), who showed how technical progress is an endogenous concept, a source of power, derived by large firms which dominate the market, an idea also shared by J. Galbraith (1973). Note that popular books of recent years on globalisation of the economy have often set power in the centre of the reasoning. However, in certain instances, the freedom these authors take with economic theory renders their publications less credible from a theoretical point of view.

In fact, the absence of power in economics, as remarked F. Perroux (1973), appears with the founders of neo-classical theory, L. Walras and V. Pareto. However, general equilibrium based on the commissar appraiser knows nothing whatever about power and the mathematics used in economics afterwards induced sophisticated models, but which were essentially based on perfect competitive equilibrium.

Since the beginning of the 1980's, imperfect competition has been developed in economic models, and as far as oligopoly is concerned the behaviour of an oligopolist depends on that of his competitors; thus, one can speak about reciprocal influence, implicitly meaning that a power relationship exists between them .

Economic interactions were substantially stimulated by game theory and seminal J. Von Neuman and O. Morgenstern's book (1944), in which power appears as important for understanding economics. Game theory transformed the foundation of economic theory in situations in which, usually, a small number of agents are involved in a conflict of interests where they rationally choose between alternative decisions, given their expectations about the behaviour of the agents with whom they interact. Contemporary economic theory uses game theory extensively, especially for imperfect competition markets. J. Nash (1950, 1953), introduced equilibrium strategies where the optimal strategies of one agent are defined given the strategies of the others. As usual, several equilibria are possible in repeated games, and a theory of strategic bargaining is proposed for choosing between the set of efficient agreements (A. Rubinstein 1982, 1985a, 1985b). Strategic bargaining theory deals with the problem of finding, not of enforcing an agreement. Nevertheless, a credible binding contract ensures the respect of the agreement in a non co-operative context. This idea gave birth to the theory of contracts. Recently, F. Stähler (1998) linked strategic bargaining and the theory of self-enforcing contracts between two parties.

As the optimal decision in game theory takes into consideration the interdependence of agents, power is also implicit in this analysis. It is present through the threat of reaction, just as much in strategic bargaining as in the theory of contracts. But behaviour is autonomous, and strategies depend only on the awaited gains or losses, imputable to their own choices and those of other agents. No decision is ever imposed on an agent by the use of power, even if the reaction could be interpreted as a threat, an element of rational choice.

In fact, the analysis of the interdependence structure as the basis of the use of power in economics uncovers the heterodoxy.

On the contrary, in what could be called the French school or the heterodoxy school (see J. Weiller and B. Carrier, 1994), under the leadership of F. Perroux (1948, 1961, 1969,1971, 1973, 1975, 1982) with J. Weiller (1948, 1989) and M. Byé (1956), power was associated with structure, even if this last concept is often confused. J. Weiller defines national preference structure, showing how national politics determine production and exchange structures on the basis of preference, not to be confused with a simple aggregation of individual preferences. M. Byé insists on the power relationship dominated by transnational firms (*Les Grandes Unités Territoriales*). F. Perroux, who had always emphasised the existence of domination in exchanges, introduced, in 1975, the *active unity* concept, showing how a nation sets up a preference structure that could be in conflict with the preference of other nations and with that of multinational firms. Thus, power is introduced at the international scale with the interference of politics in economics. After the crisis of the 1970's, a new approach appears, the " "*régulation school*", which, following F. Perroux and under the impulsion of G. de Bernis (1975), M. Aglietta(1982) and R.Boyer (1986) stress the reproduction of social relationships through Institutions which manage the accumulation regime, evolution of salaries, competitive forces, insertion in the world economy and the rule of the State. The adjustment is, of course, the result of decentralised decisions, but individual agents are not conscious of the transfer of power to Institutions.

To be competitive with orthodox economic theory, these approaches lacked a formalised methodology. In the 1970's, R. Lantner (1974) introduced graph analysis in order to study dominance in input-output analysis**;** this methodology, in particular, inspired me to extend structural analysis and introduce a formalisation of power : Gazon (1976, 1981, 1989) .

For various reasons, the basic results of structural analysis are not well known. In this paper I rework and generalise the results of a previous work (Gazon 1981) to the dictatorial structure.

After an introduction in Section 1 of the often confused notions of structure and system and basic structural vocabulary, Section 2 introduces the power structure and its importance in economics. Section 3 is devoted to the typology of power, distinguishing different types of power structures. The use of power is studied through opinion transmission. Given a power structure and an initial opinion of the agents, we study how the initial opinion changes due to the influence of power, in order to analyse the final opinion. Is a compromise possible? If it is, how are initial opinions weighted in the compromise and how does this weighting depend on the agent's position in the power structure? Thus, one can infer results, which constitute a basis to explain real economic and politic situations, including the following: terror equilibrium, oligopolists' agreements, conditions for reaching a unanimous compromise, or how a compromise resulting from only a few agents can be imposed on others, as in the current globalisation process throughout the world.

1. Definition of an economic structure and an economic system

Structure is often used in different meanings and contexts, and sometimes used in the same sense as other words. The word *structure* is used both as a homonym and a synonym. The same is true for *system*, often confused in the literature with *structure*.

As far as *structural analysis* is concerned, it is necessary to clarify these notions.

The concepts and the definitions of Gazon (1976) are used here:

1.1. Definition of an *economic structure*

The *structure* ***S*** is a triplet, including :

1°) a support, called the *structure support*, which is defined by the finite and non-empty set ***M***={1,…,M } where the element $i\in$ ***M*** is a *pole of the structure* that, in economics, generally represents an economic unit, such as a sector, a country, a firm, a person, etc., to which is generally associated one or several economic variables, such as added value, turn-over, market price, preferences, etc.

2°) a relation on the structure support, called the *structural relation* ***R,*** which includes a set of N binary relations $R_1,...,R_N$ where for *k=1,…N :*where *(i,j)*$\in M^2$ *if and only if* $\exists k$ *such as* $i\,R_k\,j$ *;* when N=1, we note $iR_1 j$ *=iRj*. For example, R_k could represent : "to be a customer of" or "to be a supplier of" in an exchange relation, or still "to pay s.o." and more generally "to influence".

3°) an application of the structural relation, called the *structural application* ***H,*** which includes the N applications $H_1,...,H_N$ where for *k=1,…N,* H_k *is defined as :*

$H_k(i,j)$ = *{(i,j)*$\in M^2$: $iR_k j$ *}*$\rightarrow \mathbb{R}$ or ***H*** : *{(i,j)*$\in M^2$ *:i**R**j}*$\rightarrow \mathbb{R}^N$. Let ${}_k h_{ji} = H_k(i,j)$. If N=1, we note ${}_1 h_{ji} = h_{ji}$.

This third concept implies that the structural relation could not be only binary. It specifies how "influence" is transmitted by associating a value to each $i\,R_k\,j$. For example, if the structural relation characterises an input-output analysis, ${}_k h_{ji}$ could represent a technical coefficient.

As a consequence, we can formally write a structure ***S*** as : ***S={M,R,H}***

2. *Graph* of the structure

To the structure ***S***, it is possible to associate an oriented graph ***G={M,***Γ***}*** for which the *support* is ***M*** and Γ, the set of the *correspondences* $\Gamma_1,...,\Gamma_N$ defined for, k=1,…,N by :

$j \in \Gamma_k(i)$ *if and only if* $iR_k j$, where j are the *following poles* of i . Notice also the reciprocal correspondence Γ^{-1} of Γ, which is defined by :

$\forall j \in \boldsymbol{M},\ \Gamma^{-1}(j) = \{i \mid i \in \boldsymbol{M},\ j \in \Gamma(i)\}$ where *i* are the *preceding poles* of *j* for $\Gamma_1,...,\Gamma_N$. As a consequence, $i \in \Gamma^{-1}(j) \Leftrightarrow$ *i**R**j*, which implies that the structure ***S*** can also be written as : ***S={M,***Γ***,H}.***

When N=1, we note $j \in \Gamma(i)$ if an only if iRj .

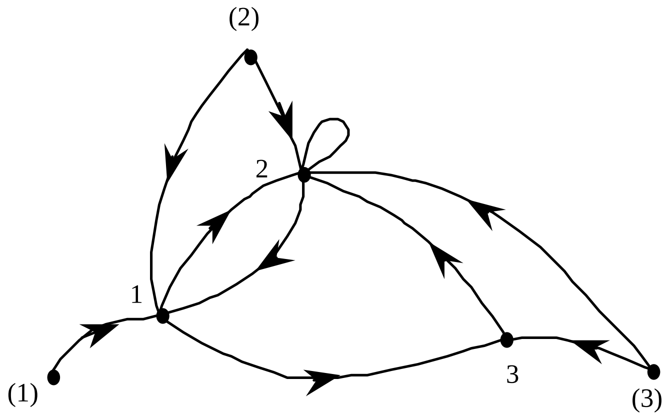

Figure 1
Structure graph

On Figure 1, the poles of the structure are represented by points called *summits of the graph* and the couples $(i,j)_k$ corresponding to $iR_k j$, by oriented lines from i to j, called *arc* $(i,j)_k$ *of the graph* $\boldsymbol{G}$ (or by extension, arc of the structure $\boldsymbol{S}$). The summit i is the *origin* of the arc, and j its *destination.* When the origin of the arc joins its destination, like in $(i,i)_k$, the arc is a *loop.*

Two arcs are *consecutive if the distribution of the first is the origin of the second.*

Because the interest of the analysis could only concern a part of the structure or of the graph, one defines a *sub-structure* (resp. a *sub-graph)* of a structure (resp. of a graph) as a structure (resp. a graph), which has for support a sub-set of the “initial” structure support, including all the arcs of the initial structure whose origin and destination belong to this sub-set. Furthermore, a *partial structure* (resp a *partial graph)* of a structure (resp. of a graph) is a structure (resp. a graph), which has the same support as the initial structure, but for which the set of the arcs is a sub-set of the initial structure arc set. In addition, a structure (resp. a graph) is *full* for the structural relation R_k if and only if $\forall (i,j) \in \boldsymbol{M}^2 \ : iR_k j$.

2.1. System associated to structure

2.1.1. Scalar system

Generally in Nature, and especially in economics, the usual problem is to explain how quantities, like consumption, production, savings, etc., vary as a consequence of the variation of other quantities, which is formally represented by a function like : $y=y(x)$. In the example of $x, y \in \mathbb{R}$, nothing prevents considering this function as an input-output scalar system, where x is an *impulse* and y the *response*. As far as the notion of system is considered, time must be introduced, because the response to an impulse usually takes time. The analysis of $y=y(x)$ then becomes $y(t) = y\left[x(t)\right]$.

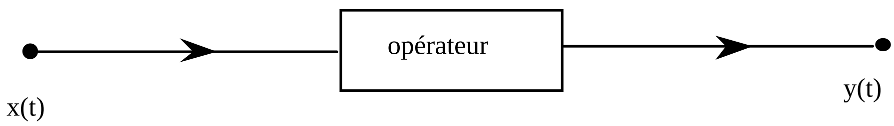

Figure 2
Scalar system

Consider now the arc $(i, j)_k$ and suppose that *x(t)* is located at the summit *i* and *y(t)* at the summit *j*. The above function, assimilated to a scalar system, can then be represented by

$x(t) \xrightarrow{k} y(t)$ *for* $i R_k j$ where the impulse or entry function *x(t)*, is transformed by an *operator* to give the response *y(t).* We consider that the operator is a *transmitter of influence* from *i* to *j*; then the quantities *x(t)* and *y(t)* will be in the generic term *influences.*

To know how the operator works, it is necessary to characterise it, as in system theory. This analysis is complete in Gazon (1976, ch 1) for the case N=1, which is the one used here. The presentation will, however, be simplified. But to have an idea of the general nature of the approach, a few more concepts are necessary.

In economics, impulses of the system are generally the decisions of economic agents. They can be instantaneous, temporary or permanent. Let's consider an increase in tax rates. The new rates could constitute temporary or permanent impulses on income. But if one takes into consideration the variation of taxation as the impulse, it is instantaneous.

The unit of measure for an instantaneous impulse is the Dirac signal $\delta(t)$,defined as :

$\delta(t) = 0$, $\forall t \neq 0$ and $\int_{-\infty}^{+\infty} \delta(t)$ =1. As $\delta(t) = 0$ almost everywhere, the integrator is not a classic one.

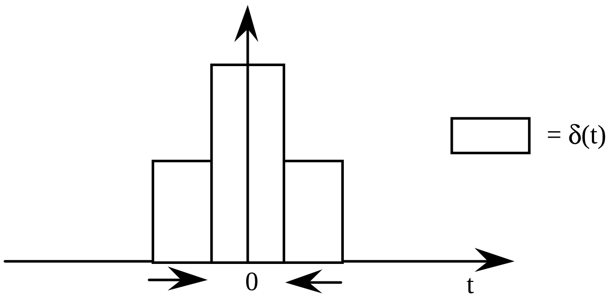

Figure 3
Dirac (Instantaneous) impulse

Then, any impulse at time *t=t*∘ is given by : $x(t_\circ) = \int_{-\infty}^{+\infty} \delta(t - t_\circ) x(t) dt$, and the permanent impulse is given by : $\int_{-\infty}^{+\infty} x(\tau)\delta(t-\tau)d\tau$.

One can now how guess how the influence transmitter will operate if one knows how it determines the response to the unit instantaneous impulse, which is the Dirac signal when the system is at z*ero-state response* at the time of the impulse. This response to the Dirac signal $\delta(t)$ is the *impulsion response h(t).* We assume, without losing the general nature, first, time-invariance, which means that the impulsion response doesn't change with time, i.e., the response to $\delta(t-\tau)$ is defined by $h(t-\tau)$, and second, the non-anticipation of the response.

