# Input-Output Analysis and Econometrics: a Discussion of Some Key Issues with an Example from the Theory of Production

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## Abstract

In this paper we compare classical econometrics, calibration and Bayesian inference in the context of the empirical analysis of factor demands. Our application is based on a popular flexible functional form for the firm's cost function, namely the Diewert's Generalized Leontief, and uses the well known Berndt-Wood's 1947-1971 KLEM data on the U.S. manufacturing sector. We illustrate how the Gibbs sampling methodology can be easily used to calibrate parameter values and elasticities on the basis of previous knowledge from alternative studies on the same data but with different functional forms. We rely on a system of mixed uninformative diffuse priors for some parameters and informative tight priors for others. Within the Gibbs sampler, we employ rejection sampling to incorporate parameter restrictions, which are suggested by economic theory but in general rejected by economic data. Our results show that values of those parameters that relate to uninformative priors are almost equal to the standard SUR estimates, whereas differences come out for those parameters to which we have assigned informative priors. Moreover, discrepancies can be appreciated in some crucial parameter estimates obtained with or without rejection sampling.

## 1. Introduction

Once input-output (I-O) analysis and macroeconometrics used to be very close to each other. In the subject index of the book by Klein *et al*. (1991), the item “I-O analysis” has 44 entries. In the same book Wassilis Leontief is quoted 5 times, whereas 16 citations are devoted to John Maynard Keynes. This evidence can be interpreted as a rough estimate of the relative importance of the two authors from the viewpoint of the founder of modern applied macroeconometrics. For some time the two traditions have cohabited: I-O analysis was dealing with long-run structural equilibria, while macroeconometrics was concentrating on business cycles and forecasting. Later on, the two styles of quantitative macro analysis have drifted apart and have communicated rarely to each other. On the one hand, Computable General Equilibrium (CGE) models, the modern continuators of the I-O tradition, have grown up both in complexity and realism, and have incorporated demand and adjustment factors. On the other hand, Vector Autoregressive (VAR) and Vector Equilibrium Correction (VECM) models have solved the dichotomy between short run and long run by means of a sophisticated analysis of the time series properties of the data. Despite their widespread success at an applied level, both traditions have been somewhat obscured, at the theoretical level, by the emergence of the new classical economics and the general stochastic equilibrium models (e.g. real business cycle, or RBC, models). This new class of models has introduced a novel fashion of estimating the parameters of interest, the so-called calibration. To those of us who are older this new methodology does not look like a terrific innovation. After all, I-O people have always calibrated the relevant parameters of their models, whereas macroeconometric people had done this at the very beginning of the discipline (remember Tinbergen’s work in the Thirties) and had always been prone to calibrate whatever could not be estimated. In any case, we believe that important indications towards a better understanding of the relative merits of I-O-CGE models and VAR-VECM econometrics can be found mostly in the debate about calibration raised by the RBC modelling approach. To illustrate this point, in this paper we develop an example from the modern theory of production. The model discussed in our example can be interpreted as one of the building-blocks of a more general CGE model. Moreover, it has the advantage of being manageable enough as to give a simple illustration of the main points involved in the comparison of the two methodologies. In this paper we suggest to resort to Bayesian inference and calibrate the parameters of the model in a systematic way according to Bayes rule, taking advantage of recent developments of the Gibbs sampling approach and related methodologies.

The paper is organized as follows. In Section 2 some problems related to the empirical analysis of factor demands are briefly discussed, which typically arise within the standard econometric approach, and some alternative methodologies are suggested. In Section 3, a very popular flexible functional form, namely the Generalized Leontief cost function, and the corresponding system of factor demand equations are presented. Moreover, the methodology followed to calibrate parameter values and elasticities based on different a priori is illustrated. In Section 4 and 5 the main empirical results are reported and commented on. Section 6 provides some concluding comments.

## 2. Aggregate factor demand analysis: classical econometrics and alternative approaches

Empirical factor demand analysis typically involves making a choice from among several competing functional forms. Each of the commonly used factor demand systems, such as Translog, Generalized Leontief, Symmetric Generalized McFadden, Symmetric Generalized Barnett, Generalized Box-Cox (see, for details, Diewert and Wales, 1987; Berndt and Khaled, 1979) and so forth, can provide a valid and useful empirical description of the underlying production structure of the multi-input neoclassical firm.

A common feature of flexible functional forms is that, as they are in general separate and there is no a priori theory suggesting that the specification of one system of derived factor demands should be preferred over another, it is not obvious how to choose among them.

A possible solution to the important task of model selection lies within the classical hypothesis testing framework and is given by formal non-nested testing procedures. Paired and joint univariate and multivariate non-nested tests of a null model against both single and multiple alternatives have been discussed and criticized at length in the literature (see, among others, Davidson and MacKinnon, 1982). One of the major drawbacks of these procedures is that the outcome of a non-nested test can be highly influenced by the type of misspecification affecting the competing models. When the alternative models are systems of factor demand equations, common forms of misspecification include violations of classical assumptions on the error terms (e.g. absence of autocorrelation) and of regularity conditions on the underlying cost function (e.g. symmetry of the cross-price effects, monotonicity and concavity).

Alternative approaches are related to calibration (see, among others, Kim and Pagan, 1995; Hansen and Heckman, 1996) and Bayesian inference (see, e.g., Box and Tiao, 1992; for an application to a flexible cost function, see Koop *et al.*, 1994). In many studies based on calibration, model parameter values are simply taken from previous empirical work. Conversely, in the

Bayesian approach the strategy is to calculate posterior moments of the parameters of a given factor demand system, taking into account and incorporating into the prior distribution some relevant information obtained from alternative functional forms (e.g. different sets of estimated input price elasticities, alternative input substitution or complementarity relations) or concerning some regularity conditions which are known to be generally violated by the data. In other words, what does calibration informally, Bayesian inference does it formally.

This paper concentrates on the Bayesian approach. To illustrate the relative merits and problems associated with incorporating relevant information into the prior distribution, we use the Generalized Leontief flexible functional form as discussed in Diewert and Wales (1987) and a variety of priors to compute posterior input price elasticities. The data set used in our empirical application is very popular in the applied production literature (see, just to quote a few studies which use the same data set, Berndt and Khaled, 1979; Terrell, 1996; Thomsen, 2000), and is based on annual data on aggregate output of U.S. manufacturing industries, and prices and quantities for a capital-labour-energy-materials (KLEM) technology over the period 1947-1971 (see Berndt and Wood, 1975).

## 3. Handling the Generalized Leontief cost function in the context of Gibbs sampling

In this paper attention is focused on one of the most widely used flexible functional forms in the context of cost function estimation, namely the Generalized Leontief. The theoretical framework is well known and can be summarized as follows. Let the firm's technology be represented by the production function:

$$Y = F\left[X_1, X_2, ..., X_n\right] \tag{3.1}$$

where $X=[X_1, X_2, \ldots, X_n]'$ is the vector of inputs and Y is the maximal output that can be produced using this input vector in any period. Given a positive vector of input prices, $P=[P_1, P_2, \ldots, P_n]'$, for any period the cost function dual to equation (3.1) can be defined as:

$$C(Y, P, t) = \min_X \left[P'X : F(\cdot) \geq Y, X \geq 0_n\right]. \tag{3.2}$$

In (3.2), C(·) satisfies various regularity conditions, depending on the assumptions placed on equation (3.1). Following Diewert and Wales (1987, p. 45), we are primarily concerned with linear homogeneity and concavity of the cost function in input prices. A necessary and sufficient condition for a twice continuously differentiable cost function to be concave in prices P is negative semi-definiteness of the matrix of second-order partial derivatives of the cost function with respect to factor prices. Moreover, a matrix is negative semi-definite if all its odd-numbered principal minors are non-positive and all its even-numbered principal minors are non-negative (see Morey, 1986). One way to check whether the estimated cost function satisfies the theoretical concavity property is to calculate all the principal minors and leading principal minors and to evaluate them over the sample period.

