An Oligopoly Model in a Leontief Framework

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The price dual in the Leontief input-output system has played the Cinderella role in practical applications of the model. This results largely from the necessity of using the dollar as the homogenous unit of aggregation for inputs and outputs of sectors containing quite diverse products. Prices are assumed fixed and are used, therefore, to convert naturally calibrated input coefficients to cents-worth of input per dollar's worth of output, effectively neutralizing the price dual as an analytical tool. The "dollar's worth" is the homogenous physical unit of all goods as long as prices do not change. Of course, the choice of this unit masks the heterogeneity of the natural physical units of the sectoral outputs, given the wide variety of products aggregated in the sectors of even the largest input-output models. Constant prices are "virtual" prices of a conglomerate of disparate sectoral products. But in empirical and theoretical applications of the output primal the values of outputs and inputs are meaningful units in short-run periods of stable prices and product mixes. They are operationally interpretable.

This analytical device can be used only if the gross output primal model is independent of the price dual. Constant returns to scale, perfect complementarity of inputs, and exogenization of the bill of final goods achieves this prerequisite. Gross output vectors cannot affect prices and the "dollar's-worth" metric of the output primal is defensible.

When we turn to effective usage of the price model, however, we must employ natural units of sectoral outputs, and aggregation effects become daunting. In empirical and theoretical work we must estimate the "virtual" output numbers from the empirically derived values of gross products, bills of goods and production coefficients. We cannot logically assume the existence of unambiguous natural units in the face of heterogeneous products in the sectors.

At a deeper level, however, we may identify another deficiency in the potential usage of the price dual to cope with the determination of market prices. Even were each sector to possess reasonably homogeneous outputs the oligopolistic interactions among firms within each sector is eliminated by the effective assumption of short-run perfect competition. Leontief, like Walras before him, chose the industry as the unit of analysis, rather than the firm, or more realistically, the dominant firms. Perfect competition was an aggregative tool in the production segment of their models. Of course, given the analytical ambition of their models and the necessity to focus on intersectoral interdependence, the simplification was a necessity, and their choices cannot be faulted. But nonetheless it does effectively eliminate the oligopoly-inspired interplay between prices, outputs and profits within the sectors, with concomitant deficiencies in the output solutions of the model as well as prices.

1. The Rivalrous Consonance Approach to Oligopolistic Decision Making

This paper will use the input-output framework to illustrate in a simplified way one form of oligopolistic decision-making that I have termed rivalrous consonance in previous publications.[1] In brief, this framework applies to mature oligopolies that have formed relatively stable communities whose relations are a mixture of the competitive and the cooperative. They have developed a power structure reflecting patterns of dominance and deference, leadership and followership, self-interest and group-interest. In short, as in all human communities they have formed a group of tacit mores or a rivalrous consonance of interests, incorporating both competitive and cooperative behavior. The proactive and reactive patterns of conduct result in industry decision making as a mixture of the harshly competitive and the tacitly collusive, and my hypothesis is that its structure can be at least partially captured in a set of consonance coefficients, which I will discuss below.

Within the last ten years of so economists have become increasingly interested in such ambivalent relations among oligopolistic rivals. The term “co-opetition” has been used to signify such relations[2] that are formalized in agreements short of outright merger: joint ventures, licensing technologies to rivals, alliances, risk-sharing partnerships in such areas as research and development, outsourcing, and joint sales of rivals’ competing products with one’s own. The pressures of globalization, the drive to concentrate on core competencies, the need to enhance flexibility in production and marketing, and the large amount of funds required in many industries to engage in research are driving forces behind these developments. Antitrust authorities have been permissive –and indeed encouraging—to such arrangements when they perceive them to be advantageous to consumers and to competition; they have frowned on them consistently when such joint ventures include marketing apparatus and agreements.

These arrangements may be viewed as recent extensions of rivalrous consonance, but I have used that term more narrowly to denote the tacit cooperation in price and nonprice competition that tempers the latter, as described above. It is the implications of these more informal but realistically pervasive arrangements that I seek to model within the limitations of the Leontief price dual in this paper. The more direct price implications of such behavior may be illustrated thereby, as well as the impacts of the bill of goods (final demand) on gross output although the independence of the output model will eliminate effects that operate on the demand side.

2. The Rivalrous Consonance Model in the Leontief Price Dual Context

We will make a primary distinction between industries which are oligopolistic in structure and those that may be treated as effectively purely competitive. In the former case, the dominant firms will be identified and each will be treated as a Leontief sector with input-output coefficients. Once prices are determined we will introduce them into demand functions for the output model's bill of final goods and determine the implied gross outputs for the sectors from that model. Of course, there is no feedback from the primal solutions to the price dual, given its independence even after endogenizing the bill of goods. It follows, therefore, that in our treatment of rivalrous consonance we are accepting the Leontief elimination of output as a determinant of price and hence limiting the general equilibrium aspects of the model. Moreover, such acceptance also means that we must sacrifice profit maximization behavior by the rival firms in the oligopoly.