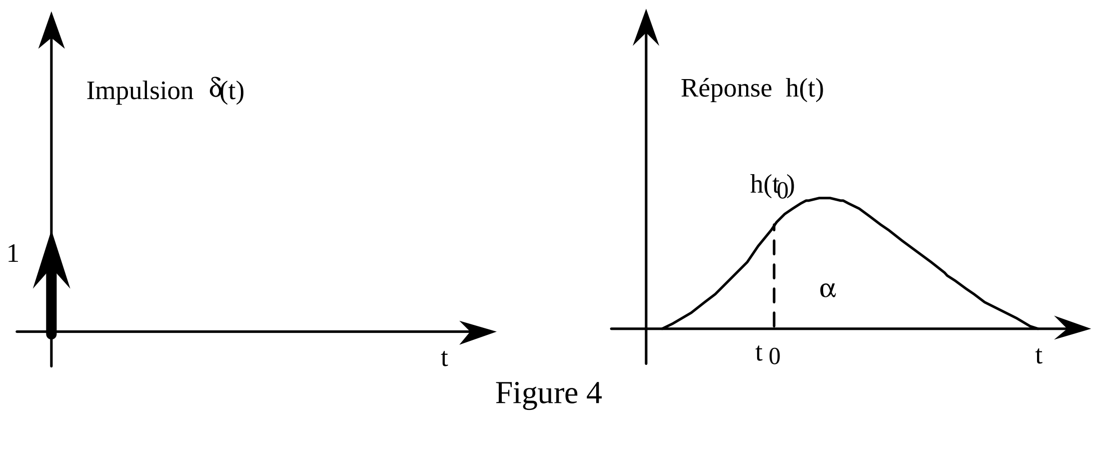

Figure 4

Instantaneous impulse and continuous response

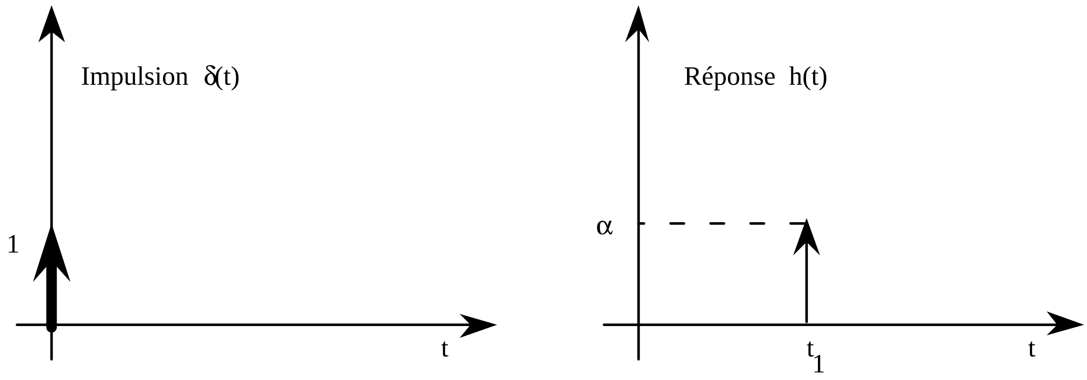

Figure 5
Instantaneous impulse and instantaneous response

In economics, like in physics, it is interesting, not only to know the response at a particular moment *t*, but also the cumulated response at *t* and the total response to the impulse.

So if *h(t)* is the response to $\delta(t)$, the cumulated response at $t_\circ$ is $\int_0^{t_0} h(t)dt$ and the total response, which is called the *intensity of the impulsion response* is $\int_0^{+\infty} h(t)dt = \alpha \in \mathbb{R}$.
The scalar system is stable if and only if $\int_0^{+\infty} |h(t)|dt = \overline{\alpha}$ is finite.

Now as a consequence, if the scalar system is linear, we can specify how response *y(t)* to impulse *x(t)* will be a function of the impulsion response *h(t).*

Notice the instantaneous impulse at *t=0* as *x(0).*$\delta(t)$, then the response at time $t_\circ$ is $y(t_\circ)=x(o).h(t_\circ)$, the cumulated response at $t = t_\circ$ is $y^*(t_\circ) = x(o)\int_0^{t_0} h(t)dt$ and the total response is $y = x(0)\int_0^{+\infty} h(t)dt = \alpha\, x(o)$.

If the impulse is permanent, for example *x(t)* is defined for $t \in [0, \infty[$, then for the linear scalar system, the response at $t_\circ$ is given by : $y(t_\circ) = \int_0^{t_\circ} x(t)h(t_\circ - t)dt$; the cumulated response is : $y^*(t_\circ) = \int_0^{t_\circ} y(\tau)d\tau = \int_0^{t_\circ} x(t)dt \int_t^{t_\circ} h(\tau - t)d\tau$, and the total response is

$$y = \int_0^{+\infty} y(t)dt = \int_0^{+\infty} x(\tau)d\tau \int_\tau^{+\infty} h(t-\tau)dt = \alpha \int_0^{+\infty} x(\tau)d\tau$$

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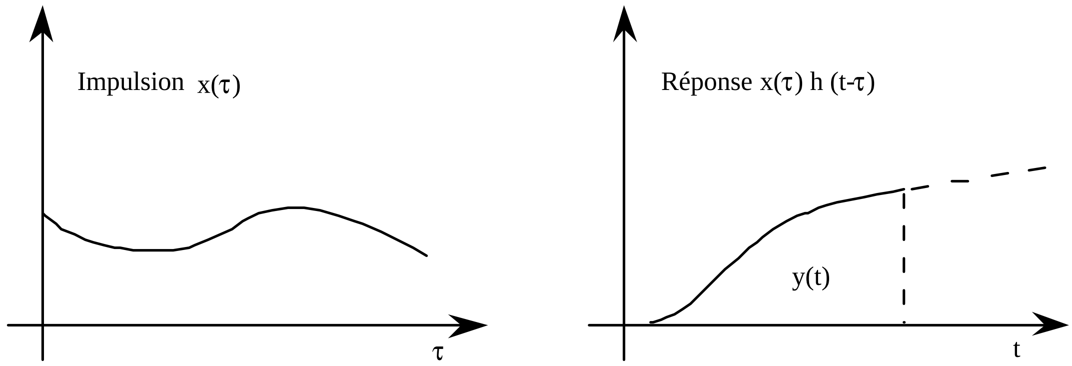

Figure 6
Continuous impulse and continuous response

In order to simplify the calculation, especially for the response at a given moment $t_{\circ}$, the Laplace-transformation is very useful in linear systems. The Laplace-transformation of a scalar function *f(t)* locally integrable**,** for $t \in [0, \infty[$ is given by:

$$L[f(t)] = \int_0^{+\infty} f(t) e^{-st} dt = F(s).$$

Supposing for our problem of influence transmission that $L[f(t)]$ is absolutely convergent and that the scalar system is stable, without discussing here the convergence problem, one can define the Inverse Laplace-transformation as $f(t) = L^{-1}[F(s)]$.

Now write $y(t_{\circ}) = h(t) \circ x(t)$, called in system theory the *convolution product*, the Laplace-transformation of this equation is *Y(s)=H(s).X(s),* where *H(s)* is the *transfer function.* What is important to observe is that the convolution product defined on time-space becomes a multiplication on Laplace space, which simplifies the calculation. As it is generally difficult to specify *h(t),* but relatively easy to find *H(s)* through the observation of the impulse *X(s)* and the response *Y(s*), one has the impulsion response *h(t)=* $L^{-1}[H(s)]$ from *H(s)=Y(s)/X(s),* knowing that conversion tables exist for this calculation.

Furthermore, one demonstrates that in Laplace space, the cumulated response at time *t* is defined by : $y^*(t) = Y^*(s) = Y(s)/s$ and the total response $y = \lim_{s \to 0} Y(s) = \lim_{s \to 0} H(s).X(s)$ as

$\lim_{s \to 0} H(s) = \lim_{s \to 0} \int_0^{+\infty} h(t) e^{-st} dt = \int_0^{+\infty} h(t) \lim_{s \to 0} e^{-st} dt = \alpha$ EMBED because $|h(t)|$ is finite. Then

$$y=H(0)\ X(0)=\alpha\, X(0).$$

1.3.2. Vectorial system

Coming back to the structure ***S,*** the structural relation in economics is often based on a model for which one or several economic quantities are attached at the poles, linked together on the basis of the structural relation. To simplify the analysis, suppose that N=1, which means that from one pole *i* to another one *j*, there exists only one relation, *iRj*. In addition, suppose that only one quantity is attached at a pole. If a particular problem implies that several quantities must be taken in consideration for one pole, it could be duplicated, which would increase structural support and introduce additional adequate relations.

Thus, it is possible to associate to the structure ***S*,** a *vectorial system of influence transmission*.

In fact, starting from the scalar system $i \to j$ corresponding to the relation *iRj* and characterised by the impulsion response $h_{ji}(t)$, which also defines the structural application on *(i,j)*, one must define how influence is transmitted from pole *i* to pole *k* through the articulation of the two scalar systems corresponding to the consecutive relations *iRj* and *jRk*, i.e.: Construction buys iron from Steel industry and the latter buys petroleum from Energy sector in inter-industrial relations, and, in general how the influence received by a pole *j* from the poles *i preceding* of *j*, $i \in \Gamma^{-1}(j)$ is transmitted to the poles *k consecutive* to *j*, $k \in \Gamma(j)$.

Because of the hypothesis of vectorial system linearity, the responses of *j* to the impulses coming from *i* in the scalar systems $i \to j$ are first added; secondly, the cumulated response of *j* at each time *t*, constitutes an impulse of *j* for the scalar systems $j \to k$. In this example, poles like *i* which are origin at one arc of the graph ***G*** at least, are called *transmitters*, poles like *k* destination of one arc at least are *receivers*, and poles like *j*, which are at the same time both transmitters and receivers, are called *relay poles.*

Notice that in the case of N>1, it could be more complicated to analyse the transmission of influence, because it would then be necessary to explain how to combine the transmission between two scalar systems $i \overset{a}{\to} j$ and $j \overset{b}{\to} k$, corresponding to different relations.

It is now possible to precisely define the linear vectorial system associated to the structure ***S*.**

Associate to each pole *i* of the structural support ***M***, the time-variable quantity $y_i(t)$, the convolution products resulting from *iRj*, $\forall j \in$ ***M*,** are given by :

$$y_j(t) = \int_0^t h_{ji}(t-\tau)\cdot y_i(\tau)d\tau = h_{ji}(t) \circ y_i(t)$$

and the vectorial system at time *t*, $\forall j \in$ ***M*,** is :

$$y_j(t) = \sum_{i=1}^{M} h_{ji}(t) \circ y_i(t) .$$

Finally, one writes the equations system as a matricial convolution product :

$y(t) = H(t) \circ y(t)$,

where y(t) is the response-vector at time *t* and *H(t)* the impulsion response matrix.

As a consequence, at the *structure* ***S*** is associated the linear *system* $\boldsymbol{S}_{+,\circ}$ describing pole responses at time *t*. Furthermore, concepts such as the notation of structure and system must not be confused : *structure* characterises the permanent links between the poles of its support, while *system* describes the influence transmission inside the support.

Notice now that by using the above description, it is easy to infer the cumulated response-vector $y^*(t)$ and total response-vector *y*.

Under the convergence conditions of Laplace transformation :

$y^*(t) = L^{-1}\left[Y(s)/s\right]$ where $Y(s)/s = H(s).Y(s)/s$ and *y=A.y,* where *A* is the intensity matrix of impulse responses or intensity matrix. As an extension, the system of total responses associated to the structure $\boldsymbol{S}$ is noted as $\boldsymbol{S}_{+,\bullet}$ ***EMBED***. This system looks like a static system, which will be consistent if the dynamic system $\boldsymbol{S}_{+,\circ}$ is stable.

In order to distinguish endogenous and exogenous variables within the system (Gazon, 1976), a distinction is made between an *impulse structure* and a *response structure*. An impulse structure contains only impulse poles, while the response structure includes receiver poles and relay poles. Thus, one can separate in the support, the set of *impulse-sources* $\boldsymbol{N}^{\circ} = \{i \in \boldsymbol{M} \mid \Gamma^{-1}(i) = \varnothing\}$ from the set of *receivers* $\boldsymbol{N} = \{i \in \boldsymbol{M} \mid \Gamma^{-1}(i) \neq \varnothing\}$ and $\boldsymbol{N}^{\circ} \cup \boldsymbol{N} = \boldsymbol{M}$.

With a view to having a notation close to input-output analysis, the most useful application of structural analysis and which generally considers a total response-system, the notation A is reserved for the intensities-matrix of the response structure, and the notation D is chosen for the intensity matrix of the set: $\{ (i, j) \mid i \in \boldsymbol{N}^{\circ} \text{ and } j \in \boldsymbol{N}\}$. In addition, the vector of variable quantities corresponding to the impulse-source poles is noticed *u(t)* at time *t* and *u* for the total impulses. Without reducing the analysis, one supposes that the number *N* of impulse-source poles is equal to the number of receiver-poles. So *A* and *D* are *NxN* matrixes, while y and *u* are *Nx1* vertical vectors.

Then, the system of equations giving the total response can be written:

$$y = Ay + Du .$$

As an example, if the impulse-sources are the final demand vector *u*, *D* is the unit-diagonal matrix, *A*, the technical coefficient matrix and *y*, the output-vector.

1.4. Influence graph

The graph of the structure associated to the vectorial system giving the total response is called the *influence graph,* introduced by Lantner (1974) for input-output analysis and generalised by Gazon (1976), as summarised above. In fact, as far as total response is concerned, each arc is valued by the impulsion response intensity of the associated scalar system. This valuation is the arc *intensity* of the influence graph. Influence transmission is governed by the same rules as in the vectorial system. If *consecutive arcs* are defined as two arcs for which the destination of one is the origin of another, and a *path* as a sequence of consecutive arcs, the paths are the supports of the influence transmission, and are called *influence paths.* The *path origin* is the first arc origin of the sequence, and the *path destination*, the destination of the last arc. When the path origin and the path destination are similar, the path is a *circuit* and the arc associated to the element- relation *(i,i)* is a *loop.* The *path intensity,* which is the response of the path destination to a unit-impulse of the path origin, is equal to the product of the path arc intensity. It is also called the *direct influence* of the path. Recall the two other important structural concepts introduced by Lantner(1974), *total influence* and *global influence*, which are at the basis of the important *Influence theorem.* As these notions are not necessary in this paper, they will not be developed further.

On the other hand, for power analysis, one does need another concept of graph theory, reflecting the interdependence of the poles. When all the poles of the (influence) graph associated to the structure $\boldsymbol{S}$ are linked by at least a circuit, the structure $\boldsymbol{S}$ is a *strong connected component.* This means that each impulser pole of the structure receives, in return, influences from the other poles which have responded to this initial impulse.

