A Nonlinear Approach for the Adjustment and Updating of IO Accounts

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Abstract

Many structural relations should be taken into account in any reasonable updating e¤ort. These structural relations are mainly represented by coe¢ cients of di¤erent types like technical coe¢ cients or the proportion of the value of a cell in relation to its row or column total. These coe¢ cients can normally be introduced in an optimization framework by using nonlinear programming approaches. Standard approaches concentrate in using distance measures that minimize the absolute or relative di¤erence in technical or other types of coe¢ cients. However, these approaches show a tendency to concentrate the changes in the biggest cells and therefore produce a non-homogeneous pattern of coe¢ cient adjustment.

This study has two main objectives. First, we propose a formulation that tries both to obtain a more homogeneous relative adjustment of the structural coe¢ cients and to reduce the nonlinearities of the programms in order to facilitate obtaining a solution. Second, we try to test the usefulness of this proposal by comparing its results with the ones obtained with more standard approaches.

This is a preliminary version of an ongoing research that aims to be …nished by the end of this year. Next steps in the near future will include testing these approaches in a more broad variety of scenarios, such as allowing changes to the initially …xed vectors, including imports, and trying to compare and combine our methods with those recently presented by authors like Robinson, Cattaeno and El-Said (1998). Any suggestions or reccomendations will be sincerely welcome. (casiano@empresariales.ulpgc.es)

1 Introduction

Many structural relations should be taken into account in any reasonable updating effort. These structural relations are mainly represented by coefficients of different types like technical coefficients or the proportion of the value of a cell in relation to its row or column total. These coefficients can normally be introduced in an optimization framework by using nonlinear programming approaches. Standard approaches concentrate in using distance measures that minimize the absolute or relative difference in technical or other types of coefficients. However, these approaches show a tendency to concentrate the changes in the biggest cells and therefore produce a non-homogeneous pattern of coefficient adjustment.

On the other hand, most practical efforts to update IO matrices would generate very complicated nonlinear programms for which even obtaining a solution could prove to be very difficult, especially when updating very disaggregated accounts. This is especially the case when we introduce more than one coefficient in the objective function (e.g.: technical coefficients, row and column coefficients or some combination of all three). In many occasions this forces practitioners to introduce exogenous bounds to the different elements of the IO matrix that naturally biases the results in an artificial manner.

This study has two main objectives. First, we propose a formulation that tries both to obtain a more homogeneous relative adjustment of the structural coefficients and to reduce the nonlinearities of the programms in order to facilitate obtaining a solution. Second, we try to test the usefulness of this proposal by comparing its results with the ones obtained with more standard approaches.

This approach was developed during the process of updating and adjustment of the IO Accounts and an aggregated SAM of the Canary Islands for the year 1990 using the IO Table of 1985 as a benchmark (Manrique de Lara Peñate, 1999). This exercise covered all the elements of the IO Table and considered simultaneously the incorporation of trade flows with other regions and the rest of the world. The final programm used 18 different sectors and its different parts summed up to 6.130 restrictions and 10.875 variables.

Next section presents a brief summary of the main contributions found in the literature about the adjustment of IO accounts with mathematical programming. Section three summarizes our proposal in mathematical terms. Finally the results of the different comparisons done to evaluate the usefulness of our approach as well as a short section with our main conclusions are going to be presented.

2 The Adjustment and Updating of IO Accounts with Mathematical Programming

A way of solving the problem of adjustment and updating of IO accounts consists in using mathematical programming. One of the approaches that can be expressed in terms of a mathematical problem is the RAS algorithm, as demonstrated by Macgill (1977). The RAS algorithm solves the following problem:

$$\min \sum_i x_{ij}^t \ln\left(\frac{x_{ij}^t}{x_{ij}^0}\right)$$

subject to:

$$\sum_i x_{ij}^t = \gamma_j^t \qquad (j = 1, \ldots, n)$$
$$\sum_j x_{ij}^t = \rho_i^t \qquad (i = 1, \ldots, n-1)$$

being:

X^0 : initial IO matrix
X^t : updated IO matrix
γ^t : new vector of intermediate inputs in t
ρ^t : new vector of intermediate outputs in t

Morrison and Thuman (1980) proposed to minimize the sum of the weighted squared deviations:

$$\min \sum_{i;j} \frac{(x_{ij}^t - x_{ij}^0)^2}{\pi_{ij}}$$

subject to:

$$\sum_i x_{ij}^t = \gamma_j^t \qquad (j = 1, \ldots, n)$$
$$\sum_j x_{ij}^t = \rho_i^t \qquad (i = 1, \ldots, n-1)$$

The objective function could represent a χ^2 (when $\pi_{ij} = x^0_{ij}$) and different weights could be applied to the elements of the matrix depending on the interest in favoring their change in the matrix X^t to be obtained (e.g.: $\pi_{ij} = \left(x^0_{ij}\right)^2$ or $\pi_{ij} = \sqrt{x^0_{ij}}$).

Other examples include the use by Mankinen (1993) of generalized or conditioned least squares and the efforts of Cole (1992) to introduce additional restrictions on certain groups of elements.

Matuszewski, Pitts and Sawyer (1964) were the first to propose an adjustment technique based on linear programming. Their problem was formulated as follows:

$$\min \sum_{(i,j)/a^0_{ij} \neq 0} \left| \frac{a^t_{ij}}{a^0_{ij}} - 1 \right|$$

subject to:

$$\sum_{i/a^0_{ij} \neq 0} a^t_{ij} p^t_j = \gamma^t_j$$

$$\sum_{j/a^0_{ij} \neq 0} a^t_{ij} p^t_j = \rho^t_i$$

$$\frac{1}{2} \leq \frac{a^t_{ij}}{a^0_{ij}} \leq 2 \qquad \forall (i,j)/a^0_{ij} \neq 0$$

being:

A^0 : technical coefficients matrix obtained from X^0
A^t : technical coefficients matrix obtained from X^t
P^0 : vector of effective production in 0
P^t : vector of effective production in t

Their last restriction was introduced to avoid the fact that the changes in the coefficients tended to concentrate in the larger elements of the intermediate transaction matrix. It is clearly arbitrary but it helped to increase the number of basic variables giving more realistic solutions.

Since the new vector of production was known to them, they switched from using coefficients to flows taking the inverse of the new known values of effective production as weights.They converted this nonlinear formulation into a linear one by including two new positive variables for each of the elements to be updated, avoiding the nonlinearity in the objective function due to the calculation of absolute values. The final formulation ended up looking very much the same to the classical linear programming problem with upper bound constraints.