In our presentation of the model we will assume one oligopolistic industry and aggregate all "purely competitive" industries into a single sector. The oligopolistic industry is assumed to have three dominant firms that warrant identification as sectors 1, 2 and 3, with the "all other" sector denoted as 4. Ideally we would like to assume the availability of the Leontief input-output matrix in natural (i.e., physical) units:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}.$$

Of course, the practical problem is that we do not have this matrix, but rather must deal with the value-based matrix of input-output coefficients. Indeed, given the rather heterogeneous product mixes of the typical Leontief sector it would be difficult to interpret A if we had it in other than index terms. In terms of the present sectors, however –other than the “all other goods” sector 4 – we have reasonably homogenous units for products, and so would be able to escape the indexing problem, were the A elements available.

Because we need A for our treatment of oligopoly pricing it will be necessary to estimate it from the value-based coefficients. The customary value basis is the matrix A’:

$$(1) \qquad A' = \begin{bmatrix} a_{11}\frac{p_1}{p_1} & a_{12}\frac{p_1}{p_2} & a_{13}\frac{p_1}{p_3} & a_{14}\frac{p_1}{p_4} \\ a_{21}\frac{p_2}{p_1} & a_{22}\frac{p_2}{p_2} & a_{23}\frac{p_2}{p_3} & a_{24}\frac{p_2}{p_4} \\ a_{31}\frac{p_3}{p_1} & a_{32}\frac{p_3}{p_2} & a_{33}\frac{p_3}{p_3} & a_{34}\frac{p_3}{p_4} \\ a_{41}\frac{p_4}{p_1} & a_{42}\frac{p_4}{p_2} & a_{43}\frac{p_4}{p_3} & a_{44}\frac{p_4}{p_4} \end{bmatrix} = \begin{bmatrix} a^*_{11} & a^*_{12} & a^*_{13} & a^*_{14} \\ a^*_{21} & a^*_{22} & a^*_{23} & a^*_{24} \\ a^*_{31} & a^*_{32} & a^*_{33} & a^*_{34} \\ a^*_{41} & a^*_{42} & a^*_{43} & a^*_{44} \end{bmatrix}$$

The a’ coefficients are derived from value figures in A which meld price and quantities in dissoluble aggregates. The elusive prices that convert the natural units to the homogeneous dollar unit cannot be expected to be equilibrium prices, and in our modeling they will have to be converted to the sectoral solution prices of the model. That is, the a_{ij} will have to be estimated in an iterative process from the a'_{ij} by multiplication by successive $\frac{p_j}{p_i}$ ratios deriving from our iterative algorithm. Hence, the impact of oligopolistic pricing policies will be effected in three paths of causation: by changing the estimated values of the a_{ij}, via the direct impacts of tacit collusion on prices and by determining the quantities of the bill of goods and the gross outputs in physical units. Matrix A* is the result of the estimation process:

$$(2) \quad A^* = \begin{bmatrix} a_{11}^{'} \cdot \frac{p_1}{p_1} & a_{12}^{'} \cdot \frac{p_2}{p_1} & a_{13}^{'} \cdot \frac{p_3}{p_1} & a_{14}^{'} \cdot \frac{p_4}{p_1} \\ a_{21}^{'} \cdot \frac{p_1}{p_2} & a_{22}^{'} \cdot \frac{p_2}{p_2} & a_{23}^{'} \cdot \frac{p_3}{p_1} & a_{24}^{'} \cdot \frac{p_4}{p_2} \\ a_{31}^{'} \cdot \frac{p_1}{p_3} & a_{32}^{'} \cdot \frac{p_2}{p_3} & a_{33}^{'} \cdot \frac{p_3}{p_1} & a_{34}^{'} \cdot \frac{p_4}{p_3} \\ a_{41}^{'} \cdot \frac{p_1}{p_4} & a_{42}^{'} \cdot \frac{p_2}{p_4} & a_{43}^{'} \cdot \frac{p_3}{p_1} & a_{44}^{'} \cdot \frac{p_4}{p_4} \end{bmatrix} = \begin{bmatrix} a_{11}^{*} & a_{12}^{*} & a_{13}^{*} & a_{14}^{*} \\ a_{21}^{*} & a_{22}^{*} & a_{23}^{*} & a_{24}^{*} \\ a_{31}^{*} & a_{32}^{*} & a_{33}^{*} & a_{34}^{*} \\ a_{41}^{*} & a_{42}^{*} & a_{43}^{*} & a_{44}^{*} \end{bmatrix}$$

We now make the Leontief assumptions that wage (W) and capital costs (K) per gross unit of output are constant, and that gross profit margins (M) are a fixed proportion of price. The profit margins are the “normal” profit proportions of price that must be recovered as a component of costs.

$$W = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}, \quad K = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix}, \quad M = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix}. \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then, treating prices P as a column vector, we obtain the straightforward Leontief price dual:

$$(3) \qquad P^T\left[I - A^* - IM\right] = W + K \ ,$$

where I is a 4×4 identity matrix.

To build in the oligopolistic price interdependence we now determine the power structure of the oligopoly by defining a matrix of consonance coefficients, whose significance will be made clear below:

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} .$$

In accordance with the rivalrous consonance approach to oligopoly as outlined in section 1, the power structure within an industry is a complicated web of subtle bilateral relations among incumbents that manifests itself in patterns of deference and leadership, proactive and reactive roles in price and nonprice policy decisions, personal relationships among managements, and tacit or explicit cooperative ventures. In a mature oligopoly these relationships are immanent in a historically established body of cultural mores, which evolve over time as competitive successes or failures dictate, but which in any short run period may be taken as given. Such an industry, like any other community, establishes rules of conduct and role expectations governing competitive and cooperative behavior, with effective sanctions for trespass. In the corporate area, of course, one expects the rivalrous relations to dominate the cooperative: but economists tend to overlook the latter in stressing the former. New entrants into the industry that succeed are absorbed in the culture, taught the mores tacitly or explicitly, and over time assume their position in the power structure. In some industries there develops a real sense of community pride that reinforces the tacit limitations on selfish behavior –whether at the club, on the golf course, or through the market.