When a functional form for the cost function has been specified to satisfy the regularity conditions, the system of conditional factor demands can be derived by applying Shephard's lemma:

$$X_i = \frac{\P C(\cdot)}{\P P_i} \quad \text{(i=1,...,n).} \tag{3.3}$$

The traditional Generalized Leontief cost function is a functional form in the square roots of input prices. In this paper, we consider the following version (see Diewert and Wales, 1987, p. 49):

$$C(\cdot) = Y\left(\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\sqrt{P_i P_j}\right) + \sum_{i=1}^{n} b_i P_i + \sum_{i=1}^{n} b_{it} P_i t Y \tag{3.4}$$

$$+ b_t\left(\sum_{i=1}^{n} \mathbf{a}_i P_i\right)t + b_{yy}\left(\sum_{i=1}^{n} \mathbf{b}_i P_i\right)Y^2 + b_{tt}\left(\sum_{i=1}^{n} \mathbf{g}_i P_i\right)t^2 Y$$

Setting $b_t = b_{yy} = b_{tt} = 1$, the system of factor demands is derived in the usual way via Shephard's lemma (3.3), namely:

$$\frac{X_i}{Y} = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} P_i^{-1/2} P_j^{1/2} + \frac{b_i}{Y} + b_{it} t + \mathbf{a}_i \frac{t}{Y} + \mathbf{b}_i Y + \mathbf{g}_i t^2 \quad \text{(i=1,...,n).} \tag{3.5}$$

Following Diewert and Wales (1987, pp.49-50), the Generalized Leontief cost function (3.4) is linearly homogeneous in Y if the following restrictions are satisfied:

$$b_i = 0; b_t = 0; b_{yy} = 0\,. \tag{3.6}$$

Moreover, (3.4) does not depend on time if the following restrictions are true:

$$b_{it} = 0; b_t = 0; b_{tt} = 0 \,. \tag{3.7}$$

Finally, non-negative restrictions on the parameters $b_{ij}$ in (3.4), for i≠j, are sufficient for (3.4) to be globally concave, even if they rule out complementarity between all pairs of inputs.

The system of Generalized Leontief factor demands (3.5) has been estimated with SUR using Berndt and Wood's (1975) classical annual data set for the U.S. manufacturing sector over the period 1947-1971. It is assumed that U.S. manufacturing can be described by a regular aggregate production function relating the flows of gross output Y to the services of four inputs, namely capital (K), labour (L), energy (E) and materials (M). Corresponding to such a production function, there exists a dual cost function summarizing all the characteristics of the representative firm's technology. When output quantity and input prices are exogenous, the dual cost function can be written as $C = C(Y, P_k, P_l, P_e, P_m, t)$, where $C(\cdot)$ represents total input costs, $P_i$, i=K,L,E,M, are the factor prices, and $t$ is an index of technical progress. For purposes of empirical implementation, the existence of random errors in the cost minimizing behaviour of the firm is such that each equation in each demand system has an additive disturbance term which reflects the firm's errors in deciding the optimal level of inputs. In equations (3.5), the dependent variables are input levels divided by output, as this makes the assumption of homoskedasticity of the disturbances more plausible.

A SUR model can be witten as (see Bauwens *et al.*,1999):

$$y = Z\boldsymbol{b} + \boldsymbol{e} \,,$$

where $y' = (y_1', y_2', ..., y_n')$, $\boldsymbol{b}' = (\boldsymbol{b}_1', \boldsymbol{b}_2', ..., \boldsymbol{b}_n')$, $\boldsymbol{e}' = (\boldsymbol{e}_1', \boldsymbol{e}_2', ..., \boldsymbol{e}_n')$ and $Z = diag(Z_1, Z_2, ..., Z_n)$. Each element of $y$ and $\boldsymbol{e}$ is a Tx1 vector, each element of $\boldsymbol{b}$ is a kx1 vector, whereas each element of $Z$ is a Txk matrix. Given the Generalized Leontief system of factor demands (3.5) and the sample of data used for our empirical application, n = 4, k = 9, and T = 25. The total number of $\boldsymbol{b}$ parameters in the model is then K = kxn = 36.

The distribution of the Tnx1 vector $\boldsymbol{e}$ is assumed to be $N_{Tn}(0, \Sigma \otimes I_T)$. Indicating with $IW_n$ a n-dimensional Inverted Wishart distribution (see, e.g., Press, 1982), we assume that the prior distribution for $S$ is $IW_n\left(n + n, R_1^{-1}\right)$, where $n$+$n$ are the degrees of freedom and $R_1$ is the scale matrix. Moreover, we assume that the prior distribution for $b$ is $N_K\left(m, R_0\right)$ (see Tiao and Zellner, 1964). Although posterior marginal densities for the SUR model cannot be derived analytically, full conditional densities of $b$ and $S$ are available and can be used to define a Gibbs-sampling technique to generate samples from the joint posterior distribution of $(b, \Sigma)$. It is possible to show that the posterior distribution of $\Sigma \mid b$ is $IW_n\left(n + n, R_n^{-1}\right)$, with $R_n^{-1} = R_1^{-1} + \Omega$, $\Omega = e'e$. In addition, the posterior distribution of $b \mid \Sigma$ is $N_K\left(\hat{b}, Q\right)$, where $Q = \left[R_0^{-1} + Z'\left(\Sigma^{-1} \otimes I_T\right)Z\right]^{-1}$ and $\hat{b} = Q\left[R_0^{-1}m + Z'\left(\Sigma^{-1} \otimes I_T\right)y\right]$. In the empirical exercise we set $v$+$n$ = 16, $R_1$ = $I_4$x$10^6$ and $R_0 = I_{36} \times \begin{cases} 0.05 \\ 0.0001 \end{cases}$, depending on whether the prior is uninformative or informative (see Section 4 for details).

The Gibbs sampling approach and its applications to SUR models are discussed in a number of papers (see, among others, Albert and Chib, 1993; Casella and George, 1992; Chib, 1993; Chib and Greenberg, 1994, 1995; Percy, 1992; Kim and Nelson, 1999). Efficient sampling techniques to generate random numbers from Multivariate Normal and Inverted Wishart distributions can be found in Rubinstein (1981) and Bauwens *et al.* (1999). Basically, given the posterior distributions for $\Sigma \mid b$ and $b \mid \Sigma$, the Gibbs sampling algorithm is started with arbitrary starting values for $S$, say $S^{(0)}$. Then iterations are for $r$ = 1, 2, …., L+M, according to the following scheme: i) conditional on $S$ generated at replication $r$-1, $b$ is generated from the conditional posterior distribution of $b \mid \Sigma$ at replication $r$; ii) conditional on $b$ generated at replication $r$, $S$ is generated from the conditional distribution of $\Sigma \mid b$ at replication $r$; iii) set $r$ = $r$-1 and go to i). A well known result is that the joint and marginal distributions of generated $\left\{b^{(1)}, b^{(2)}, ..., b^{(r)}, ..., b^{(L+M)}\right\}$ and $\left\{\Sigma^{(1)}, \Sigma^{(2)}, ..., \Sigma^{(r)}, ..., \Sigma^{(L+M)}\right\}$ converge at an exponential rate to the joint and marginal distributions of $b$ and $S$ as $r \rightarrow \infty$, provided L, the first generated parameter values, and M are large enough (see Geman and Geman, 1984; Kim and Nelson, 1999, pp. 179-180). In our empirical application, L and M have been set equal to 300 and 3000, respectively. In generating $b$ from $b \mid \Sigma$ it is also possible to use “rejection sampling” in order to incorporate sets of restrictions which are required by specific informative priors (see Terrell, 1996).