This need to set bounds to the variables is present in many other examples. From the more open formulations of Harrigan and Buchanan (1984) to the ones proposed by Zenios, Drud and Mulvey (1989) and Schneider and Zenios (1990) or Callealta (1993). In fact the need of these bounds is twofold. First, it helps the programming solver to find a solution, and second, it helps to avoid too extreme corner solutions (zero values). However, it is very easy to remain at the minimum or maximum values imposed, reducing therefore the freedom to find the optimal solution. Our proposal shows there is an alternative way to find new coefficients without imposing such strong restrictions to the updating process. Our main emphasis lies indeed in trying to respect, as much as possible, the initial relative structure (i.e. coefficients) of the accounts to be updated. It also tries to obtain more linear formulations that are particularly useful in cases where the row and column totals are unknown and several structural coefficients are simultaneously considered.

3 Notation, Definitions and Adjustment Criteria

Let $X = (x_{ij})_{1 \le i \le m; 1 \le j \le n}$, $P = (p_i)_{1 \le i \le m}$ and $Q = (q_i)_{1 \le i \le m}$. We consider the following sets, matrices and functions.

- SETS:
  - $I = \{1, 2, \ldots, m\}$, the row index set.
  - $J = \{1, 2, \ldots, n\}$, the column index set.
  - $I_+ = \{i \in I / \sum_k x_{ik} \ne 0\}$.
  - $J_+ = \{j \in J / \sum_k x_{kj} \ne 0\}$.
  - $I_j = \{i \in I / x_{ij} \ne 0\}$ and $n_j = |I_j|$ is the cardinal of set I_j, $\forall j \in J$.
  - $J_i = \{j \in J / x_{ij} \ne 0\}$ and $m_i = |J_i|$ is the cardinal of set J_i, $\forall i \in I$.
  - $S_X = \{(i, j) / x_{ij} \ne 0; i \in I; j \in J\}$
- MATRICES:
  - P- coefficients matrix, $A_X = (a_{ij})_{1 \le i \le m; 1 \le j \le n}$:

$$a_{ij} = \begin{cases} \frac{x_{ij}}{p_j} & \text{if} \quad p_j \ne 0 \\ 0 & \text{otherwise} \end{cases}$$

    (In the applications presented in this paper, P is the vector of effective production).

– Row coefficients matrix, $B_X = (b_{ij})_{1 \le i \le m; 1 \le j \le n}$:

$$b_{ij} = \begin{cases} \frac{x_{ij}}{\sum_k x_{ik}} & \text{if } \sum_k x_{ik} \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

– Column coefficients matrix $C_X = (c_{ij})_{1 \le i \le m; 1 \le j \le n}$:

$$c_{ij} = \begin{cases} \frac{x_{ij}}{\sum_k x_{kj}} & \text{if } \sum_k x_{kj} \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Note that $S_X = S_{A_X} = S_{B_X} = S_{C_X}$.

- FUNCTIONS: Given the $m \times n$ matrix $\hat{X} = (\hat{x}_{ij})$ with $S_X = S_{\hat{X}}$, we define

  –
$$F_1(X) = \sum_{(i,j) \in S_X} \left| \frac{x_{ij} - \hat{x}_{ij}}{\hat{x}_{ij}} \right|.$$

  –
$$F_2(X) = \sum_{(i,j) \in S_X} \left| \frac{x_{ij}}{\hat{x}_{ij}} - \mu_i \right|$$

  where
$$\mu_i = \frac{1}{m_i} \sum_{j \in J_i} \frac{x_{ij}}{\hat{x}_{ij}}.$$

  –
$$F_3(X) = \sum_{(i,j) \in S_X} \left| \frac{x_{ij}}{\hat{x}_{ij}} - \nu_j \right|$$

  where
$$\nu_j = \frac{1}{n_j} \sum_{i \in I_j} \frac{x_{ij}}{\hat{x}_{ij}}.$$

  –
$$F_4(X) = \sum_{(i,j) \in I \times (J - \{n\})} \big| |d_{ij} - 1| - |d_{ij+1} - 1| \big|$$

  where
$$d_{ij} = \begin{cases} \frac{x_{ij}}{\hat{x}_{ij}} & \text{if } (i,j) \in S_X \\ 1 & \text{otherwise} \end{cases}$$

–

$$F_5(X) = \sum_{(i,j)\in(I-\{m\})\times J} \big||d_{ij} - 1| - |d_{i+1j} - 1|\big|.$$

–

$$F_6(X) = \sum_{(i,j)\in S_X} x_{ij} \ln \frac{x_{ij}}{\bar{x}_{ij}}.$$

–

$$F_7(Q) = \sum_{i\in I} |q_i - 1|.$$

Henceforth, when we write $F_i(A_X)$ we assume that X^0 is replaced by A_{X^0}, analogously for B_X and C_X.

Given the matrix $X^0 = (x^0_{ij})_{1\le i\le m; 1\le j\le n}$, the adjustment and updating problem is to determine a matrix $X^t = (x^t_{ij})_{1\le i\le m; 1\le j\le n}$ with a structure similar to X^0 satisfying certain constraints. The problem is formulated as an optimization problem where the objective function is a linear combination of the functions F_i applied to particular matrices, replacing X by X^t and $\bar{X}$ by X^0. We consider the following adjustment criteria formulation (the non-zero weights determine different criteria).

- ADJUSTMENT CRITERIA FORMULATION:

  – Formulation 1:

$$\begin{aligned} \min\ G_1(X^t; Q) = {} & \pi_{11}F_1(A_{X^t}) + \pi_{12}F_2(A_{X^t}) + \pi_{13}F_3(A_{X^t}) + \\ & \pi_{14}F_1(B_{X^t}) + \pi_{15}F_2(B_{X^t}) + \\ & \pi_{16}F_1(C_{X^t}) + \pi_{17}F_3(C_{X^t}) + \\ & \pi_{18}F_7(Q). \end{aligned}$$

  – Formulation 2:

$$\begin{aligned} \min\ G_2(X^t; Q) = {} & \pi_{21}F_1(A_{X^t}) + \pi_{22}F_4(A_{X^t}) + \pi_{23}F_5(A_{X^t}) + \\ & \pi_{24}F_1(B_{X^t}) + \pi_{25}F_4(B_{X^t}) + \\ & \pi_{26}F_1(C_{X^t}) + \pi_{27}F_5(C_{X^t}) + \\ & \pi_{28}F_7(Q). \end{aligned}$$

– Formulation 3:

$$\min\ G_3(X^t; Q) = \pi_{31} F_6(A_{X^t}) + \pi_{32} F_6(B_{X^t}) + \pi_{33} F_6(C_{X^t}) + \pi_{34} F_7(Q).$$

Note that the formulation 1 with $\pi_{11} = 1$ and $\pi_{1k} = 0$, $\forall k \neq 1$ corresponds to the method of Matuszewski, Pitts and Sawyer (1964) for the input-output coefficient estimation problem, and the formulation 3 with $\pi_{31} = 1$ and $\pi_{3k} = 0$, $\forall k \neq 1$, gives the information theory criterium.