It is admittedly difficult—nay, impossible-- to capture all of the subtlety of this ethos in its implications for corporate policy in any scalar encapsulation. What I have done is to attempt to capture a portion of it in the form of the deference that firms display by taking into account in their price setting their impact on the profits of rivals. In this scheme of consonance coefficients, C, each c_{ij} is the proportion of firm j's profit margin which firm i treats as a cost coordinate with its own profit margin. Such phantom costs raise the price that firm i charges and hence reduce its own bill of goods sales and increase those of firm j. If cij is .10 firm i will consider firm j's loss (gain) of $1 in firm j's normal profit ($m_j \cdot p_j$) as the equivalent of a loss (gain) of $.10 in its own profit, and make its price decision as if its expected profits are lowered (raised) by that amount.

The size of c_{ij} will vary in largest part, of course, with firm i's view of the power of firm j to retaliate and thereby to reduce the attractiveness of its policy initiation. But it will also be affected by the mores of the industry as discussed above.

These consonance costs that firms introduce into their pricing decisions are phantom costs: they are not actually coordinate with labor and capital costs although they enter the firms' calculations as such, but they may be suffered as reduced profits if outputs x_i are reduced by effects on final demand. We do not, of course, assert that firms actually calculate the consonance coefficients; rather they are analytical constructs meant to capture the pricing decision implications of industry structure and mores. Note also that C is not a symmetrical matrix: c_{ji} is generally not equal to c_{ij}. We set $c_{ii} \equiv 1$ although it is possible to imagine cases where the weakness of a firm leads to self-abasement that implies discounting its own profits.

We also set boundaries on the values of $c_{ij,}$ such that $0 \leq c_{ij} \leq 1$. When $c_{ij} = 1$ firm i values firm j's losses or profits as equal to its own, an extreme form of deference we should not expect to find in isolation. However, if the mores lead all firms to discount all rivals' profits at this value we should have the case of near-joint-profit maximization.[3] This would be a case of perfect tacit collusion in price setting. If $c_{ij} = 0$ firm i would act in total disregard of its impacts on firm j's outputs and profits, an aggressive act that may well be engendered by its dominant position but would violate any competition tempering tenets in the industry ethos. If all firms set their consonance coefficients to zero we would have the "Cournot myopia" solution (albeit with respect to prices, not quantities), in which all firms ignore their impacts on their rivals' profits and losses. In a social welfare sense this "Cournot price solution" is the most socially desirable pricing state that it is possible to contemplate in oligopoly, where firms are acting at the lower limit of rationality by ignoring the welfare of their rivals. Finally, if $c_{ij} < 0$ firm i is willing to sustain losses to inflict losses upon rival j and we have the makings of a price war. In the brackets set above we avoid such short-run rivalrous actions that are not conformant to long-run behavior in a mature industry.

To introduce the consonance coefficients into the model, system (3) is changed to the following:

$$(4)\quad P^T\begin{bmatrix} 1-a_{11}^*-c_{11}\cdot m_1 & -a_{12}^* & -a_{13}^* & -a_{14}^* \\ -a_{21}^* & 1-a_{22}^*-c_{22}\cdot m_2 & -a_{23}^* & -a_{24}^* \\ -a_{31}^* & -a_{32}^* & 1-a_{33}^*-c_{33}\cdot m_3 & -a_{34}^* \\ -a_{41}^* & -a_{42}^* & -a_{43}^* & 1-a_{44}^*-c_{44}\cdot m_4 \end{bmatrix} = \begin{bmatrix} w_1+k_1+c_{12}\cdot m_2\cdot p_2+c_{13}\cdot m_3\cdot p_3+c_{14}\cdot m_4\cdot p_4 \\ w_2+k_2+c_{21}\cdot m_1\cdot p_1+c_{23}\cdot m_3\cdot p_3+c_{24}\cdot m_4\cdot p_4 \\ w_3+k_3+c_{31}\cdot m_1\cdot p_1+c_{32}\cdot m_2\cdot p_2+c_{34}\cdot m_4\cdot p_4 \\ w_4+k_4+c_{41}\cdot m_1\cdot p_1+c_{42}\cdot m_2\cdot p_2+c_{43}\cdot m_3\cdot p_3 \end{bmatrix}$$

In our example we assume that the oligopolistic firms have no regard for sector 4's fate and set all c_{i4}, i = 1, 2, 3 to 0. In a larger model, however, some of the sectors external to the oligopoly may be suppliers to the oligopolistic firms, and the latter may exhibit concerns for the formers' profits by discounting such profits at positive values and incorporating them as own-costs.