## 4. Results of the Bayesian estimation via the Gibbs sampler

Tables 1-3 report a variety of estimates of input price elaticities obtained in previous well know studies which use alternative flexible functional forms on the same data set employed in our empirical application.

[INSERT TABLES 1-3 ABOUT HERE]

Table 4 presents our SUR estimates of the same elasticities calculated on the basis of the Generalized Leontief cost function. Our results replicate those of Diewert and Wales (1987). We regard these elasticities values as the basic information set upon which the subjective belief of the investigator is naturally formed.

[INSERT TABLE 4 ABOUT HERE]

The tables reveal some regularities. All studies suggest the existence of a significant substitution relationship between capital and labour. However, not all inputs are substitutes, in fact the main result of this empirical literature is the complementarity between capital and energy (see, for further empirical evidence on this point, also Chung, 1994). The other factors of production are generally substitutes, but many price elasticities are quite small, implying that in the period of observation a relevant number of inputs are used mostly in fixed proportions.

These stylized facts form the basis for the specification of our Bayesian priors. Accordingly, we have devised to use a system of mixed uninformative diffuse priors for some parameters and informative tight priors for others. We have assumed uninformative priors for own price elasticities, given that, while they are significant and with the correct sign, in most studies they exhibit wide variability across different functional forms. Also, we have used uninformative priors for constant terms, scale and trend parameters, since these are obviously better determined by the likelihood. All uninformative priors have been set to zero. Instead, we have used informative and tight priors for the most relevant economic parameters, namely those pertaining to cross price elasticities. In other words, we have chosen as prior for the mean of the distribution of each relevant parameter a non-zero value approximating the means of the values reported for year 1971 (the end point of the sample) by the studies quoted above. Then, in order to give tightness to this prior, we have selected an a priori diagonal variance-covariance matrix whose elements are in the ratio of 0.05 to 0.0001,

or, alternatively, given a scale matrix of 0.01, informative priors have a variance that is 50 times less than that of uninformative priors. Of course, this has involved an informal searching procedure to individualize appropriate values for this ratio.
Results for both parameters and implied elasticities are reported in Tables 5a and 5b.

[INSERT TABLES 5a-5b ABOUT HERE]

The same results are also displayed in the following graphs.

[INSERT GRAPHS 1a-1b ABOUT HERE]

It can seen by inspection of Graphs 1a-1b that, as expected, values of those parameters that relate to uninformative priors are almost equal to the SUR estimates, whereas differences come out for those parameters to which we have assigned informative priors. A practical implication of this result for the investigator is that economically relevant parameters are more suitable to be employed in simulation experiments, given the fact that they reflect not only sample information but also previous economic knowledge. We stress that this approach can provide the investigator with a systematic but not arbitrary way of calibrating parameters which are crucial for policy evaluations.

## 5. An extension: revised estimation via rejection sampling

It is also evident by inspection of Tables 5a-5b that symmetry restrictions are not satisfied by our estimates. This may constitute a problem if one thinks that it is crucial to satisfy parameter restrictions which are suggested by economic theory. On the other hand, since a flexible functional form is only a second-order approximation to an unknown cost function, it is not possible to rule out misspecification problems, which typically prevent the model from the exact fulfilment of theoretical restrictions. Actually, in most factor demand studies symmetry restriction are seldom satisfied by the unrestricted estimates, but they are imposed anyway. On the contrary, we have decided to run the Gibbs sampler on the unrestricted demand system. In fact, even in presence of symmetric priors, we have not been able to find exactly matching cross parameters. However, we can refine our estimates in order to bring them closer to theory by using a technique known as “rejection sampling”. Rejection sampling can be readily implemented within the Gibbs sampling framework, since it requires the sampler to discard those iterations whose generated parameters are too far apart to be considered as respecting the restrictions suggested by economic theory. This approach obviously involves some judgment on the difference between generated parameters. On this respect, we have decided to use a fixed value for the absolute distance of the estimated parameters, setting it at 0.1.

Results are reported in Tables 6a-6b, whereas Graph 2 reports the whole set of price elasticities estimated with and without rejection sampling.

[INSERT TABLES 6a-6b and GRAPH 2 ABOUT HERE]

Values are, in same cases (see, e.g., the price elasticities between capital and energy and capital and labour, respectively), different from those obtained without rejection sampling. In addition, these values are not very far from the fixed criterion, and it can be appropriate to use them in applications involving simulation of the estimated model.

## 6. Conclusions

In this paper we have pursued two major objectives. The first is methodological, and consists in appreciating the practical possibilites of the Gibbs sampler applied to a problem of Bayesian inference. The context, namely factor demand analysis, is very common in classical econometrics, but, given the rather rich parametrization of the involved models as well as the related computational problems, it has not been considered in Bayesian econometrics until very recently. On this respect, we can report a remarkable stability of the algorithm and no significant convergence problems.

The second, more substantial, goal is to evaluate if the Gibbs sampling technique can be proposed to calibrationists and I-O specialists as a valid tool to estimate models which be employed in policy exercises or in structural interpretations that are, at the same time, not entirely based on (always limited) sample evidence and not entirely based on a priori or conventional belief. We have shown that Bayesian inference can offer a natural way to combine a priori information with sample evidence in models of factor demands, and, given recent computational advances, may work very well also in practice.

## Tables

**Table 1.** Estimated input price elasticities using alternative flexible functional forms, Berndt-Wood data set, year 1947

| Elasticities | SGB | SGM | TL1 | TL2 | TL3 |
|---|---|---|---|---|---|
| KK | -0.10 | -0.18 | -0.34 | -0.96 | -0.49 |
| KL | 0.14 (s) | 0.46 (s) | 0.48 (s) | 0.29 (s) | 0.26 (s) |
| KE | -0.03 (c) | -0.13 (c) | -0.09 (c) | 0.03 (s) | -0.14 (c) |
| KM | 0.00 (s) | -0.14 (c) | -0.05 (c) | 0.64 (s) | 0.37 (s) |
| LK | 0.03 (s) | 0.10 (s) | 0.11 (s) | 0.06 (s) | 0.06 (s) |
| LL | -0.18 | -0.26 | -0.20 | -0.77 | -0.45 |
| LE | 0.06 (s) | 0.11 (s) | 0.08 (s) | 0.06 (s) | 0.03 (s) |
| LM | 0.09 (s) | 0.05 (s) | 0.01 (s) | 0.65 (s) | 0.37 (s) |
| EK | -0.04 (c) | -0.16 (c) | -0.12 (c) | 0.03 (s) | -0.17 (c) |
| EL | 0.32 (s) | 0.66 (s) | 0.44 (s) | 0.31 (s) | 0.16 (s) |
| EE | -0.52 | -0.77 | -0.62 | -0.98 | -0.47 |
| EM | 0.24 (s) | 0.27 (s) | 0.30 (s) | 0.64 (s) | 0.49 (s) |
| MK | 0.001 (s) | -0.01 (c) | -0.01 (c) | 0.05 (s) | 0.03 (s) |
| ML | 0.03 (s) | 0.02 (s) | 0.01 (s) | 0.26 (s) | 0.15 (s) |
| ME | 0.02 (s) | 0.02 (s) | 0.02 (s) | 0.05 (s) | 0.03 (s) |
| MM | -0.05 | -0.02 | -0.02 | -0.35 | -0.21 |