The adjustment problem is

$$\min\ F(X; Q) \qquad \text{subject to } X \in \mathcal{X}$$

where $F = G_i$ for some i and certain weights π_{ik}, and $\mathcal{X}$ is the feasible set defined by certain constraints on X. These constraints are such as

$$\rho_{i1} x_{i1} + \rho_{i2} x_{i2} + \dots + \rho_{in} x_{in} = (\leq)(\geq) \rho_i \tag{1}$$

$$\gamma_{1j} x_{1j} + \gamma_{2j} x_{2j} + \dots + \gamma_{nj} x_{nj} = (\leq)(\geq) \gamma_j \tag{2}$$

$$x_{ij^*} = q_i r_i \qquad \forall i \in I$$

where $R = (r_i)_{1 \leq i \leq m}$ is given (this constraint is associated to the function F_8 and j^* represents a particular column of X).

Now, we consider two cases:

1. Case 1: P, $\sum_k x_{ik}$, $1 \leq i \leq m$, and $\sum_k x_{kj}$, $1 \leq j \leq n$, are known.

   In this case, the adjustment problem can be formulated as a linear programm. It is enough to consider that each real number s satisfies $|s| = y + z$ with $s = y - z$ and $y = 0$ or $z = 0$.

2. Case 2: P, $\sum_k x_{ik}$, $1 \leq i \leq m$, and $\sum_k x_{kj}$, $1 \leq j \leq n$, are not known.

   In this case, there are adjusment problems which can be formulated as linear problems. In the situations for which the linearization is not so evident, we have applied a change of variables to reduce the difficulty of the problem.

To illustrate the procedure used in this work for case 2, we present two particular problems.

• Problem 1:

$$\min\ G_1(X) \text{ subject to constraints (2)}$$

with $\pi_{16} = 1; \pi_{1j} = 0;\ j \neq 6$, and $\sum_k x_{kj}$, $1 \leq j \leq n$, not totally known. That is

$$\min\ F_1(C_X) \text{ subject to constraints (2)}$$

This problem can be formulated as

$$\min \sum_{(i,j)\in S_X} (y_{ij} + z_{ij})$$

$$\frac{x_{ij}}{\sum_{1\leq k\leq n} x_{kj}} - c^0_{ij} = c^0_{ij}(y_{ij} - z_{ij}) \qquad \forall (i,j) \in S_X$$

$$X \in \mathcal{X} \text{ (constraints (2))}$$

$$y_{ij}, z_{ij} \geq 0 \qquad \forall (i,j) \in S_X.$$

We define the variables

$$t_j = \frac{1}{\sum_{1\leq k\leq n} x_{kj}};\ \forall j \in J_+ \qquad u_{ij} = x_{ij} t_j;\ \forall (i,j) \in S_X.$$

Then

$$\sum_{1\leq i\leq n} u_{ij} = 1 \qquad \forall j \in J_+.$$

For $(i,j) \notin S_X$; $u_{ij} = 0$. Using these variables, problem 1 can be transformed into the following linear problem

$$\min \sum_{(i,j)\in S_X} (y_{ij} + z_{ij})$$

$$u_{ij} - c^0_{ij} = c^0_{ij}(y_{ij} - z_{ij}) \qquad \forall (i,j) \in S_X$$

$$\sum_{1\leq k\leq n} \gamma_{kj} u_{kj} = (\leq)(\geq) \gamma_j t_j \qquad \forall j \in J' \subset J$$

$$\sum_{1 \le i \le n} u_{ij} = 1 \qquad \forall j \in J_+$$

$$y_{ij}, z_{ij} \ge 0 \qquad \forall (i,j) \in S_X$$

If the optimal solution is (u^*_{ij}, t^*_j), then the optimal X is $X^t = (x^t_{ij})$ where

$$x^t_{ij} = \frac{u^*_{ij}}{t^*_j},\ \forall (i,j) \in S_X; \qquad x^t_{ij} = 0,\ \forall (i,j) \notin S_X.$$

- Problem 2:

$$\min\ G_1(X) \text{ subject to constraints (1) and (2)}$$

with $\pi_{14} = \pi_{16} = 1$; $\pi_{1j} = 0$; $j \neq 4, 6$, and $\sum_k x_{ik}$, $1 \le i \le m$, $\sum_k x_{kj}$, $1 \le j \le n$, not totally known. That is

$$\min\ F_1(B_X) + F_1(C_X) \text{ subject to constraints (1) and (2)}$$

Now, we introduce the variables

$$v_i = \frac{1}{\sum_{1 \le k \le n} x_{ik}},\ \forall i \in I \qquad w_{ij} = x_{ij} v_i, \forall (i,j) \in S_X.$$

Then

$$\sum_{1 \le j \le n} w_{ij} = 1 \qquad \forall i \in I_+.$$

For $(i,j) \notin S_X$; $u_{ij} = w_{ij} = 0$. Using the variables u_{ij}, t_j, w_{ij} and v_i, the problem 2 can be converted into the problem

$$\min \sum_{(i,j) \in S_X} (y_{\rho ij} + z_{\rho ij} + y_{\gamma ij} + z_{\gamma ij})$$

$$w_{ij} - b^0_{ij} = b^0_{ij}(y_{\rho ij} - z_{\rho ij}) \qquad \forall (i,j) \in S_X$$

$$\sum_{1 \le k \le n} \rho_{ik} w_{ik} = (\le)(\ge) \rho_i v_i \qquad \forall i \in I' \subseteq I$$

$$\sum_{1 \le j \le n} w_{ij} = 1 \qquad \forall i \in I_+$$

$$u_{ij} - c^0_{ij} = c^0_{ij}(y_{\gamma ij} - z_{\gamma ij}) \qquad \forall (i;j) \in S_X$$

$$\sum_{1 \le k \le n} \gamma_{kj} u_{kj} = (\le)(\ge) \gamma_j t_j \qquad \forall j \in J' \subseteq J$$

$$\sum_{1 \le i \le n} u_{ij} = 1 \qquad \forall j \in J_+$$

$$u_{ij} v_i - w_{ij} t_j = 0 \qquad \forall (i;j) \in S_X \tag{3}$$

$$y_{\rho ij}; z_{\rho ij}; y_{\gamma ij}; z_{\gamma ij} \ge 0 \qquad \forall (i;j) \in S_X:$$