Consider, now, the limitations that the Leontief system exercises on our model. Most importantly, the impacts of rivals' consonance decisions on their own and their competitors' profits play no role in their decision-making. In more sophisticated models we introduce profit maximization constrained by the consonance coefficients into the model. Prices may be raised or lowered by rivals on the basis of the effects on their profits via their total demand functions. In the Leontief model, however, the consonance decisions must raise prices as determined by the price dual, with gross output outcomes that will not reflect back upon their own-price choices. The gross outputs may rise or fall, with consequent increases or decreases in their actual profits (i.e., "normal" profits plus rents occurring through rivalrous consonance). This is, of course, a serious deficiency in the attempt to study general equilibrium results of tacit collusion. Nonetheless, there are valuable insights to be gained from the simplicity of the model.

First, we may study the absolute and relative behavior of oligopolistic prices as the consonance parameters are varied. Most particularly of interest are the ranges of prices determined between a base case when all $c_{ij} \equiv 0$, $j \neq i$, and those resulting from extreme tacit collusion and all $c_{ij} = 1$.

Second, it is interesting to note the behavior of prices in sectors external to the oligopolistic sectors, which are only indirectly affected by tacit collusion via their intermediate goods requirements.

Third, it is a valuable exercise, quite apart from the oligopoly price and output implications, to study the degree to which estimates of the natural unit coefficients underlying our analysis differ as the consonance coefficients change and affect prices of oligopoly products.

In section 3 we present the model formally before performing simulations with it to derive insights that cast light on the questions raised above.

3. The Model

We are given the matrix A' , defined in (1) as the production coefficients dollar value form. From it we derive estimates of the production coefficients in natural units, as depicted in (2) in A*. From it we obtain

$$(5) \qquad S=\begin{bmatrix} 1-a_{11}^{*}-c_{11}\cdot m_1 & -a_{12}^{*} & -a_{13}^{*} & -a_{14}^{*} \\ -a_{21}^{*} & 1-a_{22}^{*}-c_{22}\cdot m_2 & -a_{23}^{*} & -a_{24}^{*} \\ -a_{31}^{*} & -a_{32}^{*} & 1-a_{33}^{*}-c_{33}\cdot m_3 & -a_{34} \\ -a_{41}^{*} & -a_{42}^{*} & -a_{43}^{*} & 1-a_{44}^{*}-c_{44}\cdot m_4 \end{bmatrix},$$

which is simply [I-A*] enhanced along the main diagonal by subtracting c_{ii} ($\equiv$1) times the profit margin of the row and column sectors.

The remaining "costs"—both real and phantom – other than the own profit margins are defined as:

$$(6) \qquad V=\begin{bmatrix} w_1+k_1+c_{12}\cdot m_2 p_2+c_{13}\cdot m_3\cdot p_3+c_{14}\cdot m_4\cdot p_4 \\ w_2+k_2+c_{21}\cdot m_1 p_1+c_{23}\cdot m_3\cdot p_3+c_{24}\cdot m_4\cdot p_4 \\ w_3+k_3+c_{31}\cdot m_1 p_1+c_{32}\cdot m_2\cdot p_2+c_{34}\cdot m_4\cdot p_4 \\ w_4+k_4+c_{41}\cdot m_1 p_1+c_{42}\cdot m_2\cdot p_2+c_{43}\cdot m_3\cdot p_3 \end{bmatrix},$$

The price system is then

$$(7) \qquad P=(S^T)^{-1}\cdot V \quad .$$

Given the price system the bill of goods for the primal problem is then determined:

(8) $$Y = \begin{bmatrix} g_1 - f_{11} \cdot p_1 + f_{12} \cdot p_2 + f_{13} \cdot p_3 + f_{14} \cdot p_4 \\ g_2 + f_{21} \cdot p_1 - f_{22} \cdot p_2 + f_{23} \cdot p_3 + f_{24} \cdot p_4 \\ g_3 + f_{31} \cdot p_1 + f_{32} \cdot p_2 - f_{33} \cdot p_3 + f_{34} \cdot p_4 \\ g_4 + f_{41} \cdot p_1 + f_{42} \cdot p_2 + f_{43} \cdot p_3 - f_{44} \cdot p_4 \end{bmatrix} .$$

Finally, gross outputs are then calculated:

(9) $$X = [I - A^*]^{-1} \cdot Y \ .$$

The system is iterated until P and X converge to stationary solutions.

4. Some Illustrative Simulations

Table 1 lists the parameter values for the four cases we will solve as illustrations and to gain insights into the impacts of tacit collusion within the Leontief context. The four cases are: 1) a base case with no rivalrous consonance; 2) moderate rivalrous consonance; 3) high rivalrous consonance; and 4) extreme rivalrous consonance. The only case-specific parameters are the elements of C, the matrix of consonance coefficients. Other parameters are common to all four runs.

1. Sector Profiles

Firm 1 has low basic demand (g_1) and a high own-price coefficient as well as low other-price coefficients in its final demand equation. It is therefore sensitive to situations where firms 2 and 3 do not raise prices much (i.e., low c_{2j} and c_{3j}) and it has a high c_{11}. On the other hand, its primary factor costs are the lowest of the three rivals and its imports of inputs from sector 4 are also the lowest, Its input coefficients for imports from rivals are the highest among them, but their relatively small values do not materially offset its advantages from low sector 4 imports. Finally, as the largest exporter of industry products to sector 4, it benefits more than its rivals from increases in that sector's gross outputs. It is, therefore, the low-cost producer among the rivals, which counteracts to some degree its disadvantages on the demand side.