*Notes*: 1. ij (i,j=K,L,E,M) means the elasticity of demand for input i with respect to the price of input j. 2. c = inputs i and j are complements; s = inputs i and j are substitutes. 3. SGB = Symmetric Generalized Barnett cost function; SGM = Symmetric Generalized McFadden cost function; TL1 = Translog cost function; TL2 = Translog cost function constrained to be concave for non-negative share values. See Diewert and Wales (1987) for details. The values of elasticities in the first four columns of the table are taken from Table II reported in Diewert and Wales (1987), pag. 61. 4. TL3 =Translog cost function with constant returns to scale and no technical progress. See Berndt and Wood (1975) for details. The values of elasticities in the last column of the table are taken from Table 5 in Berndt and Wood (1975), pag. 265.

**Table 2.** Estimated input price elasticities using alternative flexible functional forms, Berndt-Wood data set, year 1959

| Elasticities | GBC1 | GBC2 | GBC3 | GBC4 | GBC5 |
|---|---|---|---|---|---|
| KK | -0.35 | -0.48 | -0.37 | -0.51 | -0.55 |
| KL | 0.57 (s) | 0.24 (s) | 0.21 (s) | 0.24 (s) | 0.59 (s) |
| KE | -0.11 (c) | -0.12 (c) | -0.10 (c) | -0.13 (c) | -0.06 (c) |
| KM | -0.11 (c) | 0.36 (s) | 0.25 (s) | 0.40 (s) | 0.02 (s) |
| LK | -0.12 (c) | 0.05 (s) | 0.04 (s) | 0.05 (s) | 0.12 (s) |
| LL | -0.17 | -0.46 | -0.44 | -0.46 | -0.37 |
| LE | 0.11 (s) | 0.02 (s) | 0.02 (s) | 0.02 (s) | 0.09 (s) |
| LM | -0.06 (c) | 0.39 (s) | 0.37 (s) | 0.39 (s) | 0.16 (s) |
| EK | -0.14 (c) | -0.15 (c) | -0.12 (c) | -0.17 (c) | -0.08 (c) |
| EL | 0.66 (s) | 0.13 (s) | 0.16 (s) | 0.12 (s) | 0.55 (s) |
| EE | -0.71 | -0.59 | -0.45 | -0.61 | -0.55 |
| EM | 0.20 (s) | 0.62 (s) | 0.42 (s) | 0.66 (s) | 0.08 (s) |
| MK | -0.1 (c) | 0.03 (s) | 0.02 (s) | 0.04 (s) | 0.002 (s) |
| ML | -0.03 (c) | 0.17 (s) | 0.16 (s) | 0.17 (s) | 0.07 (s) |
| ME | 0.01 (s) | 0.04 (s) | 0.03 (s) | 0.05 (s) | 0.006 (s) |
| MM | 0.02 | -0.25 | -0.22 | -0.25 | -0.08 |

*Notes*: 1. ij (i,j=K,L,E,M) means the elasticity of demand for input i with respect to the price of input j. 2. C = inputs i and j are complements; s = inputs i and j are substitutes. 3. c = inputs i and j are complements; s = inputs i and j are substitutes. 4. GBC1 = Non-homothetic generalized Box-Cox functional form with non-neutral technical change; GBC2 = Homogeneous generalized Box-Cox functional form with neutral technical change; GBC3 = Constant return-to-scale generalized Box-Cox functional form with neutral technical change; GBC4 = Homogeneous generalized Box-Cox functional form with no technical change. GBC5 = Constant return-to-scale generalized Box-Cox functional form with no technical change. See Berndt and Khaled (1979) for details. The values of elasticities in the table are taken from Table 5 reported in Berndt and Khaled (1979), pag. 1236.

**Table 3.** Estimated input price elasticities using alternative flexible functional forms, Berndt-Wood data set, year 1971

| Elasticities | SGB | SGM | TL1 | TL2 | TL3 | LRGLT |
|---|---|---|---|---|---|---|
| KK | -0.38 | -0.13 | -0.26 | -0.96 | -0.44 | -0.49 |
| KL | 0.51 (s) | 0.56 (s) | 0.56 (s) | 0.33 (s) | 0.30 (s) | 0.50 (s) |
| KE | -0.13 (c) | -0.13 (c) | -0.11 (c) | 0.03 (s) | -0.16 (c) | -0.34 (c) |
| KM | 0.00 (s) | -0.30 (c) | -0.19 (c) | 0.60 (s) | 0.30 (s) | 0.33 (s) |
| LK | 0.08 (s) | 0.09 (s) | 0.09 (s) | 0.06 (s) | 0.05 (s) | 0.11 (s) |
| LL | -0.39 | -0.42 | -0.24 | -0.72 | -0.45 | -0.31 |
| LE | 0.18 (s) | 0.14 (s) | 0.07 (s) | 0.05 (s) | 0.03 (s) | 0.04 (s) |
| LM | 0.13 (s) | 0.18 (s) | 0.07 (s) | 0.61 (s) | 0.37 (s) | 0.17 (s) |
| EK | -0.14 (c) | -0.14 (c) | -0.12 (c) | 0.04 (s) | -0.17 (c) | -0.34 (c) |
| EL | 1.16 (s) | 0.93 (s) | 0.48 (s) | 0.35 (s) | 0.20 (s) | 0.16 (s) |
| EE | -1.07 | -0.74 | -0.63 | -0.98 | -0.49 | -0.41 |
| EM | 0.04 (s) | -0.05 (c) | 0.27 (s) | 0.60 (s) | 0.46 (s) | 0.59 (s) |
| MK | 0.00 (s) | -0.02 (c) | -0.01 (c) | 0.06 (s) | 0.02 (s) | 0.04 (s) |
| ML | 0.06 (s) | 0.09 (s) | 0.04 (s) | 0.30 (s) | 0.18 (s) | 0.09 (s) |
| ME | 0.00 (s) | 0.00 (s) | 0.02 (s) | 0.04 (s) | 0.03 (s) | 0.07 (s) |
| MM | -0.07 (c) | -0.06 (c) | -0.04 (c) | -0.39 (c) | -0.24 (c) | -0.19 (c) |

*Notes*: 1. ij (i,j=K,L,E,M) means the elasticity of demand for input i with respect to the price of input j. 2. c = inputs i and j are complements; s = inputs i and j are substitutes. 3. SGB = Symmetric Generalized Barnett cost function; SGM = Symmetric Generalized McFadden cost function; TL1 = Translog cost function; TL2 = Translog cost function constrained to be concave for non-negative share values. See Diewert and Wales (1987) for details. The values of elasticities in the first four columns of the table are taken from Table II reported in Diewert and Wales (1987), pag. 61. 4. TL3 =Translog cost function with constant returns to scale and no technical progress. See Berndt and Wood (1975) for details. The values of elasticities in the last column of the table are taken from Table 5 in Berndt and Wood (1975), pag. 265. 5. LRGLT = long-run Generalized Leontief cost function, with K and L quasi-fixed and E and M flexible. See Thomsen (2000) for details.