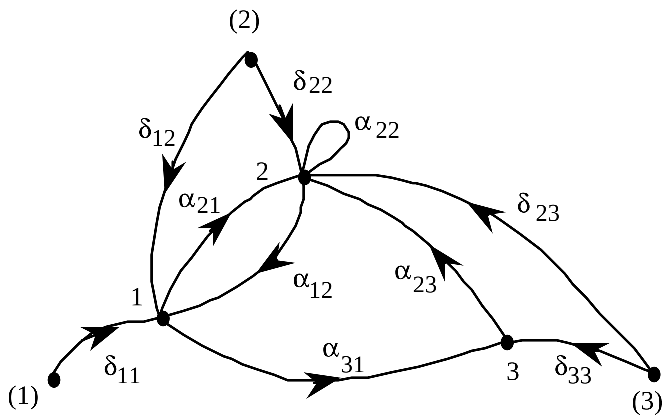

Figure 7
Influence graph

2. Power structure

The aim here is not to present a power theory nor an epistemological study of power, but only to show how an economic structure, when it is formalised, could infer power, which could be used to influence the opinion of the economic agents involved in the economic structure.

2.1. Power in economics

Many economic quantities arise from the decision of economic agents or actors, such as production, consumption, employment, salary and income, savings, prices, interest rates, etc. and more generally, supply and demand, with the resulting price. In the view of system theory, the dependence of one quantity on another could be interpreted as a transmission of influence from the second to the first. If each quantity level results from the decision of an agent, does this imply that the link between them signifies that agents could influence the behaviour of others, i.e., meaning that agents would have power over other agents?

Suppose that the economic link between agents is governed by the market. If it is perfectly competitive and equilibrated, no one has the possibility of changing the behaviour of others: one supplier or one buyer can decide to leave the competitive market without others reacting, and without influencing the equilibrium price. The characteristic of a perfect equilibrated competitive market is precisely not to induce power. As a consequence, the existence of systemic relations between decision-variables is not enough to induce a power relation.

But the above description of the competitive market is assimilated to the Walras equilibrium, which is defined by an official appraiser. In fact, even when there are many suppliers and many buyers at a given fixed price of a good, there could exist a disequilibrium between supply and demand. If, as is usually the case, agents wish nevertheless to make exchanges, different situations are possible where certain agents will be rationed, as in these transactions, the effective demands could be superior or inferior to effective supplies. This generally results from being below or exceeding production capacities. Lantner (1974) calls these situations *demand-dominance* if supply is greater than demand, and *supply-dominance* in the inverse case. The aim here is not here to review the vast neo-classical literature on market equilibrium, but to emphasise that the adjustment based on prices or on quantities depends on a power relation .

Furthermore, if competition in the market is imperfect, as in the case of monopoly, it is evident that profit-maximisation behaviour implies for the monopolist, the capacity to fix price and quantity; indeed, in terms of demand, but, with the power to choose a limitation of the production, in order to ration the demand, and so increasing the price until reaching the level which maximises profit, with, in addition, the possibility to discriminate one market from another, if demand-price elasticities differ. On the demand side, the buyer has the choice either to renounce or to accept the price. Clearly, he is under the influence of the monopolist if he needs to buy.

If now one considers oligopoly, where the behaviour of each supplier depends on that of the other, by definition there is reciprocal influence between the oligopolists. If one supposes a Cournot production adjustment, where from period to period each oligopolist determines his supply as a monopolist, given the supplies of the others, or if one has a Bertrand adjustment process through prices, or still, if both behaviours exist simultaneously, a power relation is inherent to the supply structure and its use depends on the capacity of each to influence the others. Strategies based on a hypothesis or anticipations of competitors' reactions will be set up by the oligopolists. Game theory analyses many cases with or without co-operation or collusion between producers, and this constitutes one approach to power. Of course, if the demand side is introduced in the analysis, the supply power on demand is the same type as in the case of monopoly.

Now as far as macroeconomics is concerned, if one analyses the Keynesian circuit demand-production-revenue-demand, an interdependence of influence between these aggregates can be observed, where the main exogenous impulse is the investment or, when the foreign country is exogenous to the analysis, the export demand. One knows that growth will depend on the multiplier and on the accelerator, which themselves depend on the behaviour of consumers and producers. Here it is probably difficult to speak about power, as the aggregates are not agents, but rather only the result of a conglomerate of agents. Nevertheless, the structural analysis of this interdependence could enrich understanding, and the power analysis becomes more feasible when the economic model distinguishes several categories for each macroeconomic variable. This is especially the case in the partial model of the well known input-output analysis or in the global one, when one considers the circuit through income distribution.

In input-output analysis at the sectorial level, the initial impulse comes from final demand; the sectors usually diffuse production activity between them on the basis of technical coefficients. If all producers have exceeded capacities, demand influences supply, and it is the buyer who has power over the supplier. But because of the evolution of technology and competition, the products' position in their cycle are different from one another. If the sectors are defined at a level disaggregated enough to include products which have the same cycle position, the relation with suppliers and with customers will imply either demand dominance or supply dominance. The orientation of power will depend on the position in product cycle. Suppose the production flow : *steel* - *special steel*- *building*, and consider that *special steel* is in growth phases in its product cycle, while *steel* is in maturity and *building* depends on special steel because of customer preferences. Then, one can consider that *special steel* influences the other two sectors much more than the inverse. The orientation of power is neither that of the production flow nor that of the demand. This example shows that the inference of a power structure from an economic structure based on a formal model is not necessarily automatic. Rather, a specific analysis of the power induced by the economic structure is necessary.

Finally it is important to notice that the aim of this article is not to help the economic agent choose between possible strategies, as is the case in game theory. As already mentioned, in game theory, reactions are autonomous and rational, without directly considering the use of power to influence choice. But as decisions are determined by anticipation of the reaction, given the possibilities of gain, this situation can also be considered as the use of potential threat, a component of power. Furthermore, when the possibilities of gain are not the same for all agents, the difference can be interpreted as a result of a power relation. But in game and bargaining theories, these differences are exogenous facts, which influence rational behaviour. And in the theory of contracts, there is no use of power to explain the terms of the contract in another way than the rational one.

On the contrary, in our analysis, power is endogenous to the structure, which determines the consequences of the use or potential use of power. Thus, our reflection is anterior to game theory decision. The goal is to infer a power structure from an economic structure to which a vectorial system is linked. This power structure should be the basis of strategies, as in game theory, as far as the use of power constitutes the most important foothold of strategies. But, first of all, it is necessary to set up a typology of power structures.

2.2. Power structure associated to economic structure

Let us consider an economic structure ***S={N,R,H},*** for which one assumes that the *N* poles of the structure are agents, capable of making decisions, or at least of having opinions. The power structure ^{P}***S={N,P,*** ^{P}***H}*** where ***:***

- the *support* ***N*** is the same as that of the economic structure; of course the concept defining the poles of the economic structure could be different from the agents of the power structure, who constitute *decision-units* attached to the poles;

- the *power relation* ***P*** characterises the capability of each agent $i \in \boldsymbol{N}$ to force more or less his/her opinions on the other agents $j \in \boldsymbol{N}$**,** and in addition to maintain his/her opinion as much as possible; power is interpreted as a persuasion force, which appears as the ability or capacity to punish or make reprisals, as well as to resist. In fact, power is not the use of the force, but the capability to use it, formally :

  $\boldsymbol{P} = \{(i, j) \in \boldsymbol{N}^2 \mid$ *iPj, if and only if i influences the j opinion}.*

  Both influence diffusion and resistance result from the economic structure. If the influence orientation in the power structure is the same as the influence transmission in the economic structure, ***P***=***R.*** But this is not necessarily the case, as shown in the example of the product flow, *steel-special steel-building :* the economic relation can represent the product flow " to be supplier of" or inversing the sense " to be customer of". The power relation will be the same as the economic relation, if the structure is supply dominant in the first case and demand dominant in the second. But if, in order to be more realistic, one introduces the position of sectors in the business cycle, the power relation is a third one, where *special steel* influences both *steel* and *building;*

- the *application* ^{P}***H*** is an application on ***P*** defined by :

$${}^{P}\boldsymbol{H}:\{(i,j)\in \boldsymbol{N}^2 \mid iPj\}\rightarrow p_{ji}\in \mathbb{R}.$$

The application ${}^{P}\boldsymbol{H}$ is different from the application ***H,*** even if ***P=R***. To specify this notion, it is necessary to introduce the power system associated to the power structure. As a result of the power relation definition, one considers the impulse and response functions as quantifiable opinions. Some opinions, especially when they concern economic variables, are naturally quantifiable, like prices, quantities to produce or buy, etc. Other opinions are sometimes more difficult to quantify, e.g., national nuclear weapons capacity or adhesion to a political regime. One possible method is to define the opinion on an interval $[0,1]\subset\mathbb{R}$ EMBED. Of course, this will imply statistical studies and surveys to define it.

Moreover, " influencing an opinion" takes time. This is why the opinion of an agent $i\in \boldsymbol{N}$ is defined at time *t* and will be noted $x_i(t)\in\mathbb{R}$ or, in this case by :

$$x_i(t)\in[0,1].$$

Therefore, the scalar power system, also called *opinion transmission*, is *:*

$\forall(i,j)\in \boldsymbol{N}^2$***,*** *iPj if and only if* $x_j(t)=p_{ji}x_i(t-1)$ with $p_{ji}\in[0,1]$,

which means that in this scalar system, the *j* opinion at time *t* is equal to the proportion p_{ji} of the *i* opinion at time *t-1*. The *vectorial power system* is a linear system, as defined above, but here the convolution product which defines the response at time *t* is a simple multiplication. As a consequence, to the power structure ${}^{P}\boldsymbol{S}$***,*** is associated the power system ${}^{P}\boldsymbol{S}_{+,\bullet}$

EMBED .

So, one reduces opinion transmission on a discrete basis, not to be more realistic, nor to simplify, but only because power which seeks to change opinions generally implies negotiations, which are interesting to follow from period to period. For pedagogical reasons, one just simplifies by introducing the same lag for each scalar system, which means that the equation system associated to the power structure is :

x(t)=Px(t-1) where *x(t)* is the *Nx1* opinion vector at time *t* of the agents *i*∈ ***N*** and *P* is the *NxN* power matrix, *x(0)* being the *initial opinion vector* with *P* temporal invariable. The power matrix *P* is a matrix where the sum of line-elements is equal to the unity:

$$\sum_{j=1}^{N} p_{ij}=1,\ \forall i\in N$$

In fact *P* is a *stochastic matrix*, but, even if the elements p_{ij} could be interpreted as probabilities in the power structure, one considers that they are the *permanent power* of the *j* agent over the *i* agent, also called *resistance* if *i=j*. It is the relative part of the *j* agent opinion $x_j(t-1)$ in the *i* agent opinion $x_i(t)$, because one has :

$$x_i(t)=\sum_{j=1}^{N} p_{ij}x_j(t-1),\ \forall i\in N\ .$$

If the problem is to infer a power structure coming from an economic structure, like in input-output analysis defined by the equation system, *y=Ay+d,* where *y* and *d* are (*Nx1)* vectors and *A* is a (*NxN)* matrix, considering only the power problem inside the endogenous inter-industrial part, if suffices to determine the power matrix *P* coming from *A*, to norm each row, in order to have *1* for the sum of the row elements. So, with a_{ij} and p_{ij} as generic terms of the matrixes, one has:

$$p_{ij} = \frac{a_{ij}}{\sum_{j=1}^{N} a_{ij}} \text{ and } \sum_{j=1}^{N} p_{ij} = 1 .$$

Furthermore, one supposes that each agent $i \in N$ has permanent power over at least one agent $j \in N$, and is submitted to the permanent power of at least one agent $j \in N$, knowing that *j* could be *i*.

Of course, it is easy to determine the opinion vector at time *t* as a result of the initial opinion vector, because : $x(t) = P^t x(0)$.

2.3. Power graph

As the influence graph has been associated to the graph of the economic structure, the valuated power graph can be associated to the graph of the power structure. As an example for the power structure ${}^{P}\boldsymbol{S}=\{\boldsymbol{N},\boldsymbol{P}, {}^{P}\boldsymbol{H}\}$, and the power matrix *P*, where :

$\boldsymbol{N}=\{1,2,3\}$

$\boldsymbol{P}=\{(1,1), (1,2), (2,2), (2,1), (2,3),(3,3), (3,1)\}$

$$P = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.4 & 0.6 & 0 \\ 0 & 0.7 & 0.3 \end{bmatrix}$$

The power graph ${}^{P}\boldsymbol{G} = \{\boldsymbol{N}, {}^{P}\Gamma\}$ is illustrated by Figure 8.

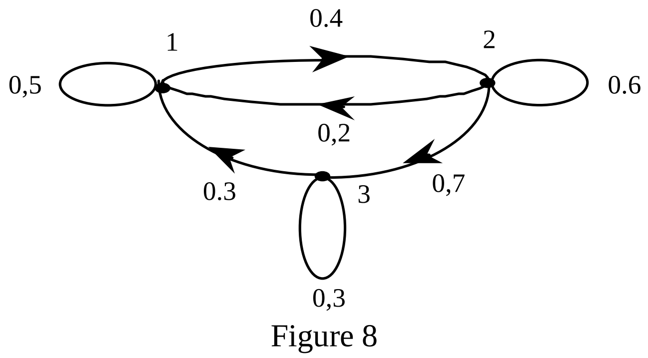

Figure 8
Power graph

As power analysis consists in following the opinion vector from period to period, it is interesting to associate to the *t- power matrix,* P^t, the *t-power graph* ${}^{P}\boldsymbol{G}^t = \{\boldsymbol{N}, \Gamma^t\}$, where the correspondence Γ^t is defined as follows: $\forall i, j \in \boldsymbol{N}$, $j \in \Gamma^t(i)$ *if and only if there is, in the* *power graph* ^{P}***G****, at least one path from the origin i to the destination j whose the length is equal to t* . So the element *i,j* of the *t*-power matrix P^t, noticed $p_{ij}^{\ t} = \sum_{k=1}^{L} {}_k p_{ij}^{\ t}$ where ${}_k p_{ij}^{\ t}$ is the *power intensity* of the path $c_k^{\ t}$ which links *j* to *i* with a length equal to *t.* These opinion transmissions through the *t*-length paths from *j* to *i* determine the proportion $p_{ij}^{\ t}$ of the *j*-agent initial opinion in the *i*-agent opinion at time *t.*

3. Structural typology of power

To explain how the power structure determines the use of power, the evolution of the agents' initial opinions due to the power system, resulting from the power structure will be studied.