Table 1

Parameter Values for the Simulations

1. Common Parameter Values

$$A' = \begin{bmatrix} .03 & .03 & .01 & .10 \\ .02 & .02 & .01 & .06 \\ .04 & .03 & .04 & .07 \\ .25 & .30 & .28 & .28 \end{bmatrix}, \quad W = \begin{bmatrix} 50 \\ 60 \\ 65 \\ 47 \end{bmatrix}, \quad K = \begin{bmatrix} 2.7 \\ 4.0 \\ 3.4 \\ 2.8 \end{bmatrix}, \quad M = \begin{bmatrix} .10 \\ .08 \\ .09 \\ .09 \end{bmatrix}$$

$$F = \begin{bmatrix} 10 & 4 & 5 & .5 \\ 3 & 12 & 13 & .3 \\ 7 & 12 & 18 & .4 \\ .05 & .06 & .04 & 10 \end{bmatrix}, \quad G = \begin{bmatrix} 250 \\ 300 \\ 500 \\ 2000 \end{bmatrix}$$

2. Case-Specific Parameter Values

a. Case 1: Zero Rivalrous Consonance

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b. Case 2: Low Rivalrous Consonance

$$C = \begin{bmatrix} 1 & .05 & .10 & 0 \\ .06 & 1 & .15 & 0 \\ .05 & .05 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c. Case 3: High Rivalrous Consonance

$$C = \begin{bmatrix} 1 & .15 & .10 & 0 \\ .25 & 1 & .15 & 0 \\ .30 & 0.20 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d. Case 4: Extreme Rivalrous Consonance

$$C = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Firm 2 is also a low basic demand producer with high final demand sensitivity to rival prices (especially to p_3), somewhat offset by a high own-price coefficient. Rises in p_1 benefit its final demand only slightly, but high values of p_3 (caused by large c_{31} and c_{32}) benefit it greatly. On the cost side its intermediate goods costs are the highest among the rivals and its primary factor costs are intermediate among the rivals. Given the narrow variance of intermediate costs among the rivals we must rank it as an intermediate cost firm on the basis of its primary factor costs.

Firm 3 has high own-price and other-price coefficients and high basic demand in its final demand equation, so that willingness to participate in tacit collusion raises its own price significantly and lowers its final demand, tempered slightly by its high basic demand. At the same time it puts itself at the mercy of its rivals' willingness to reciprocate, or alternatively to benefit from their willingness to defer to it. Its primary factor costs are the highest of the rivals. It is not a great exporter of inputs to sector 4, and therefore does not benefit greatly from output expansion from the latter, although this is true for the other rivals as well.

Sector 4 does not participate in the tacit collusion of the oligopolistic industry and therefore does not actively change prices. It is wholly passive on price account and its gross output is affected by the rival firms' price changes and the imports of its product induced by their gross output changes. As a large aggregate sector its inputs into the three firms are large as is its own absorption of product. Its basic final demand is large and its own-price coefficient large, but imports from the rivals are small. It is a low cost producer on primary factor account, although its intermediate account costs are on the larger side. In short, it is a reactive sector whose primary stimuli are the gross outputs of the rival firms.

2. Case Solutions

Table 2 lists the values of the state variables in each of the four case iterations.

Table 2
State Variable Solutions to Models

(With Percentage Changes From Base Case Values in Parentheses)

Model Description	Sectors	Prices	Gross Output	Final Demand	Actual Profits per Unit	Total Profits	Estimated Natural Unit Production Coefficients (a_{ij})
1. Base Case Zero Rivalrous Consonance	1.	$94.11	762	445	$9.34	$7,117	a_{11}=.03, a_{12}=.038,a_{13}=.013, a_{14}=.132
	2.	$118.52	914	757	$9.29	$8,491	a_{21}=.016, a_{22}=.02, a_{23}=.01, a_{24}=.063
	3.	$120.00	687	471	$10.86	$7,461	a_{31}=.031, a_{32}=.03, a_{33}=.04, a_{34}=.073
	4.	$124.50	1891	772	$11.01	$20,820	a_{41}=.189, a_{42}=.286, a_{43}=.27, a_{44}=.28
2. Low Rivalrous Consonance	1.	$96.94 (3.0)	754 (-1.1)	442 (-.7)	$11.10	$8,369 (17.6)	a_{11}=.03, a_{12}=.038,a_{13}=.013, a_{14}=.128
	2.	$122.64 (3.5)	893 (-2.3)	739 (-2.4)	$11.92	$10,640 (25.3)	a_{21}=.016, a_{22}=.02, a_{23}=.01, a_{24}=.061
	3.	$121.71 (1.4)	727 (5.8)	509 (8.1)	$11.75	$8,542 (14.5)	a_{31}=.032, a_{32}=.03, a_{33}=.04, a_{34}=.072
	4.	$124.50 (0.0)	1919 (1.5)	772 (0.0)	$11.34	$21,760 (4.5)	a_{41}=.195, a_{42}=.296, a_{43}=.274, a_{44}=.28
3. High Rivalrous Consonance	1.	$98.88 (5.1)	786 (3.2)	472 (6.1)	$12.33	$9,691 (36.2)	a_{11}=.03, a_{12}=.038,a_{13}=.013, a_{14}=.126
	2.	$126.32 (6.6)	945 (3.4)	792 (4.6)	$14.37	$13,580 (59.9)	a_{21}=.016, a_{22}=.02, a_{23}=.01, a_{24}=.059
	3.	$128.75 (7.3)	650 (-5.4)	440 (-6.6)	$16.46	$10,700 (43.4)	a_{31}=.031, a_{32}=.029, a_{33}=.04, a_{34}=.068
	4.	$124.50 (0.0)	1951 (3.2)	773 (.1)	$11.08	$21,620 (3.8)	a_{41}=.199, a_{42}=.304, a_{43}=.29, a_{44}=.28
4. Extreme Rivalrous Consonance	1.	$146.29 (55.5)	666 (-12.6)	396 (-11.0)	$43.76	$29,140 (309.4)	a_{11}=.03, a_{12}=.036,a_{13}=.012, a_{14}=.085
	2.	$173.96 (46.8)	1042 (14.0)	900 (18.9)	$43.70	$45,540 (436.3)	a_{21}=.017, a_{22}=.02, a_{23}=.01, a_{24}=.043
	3.	$170.08 (41.8)	810 (17.9)	600 (27.4)	$43.61	$35,320 (373.4)	a_{31}=.034, a_{32}=.031, a_{33}=.04, a_{34}=.051
	4.	$124.50 (0.0)	2391 (26.4)	780 (1.0)	$11.16	$26,680 (28.1)	a_{41}=.294, a_{42}=.419, a_{43}=.383, a_{44}=.28