**Table 4.** Estimated input price elasticities using the Generalized Leontief cost function, years 1947, 1959, 1971 (restricted SUR)

| Elasticities | 1947 | 1959 | 1971 |
|---|---|---|---|
| KK | -0.09261 | -0.13541 | -0.23236 |
| KL | 0.46604 | 0.45342 | 0.59117 |
| KE | -0.13204 | -0.11407 | -0.12937 |
| KM | -0.2414 | -0.20394 | -0.22944 |
| LK | 0.09718 | 0.09903 | 0.09536 |
| LL | -0.1787 | -0.18429 | -0.18948 |
| LE | 0.0952 | 0.09921 | 0.10936 |
| LM | -0.01368 | -0.01394 | -0.01525 |
| EK | -0.16081 | -0.15179 | -0.13652 |
| EL | 0.55605 | 0.60444 | 0.71543 |
| EE | -0.62965 | -0.67392 | -0.80489 |
| EM | 0.23441 | 0.22126 | 0.22599 |
| MK | -0.01894 | -0.02007 | -0.01777 |
| ML | -0.00515 | -0.00628 | -0.00732 |
| ME | 0.0151 | 0.01637 | 0.01658 |
| MM | 0.00899 | 0.00999 | 0.0085 |

*Notes*: 1. ij (i,j=K,L,E,M) means the elasticity of demand for input i with respect to the price of input j. 2. c = inputs i and j are complements; s = inputs i and j are substitutes. 2. The numerical values of the elasticities reported in the table replicate exactly those presented in Diewert and Wales (1987), Tables II and IV.

**Table 5a.** Classical vs Bayesian inference: classical parameter estimates with restricted SUR compared with Bayesian parameter estimates of unrestricted SUR using a mixture of uninformative diffuse priors and informative tight priors

| Parameters | SUR (restricted) | SUR (se) | SUR (t stat) | Priors (means) | Priors (sd) | Gibbs (means) | Gibbs (sd) | Gibbs (pseudo t stat) |
|---|---|---|---|---|---|---|---|---|
| $b_{KK}$ | -0.203650 | 0.021400 | -9.518400 | 0.000000 | 0.050000 | -0.205630 | 0.011200 | -18.36445 |
| $b_{KL}$ | 0.043980 | 0.011080 | 3.969720 | 0.037200 | 0.000100 | 0.048510 | 0.009210 | 5.264950 |
| $b_{KE}$ | -0.012460 | 0.002970 | -4.190090 | -0.048160 | 0.000100 | -0.014340 | 0.005600 | -2.563050 |
| $b_{KM}$ | -0.022780 | 0.013790 | -1.651930 | 0.000000 | 0.050000 | -0.024950 | 0.009470 | -2.633690 |
| $b_K$ | 0.134140 | 0.011450 | 11.72012 | 0.000000 | 0.050000 | 0.135730 | 0.006550 | 20.73322 |
| $b_{Kt}$ | -0.013170 | 0.001620 | -8.139820 | 0.000000 | 0.050000 | -0.012950 | 0.000880 | -14.74188 |
| $\alpha_K$ | 0.017040 | 0.001710 | 9.976260 | 0.000000 | 0.050000 | 0.016820 | 0.000920 | 18.38463 |
| $\beta_K$ | 0.103960 | 0.011180 | 9.298580 | 0.000000 | 0.050000 | 0.103780 | 0.005770 | 17.97900 |
| $\gamma_K$ | 0.000130 | 2.00E-05 | 5.449530 | 0.000000 | 0.050000 | 0.000120 | 1.00E-05 | 8.709600 |
| $b_{LL}$ | -0.029630 | 0.124780 | -0.237460 | 0.000000 | 0.050000 | -0.188040 | 0.036900 | -5.096430 |
| $b_{LK}$ | 0.043980 | 0.011080 | 3.969720 | 0.115310 | 0.000100 | 0.048000 | 0.008440 | 5.684800 |
| $b_{LE}$ | 0.043090 | 0.013960 | 3.086090 | 0.000000 | 0.000100 | 0.027910 | 0.009910 | 2.817820 |
| $b_{LM}$ | -0.006190 | 0.079550 | -0.077850 | 0.101540 | 0.000100 | 0.142590 | 0.010620 | 13.42205 |
| $b_L$ | 0.122000 | 0.048450 | 2.518370 | 0.000000 | 0.050000 | 0.134430 | 0.016560 | 8.116870 |
| $b_{Lt}$ | -0.006380 | 0.006040 | -1.055490 | 0.000000 | 0.050000 | -0.006520 | 0.002340 | -2.789230 |
| $\alpha_L$ | 0.007770 | 0.006490 | 1.197500 | 0.000000 | 0.050000 | 0.009380 | 0.002500 | 3.749820 |
| $\beta_L$ | 0.051600 | 0.045300 | 1.139100 | 0.000000 | 0.050000 | 0.061890 | 0.016260 | 3.806310 |
| $\gamma_L$ | 4.00E-05 | 8.00E-05 | 0.532590 | 0.000000 | 0.050000 | 6.00E-05 | 3.00E-05 | 1.876500 |
| $b_{EE}$ | -0.032800 | 0.020710 | -1.583550 | 0.000000 | 0.050000 | -0.022340 | 0.014320 | -1.559900 |
| $b_{EK}$ | -0.012460 | 0.002970 | -4.190090 | -0.045640 | 0.000100 | -0.014620 | 0.002630 | -5.553670 |
| $b_{EL}$ | 0.043090 | 0.013960 | 3.086090 | 0.000000 | 0.000100 | 0.027350 | 0.008090 | 3.381060 |
| $b_{EM}$ | 0.018160 | 0.011890 | 1.527910 | 0.047420 | 0.000100 | 0.030330 | 0.007360 | 4.122620 |
| $b_E$ | 0.023000 | 0.009470 | 2.428610 | 0.000000 | 0.050000 | 0.020690 | 0.005660 | 3.657920 |
| $b_{Et}$ | -0.001290 | 0.001210 | -1.064900 | 0.000000 | 0.050000 | -0.000760 | 0.000830 | -0.921730 |
| $\alpha_E$ | 0.002150 | 0.001290 | 1.673940 | 0.000000 | 0.050000 | 0.001710 | 0.000860 | 1.981070 |
| $\beta_E$ | -0.001120 | 0.008700 | -0.129090 | 0.000000 | 0.050000 | -0.003630 | 0.005380 | -0.675370 |
| $\gamma_E$ | 2.00E-05 | 2.00E-05 | 1.013290 | 0.000000 | 0.050000 | 2.00E-05 | 1.00E-05 | 1.275080 |
| $b_{MM}$ | 0.926780 | 0.190690 | 4.860050 | 0.000000 | 0.050000 | 0.735070 | 0.062710 | 11.72076 |
| $b_{MK}$ | -0.022780 | 0.013790 | -1.651930 | 0.000000 | 0.050000 | -0.024680 | 0.014250 | -1.731780 |
| $b_{ML}$ | -0.006190 | 0.079550 | -0.077850 | 0.211500 | 0.000100 | 0.142630 | 0.010720 | 13.30569 |
| $b_{ME}$ | 0.018160 | 0.011890 | 1.527910 | 0.000000 | 0.000100 | 0.030470 | 0.010320 | 2.952100 |
| $b_M$ | -0.193350 | 0.088390 | -2.187470 | 0.000000 | 0.050000 | -0.176350 | 0.028510 | -6.184590 |
| $b_{Mt}$ | -0.006900 | 0.010910 | -0.632340 | 0.000000 | 0.050000 | -0.007940 | 0.004340 | -1.831550 |
| $\alpha_M$ | -0.000780 | 0.011720 | -0.066300 | 0.000000 | 0.050000 | -0.000530 | 0.004560 | -0.116900 |
| $\beta_M$ | -0.113910 | 0.082280 | -1.384430 | 0.000000 | 0.050000 | -0.102210 | 0.029210 | -3.499600 |
| $\gamma_M$ | 0.000280 | 0.000150 | 1.873960 | 0.000000 | 0.050000 | 0.000240 | 6.00E-05 | 3.973830 |