If the optimal solution is $(u^*_{ij}; t^*_j; w^*_{ij}; v^*_j))$, then the optimal X is $X^t = (x^t_{ij})$ where

$$x^t_{ij} = \frac{u^*_{ij}}{t^*_j} \qquad \forall (i;j) \in S_X; \qquad x^t_{ij} = 0; \forall (i;j) \notin S_X:$$

Note that, by constraint (3),

$$x_{ij} = \frac{u_{ij}}{t_j} = \frac{w_{ij}}{v_i} \ \forall (i;j) \in S_X$$

To facilitate obtaining a solution, equation 3 could be reformulated as follows:

$$u_{ij} v_i - w_{ij} t_j = e1(i;j) - e2(i;j) \qquad \forall (i;j) \in S_X$$

where

$$e1(i,j), e2(i,j) > 0 \qquad \forall (i,j) \in S_X$$

and the sum

$$\pi \sum (e1(i,j) + e2(i,j))$$

with a sufficiently high weight π, would be added to the objective function.

- COMPARISON MEASURES: The following measures will be used to compare the matrices X and X^0.

  1. Standardized total error:

  $$STE(X^0, X) = \frac{\sum_{(i,j)} |x_{ij} - x^0_{ij}|}{\sum_{(i,j)} x^0_{ij}}$$

  2. Correlation coefficient:

  $$CC(X^0, X) = \frac{\sum_{(i,j)} (x_{ij} - \mu)(x^0_{ij} - \mu^0)}{\sigma \sigma^0}$$

  where μ, μ_0, and σ, σ^0, are the means and standard deviations of the sets $\{x_{ij}\}$ and $\{x^0_{ij}\}$, respectively.

  3. Mean absolute difference:

  $$MAD(X^0, X) = \frac{\sum_{(i,j)} |x_{ij} - x^0_{ij}|}{m \times n}$$

  4. Mean relative difference:

  $$MRD(X^0, X) = \frac{\sum_{(i,j) \in S^0_X} \left|\frac{x_{ij} - x^0_{ij}}{x^0_{ij}}\right|}{m \times n}$$

  5. Index of inequality (Theil's U):

  $$TII(X^0, X) = \left(\frac{\sum_{(i,j)} (x_{ij} - x^0_{ij})^2}{\sum_{(i,j)} (x^0_{ij})^2}\right)^{\frac{1}{2}}$$

  6. Root mean squared error:

  $$RMSE(X^0, X) = \frac{\left(\sum_{(i,j)} (x_{ij} - x^0_{ij})^2\right)^{\frac{1}{2}}}{m \times n}$$

  7. Root mean squared relative error:

  $$RMSRE(X^0, X) = \frac{\left(\sum_{(i,j)} \left(\frac{x_{ij} - x^0_{ij}}{x^0_{ij}}\right)^2\right)^{\frac{1}{2}}}{m \times n}$$

8. Maximal absolute difference:

$$MXAD(X^0; X) = \max_{(i,j)} |x_{ij} - x^0_{ij}|$$

9. Maximal relative difference:

$$MXRD(X^0; X) = \max_{(i,j) \in S_{X^0}} \left|\frac{x_{ij} - x^0_{ij}}{x^0_{ij}}\right|$$

10. Weighted absolute difference

$$WAD(X^0; X) = \frac{\sum_{(i,j)} (x_{ij} + x^0_{ij})|x_{ij} - x^0_{ij}|}{\sum_{(i,j)} (x_{ij} + x^0_{ij})}$$

11. Information measure:

$$IM(X^0; X) = \sum_{(i,j) \in S_{X^0}} x_{ij} \ln \frac{x_{ij}}{x^0_{ij}}$$

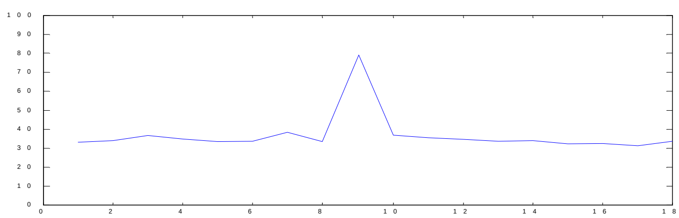
In our problems the matrices compared are always matrices of coefficients.

4 Analysis of the models proposed

In this section we proceed to present and analyse the results of the different comparisons prepared to measure the usefulness of the models proposed. All the applications presented in this work used the IO Table of the Canary Islands for 1985 as a benchmark (ISTAC, 1995). All the models have been solved combining the optimization and computational capabilities of GAMS and MATLAB, respectively.

4.1 Cases formulated

The two cases prepared correspond themselves to case 1 and case 2 described in the previous section. In case 1 we considered the updating of the intermediate requirements matrix, where the row and column totals of this matrix and the vector of effective production are known. Figure 1 shows the percentage changes introduced in the vectors of effective production, total intermediate inputs and outputs, by this order.



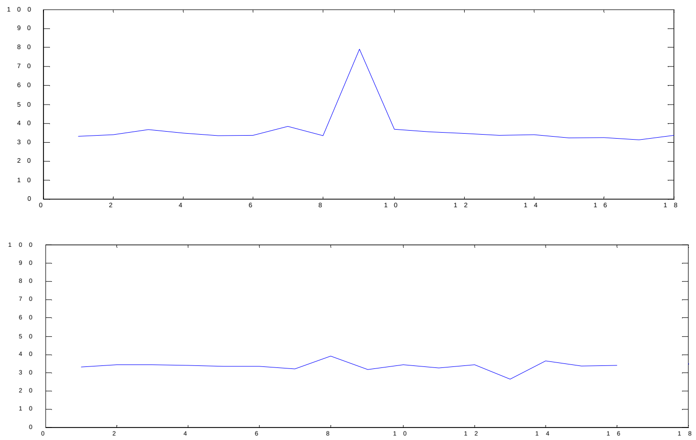


Figure 1. Total percentage change of production and intermediate inputs and outputs.