Case 1: The Base Case. Without any rivalrous consonance this case solution conforms well to the sector profiles. Firm 1 has the lowest price and its bill of goods or final demand sensitivities as well as its low basic demand penalize its sales to final users. Although it is the lowest cost rival, its large sector 4 input per unit reduces its actual profit margin below its "normal" margin. On the other hand, its high ratio of gross output to final demand (1.712) occurs because of its high value of exports to Sector 4.

Firm 2's performance is initially something of a surprise. It is intermediate in its cost structure and its price reflects that, and it benefits output-wise somewhat from firm 3's higher price. But its exports to sector 4 are the least of the rivals', so that its gross output is only 21% above its final demand, and its profits suffer accordingly. The surprising aspect of its solution is its high market share and resulting profit, both of which are the highest of the three rivals. The latter occurs despite the shortfall of its actual profit margin from its "normal" value, the high input coefficient for sector 4's product in the production of firm 2's output is largely the cause of this. This large absolute value of profits nonetheless must be attributed to its large final demand, in turn the result of the high other-demand coefficient for firm 3 and the latter's high price.

Firm 3 has the lowest sales of the industry although relatively high exports to its rivals and to sector 4 raise gross output 46% above its final demand. But its profits are only slightly above firm 1's. Its high own-price final demand coefficient and the low prices of firm 1 are the culprits causing its low final demand, and its high primary factor costs contribute to its disappointing profit performance. This occurred despite the fact that its actual profit margin was \$.06 above its "normal" margin, the only positive difference of the four sectors.

For a sector with high basic final demand sector 4's total final demand is relatively small, because it benefits only slightly from the rivals' prices. However, it does enjoy large enhancements from intermediate good contributions to the oligopolistic industry, so that its ratio of gross output to final demand is 2.45, the highest of the four sectors. Its low primary factor costs are largely offset by the cost of its inputs from the three rivals and from itself, so that although its profit margin is the highest of the sectors it is about $.20 below its normal profit margin, about matching the shortfall of firm 2 for the largest value of the four sectors.

Case 2: Low Rivalrous Consonance. In this case firm 3 is the recipient of the largest deference, with $c_{13} + c_{23} =.25$, whereas firm 1 receives a total consonance deference of .11 and firm 2 of .10 from their rivals. Firm 3, however, grants only a miserly .05 to each of its rivals. The result is that p_3 rises by a small amount, whereas p_1 and p_2 raise their prices most because of their more generous consonance coefficients. The net result is a fall in both final and gross outputs for firms 1 and 2, whereas firm 3 gains on both accounts. On the other hand, firm 3 raises price only 1.4% above the base case, and sees its final demand rise 8.1% and its gross output 5.8%. Its profits rise 14.5% over Case 1 levels, but the greatest benefit accrues to the most deferential rivals. Firm 1's price rise of 3.0 and firm 2's of 3.5% more than offset sales declines and their profits rise fully 17.6% and 25.3% respectively. With zero consonance coefficients in its C row, sector 4's price does not change but its total sales rise from its increased sales to firm 3 offset by losses from the fall in sales of the other two rivals.

Also note that the estimated quantities of sector 4's export coefficients rise by virtue of the rises in prices of the three firms, and that by the same token its import coefficients in the fourth column of A* fall from their base case values (see the definition of A* in (2) above. The result of these occurrences is to boost the sector's actual profit margins over their normal values on its increased sales, and its total profits rise by a modest 4.5% over the base case. This last result is suspect, of course, because had we the actual coefficients in natural units sector 4's import and export coefficients would remain constant, as would the value of export coefficients with unchanged p_4, while the higher prices of its imports on intermediate account would raise their value. Hence, its costs would rise and with constant price in the Leontief model its profit margin would fall, so that sales would have to raise more than in Case 2 to obtain higher profits. Note that the small proportional rises in p_1, p_2, and p_3 resulted in ignorable differences in intra-industry a* coefficients for the rivals compared with base case levels.