For this, the question is to know how, owing to their position in the power structure, some agents are able to impose their opinions on other agents .

From period to period the opinion vector is transformed by the power-matrix as negotiation rounds continue. Will there be a compromise, and if so, how will it integrate the initial opinions of each agent? The aim is to link the nature of the compromise to the configuration of the power structure.

3.1 Dominant situation relation

Let us consider a power structure ^{P}***S*** and its associated power graph ^{P}***G*** and define the *dominance relation* $\triangleright$ as Gazon (ch..7, 1976). In the power structure ^{P}***S,*** one has *:* $i \triangleright j$ *if and only if j is the destination of a path having i as origin.* This relation is a pre-order relation in the support ***N.*** Now one determines an equivalence relation " *to be equivalent* " $\approx$, which means that $i \approx j$ if and only if *i* and *j* belong to the same strong connected component of ***N.*** Then the set of the strong connected components of ***N*** constituted classes of the partition $\boldsymbol{N}_{/\approx}$ induced by $\approx$, if one considers that an isolated agent *i* for whom $(i,i) \notin \boldsymbol{P}$ doesn't belong to a strong connected component, is also a class of $\boldsymbol{N}_{/\approx}$ **.** It is thus possible to infer a *reduction power graph* or *quotient power graph* $^P\boldsymbol{G}_{/\approx}=\{\boldsymbol{N}_{/\approx},\ ^P\Gamma_{/\approx}\}$ to the power graph in order to introduce a *domination situation relation* between *the classes* of agents. So the classes are the strong connected components (and the isolated agent, defined as before) of the power graph where the agents are called equivalent($\approx$) because, inside a class, each agent influences the others and is influenced by them. With each class agrees one unique summit of the quotient power graph, and there exists one and only one arc from a class C_i to a class C_j if and only if there is at least one arc from one agent $i \in C_i$ to one agent $j \in C_j$. By convention, no loop is associated to the summits of the quotient power graph $^P\boldsymbol{G}_{/\approx}$. From this, the ordinal function, called *dominant position relation* $\succ$ in $^P\boldsymbol{G}_{/\approx}$ is defined by: $C_i \succ C_j$ if and only if C_j is the destination of a path having C_i as origin in $^P\boldsymbol{G}_{/\approx}$.

Furthermore, let us define the rank $r_i (i = 0,...,L)$ of equivalent classes C_k in the quotient power graph as follows :

$$r_k = \{C_k \in N_{/\approx} \,\big|\, ^P\Gamma_{/\approx}^{-1}(C_k) \in \bigcup_{j=1}^{k-1} r_j\}$$

$$r_0 = \{C_k \in N_{/\approx} \,\big|\, ^P\Gamma_{/\approx}^{-1}(C_k) = \varnothing\}$$

In Gazon (1976), an algorithm is proposed to find the rank r_i. The quotient power graph of Figure 9.2 is presented in the hierarchical order of the dominant position relation and rank. Agents of the r_0 rank-classes are the *basic agents* of the power structure, while the others are *secondary agents.*

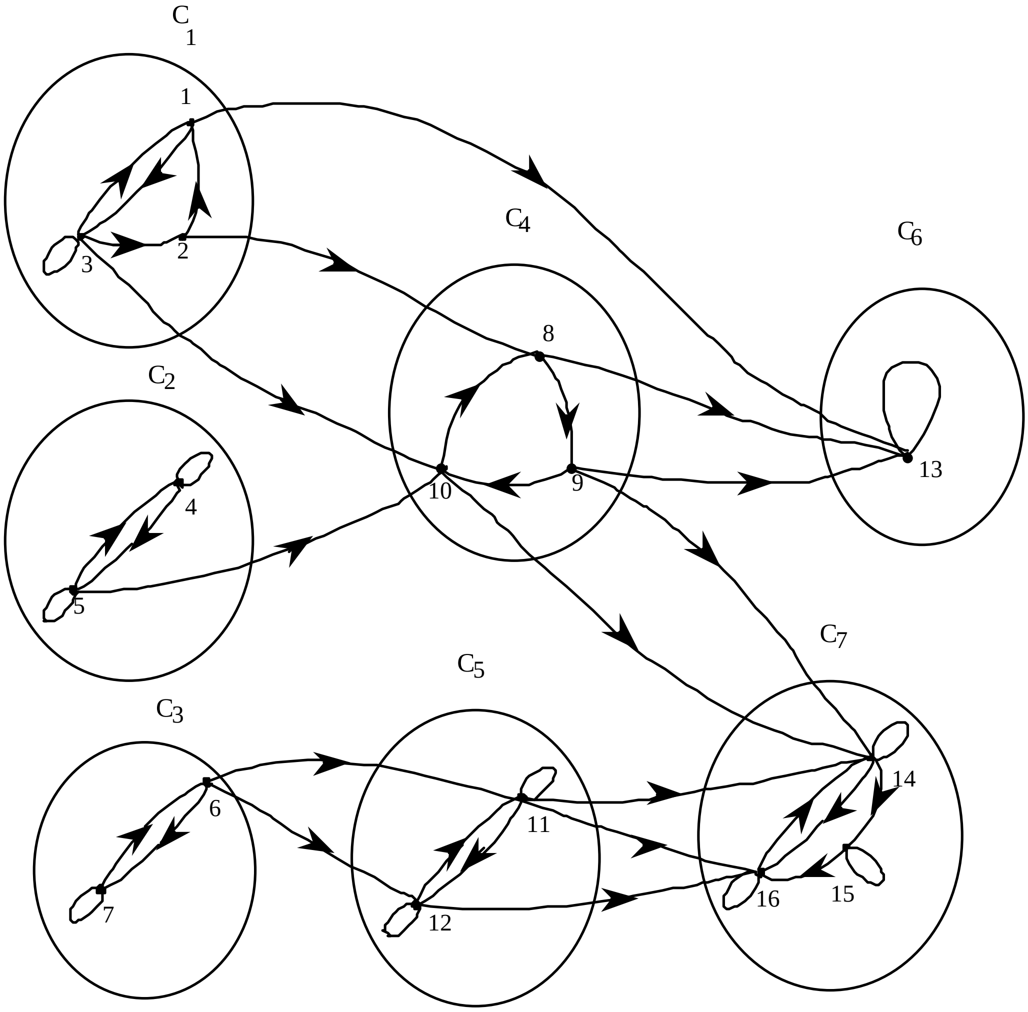

Figure 9.1
Power graph (not indicating permanent powers)

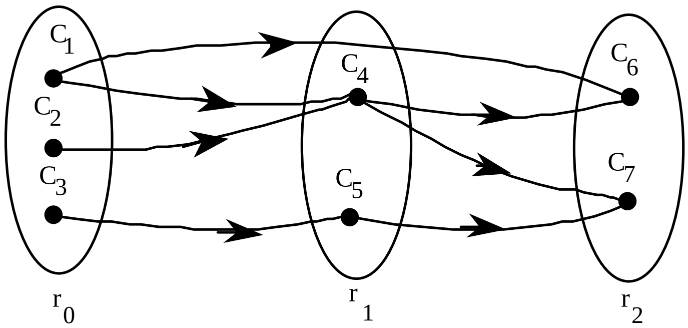

Figure 9.2
Quotient power graph

From the quotient power graph one can induce the following important results:

1. Basic agents of one r_0 rank-class are not influenced either by secondary agents or by basic agents of the other r_0 rank-classes.

2. Secondary agents are only influenced by agents of the same class and by agents of classes which are the origin of at least one path in the quotient power graph, having the considered secondary agents' classes as destination.

For instance, agents 11 and 12 of C_5 are influenced outside themselves only by agents 6 and 7 included in the basic agent's class C_3. And agents 8, 9, 10 and 13 are never influenced by the basic agents of C_3.

One will see that these results play an important part in the dictatorial power structure.

3.2. Equilibrium opinion

Because P is stochastic, the power system transforms the opinion vector from period to period as the Markov-chain. In fact, one has $x(t)=Px(t-1)$ and $x(t)=P^t x(0)$.

One calls *equilibrium opinion* or *final opinion* of an *i* agent, the opinion $\overline{x_i} \in \mathbb{R}$; as for $\varepsilon \in \mathbb{R}$ arbitrarily small, there exists $t \in \mathbb{N}$ for which $\left|\overline{x_i} - x_i(\theta)\right| < \varepsilon, \forall \theta > t$. The *equilibrium opinion vector* or the *final opinion vector* is given by $\overline{x} = \left[\overline{x}_i\right]$.

Particular case of an instantaneous compromise

Proposition 1

If the initial opinion is unanimous, the compromise is instantaneous.

Demonstration

A unanimous initial opinion means that $x_1(0) = x_2(0) = ... = x_N(0)$, then

$$\forall i \in N, x_i(1) = \sum_{k=1}^{N} p_{ik} x_k(0),$$

$$x_i(1) = x_i(0) \sum_{k=1}^{N} p_{ik}$$

and as a consequence, $x_i(1) = x_i(0)$, because $x_i(0) = x_k(0)$ by hypothesis, and $\sum_{k=1}^{N} p_{ik} = 1$, because the power matrix *P* is stochastic.

Then, initial unanimity is held in position and compromise is instantaneous. In this particular case, the final opinion is independent of the power structure.

This proposal is evident, but it is interesting to know that in this formulation of power, negotiations do not change a compromise that has already been reached.

But generally, the nature of the equilibrium vector, i.e., the relation between the initial vector and the equilibrium vector depends, of course, on the power matrix *P*, which is stochastic, and the evolution of the opinion vector is isomorphic to the Markov-chain. This is why results coming from Markov-chain theory are necessary.

3.3.1. Markov chain results adapted to the power structure

Considering the power matrix *P* as a Markov matrix, where $\lambda \in \mathbb{C}$ (set of the complex numbers) is an eigenvalue of *P*, if and only if $Px=\lambda x$ and the *Nx1* column vector *x* is an eigenvector of λ with dtm($P-\lambda I)=0$, where *I* is the unit diagonal matrix *NxN*, is the characteristic equation whose roots are the *N* eigenvalues of *P*, the main Markov chain results adapted to the power matrix *P* and to the power graph one needs, are the following: [2]

(i) If the power matrix is positive (*P>0)*, which means that all its elements are positive, then $\lim_{t\to\infty} P^t = \overline{P}$ which is the *equilibrium matrix*, what is if and only if $\lim_{t\to\infty} P^t x(0) = \overline{x}$ where $\overline{x}$ is the equibrium opinion vector; so the existence and the unicity of the equilibrium opinion vector $\overline{x}$ is assured; moreover, the equilibrium matrix $\overline{P}$ exists and is positive and *ergodic*[3], which means that its rows are identical : $\overline{P} = \begin{bmatrix} \pi \\ \cdot \\ \cdot \\ \pi \end{bmatrix}$, where π is the row-vector *1xN :* $\pi = (\pi_1, ..., \pi_N)$ with the $\pi_i > 0$.

(ii) The unity 1 is an eigenvalue of the power matrix *P*.

(iii) The modulus of any eigenvalue of the power matrix *P* is inferior or equal to the unity $(|\lambda| \le 1)$

(iv) If λ is an eigenvalue of *P,* λ^t, $t \in \mathbb{N}$, is an eigenvalue of P^t.

(v) If all the elements of *P* are positive (*P>0)* (the power graph is a full graph), 1 is a single eigenvalue of *P*.

(vi) If the power graph is an *aperiodic strong connected component,* there exists $t \in \mathbb{N}$, such as $P^t > 0$ and $P^{t+k} > 0, \forall k \in \mathbb{N}$. As a consequence, the (*t+k*)-power graph ${}^{P}\boldsymbol{G}^{t+k}$ is a full graph.

(vii) The unity 1 is single eigenvalue of *P* and the moduli of the other eigenvalues are inferior to 1 if and only if the power graph is an *aperiodic strong connected component),* meaning that the length of all its circuits are prime numbers.

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[2] It is important to recall that the Markov chain theory is usually based on probabilities with a stochastic Markov matrix, where the *i,j* element means the probability to have event *j* after having had event *i*, while in structural power analysis, the *i,j* element p_{ij} of the power matrix *P,* is the permanent power influence from *j* on *i.* The graphs associated to each analysis have opposite orientations for the *i,j* element. This inversion of the graph implies adaptations for the results of Markov chains.

[3] More precisely, it is the Markov chain which is ergodic, meaning that $\overline{P}$ has identical rows; this is why $\overline{P}$ is called ergodic.

(viii) P has m single eigenvalues, which are the m mth-complex roots θ_k of the unity 1 ($\theta_k = e^{2ik\pi/m}$, with *k=0,1,…,m-1* or the θ_k are the m complex zeros of the $\lambda^m - 1$ polynomial, i.e., the modulus of these m eigenvalues is equal to the unity 1 if and only if the power graph is a *m-order periodic strong connected component,* which means that the length of all its circuits is an integers set whose the highest common factor is *m.*

(ix) If the power graph is a m-order periodic strong connected component, the relation defined by $\boldsymbol{P}^m$ = $\{ C_k \subset N \| \forall (i,j) \in N^2 : iP^m j$ *if and only if there exists a m-multiple length path from i to j* $\}$ determines a partition on $\boldsymbol{N}$ of m classes $C_0, C_1, ..., C_{m-1}$, between which exists a "cycle order" as all the following agents of one class constitute the agents of another class, called *consecutive class.*

(x) The power graph is a m-order periodic strong connected component if and only if the unity 1 is a m-multiple eigenvalue of the m-power matrix P^m. In this case, P^m, after rearranging the rows and columns in order to group the agents in the "cycle order" of the $\boldsymbol{P}^m$ relation, assumed that $C_0, C_1, ..., C_m$, is a block diagonal matrix of m blocks, and that each of them is a power-matrix. As a consequence, at each block corresponds, in m-power graph ${}^P\boldsymbol{G}^m$, an aperiodic strong connected component which is a full graph. Furthermore the m m-sub-power graphs ${}^P\boldsymbol{G}_k^m$ *(k=1,…,m)* are disjoined. The same conclusion is valid for $P^{tm}, {}^P\boldsymbol{G}^{tm}$ and ${}^P\boldsymbol{G}_k^{tm}$, where $t \in \mathbb{N}$. Furthermore, with the same rearrangement of the rows and columns, P^r and P^{m+r}, with $r = 1, ..., m-1$, are block matrices, but not diagonal matrices, for which in the associated r-power graph, the path- length is r between agents of consecutive classes.