In this …rst case, we considered the RAS algorithm and the di¤erent problems described in Table 1. Cases 1-1 to 1-3 refer themselves to the adjustment of the technical coe¢ cients of the IO table. In Cases 1-4 to 1-9 we used the same intermediate requirements matrix as in the previous subcases, but the coe¢ cients were calculated against the vectors of total input and output intermediate requirements (column and row sums). Cases 1-4 and 1-5 deal only with column coe¢ cients while cases 1-6 to 1-9 combine the use of column and row coe¢ cients. Our adjustment proposal is included only in cases 1-2, 1-3, 1-5, 1-7, 1-8 and 1-9.

Table 1.: Problems considered in Case 1
for i= 1,2;

	C 1-1	C 1-2	C 1-3	C 1-4	C 1-5	C 1-6	C 1-7	C 1-8	C 1-9
¼$_{i1}$	1	1	1	0	0	0	0	0	0
¼$_{i2}$	0	0	1	0	0	0	0	0	0
¼$_{i3}$	0	1	1	0	0	0	0	0	0
¼$_{i4}$	0	0	0	0	0	1	1	1	1
¼$_{i5}$	0	0	0	0	0	0	1	0	1
¼$_{i6}$	0	0	0	1	1	1	1	1	1
¼$_{i7}$	0	0	0	0	1	0	0	1	1

In case 2, we proceed to update a matrix that includes the vector of intermediate outputs and all the elements of the …nal demand. We impose known values for private and public consumption and exports to the rest of the world.

The programm obtains the vectors of intermediate outputs, investment (…xed and inventory changes) and the vector of total resources as the sum of the different elements considered. Table 2 shows the values of the weights that de…ne the di¤erent problems solved in relation to the second case. Figure 2 shows the percentage changes introduced in the vectors of private consumption and exports, by this order. The value of public consumption was increased in a 45%.

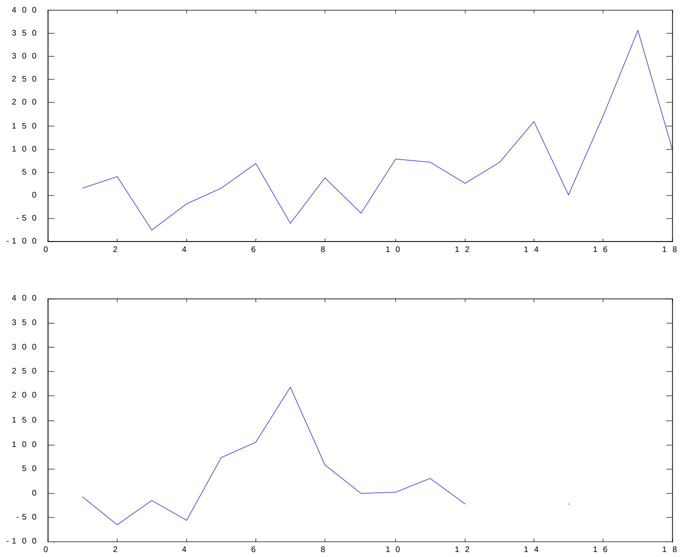


Figure 2. Total percentage change of private consumption and exports.

Table 2.: Problems considered in Case 2
for i= 1,2;

	C 2-1	C 2-2	C 2-3	C 2-4	C 2-5	C 2-6	C 2-7	C 2-8
¼i4	1	0	1	0	1	1	1	1
¼i5	0	0	1	0	0	1	0	1
¼i6	0	1	0	1	1	1	1	1
¼i7	0	0	0	1	0	1	1	0
¼32	1	0	1	0	1	1	1	1
¼33	0	1	0	1	1	1	1	1

4.2 Results

Appendix A includes the tables of the results obtained from our di¤erent problems. The results for each of the problems are presented in two types of tables, tables A and B. Tables A show the position obtained by each of the methods used according to the comparison measures described in section 3. The di¤erent methods are positioned in increasing order according to the value of the distance measure considered. Tables B show the values of the distance measures obtained by each of these methods.

Tables 1A and 1B report the results obtained for cases 1-1 to 1-3. The methods proposed in this paper clearly provide better values when the distance measure re‡ects relative di¤erences, respecting therefore better the previous relative structure of the technical coe¢cients. In the cases where RAS gets better positions, the values are not signi…cantly di¤erent from those achieved by the other methods. Figures 3 and 4 show the relative change in the technical coef…cients obtained from RAS and method G1, respectively. Our method clearly tends to globally maintain the previous relative structure of the technical coef…cients, even if for some cells the relative change is higher than the one shown by RAS

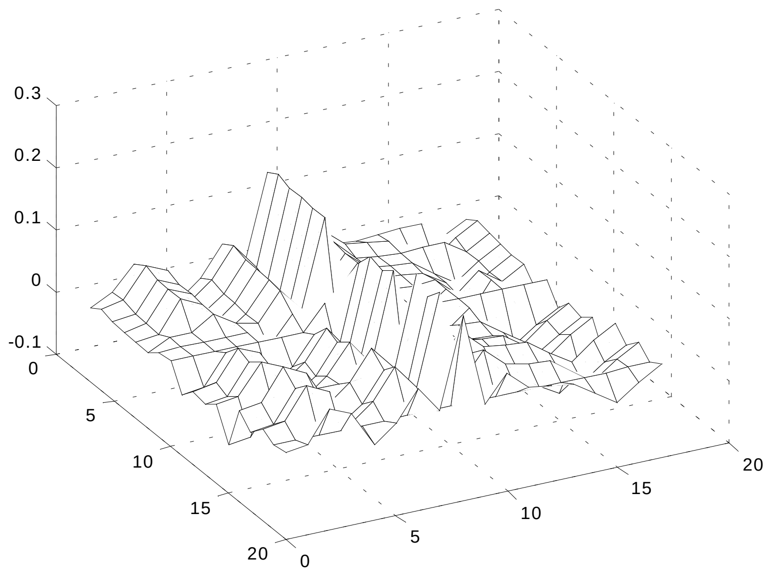


Figure 3. Relative change in the technical coe¢cients. Ras algorithm.