In this Leontief framework rises in consonance coefficients benefit all sectors, including those that suffer reduced sales as a consequence. This need not happen in a richer model in which firms maximize profits facing downward sloping total demand functions. However, this universal profit gain frequently happens in such richer models, so the results are not unusual. The tide of tacit collusion can raise all ships through higher profits – even nonparticipating sectors which are constrained from raising prices.

Case 3: High Rivalrous Consonance. In this case the three rivals institute high levels of tacit price collusion that we would expect to see in a mature oligopoly with developed cooperative institutions. Firm 1 in this case becomes the rival other firms defer to while it nonetheless increases the sum of its coefficients to .25. Firm 2 raises its sum, when compared with Case 2, to .40, most of it in deference to firm 1, while firm 3 increases the sum of the c-coefficients to .50, again with most of it favoring firm 1. As would be expected price rises among the rivals are higher increases over the base case than those experienced in Case 2. Firm 3 suffers from its large price increase by a reduction in final demand below its base case level and the induced gross output. Nonetheless, the 7.3% rise in price overbalances the –5.4% fall in final output, and its profits rise 43.4% over base case level.

Firm 1 is led to increase price by 5.1% over base case level, but this is a relatively small increase over Case 2. Its small other-price coefficients in its final demand equation relative to its substantial own-price coefficient is to blame for its failure to increase its final demand and gross output by much, while its price increase is held back by its moderate c-coefficients. Nonetheless its profits double over the Case 2 figure and rise by 36.2% over their base case level, but this is the smallest of both percentage rises among rivals.

Firm 2 continues its record established in the base and low consonance case as the highest profit earner among the rivals. Its final demand is enhanced by firm 3's large price increase, and its gross output rises by 3.4% over the base case level. With the 6.6% price rise over the base case, profit jump almost 60% over the base case, the best performance of the rivals once more.

Finally, sector 4 continues its reactive record, with no increase in price and negligible increase in final demand from the price rises of the rivals. The increases in firms 1 and 2 gross outputs offset the decline in that of firm 3 to permit sector 4's sales to rise 3.2% over base case levels, and profits to rise 3.8% over the base case, far less than the oligopolistic firms. Its actual profit margin also rises slightly over the base case value for reasons discussed in our discussion of Case 2. It remains a reactive beneficiary from rivalrous consonance.

Case 4: Extreme Rivalrous Consonance. This last case carries us into what we have defined as "extreme rivalrous consonance" or near-joint profit maximization, in which each rival counts its rivals' profits on a par with its own. The results are dramatic instances of tacit price collusion.

Each rival's profits rise between 309 and 436% over the base case levels, firm 2 once more leading the pack while even sector 4 profits rise 28% over base case levels. Prices rise between 42 and 56% in the oligopoly sectors, but remain constant in sector 4, even though the estimated natural unit values of its intermediate goods fall significantly below base case levels. Note that all of the oligopolistic firms are harmed a bit by the rise in the estimated amounts of sector 4's exports to them, but in line with our conclusion above actual profit margins are between 28 and 30% above the built-in m-values for all rivals. One of the comforting results of this body of simulations is that the estimates of the a*-coefficients do not materially affect the broad results with respect to prices and profits.

The large rise in p_1, given firm 1's large own-price sensitivity and relatively low other-price sensitivities, lowers its final demand below case 1 levels, and sales to other sectors do not prevent a fall in total sales below that benchmark. But a 13% drop in gross output is well neutralized by a 56% rise in prices to bring about the firm's profit rise.

Firm 2 retains its record as the rival that profits most from the extreme case as the intermediate cost firm with favorable coefficients in its final demand equation. Its final demand expands by 19% above base case level, and its total output by 14%. With a 47% rise in price and its moderate cost structure, its profits rise by 436% over base case levels, continuing a theme that emerged in the base case and was accentuated with all three cases of rivalrous consonance.

Despite its punishing own-price sensitivity in its final demand equation firm 3 shows dramatic increase in final demand from base case levels because of the dramatic price increases of its rivals – especially firm 2. Enhanced by sales to sector 4, total output expands by 18% above the base case value, and with an increase of 42% in its price, profits rise by 373% over base case value.

Even plodding sector 4 benefits from the expanded sales of firms 2 and 3 as well as the larger estimated values of its export coefficients. Total output rises 26% above base case value and profits rise 28% above that level. Its final demand rises by a negligible 1%, and its price is constant, so that all of its good fortune must spring from the two causes noted above.

Summary. Not surprisingly, all firms (including the non-oligopolistic sector) benefit from tacit price collusion of the rivalrous consonance variety, with their welfares rising monotonically with increasing collusion. It must be said that sector 4's welfare is innocently enhanced by the induced exports that rises in overall production of oligopolistic brands bring about. In the four cases, taken respectively, total outputs summed over the three rival brands are 2,363, 2,374, 2,381, and 2,518, a near stationary performance. But the welfare consequences of tacit price collusion are gleaned from the lack-luster performance over the four cases of final demand: 1,673, 1,690, 1,704, and 1,896. These must be compared against price rises in the cases to gauge the decline in welfare of final users.