**Table 5b.** Classical vs Bayesian inference: classical elasticities estimates with restricted SUR compared with Bayesian elasticities estimates of unrestricted SUR using a mixture of uninformative diffuse priors and informative tight priors

| Elasticities | SUR | Priors | Gibbs |
|---|---|---|---|
| KK | -0.232360 | 0.000000 | -0.251540 |
| KL | 0.591170 | 0.500000 | 0.651140 |
| KE | -0.129370 | -0.500000 | -0.148700 |
| KM | -0.229440 | 0.000000 | -0.250900 |
| LL | -0.189480 | 0.000000 | -0.532720 |
| LK | 0.095360 | 0.250000 | 0.105400 |
| LE | 0.109360 | 0.000000 | 0.071750 |
| LM | -0.015250 | 0.250000 | 0.355570 |
| EE | -0.804890 | 0.000000 | -0.667830 |
| EK | -0.136520 | -0.500000 | -0.159330 |
| EL | 0.715430 | 0.000000 | 0.451770 |
| EM | 0.225990 | 0.590000 | 0.375390 |
| MM | 0.008500 | 0.000000 | -0.176310 |
| MK | -0.017770 | 0.000000 | -0.019150 |
| ML | -0.007320 | 0.250000 | 0.167780 |
| ME | 0.016580 | 0.000000 | 0.027690 |

*Note*: ij (i,j=K,L,E,M) means the elasticity of demand for input i with respect to the price of input j.

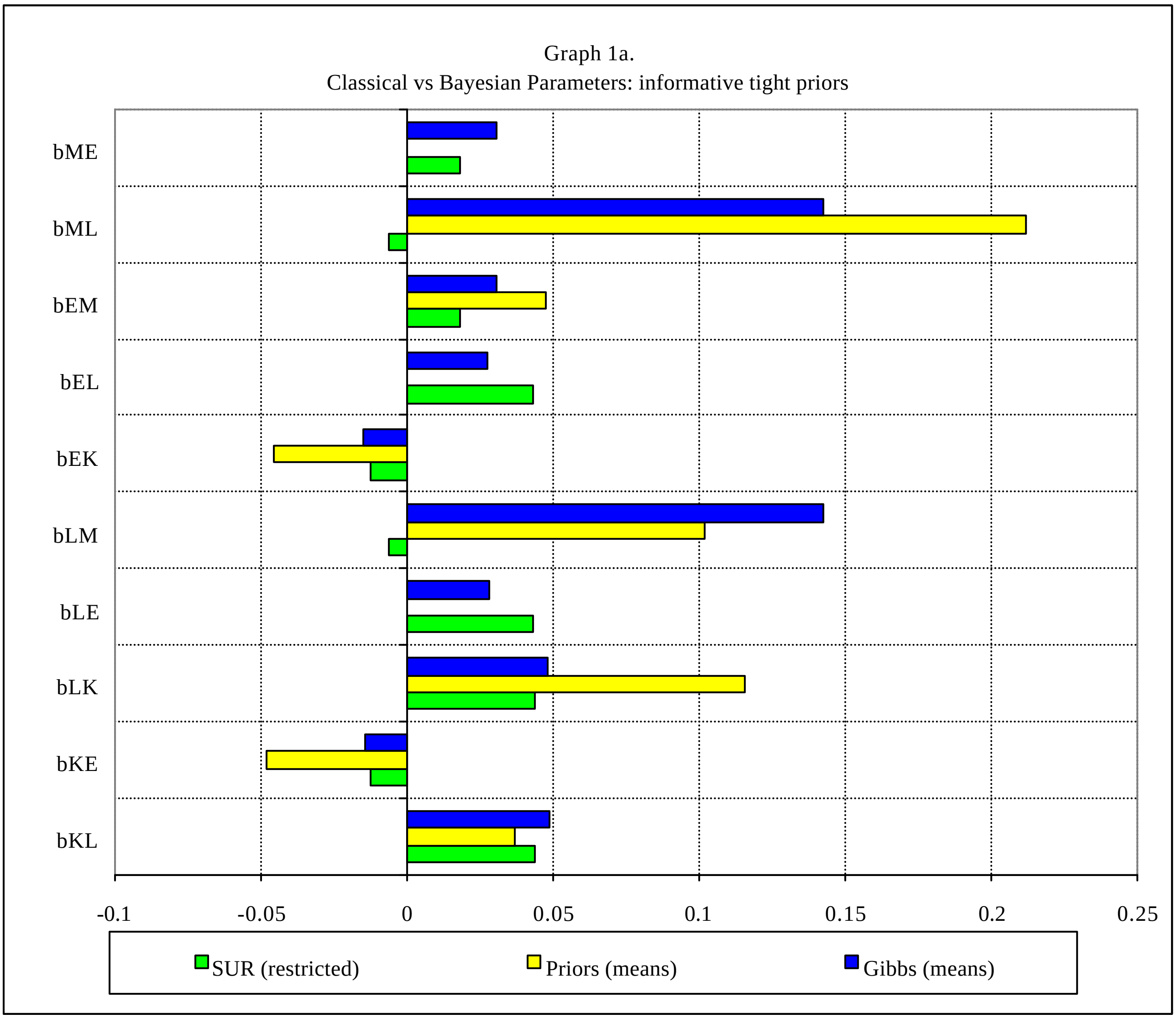
**Table 6a**. Classical vs Bayesian inference: classical parameter estimates with restricted SUR compared with Bayesian parameter estimates of unrestricted SUR trying to approximate symmetry restrictions via rejection sampling