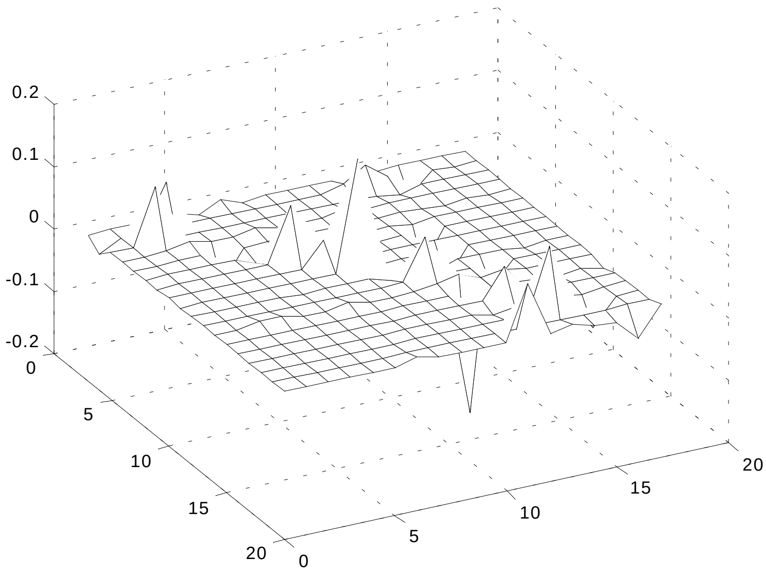


Figure 4. Relative change in the technical coe¢cients. Case 1-2. Model G1.

Tables 2A and 2B report the results obtained in cases 1-4 through 1-9. Since the RAS algorithm functioned altering the column coe¢cients, the most reasonable comparison should be done between RAS and our methods in case 1-5, where our proposals show normally better results than RAS. Obviously, the IM measure should give better results for RAS. Measures MXAD and MXRD reveal the same situation described in cases 1-1 to 1-3 where for some cells the relative change was higher than the one shown by RAS. Figures 5 and 6 show the relative change in the technical coe¢cients obtained from RAS and method G1, respectively. Our method clearly maintains the tendency to reproduce the previous technical coe¢cients, with the exception of some cells where the relative change is higher than the one shown by RAS.

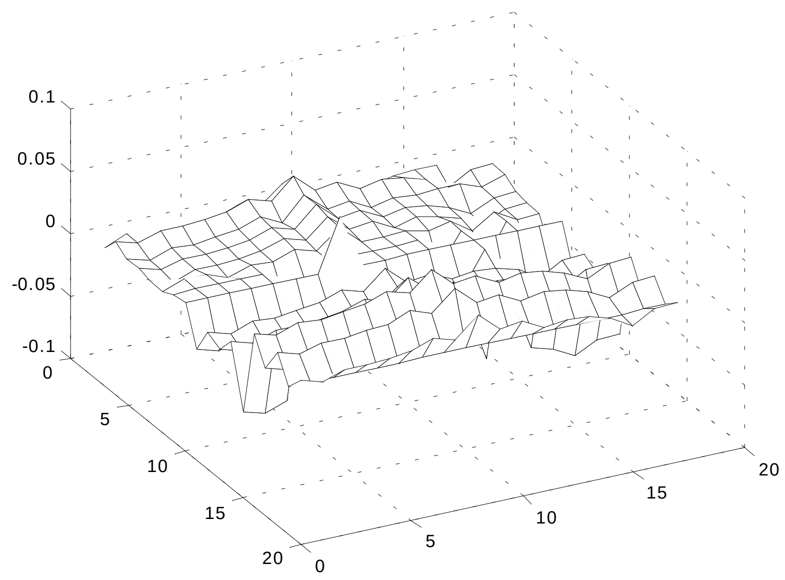


Figure 5. Relative change in the column coe¢cients RAS algorithm.

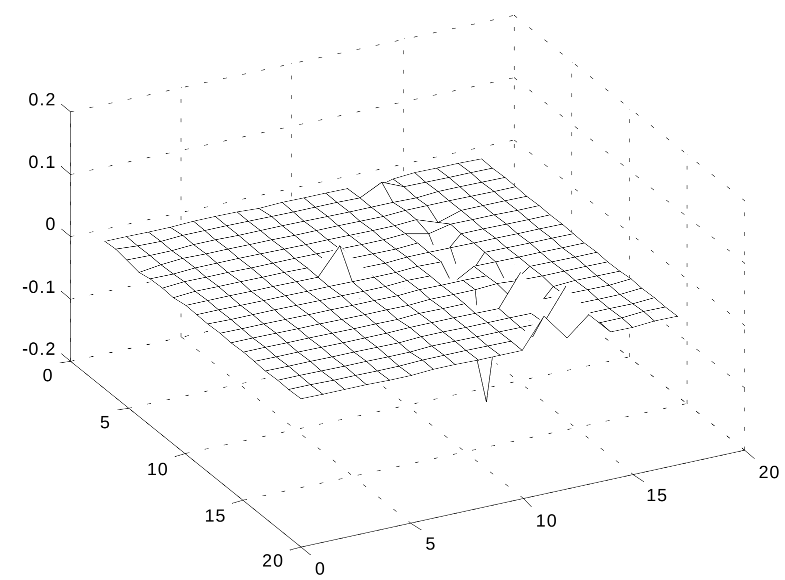


Figure 6. Relative change in the column coe¢cients. Case 1-5. Model G1.

Tables 4A to 5B, report the results obtained for all the models included in our Case 2. They follow a similar structure as the preceeding ones, and the conclusions to be extracted are also equally similar. In this case the bad results achieved by the information theory criterium may stem from the fact that it was formulated under similar bounding conditions to those imposed to the other approaches. Changing these bounds would probably generate better results. However, comparing the three methods under similar circumstances, allow us to observe that our two formulations obtain better results than those achieved by the entropy measure.

Figures 7 and 8. show the relative change in the column coe¢cients obtained from methiods G2 and G3, respectively. Taking into account the di¤erence in scaling of both pictures, our method clearly maintains the tendency to avoid concentrating the changes in some cells, what clearly has not been achieved by the simple entropy formulation.