3. The Ease of Comparative Statics Calculations

One of the advantage of the simple forms of interdependence in the Leontief model, even after endogenizing final demand, is the ease with which parametric ranging and sensitivity analysis can be conducted with the model is linear in all equations, our results are global rather than flowing from linearizations of nonlinear functions. To illustrate, we have arbitrarily chosen the model with high rivalrous consonance and firm 1's behavior within it to illustrate the points. The results are tabulated in Table 3. The default parameter results are those for the original solution to the

Table 3

Comparative Statics Operations on Parameters of Firm 1
High Rivalrous Consonance Case

Model Description	**Sectors**	**Default Parameters**	**C_{12}=.16 (.15)**	**C_{13}=.11 (.10)**	**F_{11}=11 (10)**	**F_{12}=5 (4)**	**F_{13}=6 (5)**	**F_{14}=.6 (.5)**
	1.	$98.88	$99.06	$99.09	No	No	No	No
Prices	2.	$126.32	$126.32 (+)	$126.33	Change	Change	Change	Change
	3.	$128.75	$128.76	$128.76	From	From	From	From
	4.	$124.50	$124.50 (+)	$124.50	Default	Default	Default	Default
Final Demand	1.	472	471	470	374	599	601	485
	2.	792	793	793	792	792	792	792
	3.	440	441	442	440	440	440	440
	4.	773	773 (+)	773 (+)	772	773	773	773
Gross Output	1.	786	784	784	680	922	925	800
	2.	945	946	946	941	950	950	946
	3.	650	652	652	644	658	658	651
	4.	1951	1952	1952	1918	1994	1995	1955

model with the parameters listed in Table 1. We have boosted those parameters singly by one unit or less from their default values (listed in parentheses in the column headings) and derived prices, final demands and gross outputs in each of the new solutions for comparison with the default parameter case. Figures have been rounded to two decimal places for prices and to the last digit for production values. Where a change from the default value was too small to register after rounding we have placed a sign to indicate the direction of the variable's movement above that value.

Increasing the consonance coefficients for firm 1 lifted the prices of the rivals as well as p_1. It lowered the final demand of firm 1, by virtue of the rise in p_1, and raised the final demands of rivals 2 and 3 by small amounts in reflection of the small rises in their prices offsetting the positive effects of the change in p_1. Gross output for firm 1 fell in both cases and rose for rivals. Sector 4 revealed no change in price or final demand at the two decimal level in both cases, but did gain one unit in gross output from the rise in rivals' gross outputs.

Changes in the final demand coefficients yielded some rather surprising sensitivities in final demand and gross outputs. Of course, given the lack of feedback from such quantities to prices, the latter retained their default parameter values. The unit change in own-price sensitivity for firm 1 resulted in almost a 20% fall in final demand, with, of course, no changes in final demands by its rivals. Firm 1's gross output fell about 13% from default level, and by its reduction in intermediate demand caused noticeable reductions in the gross outputs of its rivals. Most severely affected, however, was sector 4's gross output, which fell 12%. Such impacts emphasize the importance in such modeling to gaining accurate estimates of own-price coefficients in final demand equations.

Upward unit changes in f_{12} and f_{13} also resulted in dramatic increases in both final demand and gross output for firm 1. There were no changes in final demand for firms 2 and 3, but large increases in these amounts for the two rivals, and small increases in gross output benefited these two rivals from intermediate good absorption increases by firm 1.

Finally, the rise in f_{14} led to a modest but non-ignorable increase in final demand for firm 1. Final outputs for the remaining three firms rose by 4 or 5 units. The impact on sector 4 was negligible therefore.

6. In Conclusion

Economists and policy makers are becoming increasingly sensitive to the strength of cooperative urges among competing industrial units, and their real and potential tempering of competition. Increasingly, formal institutions are arising in the fields of research and development, purchasing, product design and manufacture and even marketing to facilitate such cooperation without the extremity of merger or acquisition. Less formally, the rationality among oligopolistic rivals of tacit collusion in pricing and forms of nonprice competition like new brand introduction is being accepted increasingly by economists whose professional bias is to emphasize competition.

To illustrate one manner of incorporating parameters that permit mixtures of competition and cooperation in oligopolistic pricing, this article discusses the concept of rivalrous consonance and demonstrates some of its implications for the firms involved and for external industries. The Leontief price dual and its independent output primary model permits us to present such modeling in its simplest form, abstracting from profit maximization with defined total demand functions and rising marginal costs. We have discussed the limitations of the model above, but nonetheless have illustrated through simulations some of the impacts it has on prices and outputs throughout the economy. Our results must be presented with numerous caveats because of the restricted interdependence of the Leontief system, but at least the fundamental ideas have been illustrated.

Endnotes

[1] See, most recently *Price and NonPrice Rivalry in Oligopoly: The Integrated Battleground*, London: Macmillan, 1999, and the bibliography specified there.

[22] Adam M. Brandenburger and Barry J. Nalebuff, (1996), *Co-opetition,* New York: Doubleday. The term was coined by Ray Noorda, founder of Novell.

[3]Perfect joint-profit maximization would involve each firm' inclusion of its rivals' profits in its objective function and differentiation of those profits with respect to its own price.