3.3.2. Typology of the power structure

Gazon (1981 and 1989) introduces a typology of the power structure. This paper presents the same results including a generalisation to the dictatorial structure, with the spectral analysis support of the power matrix, where the nature of the eigenvalues can help define a classification of the power structure. Comments and applications are also reworked and expanded.

3.3.3. Autarkic structure: the particular, but usual case in economic theory:

A particular situation is that where no agent is submitted to the permanent power of the others. Then the power matrix is the unit diagonal matrix *P=diag(1,…,1)* and the power graph is illustrated by Figure 10, which corresponds to an *autarkic structure.*

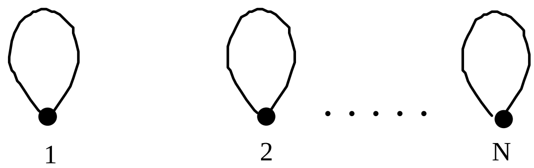


Figure 10
Autarkic structure

<u>Proposition 2</u>

In an autarkic structure the equilibrium opinion vector is permanent.

<u>Demonstration</u>

It is evident because if *P=diag(1,…,1),* one has *: x(0)=Px(0) =* $P^t x(0), \forall t \in \mathbb{N}$

This means that the agents maintain their initial opinions.

Everyone understands that this situation is very exceptional in a power structure, coming from an economic or, more generally, from a social structure. The autarkic structure means that agents are autonomous : they aren't under the influence of anyone, nor do they influence anyone. This hypothesis is useful in economics, particularly in neo-classical theory where agents, for example in consumer theory, determine their behaviour, and particularly their preferences, without considering other agents' behaviour. The resulting autarkic structure shows to what extent this assumption is very restrictive. The following proposals presented in this study reinforce this feeling.

<u>3.3.4. Aperiodic strong connected power structure</u>

An *aperiodic strong connected power structure* is a strong connected structure, where the length of the strong connected power graph circuits are prime numbers. The importance of this binary distinction between the *aperiodic* and the *periodic strong connected structure* will appear in the following proposals.

1°) The case of a full power graph

If the power graph is full, the structure is necessarily strong connected and aperiodic, because the loop of each agent is a circuit whose length is 1. In addition, the power matrix *P* is positive, meaning that all its elements are positive.

<u>Proposition 3</u>

If the power graph is full, the vector $\overline{x}$ is the equilibrium vector of a power system if and only if $\overline{x} = P\overline{x}$.

<u>Demonstration</u>

Necessary condition

If $\overline{x}$ is the equilibrium vector one has from (i) :

$$\lim_{t\to\infty} P^t x(0) = \overline{x}$$

what implies :

$$\lim_{t\to\infty} P^{t+1} x(0) = P\overline{x}$$

From (i), one knows that $\lim_{t\to\infty} P^t = \lim_{t\to\infty} P^{t+1} = \overline{P}$, then $\overline{P}x(0) = P\overline{x}$ and as $\overline{P}x(0) = \overline{x}$, one concludes that :

$$\overline{x} = P\overline{x}$$

Sufficient condition

If $\overline{x} = P\overline{x}$, one has :

$\overline{x} = P\lim_{t\to\infty} x(t) \Rightarrow \lim_{t\to\infty} P^{t+1}x(0) = \lim_{t\to\infty} P^{t}x(0)$

From (i), one concludes that $\overline{x}$ is the equilibrium opinion vector.

Corollary

$\forall t \in \mathbb{N}$**, if** $x(t) = x(t+1), x(t)$ **is the equilibrium opinion vector.**

Demonstration

It is evident because if $x(t) = x(t+1)$, one has $x(t) = Px(t)$ and *x(t)* is the equilibrium opinion vector under Proposition 2.

This property means that when the equilibrium vector is reached, future negotiations will not be able to change it.

Proposition 4

If the power structure is characterised by a full power graph, the equilibrium or final agents' opinions are unanimous.

Demonstration

From (i), with π the row of the ergodic equilibrium power matrix $\overline{P}$, one has $\forall i \in N$:

$\overline{x}_i = \pi x(0) = x$

One remarks that the final opinion *x* is a *compromise,* called *final compromise,* which is *unanimous* if the ergodic matrix $\overline{P}$ >0. This means that this unanimous compromise is equal to a weighting of initial agents' opinions; with $\pi = (\pi_1,...\pi_N)$, one has :

$$x = \pi x(o) = \sum_{i=1}^{N} \pi_i x_i(0).$$

The element π_i is the *resulting power* of the *i*-agent at the equilibrium situation, which doesn't depend on the initial opinion, but only on the power matrix *P*.

Notice that in reality, it is possible to fix an approximate value of the equilibrium opinion vector. So, if *N=2* and if *P*= $\begin{bmatrix} 0.750 & 0.250 \\ 0.338 & 0.662 \end{bmatrix}$ with an approximation at the thousandth $\overline{P} \simeq \begin{bmatrix} 0.575 & 0.425 \\ 0.575 & 0.425 \end{bmatrix} = P^{10}$. As a consequence $\overline{x} \simeq x(10) = P^{10}x(0)$.

The full power graph associated to a positive-power matrix, in fact, presents an "ideal" situation of a reciprocal use of power and must move opinions closer together. But this explanation would be true for any strong connected power structure. Nevertheless, in order to specify the nature of the power structure and of the final compromise, it will be necessary to introduce a distinction depending on the strong connexity configuration of the power graph.

2°) The general case in the aperiodic strong connected power structure

What happens when the power graph is not full, but the power structure strong connected and aperiodic?

Proposition 5

The agents' opinions in an aperiodic strong connected power structure converge to a final unanimous compromise.

Demonstration

From (vi) one knows that when the power structure is strong connected and aperiodic with a power matrix *P*, there exists $t \in \mathbb{N}$ such as $P^t > 0$. Furthermore, the *(t+k)* power matrix for $k \in \mathbb{N}$ stays positive.

From (i), because $P^t > 0$, one has $\lim_{k\to\infty} P^{t+k} x(t) = \overline{x}$, where $\overline{x}$ is the equilibrium opinion vector for which the components are equal to the final compromise *x* (Proposal 4). As a consequence, $\lim_{k\to\infty} P^k x(o) = \overline{x}$, because $x(t) = P^t x(0)$.

So the final compromise in an aperiodic strong connected power structure supposes that, from a certain moment ***on****, each agent has permanent power over all the agents, which also means that all the agents influence each agent from this moment, because of the nature of the aperiodicity. At the beginning of negotiations, each agent does not necessarily directly influence all the others. Nevertheless, the power structure* ^{P}***S*** *must be such that each agent influences all the others through the intermediary of agents on whom he/she has permanent power, with the restriction that the length of the strong connected power graph circuits be prime numbers.*

This corollary is interesting, as it is easy to calculate the eigenvalue spectrum of the power matrix, and so immediately recognise the nature of the power structure.

Corollary 1

If the power structure is strong connected and if at least one agent has a resistance to the opinions of the other agents, the agents' opinions converge to a final compromise.

Demonstration

If the agent $i \in \mathbb{N}$ resists the opinions of the other agents, this means that $p_{ii} > 0$. Then the power graph has a circuit whose length is 1, and the highest common factor of the circuits is 1. Due to (vii) and (viii), the structure is an aperiodic strong connected component, and with Proposal 4, the Corollary is demonstrated.

Corollary 2

If the matrix-power *P* has the unity 1 as a single eigenvalue and the moduli of the other eigenvalues inferior to 1, the agents' opinions converge to a final compromise.

The demonstration is evident because of (vii).

Proposition 6

A unanimous compromise is reached in one round if and only if the power matrix *P* is ergodic .

<u>Demonstration</u>

Necessary condition

In discrete analysis a unanimous compromise reached at most in one round, which means that *x(2)=x(1)* = $\begin{bmatrix} x \\ \cdot \\ \cdot \\ x \end{bmatrix}$

This means that:

$$\sum_{j=1}^{N} p_{ij} x_j(0) = \sum_{j=1}^{N} p_{kj} x_j(0) \Rightarrow \sum (p_{ij} - p_{kj}) x_j(0) = 0$$

As this last equation must be verified $\forall x_j(0) \geq 0$ with at least one $x_j(0)$ >0, one has necessarily $p_{ij} = p_{kj}, \forall i, j, k \in$ ***N.***

So, the row of the power matrix *P* are identical and *P* is ergodic.

Sufficient condition

If the power matrix *P* is ergodic, $P = \begin{bmatrix} \pi \\ \vdots \\ \pi \end{bmatrix}$ where π is a row vector *1xN*.

With $x = \pi x(0)$, one has:

$x(1) = \begin{bmatrix} x \\ \vdots \\ x \end{bmatrix}$ and $x(2) = Px(1) = \begin{bmatrix} x \\ \vdots \\ x \end{bmatrix}$, because *P* is stochastic.

As a consequence, *x(1)=x(2)* and due to the Corollary of Proposition 3, a unanimous compromise is reached in one round.

Remark that in this case the power matrix *P* is idempotent. So, on the one hand, $x(2)=P^2 x(0)$; on the other hand $x(1)=Px(0)$.
As $x(2)=x(1)$ and *P* ergodic, one has $P^2 = P$.

A power structure which implies an instantaneous unanimous compromise is called a *stationary power structure.*
One proposes to extend the notion of instantaneity to the situation where the compromise is reached in only one round. This seems realistic because in discrete analysis when a negotiation is opened, at least one round is needed. But if continuous analysis is used, what requires one round in discrete analysis would become instantaneous; and it is this kind of situation we would like to introduce.

In addition, remark first that the unanimous compromise implies the ergodicity of the equilibrium power matrix and reciprocally, whether it is instantaneous or reached after negotiations. Second, the stationnary power structure could be the result of anterior negotiations. Third because of this ergodicity, the elements of any column are identical, which induces the following corollary.

Corollary

A unanimous compromise is reached in one round if and only if each agent has identical permanent power over the others, equal to his/her resistance.

It is interesting to show that this agent repartition between permanent powers and resistance doesn't imply equality between all elements of the power matrix, as illustrated by the associated power graph of Figure 11.

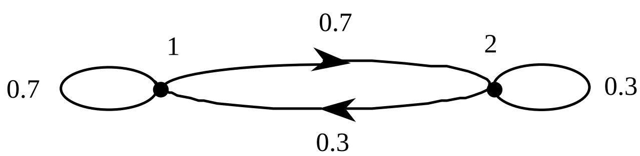


Figure 11
Power structure with an '*instantaneous*' compromise

The unanimous compromise of the Figure 11 example is given by : $x = 0,7x_1(0) + 0,3x_2(0)$
This equilibrium is possible even given the agents' permanent power inequality (0,7 and 0,3).

A stationary power structure is compatible with the inequality of the agents' permanent power, as "economic world order" stability doesn't mean equal repartition of the economic agents' forces. What is necessary is compensation between persuasion forces and resistance. In fact the Corollary illustrated by Figure 11 means that if the persuasion force of one agent (agent 2) is less than that of the other (agent 1), to have instantaneous unanimous compromise, it is necessary that his/her resistance be weak. In other words, the more an *agent having a weak persuasion force (agent 2) adheres to the opinion of a strong one (agent 1), the more probable it is that unanimity will be reached instantaneously. So, one can suppose that inside NATO, Americans have a stronger permanent power over Europeans than the inverse, but one notices that in international conflicts, like the war against Irak or the Kosovo intervention, a relatively quick compromise is reached, because the Europeans (EU) quickly adhere to the American thesis. This signifies, in structural language, that they weakly resist the American influence. But one might think that is due to an initial opinion that is close to the American one. This was probably the case in the examples cited, but if one replaces 'the Europeans' by 'the Russians', the difference between initial opinions doesn't compromise their agreement; in fact, only because the Russians were not able to resist the American influence, and they are not able to strongly influence them. Furthermore, with reference to the autarkic power structure, where resistance is at a maximum (see Figure 10), the equilibrium opinion vector doesn't constitute a unanimous compromise, except if agents have the same initial opinion. This situation shows that without a power connection between agents, no negotiation is possible and absolute resistance or indifference of each to the others authorises autonomous action without taking in consideration others' reactions. This situation is incompatible with the wish to create unanimity from different initial opinions, which postulates a minimum of reciprocal persuasion or a minimum of permeability to the other agents' opinions. For example, in the military domain after the Second World War there was a period called "terror equilibrium", which was possible only because the international power structure allowed the dissuasion strategy.*

This equilibrium is based on the 'strategy of dissuasion'. Each block lets its adversary see the possible consequences of its belligerent act, by the resulting effects, equally disastrous for both sides. In fact, none of the benefits that would come with victory would compensate for losses incurred. Each side reasons in this manner, knowing that the other side does the same. Each also has confidence in his capacity for reprisal and in the rationality of his enemy. This explains why possible dfferences are 'short-lived' and a modus vivendi was established between East and West. However, the advantage gained by a modification of such a power structure does not authorise concluding a 'lasting peace' based on fear, i.e., based on the approximative and aleatory equality between crime and punishment.

Another example, this time an 'economic' one, of this peaceful co-existence is illustrated by the 'behaviour of giant firms' in oligopolistic market. Like the atomic bomb, the arm of prices remains the supreme threat. But, conscious of the reprisal potential of their adversary, the concerned parties therefore avoid resorting to it. The equilibrium of power thus avoids combat by the lowering of prices. With regard to price increases, these are usually triggered by a 'common accord', which corresponds to a final unanimous opinion. This situation corresponds to the Chamberlin (1929) suggestion for an oligopoly producing a homogeneous product; firms would recognise their interdependence and, thefore, might be able to sustain the monopoly price with tacit collusion. In terms of our analysis, such an attitude postulates a power matrix with identical rows (ergodic), which could readily 'accept an inequal repartition of permanent power'. Price adjustments made by firms that did not take the initiative is almost immediate, corresponding to an equilibrium reached in one round. It is important not to confuse this case with the autarkic case; here, agents interact.