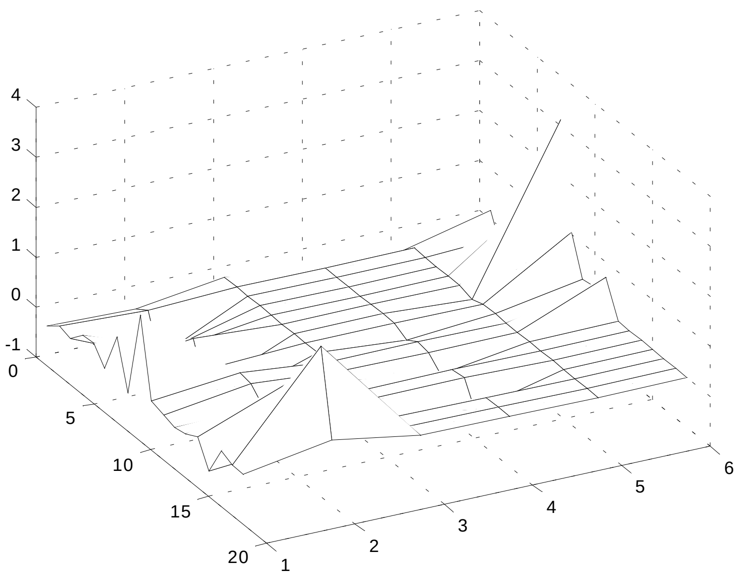


Figure 7. Relative change in the column coe¢cient. Case 2-6. Model G2.

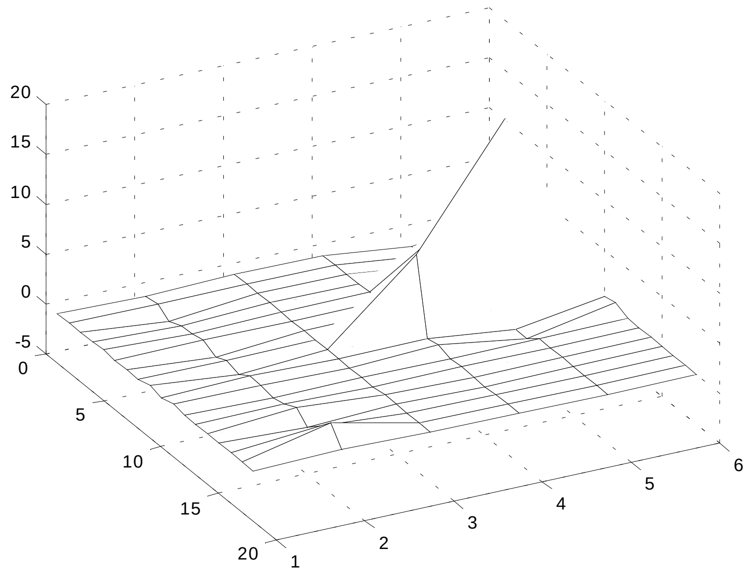


Figure 8. Relative change in the column coe¢cient. Case 2-6. Model G3.

5 Conclusions

In this paper we have proposed new formulations for the updating and adjustment problem of economic accounts. The preliminary results allow us some optimism about its usefulness. However many more experiments have to be implemented before achieving any de…nite conclusions.

Next steps in the near future will include testing these approaches in a more broad variety of scenarios, like allowing changes to the initially …xed vectors, including imports, and trying to compare and combine our methods with those presented by authors like Robinson, Cattaeno and El-Said (1998).

Appendix A

Table 1A
Estimation statistics. Case 1-1 to Case 1-3
Position obtained by each of the methods used

CASE	POSITION	STE	MAD	MRD	TII	RMSE	RMSRE	WAD	IM	MXAD	MXRD	CC
1-1	1	RAS	RAS	G2	RAS	RAS	G2	RAS	RAS	RAS	RAS	G2
	2	G2	G2	G1	G2	G2	G1	G2	G2	G2	G2	G1
	3	G1	G1	RAS	G1	G1	RAS	G1	G1	G1	G1	RAS
1-2	1	RAS	RAS	G1	RAS	RAS	G1	RAS	RAS	RAS	RAS	G1
	2	G2	G2	G2	G2	G2	G2	G2	G1	G1	G1	G2
	3	G1	G1	RAS	G1	G1	RAS	G1	G2	G2	G2	RAS
1-3	1	RAS	RAS	G2	RAS	RAS	G2	RAS	RAS	RAS	RAS	G1
	2	G2	G2	G1	G2	G2	RAS	G2	G2	G2	G2	G2
	3	G1	G1	RAS	G1	G1	G1	G1	G1	G1	G1	RAS

Table 1B
Estimation statistics. Case 1-1 to Case 1-3
Values obtained by each of the methods used

CASE	POSITION	STE	MAD	MRD	TII	RMSE	RMSRE	WAD	IM	MXAD	MXRD	CC
1-1	1	0.0230	0.0006	0.0053	0.0285	0.0001	0.0011	0.0042	0.0669	0.0298	0.1679	0.9989
	2	0.0236	0.0006	0.0065	0.0416	0.0002	0.0013	0.0053	0.0698	0.0474	0.1826	0.9990
	3	0.0239	0.0006	0.0234	0.0450	0.0002	0.0022	0.0063	0.0703	0.0475	0.1908	0.9995
1-2	1	0.0230	0.0006	0.0047	0.0285	0.0001	0.0012	0.0042	0.0669	0.0298	0.1679	0.9988
	2	0.0241	0.0006	0.0047	0.0467	0.0002	0.0012	0.0065	0.0705	0.0475	0.2052	0.9988
	3	0.0241	0.0006	0.0234	0.0467	0.0002	0.0022	0.0065	0.0705	0.0475	0.2052	0.9995
1-3	1	0.0230	0.0006	0.0068	0.0285	0.0001	0.0013	0.0042	0.0669	0.0298	0.1679	0.9820
	2	0.0239	0.0006	0.0188	0.0423	0.0002	0.0022	0.0054	0.0703	0.0475	0.2122	0.9990
	3	0.0846	0.0022	0.0234	0.1816	0.0008	0.0058	0.0159	0.3147	0.1671	1.1302	0.9995

Table 2A
Estimation statistics. Case 1-4 to Case 1-9: column coefficients.
Position obtained by each of the methods used