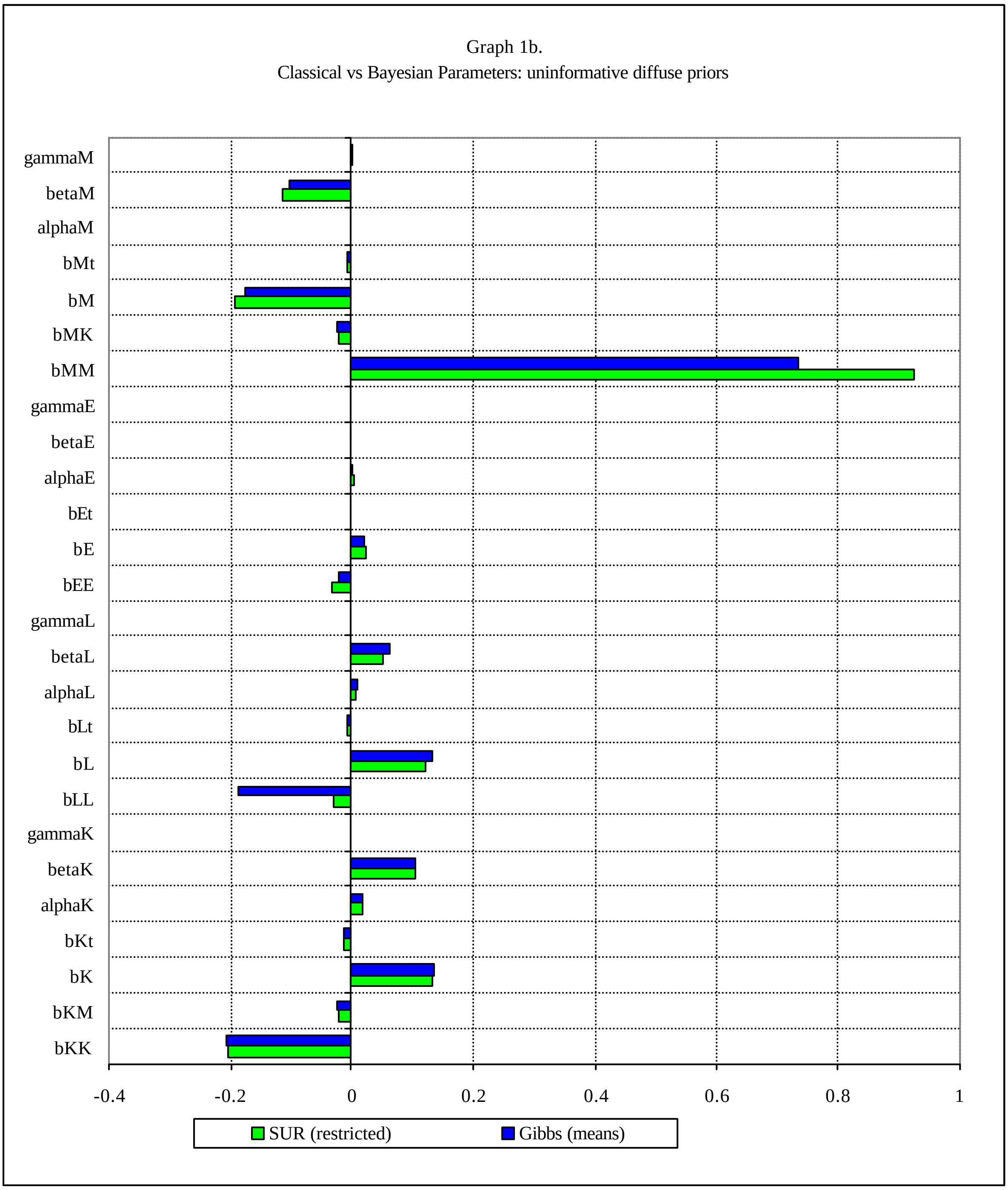
| Parameters | SUR (restricted) | SUR (se) | SUR (t stat) | Priors (means) | Priors (sd) | Gibbs_rj (means) | Gibbs_rj (sd) | Gibbs_rj (pseudo t stat) |
|---|---|---|---|---|---|---|---|---|
| $b_{KK}$ | -0.203650 | 0.021400 | -9.518400 | 0.000000 | 0.050000 | -0.215830 | 0.011400 | -18.93862 |
| $b_{KL}$ | 0.043980 | 0.011080 | 3.969720 | 0.037200 | 0.000100 | 0.049060 | 0.013180 | 3.721880 |
| $b_{KE}$ | -0.012460 | 0.002970 | -4.190090 | -0.048160 | 0.000100 | -0.023980 | 0.006410 | -3.740540 |
| $b_{KM}$ | -0.022780 | 0.013790 | -1.651930 | 0.000000 | 0.050000 | -0.018290 | 0.011590 | -1.577210 |
| $b_K$ | 0.134140 | 0.011450 | 11.72012 | 0.000000 | 0.050000 | 0.142590 | 0.006760 | 21.10322 |
| $b_{Kt}$ | -0.013170 | 0.001620 | -8.139820 | 0.000000 | 0.050000 | -0.013710 | 0.000940 | -14.58187 |
| $\alpha_K$ | 0.017040 | 0.001710 | 9.976260 | 0.000000 | 0.050000 | 0.017720 | 0.000990 | 17.86183 |
| $\beta_K$ | 0.103960 | 0.011180 | 9.298580 | 0.000000 | 0.050000 | 0.109890 | 0.005920 | 18.55247 |
| $\gamma_K$ | 0.000130 | 2.00E-05 | 5.449530 | 0.000000 | 0.050000 | 0.000130 | 2.00E-05 | 7.491690 |
| $b_{LL}$ | -0.029630 | 0.124780 | -0.237460 | 0.000000 | 0.050000 | -0.193740 | 0.037230 | -5.203690 |
| $b_{LK}$ | 0.043980 | 0.011080 | 3.969720 | 0.115310 | 0.000100 | 0.070650 | 0.008280 | 8.530770 |
| $b_{LE}$ | 0.043090 | 0.013960 | 3.086090 | 0.000000 | 0.000100 | 0.019370 | 0.010750 | 1.802080 |
| $b_{LM}$ | -0.006190 | 0.079550 | -0.077850 | 0.101540 | 0.000100 | 0.098830 | 0.008580 | 11.51951 |
| $b_L$ | 0.122000 | 0.048450 | 2.518370 | 0.000000 | 0.050000 | 0.147670 | 0.017460 | 8.456930 |
| $b_{Lt}$ | -0.006380 | 0.006040 | -1.055490 | 0.000000 | 0.050000 | -0.010710 | 0.002370 | -4.518880 |
| $\alpha_L$ | 0.007770 | 0.006490 | 1.197500 | 0.000000 | 0.050000 | 0.013460 | 0.002570 | 5.228540 |
| $\beta_L$ | 0.051600 | 0.045300 | 1.139100 | 0.000000 | 0.050000 | 0.084790 | 0.017390 | 4.876670 |
| $\gamma_L$ | 4.00E-05 | 8.00E-05 | 0.532590 | 0.000000 | 0.050000 | 0.000110 | 3.00E-05 | 3.531510 |
| $b_{EE}$ | -0.032800 | 0.020710 | -1.583550 | 0.000000 | 0.050000 | -0.034710 | 0.013930 | -2.492290 |
| $b_{EK}$ | -0.012460 | 0.002970 | -4.190090 | -0.045640 | 0.000100 | -0.010230 | 0.002790 | -3.668390 |
| $b_{EL}$ | 0.043090 | 0.013960 | 3.086090 | 0.000000 | 0.000100 | 0.018030 | 0.010260 | 1.758300 |
| $b_{EM}$ | 0.018160 | 0.011890 | 1.527910 | 0.047420 | 0.000100 | 0.039110 | 0.008950 | 4.368210 |
| $b_E$ | 0.023000 | 0.009470 | 2.428610 | 0.000000 | 0.050000 | 0.023780 | 0.005590 | 4.254660 |
| $b_{Et}$ | -0.001290 | 0.001210 | -1.064900 | 0.000000 | 0.050000 | -0.001760 | 0.000780 | -2.255830 |
| $\alpha_E$ | 0.002150 | 0.001290 | 1.673940 | 0.000000 | 0.050000 | 0.002810 | 0.000830 | 3.401180 |
| $\beta_E$ | -0.001120 | 0.008700 | -0.129090 | 0.000000 | 0.050000 | 0.002160 | 0.005240 | 0.412990 |
| $\gamma_E$ | 2.00E-05 | 2.00E-05 | 1.013290 | 0.000000 | 0.050000 | 3.00E-05 | 1.00E-05 | 2.670920 |
| $b_{MM}$ | 0.926780 | 0.190690 | 4.860050 | 0.000000 | 0.050000 | 0.635640 | 0.062580 | 10.15677 |
| $b_{MK}$ | -0.022780 | 0.013790 | -1.651930 | 0.000000 | 0.050000 | 0.003100 | 0.016860 | 0.183720 |
| $b_{ML}$ | -0.006190 | 0.079550 | -0.077850 | 0.211500 | 0.000100 | 0.191830 | 0.008470 | 22.64199 |
| $b_{ME}$ | 0.018160 | 0.011890 | 1.527910 | 0.000000 | 0.000100 | -0.011680 | 0.011330 | -1.031240 |
| $b_M$ | -0.193350 | 0.088390 | -2.187470 | 0.000000 | 0.050000 | -0.146320 | 0.026780 | -5.463680 |
| $b_{Mt}$ | -0.006900 | 0.010910 | -0.632340 | 0.000000 | 0.050000 | -0.014130 | 0.004470 | -3.158290 |
| $\alpha_M$ | -0.000780 | 0.011720 | -0.066300 | 0.000000 | 0.050000 | 0.005720 | 0.004660 | 1.226810 |
| $\beta_M$ | -0.113910 | 0.082280 | -1.384430 | 0.000000 | 0.050000 | -0.065780 | 0.029070 | -2.262950 |
| $\gamma_M$ | 0.000280 | 0.000150 | 1.873960 | 0.000000 | 0.050000 | 0.000310 | 6.00E-05 | 4.885160 |