3.2.3. Periodic strong connected power structure.

Strong connexity is not sufficient to have opinion convergence to a unique final opinion. The aperiodicity of the power structure is required. If it is periodic of period $m \in \mathbb{N}$, which means that the highest common factor of the power graph circuit lengths is *m*, the convergence to a unique opinion is compromised unless *m=1*. One then has the aperiodicity case, and thus the reason why periodicity supposes *m>1*.

<u>Proposition 7</u>

If the power structure is a *m*-order periodic strong connected component, the power structure splits itself into *m* disjointed classes $C_0, C_1, \ldots C_{m-1}$ and there exist *m* final opinions $x_0, x_1, \ldots, x_{m-1}$, each defined as an initial opinion weighting of one class of agents.

<u>Demonstration</u>

From (ix), supposing that *P* is the adequate power matrix arranged by blocks, one knows that the *m*-power matrix P^m is a block diagonal matrix of *m* blocks and as a consequence, at each block corresponds, in the *m*-power graph ${}^{P}\boldsymbol{G}^{m}$, an aperiodic strong connected component which is a full graph. Furthermore, the *m* *m*-sub-power graph ${}^{P}\boldsymbol{G}_{k}^{m}$ *(k=0,…,m-1)* are disjoined.

Write the block diagonal power matrix : $P^m = \begin{bmatrix} P_0^m & 0 & \ldots & 0 \\ 0 & P_1^m & \ldots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \ldots & P_{m-1}^m \end{bmatrix}$

Due to Proposition 4 and (ix), one has : $\lim_{t \to \infty} P^{tm} = \begin{bmatrix} \overline{P_0} & 0 & \ldots & 0 \\ 0 & \overline{P_1} & \ldots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \overline{P_{m-1}} \end{bmatrix} = \overline{P}^{tm}$

Where each $\overline{P_k}$ is an *m*-equilibrium sub-power matrix which is ergodic :

$\overline{P_k} = \begin{bmatrix} \pi_k \\ \vdots \\ \pi_k \end{bmatrix}$, and $\lim_{t \to \infty} P_k^{tm} x_{C_k}(0) = \pi_k x_{C_k}(0) = x_k$

where $x_{C_k}(0)$ is the initial opinion vector of the *k*-class agents and x_k the *m*-periodic compromise based on the *k*-class agents' initial opinions.

This Proposal means that *in fine* the agents of each *k*-class hold a compromise based on a weighting of their initial opinions, but this is only the case when the period is a multiple of the *m*-order periodicity. This is why this compromise is called the *m-multiple in fine compromise.*

What happens *in fine* but during the periods which are not a *m*-multiple ? Proposition 8 gives the answer.

<u>Proposition 8</u>

If the power structure is a *m*-order periodic strong connected component, *in fine*, each agent takes up, one after another in a cycle order, the *m* final opinions, each of them resulting from a weighting of the corresponding class agents.

<u>Demonstration</u>

Supposing that P is the adequate power matrix arranged by blocks on the basis of the partition generated by the relation $\boldsymbol{P}^m$**,** as defined in (ix). Because in the power graph ${}^{P}\boldsymbol{G}$, class C_{k+1} agents follow class C_k agents, with *k=0,…,m-1*, the power matrix P can be presented as :

$$P = \begin{bmatrix} \overbrace{0}^{C_0} & \overbrace{0}^{C_1} & \cdots & \overbrace{0}^{C_{m-2}} & \overbrace{P_{0,m-1}}^{C_{m-1}} \\ P_{1,0} & 0 & \cdots & 0 & 0 \\ 0 & P_{2,1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & P_{m-1,m-2} & 0 \end{bmatrix}$$

At time *t=1*, the transmission of the opinion from class to class is : $C_0 \to C_1 \to \cdots \to C_{m-2} \to C_{m-1} \to C_0 \to \cdots\cdots$

At time $t=2 \leq m$, *it is :*

$C_0 \to C_2 \to \cdots \to C_{m-2} \to C_0$ if *m* is an even number, and

$C_0 \to C_2 \to \cdots \to C_{m-1} \to C_1 \to \cdots \to C_{m-2} \to C_0 \to \cdots$, if *m* is an uneven number.

In these sequences of "cycle order", the path length at time *t* between two consecutive classes is *t*; this means that when *t*= an *m*-multiple, the sequence of the transmission between classes is :

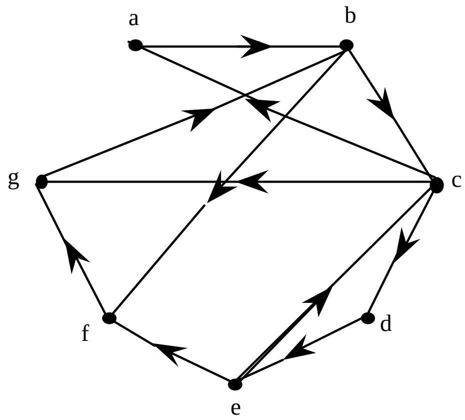
$C_0 \to C_0; C_1 \to C_1; ...; C_{m-1,m-1} \to C_{m-1,m-1}$. which corresponds to the diagonal matrices P^{tm}.

As a consequence, *in fine*, when $\overline{P}^{tm}$ is reached, the k^{th} block (*k=0,…,m-1)* is given by the ergodic stochastic power matrix $\overline{P_k} = \begin{bmatrix} \pi_k \\ \vdots \\ \pi_k \end{bmatrix}$. Now *in fine*, but at time *(t+1)m*, the block (C_{k+1}, C_k) is defined by : $\left[p_{k+1,k}\right] . \begin{bmatrix} \pi_k \\ \vdots \\ \pi_k \end{bmatrix} = \begin{bmatrix} \pi_k \\ \vdots \\ \pi_k \end{bmatrix}$, where $\left[p_{k+1,k}\right]$ is the sub-power matrix of the power matrix *P*, formed of the elements at the intersection of the rows corresponding to the agents of class C_{k+1} and the columns of class C_k. In fact at the matrix $\left[p_{k+1,k}\right]$ is associated the sub - power graph whose arcs have their origin in class C_k and their destination in class C_{k+1}. Because of the definition of the partition, the other elements of the power matrix *P* corresponding to the lines agents of C_{k+1}, are equal to zero and so the matrix $\left[p_{k+1,k}\right]$ is stochastic, which establishes that the rows of the block (C_{k+1}, C_k) are identical and equal to the row vector π_k. Notice that (C_{k+1}, C_k) is not necessarily a square block; its dimensions are the row numbers of C_{k+1} and the column numbers of C_k.

At time *(t+2)m*, for the same reason it will be the rows of block (C_{k+2}, C_{k+1}) which will take up the row vector π_k, and so on in an "order cycle", knowing that at time *tm* (*m*-multiple) *in fine*, for all *t*, one has the block diagonal matrix $\overline{P}^{tm}$.

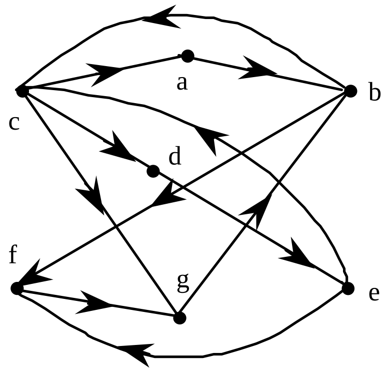
As a consequence, as $\overline{P}^{tm}$ *x(0)*= $\begin{bmatrix} \{x_0 \\ \vdots \\ \{x_k \\ \vdots \\ \{x_{m-1} \end{bmatrix}$ where $\{x_k$ is the final opinion for the agents of class C_k at time *in fine tm, one has* $\overline{P}^{tm+1} x(0) = \begin{bmatrix} \{x_{m-1} \\ \vdots \\ \{x_{k+1} \\ \vdots \\ \{x_{m-2} \end{bmatrix}$, which means that agents of class C_{k+1}, adhere *in fine (t+1)m* to the agents' final opinion of class C_k *in fine tm* and so on in the "order cycle" of the *m* classes agents' final opinions defined *in fine tm.*

Figures 12 shows the evolution, from *t=0 to t=4*, of the power graph and the associated power matrix in a periodic strong connected structure where *m=3.* When the permanent power is zero the space is empty and when it is different from zero, an *x* is put there. Moreover, Figures 12.1 and 12.2 present at time *t=0,* the power graph and the associated power matrix before and after the rearrangement due to the partition.



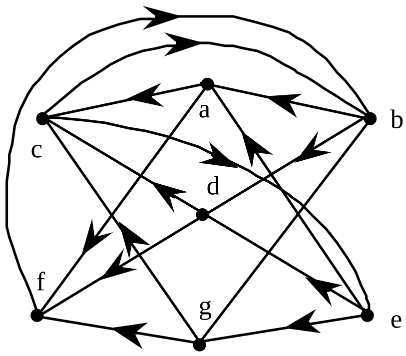
	a	b	c	d	e	f	g
a			x				
b	x						x
c		x			x		
d			x				
e				x			
f		x			x		
g			x			x	

Figure 12.1
Graph ^{P}G



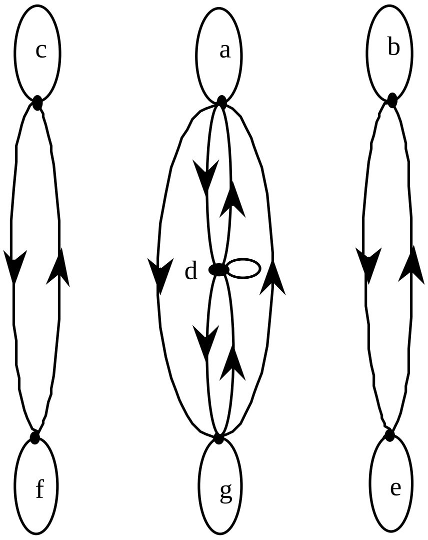
	c	f	a	d	g	b	e
c						x	x
f						x	x
a	x						
d	x						
g	x	x					
b			x		x		
e				x			

Figure 12.2
Graph ^{P}G



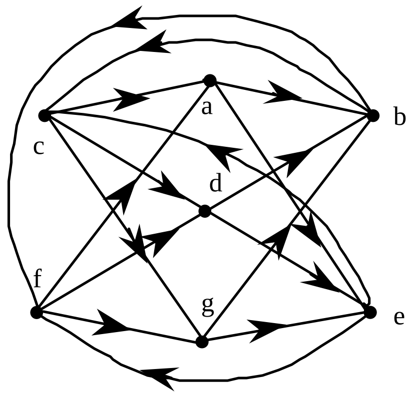
	c	f	a	d	g	b	e
c			x	x	x		
f			x	x	x		
a						x	x
d						x	x
g						x	x
b	x	x					
e	x						

Figure 12.3
Graph $^{P}G^{2}$



	c	*f*	*a*	*d*	*g*	*b*	*e*
c	*x*	*x*					
f	*x*	*x*					
a			*x*	*x*	*x*		
d			*x*	*x*	*x*		
g			*x*	*x*	*x*		
b						*x*	*x*
e						*x*	*x*

Figure 12.4
Graph ${}^{P}G^3$



	c	*f*	*a*	*d*	*g*	*b*	*e*
c						*x*	*x*
f						*x*	*x*
a	*x*	*x*					
d	*x*	*x*					
g	*x*	*x*					
b			*x*	*x*	*x*		
e			*x*	*x*	*x*		

Figure 12.5
Graph ${}^{P}G^4$

The main characteristic of a periodic strong connected power structure is that each of the class agents' opinions are influenced in a "cycle order", at time *t*, by one unique other class agents' opinions at time *t-1.* Notice that no agent has resistance to the influence of the influencing agents, which corresponds to Corollary 2 of Proposition 4. This is the reason why the agents of a periodic strong connected structure are qualified as *versatile.*

This type of structure is a-priori surprising. Nevertheless, the well-known CobWeb-model corresponds to this structure : from one period to the other, purchasers and suppliers adapt the demand and the offer to the price of the preceding period. In this model, dominance is alternatively demand dominance and supply dominance.

Corollary 1

If the power matrix P has m single eigenvalues which are the m mth-complex roots θ_k of unity 1 ($\theta_k = e^{2ik\pi/m}$, with *k=0,1,…,m-1* or the θ_k are the *m* complex zeros of the $\lambda^m - 1$ polynomial (**i.e., the modulus of these *m* eigenvalues is equal to the unity 1), there**

are *m* final opinions, which correspond to *m* classes agents' final opinions. Each agent takes up, one after another, in a cycle order, **the *m* final opinions.**

The Corollary is evident from (ix) and from Propositions 7 and 8. This facilitates recognition of the periodic strong connected structure.

3.4.4. Dictatorial structure

Up to now, this study has been limited to a structure in which the power graph was strong connected, which assumed that each agent directly or indirectly influenced the others. In economics and politics, the situation where several agents have no capacity to influence others, while they endure their influence, is an interesting one. This is the case in a market structure for price-takers or in a technical relation, when one producer totally depends on the technique of his/her supplier; this dependence relationship is evident between the parent company and its subsidiaries. In politics, using military force or financial and economic influence, one State can impose its point of view on another State, even given the formal political independence of the latter. This power relation could be unclear and make itself felt through intermediary organisms, because the permanence of the power relation is more certain when it doesn't appear obvious; thus the dominated nation does not lose face. The present relationship between the USA and Russia, through international organisms like the IMF, illustrates this notion.

This kind of power structure is shown in Figure 13.1, representing the power graph, and Figure 13.2, representing the quotient power graph, where $r_o = \{1,2,3\}$ is the basic agent's rank and $r_1 = \{4,5\}$, the dominated agent's rank.

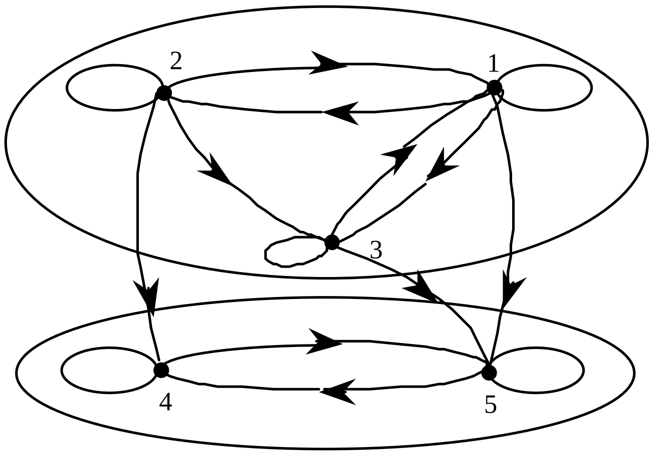


Figure 13.1
Power graph of a dictatorial structure with one basic agent class

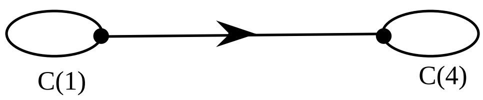


Figure 13.2
Quotient power graph of a dictatorial structure with one basic agent class

Let us define a *dictatorial power structure* as a power structure for which the dominant position relation implies at least two ranks for the quotient power graph; the agents of the classes included in rank r_O are the *basic agents*, while the others of classes of rank $r_k (k > 0)$ are the *secondary agents.* Without reducing the analysis, one supposes that the power graph is connexe which means that, at time $t=0$, there exists at least one arc between the ranks of the quotient power graph.[4]

Proposition 9

In a dictatorial power structure with only one aperiodic class of basic agents, the final equilibrium opinion is a unanimous compromise, which is only a weighting of the basic agents' initial opinion.

Demonstration

The power matrix *P* associated to the dictatorial power structure with only one class of basic agents could be written as the following stochastic matrix *NxN:*

$$P = \begin{bmatrix} P' & 0 \\ A & B \end{bmatrix}$$

where *P'* is a stochastic power matrix *GxG*, *G* being the number of basic agents,
A is a matrix *(N-G)xG*
B is a matrix *(N-G)x(N-G)*.

Furthermore :

$$P^t = \begin{bmatrix} P'^t & 0 \\ A_t & B^t \end{bmatrix}$$

where $A_t = AP'^t + BA_{t-1}$

On the other hand, because the power graph of the basic agent's class associated to *P'* is aperiodic strong connected, one has:

$$\lim_{t\to\infty} P^t = \begin{bmatrix} \overline{P'} & 0 \\ \overline{A} & 0 \end{bmatrix} = \overline{P}$$

because $\lim_{t\to\infty} B^t = 0$, due to the fact that *B* is sub-stochastic, and as a consequence its eigenvalues modulus is inferior to 1 and $\overline{A}$ is stochastic because $\overline{P}$ is stochastic.

In addition, the matrix $\overline{P'}$ is ergodic, which means that its rows are identical:

$$\overline{P'} = \begin{bmatrix} \pi_1 & \cdots & \pi_G \\ \vdots & & \vdots \\ \pi_1 & \cdots & \pi_G \end{bmatrix}$$

With *x(0)* the *Nx1* initial opinions vector, one has:

$$\lim_{t\to\infty} P^t x(0) = \overline{P}x(0) = \overline{x}.$$

The *G* first components of $\overline{x}$ are equal to $\sum_{i=1}^{G} \pi_i x_i(0) = x$, which shows that the basic agents' final opinion is unanimous and depends only on their initial opinions, because *x* is a weighting of them.

[4] In the following propositons one doesn't specify the connexity of the power graph, but it is assumed.

Furthermore $\bar{x} = \overline{P}\bar{x}$. Because each component of $\overline{P}\bar{x}$ is the weighted average of the *G*th first components of $\bar{x}$, due to the stochasticity of $\overline{P'}$ as well as this one of $\overline{A}$, as the *G*th first components of $\bar{x}$ are equal to *x*, each component of $\bar{x}$ is *x*.

Then $\bar{x} = \overline{P}x(0) = \begin{bmatrix} x \\ \vdots \\ x \end{bmatrix}$

So the final equilibrium opinion of the dictatorial structure agents is a unanimous compromise, which is a weighting of the basic agents' initial opinions.

It is interesting to remark that, due to $\bar{x} = \overline{P}\bar{x} = \overline{P}^2 x(0)$ and *P* ergodic, one concludes

that $\overline{P} = \overline{P}^2$ which implies $\overline{P}^2 = \begin{bmatrix} \overline{P'}^2 & 0 \\ \overline{AP'} & 0 \end{bmatrix} = \left[\begin{array}{ccc|ccc} \pi_1 & \cdots & \pi_G & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ \pi_1 & \cdots & \pi_G & 0 & \cdots & 0 \\ \hline \pi_1 & \cdots & \pi_G & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ \pi_1 & \cdots & \pi_G & 0 & \cdots & 0 \end{array}\right] = \overline{P}$

So, the weight of the i-basic agent in the unanimous compromise x is π_i.

Proposition 10

In a dictatorial structure with several aperiodic classes of basic agents, the final equilibrium opinion is a unanimous compromise for each basic agent of a same class and a weighting of these compromises for the secondary agents, this weighting depending on the dominant situation relation in the quotient power graph .

<u>Demonstration</u>

With the notation of the preceding demonstration, if the rank r_0 of the dominant situation relation in the quotient power graph contains *k* classes, the matrix *P'* is *:*

$$P' = \begin{bmatrix} P_1 & 0 & \cdots & 0 \\ 0 & P_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_k \end{bmatrix}$$

where the block diagonal matrix $P_i (i = 1,...,k)$ is the power matrix of the sub-power aperiodic graph having class $C_i \in r_0$ as support. Furthermore, the (*NxN*) power matrix *P*, is:

$$P = \begin{bmatrix} P' & 0 \\ A & B \end{bmatrix}$$

For the same reasons as above, one has *in fine* as each $P_i (i = 1,...,k)$ is ergodic:

$$\overline{P_i} = \begin{bmatrix} \pi^{C_i}_{G_{i-1}+1} & \cdots & \pi^{C_i}_{G_i} \\ \vdots & & \vdots \\ \pi^{C_i}_{G_{i-1}+1} & \cdots & \pi^{C_i}_{G_i} \end{bmatrix}$$

knowing that $\overline{P_1}$ has as columns the G_1 first columns of $\overline{P}$, corresponding to agents $1,\ldots,G_1$, so G_0=1, and that G_k=*G*, which is the number of basic agents for the whole of $C_i \in r_0$.

One has:

$$\overline{P} = \left[\begin{array}{ccc|c} \overline{P_1} & & 0 & \\ & \ddots & & 0 \\ 0 & & \overline{P_k} & \\ \hline & \overline{A} & & 0 \end{array}\right]$$

Of course, as for Proposition 9, the final equilibrium opinion of the basic agents' class $C_i \in r_0$, is a unanimous compromise for this class, noted x_{C_i}. But what about the final opinion of the secondary agents?

From the recurrent equation $A_t = AP'^t + BA_{t-1}$ and because of the aperiodicity of P'^t, A_t converges to $\overline{A}$. So it is possible to write *in fine:* $\overline{A} = A\overline{P'} + B\overline{A}$, which gives:

$\overline{A} = [I - B]^{-1} A\overline{P'}$

because [*I*-*B*] is invertible, due to the fact that dtm[*I*-*B*]$\neq 0$ because 1 is not an eigenvalue of *B*, which is sub-stochastic.

The matrix $\overline{A}$ does not necessarily have identical rows, because here each row depends on the rows of the $P_i (i = 1,\ldots,k)$ matrices.

If $\overline{a_{ij}}$ is the generic term of $\overline{A}$, the equilibrium matrix $\overline{P}$ is as follows:

$$\overline{P} = \left[\begin{array}{ccc|ccc|ccc|c} \pi^{C_1}_1 & \cdots & \pi^{C_1}_{G_1} & & & & & & & \\ \vdots & & \vdots & & 0 & & & 0 & & 0 \\ \pi^{C_1}_1 & \cdots & \pi^{C_1}_{G_1} & & & & & & & \\ \hline & & & \ddots & & & & & & \\ & 0 & & & \ddots & & & 0 & & 0 \\ & & & & & \ddots & & & & \\ \hline & & & & & & \pi^{C_k}_{G_{k-1}+1} & \cdots & \pi^{C_k}_{G} & \\ & 0 & & & 0 & & \vdots & & \vdots & 0 \\ & & & & & & \pi^{C_k}_{G_{k-1}+1} & \cdots & \pi^{C_k}_{G} & \\ \hline \overline{a}_{G+1,1} & \cdots & \overline{a}_{G+1,G_1} & \cdots & \cdots & \cdots & \overline{a}_{G+1,G_{k-1}+1} & \cdots & \overline{a}_{G+1,G} & \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & 0 \\ \overline{a}_{N,1} & \cdots & \overline{a}_{N,G_1} & \cdots & \cdots & \cdots & \overline{a}_{N,G_{k-1}+1} & \cdots\cdots & \overline{a}_{N,G} & \end{array}\right]$$

Then, with the initial opinion vector *x(0)*, one has: $\overline{x} = \overline{P}x(0)$, where $\overline{x}$ is the final opinion vector.

As a consequence, because $\lim_{t\to\infty} B^t = 0$, the final opinion of the secondary agents *G+1,…,N* depends only on the basic agents' initial opinion.

With $\alpha_{i,C_j} = \sum_{j\in C_i} \overline{a_{i,j}}, \forall i = G+1,...,N$ and then $\sum_{C_j\in r_0} \alpha_{i,C_j} = 1$, considering more the final unanimous opinion x_{C_i} of the basic agents class C_i and recalling that $\overline{x} = \overline{P}\overline{x}$ because as before $\overline{P'}$ and $\overline{A}$ are stochastic, the final opinion of the secondary agent *i =G+1,…,N* is given by:

$$\overline{x_i} = \sum_{j=1}^{k} \alpha_{i,C_j} x_{C_j}$$

Then the final opinion vector $\overline{x}$ is:

$$\overline{x} = \begin{bmatrix} x_{C_1} \\ \vdots \\ x_{C_1} \\ \hline \vdots \\ \vdots \\ \vdots \\ \hline x_{C_k} \\ \vdots \\ x_{C_k} \\ \hline \alpha_{G+1,C_1} x_{C_1} + ... + \alpha_{G+1,C_k} x_{C_k} \\ \vdots \\ \alpha_{N,C_1} x_{C_1} + ... + \alpha_{N,C_k} x_{C_k} \end{bmatrix}$$

As a consequence, in this general case of a connected power graph, the final equilibrium opinion is a unanimous compromise for the basic agents of the same aperiodic class, this compromise depending on the initial opinions of these agents. Moreover, the final equilibrium opinion of the secondary agents is a weighting of the basic agents classes compromises.

In order to define how this weighting of the basic agents classes is organised, one can refer to the solution of $\overline{A} = [I - B]^{-1} A\overline{P'}$, or to the quotient power graph induced by the dominant situation relation as showed infra**.**

Corollary 1

The dictatorial power structure has *k* basic agents classes if and only if the power matrix *P* has *1* as *k*-multiple eigenvalue.

The necessary and sufficient conditions result from the following expression of the P characteristic equation: $dtm[\lambda I - P] = \prod_{i=1}^{k} dtm[\lambda I - P_i] dtm[\lambda I - B]$.

Because P_i is stochastic, P_i has *1* as single eigenvalue due to (vii); furthermore, because B is sub-stochastic, the eigenvalues of B have a modulus inferior to *1*. As a consequence, *1* is *k*-multiple root of the P characteristic equation which establishes Corollary 1 .

As mentioned above, this Corollary offers a simple method to determine the basic agent classes of the power structure.

Corollary 2

If several basic agent classes are periodic, inside each periodic class there exist as many final opinions as orders of periodicity, and these final opinions are taken up, one after another, in a cycle order between sub-classes of each periodic class. The number of final opinions of the secondary agents are, for each secondary agent's class, the highest common factor of the periodicity orders of the basic agents' periodic classes which are, in the quotient power graph, the origin of at least a path which has the secondary agent's class as destination.

These periodic final opinions for secondary agents' classes depends only on the initial (and final) opinion of the basic agents' aperiodic and periodic classes, which are, in the quotient power graph, the origin of at least a path which has these secondary agents' classes as destination.

This Corollary is a direct consequence of Propositions 7, 8, 9 and 10 and the properties of the quotient power graph induced by the dominant situation relation.

But in fact as far is the power of the agents is concerned, this situation of a dictatorial structure with basic agents' periodic classes is improbable. One knows that these agents are versatile, i.e., they have no resistance to others' opinions. It is therefore difficult to imagine that these versatile agents have the capacity, not only to influence secondary agents and not be influenced by them or by the others basic agents' aperiodic classes. This is why it seems of little interest to analyse in more detail this type of power structure, more theoretical than practical.

Corollary 3

The dictatorial structure has *k* basic agents' classes, with at least one periodic basic agent's class if and only if *1* is *k*-multiple eigenvalue of *P* and *l* other eigenvalues have *1* as modulus, *l* being equal to the sum of the periodic mutiplicity orders of each periodic basic agent's class.

This Corollary is the consequence of Corollary 1, considering the property (viii).

The results testify to the basic agents' power who, due to their dominant situation in the power structure, finally impose their points of view on the rest of the structure. On the contrary, secondary agents have no other possibility than to put up with this "imperialism" in the given structure. Of course this situation is not dramatic if the secondary agents' initial opinions are very close to those of the basic agents who influence them. But if this is not the *case, they only have two possibilities. The first is to set up a reciprocal influence on the basic agents and so becoming themselves basic agents. But if this is impossible, and one can think that this is probably the case because a dominated agent has few means to change the nature of the power structure by influencing in turn the dominating agents, the only other alternative is to free him/herself of his/her dependent bonds in order to cancel any power influence coming from the basic agents, and thus finding his/her autonomy.*

In economics many examples of the dictatorial structure can be found. For instance, if in an inter-sectorial relation, several producers need the technology of a supplier who is a monopolist, or of cartelised suppliers, they have no other alternative than to accept this domination.