**Table 6b.** Classical vs Bayesian inference: classical elasticities estimates with restricted SUR compared with Bayesian elasticities estimates of unrestricted SUR trying to approximate symmetry restrictions via rejection sampling

| Elasticities | SUR | Priors | Gibbs_rj (rejection sampling) |
|---|---|---|---|
| KK | -0.232360 | 0.000000 | -0.228310 |
| KL | 0.591170 | 0.500000 | 0.665170 |
| KE | -0.129370 | -0.500000 | -0.251110 |
| KM | -0.229440 | 0.000000 | -0.185760 |
| LL | -0.189480 | 0.000000 | -0.453390 |
| LK | 0.095360 | 0.250000 | 0.155830 |
| LE | 0.109360 | 0.000000 | 0.050010 |
| LM | -0.015250 | 0.250000 | 0.247560 |
| EE | -0.804890 | 0.000000 | -0.676130 |
| EK | -0.136520 | -0.500000 | -0.112490 |
| EL | 0.715430 | 0.000000 | 0.300450 |
| EM | 0.225990 | 0.590000 | 0.488180 |
| MM | 0.008500 | 0.000000 | -0.218180 |
| MK | -0.017770 | 0.000000 | 0.002410 |
| ML | -0.007320 | 0.250000 | 0.226420 |
| ME | 0.016580 | 0.000000 | -0.010650 |

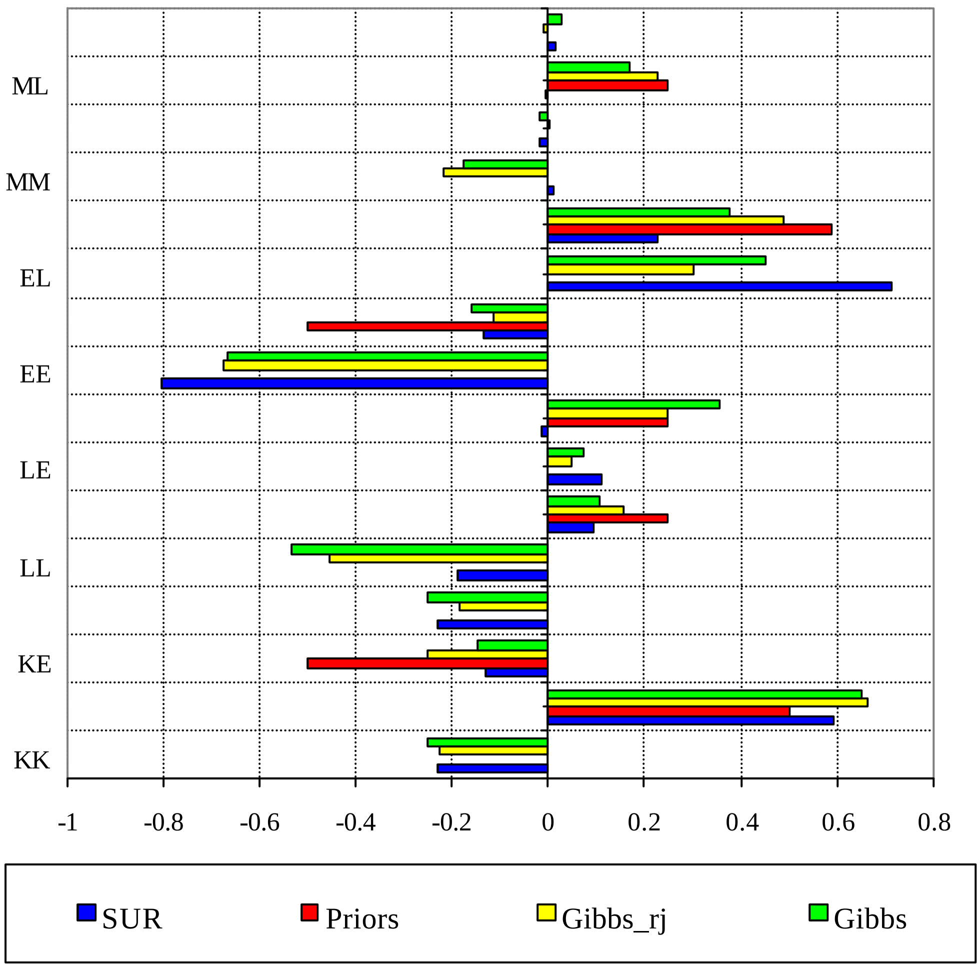
*Note*: ij (i,j=K,L,E,M) means the elasticity of demand for input i with respect to the price of input j.


Graph 1a.
Classical vs Bayesian Parameters: informative tight priors
bME
bML
bEM
bEL
bEK
bLM
bLE
bLK
bKE
bKL
-0.1
-0.05
0
0.05
0.1
0.15
0.2
0.25
SUR (restricted)
Priors (means)
Gibbs (means)



Graph 1b.
Classical vs Bayesian Parameters: uninformative diffuse priors
gammaM
betaM
alphaM
bMt
bM
bMK
bMM
gammaE
betaE
alphaE
bEt
bE
bEE
gammaL
betaL
alphaL
bLt
bL
bLL
gammaK
betaK
alphaK
bKt
bK
bKM
bKK
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
SUR (restricted)
Gibbs (means)

Graph 2. Classical vs Bayesian elasticities



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