Even if it is difficult to define exactly how power is diffused throughout the world in order to install the free market as the ideal economic organisation, often called globalisation, this situation results from a power relation, shaped essentially by politics and the ideology of neo-liberalism imposed by international firms and international organisations without consideration of the opinions of some nations and agents. The logic of the dictatorial power structure well explains the globalisation process through which national economies, in becoming more open, are more subject to supranational influences and less subjected to national control. Without here introducing the epistemological analysis of the word 'globalisation' and the close notions, mondialisation and internationalisation, the process is a dictatorial one, as the ideological opinion of "free market" doesn't mean in fact that the freedom to access to the market is the same for everyone, because perfect competition doesn't exist. Only a few basic agents, essentially multinational firms with the political and military support of their mother countries, impose the "free" process, but which implies the adhesion of secondary agents, composed of secondary firms and nations. This adhesion could be a positive or negative one. In the final equilibrium opinion vector, the adhesion of the secondary agents to the opinion of the basic agents will be positive if several secondary agents have an interest in this equilibrium, f.i., because they have access to local power and high salaries with the support of basic agents, or simply, because they find jobs and a better situation than before in another economic organisation. Negative adhesion appears when secondary agents wish to change the situation imposed by basic agents, generally because they want to have more autonomy in their social choices, sometimes with ecological claims. Then the domination of basic agents is threatened by a modification of the power structure and the basic agents must modify their initial opinion in order to better meet the opinion of the recalcitrant secondary agents, f.i., by accepting more social regulation or by respecting the environment more. But the more alternative the organisation is, the more important the risk will be for the basic agents of a structural modification. And the more basic agents' classes there are, the more numerous the alternatives will be. As the final opinion of secondary agents is a weighting of the basic agents' classes final opinions, it is easier for the secondary agents to induce a modification of the power structure by changing the weighting in favour of the basic agents' class whose opinion is closest to theirs. In the globalisation process, this theoretical conclusion is illustrated by the collapse of communism as a social system, today reducing the basic agents to only one class. Proposal 9 shows how adhesion, positive or negative, is totally submitted to this basic agents' class opinion. As a consequence, the social rights of the Welfare State could be threatened, as history of the 20th century shows that social democracy grew in the shadow of communism but also of liberalism, like a synthesis of two main basic agents' classes, liberalism and socialism. Nevertheless, inside the unique basic agents' class, there is an interdependence between several actors, where the European Union, with its greater social democratic option than that of the USA, can influence the final compromise diffused by the power structure throughout the world.

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Conclusions

Based on influence transmission, this paper studies the use of power thorough opinion transmission. Given a power structure and an initial opinion vector of the agents who constitute the support of the power structure, the influence power of agents induces an evolution of the initial opinions to reach a final opinion equilibrium vector, which is generally a compromise of the initial opinions. But the weighting of initial opinions in the compromise depends on the agent's position in the power structure and can take into consideration only the initial opinion of some agents.

The evolution of opinions in the power structure is founded on an isomorphism of Markov chains, because to the power structure a stochastic power matrix is associated, describing the interrelation between agents. Then, opinions change from period to period, like negotiation rounds, the agent opinion at time $t+1$ being a weighted average of the agents' opinions who influence him at time t, this average including the self-opinion of the agent, considered as a resistance to other opinions.

Different situations resulting from the power structure are presented.

The autarkic structure is a particular case but usual in neo-classic theory(f.i., the consumer theory), where no one is influenced by the others; here the final opinion equilibrium vector is also instantaneous, but not unanimous, because it is the initial opinion vector which doesn't change during negotiations. It is demonstrated how this situation could be conflictual if initial opinions are very different: absolute resistance to the others or the impossibility for anyone to influence anyone else leaves room for conflicts, if initial opinions are hostile

Of course, if initial opinions are unanimous, the final opinion is instantaneously unanimous.

But a unanimous compromise is possible even if initial opinions are not the same. This is the case when, in an interdependence structure (strong connected structure), persuasion forces and resistance are shared, as they compensate one another. As examples, 'terror equilibrium' during the Cold War and 'price equilibrium' in an oligopolistc structure are given. Because agreement is reached immediately after the negotiation starts, these types of power structures are called stationary structures.

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But generally, the power structure is opened to negotiations, which take time before reaching a compromise. Two main types of power structure are distinguished. The first is the strong connected structure, which means that each agent can influence the others directly or through other agents. The second is the dictatorial structure, where several basic agents' classes influence themselves, inside a same strong connected class, as well as influencing other secondary agents, without being influenced by them.

In a strong connected structure, negotiations on the basis of the initial opinion vector bring points of view closer until a unanimous compromise is reached, meaning that a pre-existing interdependence facilitates agreement. But this doesn't mean that the unanimous compromise is an equal sharing of initial opinions. The weight of some agents can be more important than that of others. Inside the strong connected structures, one distinguishes those where the unanimous compromise is unique (*aperiodic strong connected structure*) from those where several compromises are possible with a cycle adhesion of agents' sub-classes (*periodic strong connected structure*). The latter is a bit surprising. It is interesting to recall that, inside strong connected structures, the necessary and sufficient condition for this situation is that no agent has resistance: each bases his/her future opinion on that of the agents who influence him/her. These are *versatile* agents. This is a situation a-priori difficult to imagine, but one notices, as for example, the CobWeb model and the explanation of price oscillations and spread between supply and demand.

The most general case is the dictatorial structure, where each class of basic agent behaviours is itself as within a strong connected structure and reaches a unanimous or a cycle compromise, depending only on agents of the same class. But what is interesting to observe is that if only one class of basic agents exists, *in fine,* the secondary agents share the unanimous compromise of the basic agents, although this compromise doesn't take into consideration the initial opinion of these secondary agents. When there are several classes of basic agents in the power structure, the final opinion accepted by the secondary agents is a weighting of the unanimous comprises of the basic agents' classes. Many economic, social and politic examples are characterised by a dictatorial structure. This is always the case when, e.g., in a market, one class of agents can influence the others without being influenced by them. When a demand shortage exists, demanders are the basic agents, and when there is a supply shortage, it is suppliers who become the basic agents. But more generally, the power logic of this paper is able to explain the main lines of the globalisation process of economies throughout the world.

Finally, recall the reference to linear algebra, and especially to spectral analysis, which can help to distinguish the different power structures in the eigenvalue analysis of the power matrix.

Of course, this paper does not attempt to offer a complete analysis of power, but rather on the basis of rigorous results, to establish a typology of power structure and a method to approach power analysis. The main aim is certainly to heighten awareness of the importance of power, not only in politics, but in economics as well. Many topics could be reviewed in the light of the structural approach to power. Let's just consider the utility function of consumers, which is at the basis of demand function. It is not difficult to imagine how big the change in the neo-classical theory would be, if instead of an autarkic power structure between consumers, which assumes that their preferences are not influenced by the behaviour of others, one would consider that, in fact, because it is indeed a fact, that the utility of one depends on the utility of the others, and that that interdependence/dependence would result from a strong connected or a dictatorial structure.

The present approach to the power structure should be validated by applications. A priori it is easy to infer the power structure from any exchange structure and input-output analysis provides a nice field for this. But to be pertinent, one must be careful about the orientation of influence. The global hypothesis of demand dominance or supply dominance risks inferring erroneous conclusions. Surveys would be necessary to define the direction of influence inherent to each flow of products and services.

Furthermore, power is manifested in many ways. This means that between two agents, the power relation iPj is usually not unique as is the case in the inter-industrial relationships when there is political interference. The analysis here has been limited to only one relation, knowing that it can be extended to each existing relation. The problem appears when the combination of several types of relation is in question. One can imagine that it is probably impossible to aggregate multiple relations into only one, except through a survey which consists, for a given problem, in knowing how each agent in the structure weighs the "global" influence he receives from the others, including his own resistance.

But with only one relation, the main problem for applications is to determine the power application ${}^{P}\boldsymbol{H}$**,** i.e., the power matrix *P*. Nevertheless, without knowing the power matrix, but only the binary existence of permanent power between agents, due to the quotient power graph induced by the domination situation relation, it is possible to conclude with precision the following: first, who the basic agents and the secondary agents are; second, if a unanimous compromise is possible; and third, what initial opinions will be taken into consideration in a compromise. Because if one neglects weighting of the initial opinions in the compromise, the power relation suffices to know who holds the power.

REFERENCES

Aglietta, M. (1982) *'Régulation et crises du capitalisme'*, 2e éd. Calmann-Levy, Paris

Allais, M. (1981) *La théorie générale des surplus*, PUG, Grenoble

Aron, R. (1958), 'Note sur le pouvoir economique' *Revue economique*

Bierstedt, R. (1950) 'An Analysis of Social Power' *American Sociological Rewiew*, pp. 730-738.

Boyer, R. (1986) *'La théorie de la régulation: une analyse critique'* La Découverte, Paris

Boyer, R. et Mistral, J. (1983) *'Accumulation, inflation, crises'* PUF, Paris

Byé, M. (1957) *'L'autofinancement des grandes unités territoriales et les dimensions territoriales de son plan*; Travaux du congrès des économistes de langue française, pp. 5-38.

Crama, Y. Defourny, J. and Gazon, J. (1984) 'Stuctural Decomposition of Multiplier in Input-Output Social Acounting Matrix Analysis' *Economie Appliquée*, tome XXXVII, n°1, pp. 222.

Defourny, J. (1982) 'Une approche structurale pour l' analyse input-output; un premier bilan' *Economie Appliquée*, tome XXXV,n°1-2, pp. 203-230.

Defourny, J. and Thorbecke, E. (1984) 'Structural Path Analysis and Multiplier Decomposition within a Social Accounting Matrix Framework' *The Economic Journal*, n° 94, pp. 111-136.

Destanne de Bernis, G. (1975) *'Régulation ou équilibre dans l'analyse économique'* in L'idée de régulation dans les sciences, Séminaire interdisciplinaires du Collège de France, Maloine, Paris

French, J.R.P. (1966) *A Formal Theory of Democracy*, Harper and Row, New-York

Galbraith, J.K. 1973) *'La science économique et l' intérêt général'* Gallimard, Paris

Gazon, J. (1976) *'Transmission de l' influence économique. Une approche structurale'* Collection de l' lnstitut de Mathématiques Economiques, n° 13, Sirey, Paris

Gazon, J. (1979) 'Une nouvelle méthodologie: l'approche structurale de l'influence économique' *Economie Appliquée*, Tome XXXII, n°2-3, pp. 301-337

Gazon, J. (1981) 'La transmission de l'opinion. Une approche structurale du pouvoir au sein des structures fortement connexes' *Economie Appliquée*, tome XXXIV,n°4, pp. 749-784

Gazon, J. (1989) ' Analyse de l'interdépendance industrielle par la méthodologie structurale' *Economie Appliquée*, tome XLII, n°4, pp. 133-163.

Gazon, J and Tits, A (1978) 'A Stuctural Appraoch to Stability in Linear Systems. A Sufficient Condition' *International Journal of Systems Science*. Vol. 9, n°6, pp. 681-694.

Harary, F. (1959) *'A Criterion for Unanimity in French's Theory of Social Power*, in D. Carthwrigght, *Sudies in Social Power*. Arbor University of Michigan.

Huriot, J.M. (1974) *'Dépendence et hiérarchie dans une structure interindustrielle'* Collection de l'Institut de Mathématiques Economiques, n° 8, Sirey, Paris

Lantner, R; (1974) *Théorie de la dominance économique*, Dunod, Paris

Lesage, A. (1984) 'Définition structurale d'une filière de production' *Mondes en Développement.*

Lhomme, J. (1966) *Pouvoir et Société économique*. Ed Cujas, Paris

Marée M, and Defourny, J. (1978) 'La circularité comme aspect essentiel de l'interdépendance entre les secteurs: une approche structurale' *Mondes en Développement*, n°22, pp. 283-314.

Nash, JF. (1950) 'Bargaining problem' *Econometrica*, 18, pp. 155-162

Nash, J.F. (1953) 'Two-person cooperative games' *Econometrica*, 21, pp. 128-140

Perroux, F. (1948) 'Esquisse d'une théorie de l'économie dominante' *Economie Appliquée*, Archives de l'ISMEA, n° 2-3.

Perroux, F. (1961) *'L'économie dominante' in L'économie du 20e siècle*, PUF. 2e éd. 1969, pp. 61-144

Perroux, F. (1969) *'Indépendance de la nation et interdépendance des nations'* Aubier-Montaigne, Paris

Perroux, F. (1971) 'Structuralisme, modèles économiques, structures économiques' *Economie Appliquée*, tome XXIV, n°3

Perroux, F.. (1973) *'Pouvoir et Economie'* Bordas, Paris

Perroux, F. (1975) ' *Unités actives et mathématiques nouvelles'* Dunod, Paris

Perroux, F. (1982) ' *Dialogue des monopoles et des nations, "Equilibre" ou dynamique des unité actives*, PUG, Grenoble.

Roth, A.E. (ed) (1985) *'Game-theoritic Models of Bargaining'* Cambridge University Press

Rubinstein, A. (1982) 'Perfect equilibrium in a bargaining model' *Econometrica*, 50, pp. 97-109

Rubinstein, A. (1985) 'A bargaining model with incomplete information about time preferences' *Econometrica*, 53, pp. 151-172

Rubinstein, A, (1990) *'Game Theory in Economics'* The International Library of Critical Writings in Economics, Edward Elgar Publishing Limited, Adeershot and Brookfield.

Schumpeter, J.A. (1942) *'Capitalism, Socialism and Democracy'*, Urwin London.

Spengler, J.J. (1950) 'Economic power and American Capitalism' *American Economic Review*, tome XL, pp. 413-434.
Stähler, F. (1998) ' *Economic Games and Strategic Behaviour: Theory and Application'* Edward Elgar Publishing Limited', Cheltemham and Northampton.
Tirole, J. (1988) *'The Theory of Industrial Organization'* MIT Press, Cambridge
Von Boehm-Bawerk, E. (1914) 'Macht oder oekonomisches Gesetz?' *Zeitschrift für Volkswirtschaft*, Bd XXIII, pp. 205-271.
Von Neuman, J. and Morgenstern, O. (1944) ' *Theory of Game and Economic Behaviour'* Princeton University Press, Princeton
Von Wieser, M. (1926) *'Das Gesetz der Macht'* ,Vienna
Weiller, J. (1948)'Préférences nationales de structure et déséquilibre structurel' *Revue d'Economie politique*.
Weiller, J. and Carrier, B. (1994) *'L'Economie Non Conformiste en France au XXe Siècle*' PUF, Paris