CASE	POSITION	STE	MAD	MRD	TII	RMSE	RMSRE	WAD	IM	MXAD	MXRD	CC
1-4	1	G2	G2	G2	G1	G1	G1	G2	RAS	RAS	RAS	RAS
	2	G1	G1	G1	G2	G2	G2	G1	G1	G1	G1	G2
	3	RAS	RAS	RAS	RAS	RAS	RAS	RAS	G2	G2	G2	G1
1-5	1	G1	G1	G1	G1	G1	G1	G1	RAS	RAS	RAS	G2
	2	G2	G2	G2	RAS	RAS	G2	G2	G1	G1	G1	RAS
	3	RAS	RAS	RAS	G2	G2	RAS	RAS	G2	G2	G2	G1
1-6	1	RAS	RAS	RAS	RAS	RAS	RAS	RAS	RAS	RAS	RAS	G2
	2	G1	G1	G1	G2	G2	G1	G1	G1	G2	G1	G1
	3	G2	G2	G2	G1	G1	G2	G2	G2	G1	G2	RAS
1-7	1	RAS	RAS	RAS	RAS	RAS	RAS	RAS	RAS	RAS	RAS	G1
	2	G2	G2	G2	G2	G2	G2	G2	G2	G2	G2	G2
	3	G1	G1	G1	G1	G1	G1	G1	G1	G1	G1	RAS
1-8	1	G1	G1	G1	G1	G1	G1	G1	G1	G1	RAS	G2
	2	G2	G2	G2	RAS	RAS	RAS	RAS	RAS	RAS	G1	RAS
	3	RAS	RAS	RAS	G2	G2	G2	G2	G2	G2	G2	G1
1-9	1	G1	G1	G1	RAS	RAS	RAS	G1	RAS	RAS	RAS	G2
	2	RAS	RAS	RAS	G1	G1	G1	RAS	G1	G1	G1	G1
	3	G2	G2	G2	G2	G2	G2	G2	G2	G2	G2	RAS

Table 2B
Estimation statistics. Case 1-4 to Case 1-9: column coefficients
Values obtained by each of the methods used

CASE	POSITION	STE	MAD	MRD	TII	RMSE	RMSRE	WAD	IM	MXAD	MXRD	CC
1-4	1	0.0025	0.0001	0.0019	0.0051	0.0000	0.0007	0.0004	0.0008	0.0069	0.0631	1.0000
	2	0.0025	0.0001	0.0019	0.0051	0.0000	0.0007	0.0004	0.0010	0.0078	0.1857	1.0000
	3	0.0063	0.0003	0.0103	0.0061	0.0000	0.0009	0.0015	0.0010	0.0078	0.1857	1.0000
1-5	1	0.0025	0.0001	0.0019	0.0050	0.0000	0.0006	0.0004	0.0008	0.0069	0.0631	1.0000
	2	0.0034	0.0002	0.0024	0.0061	0.0000	0.0008	0.0007	0.0009	0.0073	0.1654	1.0000
	3	0.0063	0.0003	0.0103	0.0064	0.0000	0.0009	0.0015	0.0015	0.0076	0.1785	1.0000
1-6	1	0.0063	0.0003	0.0103	0.0061	0.0000	0.0009	0.0015	0.0008	0.0069	0.0631	0.9997
	2	0.0088	0.0005	0.0150	0.0231	0.0002	0.0028	0.0027	0.0094	0.0502	0.2609	0.9997
	3	0.0088	0.0005	0.0151	0.0231	0.0002	0.0028	0.0027	0.0094	0.0502	0.2609	1.0000
1-7	1	0.0063	0.0003	0.0103	0.0061	0.0000	0.0009	0.0015	0.0008	0.0069	0.0631	0.9993
	2	0.0154	0.0009	0.0180	0.0346	0.0003	0.0029	0.0047	0.0154	0.0749	0.2571	0.9993
	3	0.0162	0.0009	0.0187	0.0359	0.0003	0.0030	0.0051	0.0165	0.0776	0.2609	1.0000
1-8	1	0.0029	0.0002	0.0024	0.0044	0.0000	0.0005	0.0005	0.0007	0.0057	0.0631	0.9999
	2	0.0057	0.0003	0.0076	0.0061	0.0000	0.0009	0.0015	0.0008	0.0069	0.0875	1.0000
	3	0.0063	0.0003	0.0103	0.0126	0.0001	0.0020	0.0015	0.0045	0.0245	0.2507	1.0000
1-9	1	0.0047	0.0003	0.0090	0.0061	0.0000	0.0009	0.0010	0.0008	0.0069	0.0631	0.9994
	2	0.0063	0.0003	0.0103	0.0064	0.0000	0.0016	0.0015	0.0014	0.0110	0.2138	1.0000
	3	0.0149	0.0008	0.0157	0.0342	0.0003	0.0028	0.0046	0.0150	0.0744	0.2539	1.0000

Table 3A
Estimation statistics. Case 1-4 to Case 1-9: row coefficients.
Position obtained by each of the methods used

CASE	POSITION	STE	MAD	MRD	TII	RMSE	RMSRE	WAD	IM	MXAD	MXRD	CC
1-4	1	RAS	RAS	G1	RAS	RAS	G1	RAS	RAS	RAS	G1	G1
	2	G1	G1	G2	G1	G1	G2	G1	G1	G1	G2	G2
	3	G2	G2	RAS	G2	G2	RAS	G2	G2	G2	RAS	RAS
1-5	1	RAS	RAS	G1	RAS	RAS	G1	RAS	RAS	RAS	G1	G2
	2	G1	G1	G2	G1	G1	G2	G2	G1	G1	G2	G1
	3	G2	G2	RAS	G2	G2	RAS	G1	G2	G2	RAS	RAS
1-6	1	RAS	RAS	G2	RAS	RAS	G2	RAS	RAS	RAS	RAS	G1
	2	G2	G2	G1	G2	G2	G1	G1	G2	G2	G1	G2
	3	G1	G1	RAS	G1	G1	RAS	G2	G1	G1	G2	RAS
1-7	1	RAS	RAS	G1	RAS	RAS	G2	RAS	RAS	RAS	RAS	G2
	2	G1	G1	G2	G1	G1	G1	G1	G1	G1	G2	G1
	3	G2	G2	RAS	G2	G2	RAS	G2	G2	G2	G1	RAS
1-8	1	RAS	RAS	G2	RAS	RAS	G2	RAS	RAS	RAS	G1	G2
	2	G1	G1	G1	G1	G1	G1	G1	G1	G1	RAS	G1
	3	G2	G2	RAS	G2	G2	RAS	G2	G2	G2	G2	RAS
1-9	1	RAS	RAS	G2	RAS	RAS	G2	RAS	RAS	RAS	G1	G2
	2	G1	G1	G1	G1	G1	G1	G1	G1	G1	RAS	G1
	3	G2	G2	RAS	G2	G2	RAS	G2	G2	G2	G2	RAS

Table 3B
Estimation statistics. Case 1-4 to Case 1-9: row coefficients
Values obtained by each of the methods used

CASE	POSITION	STE	MAD	MRD	TII	RMSE	RMSRE	WAD	IM	MXAD	MXRD	CC
1-4	1	0.0082	0.0004	0.0254	0.0097	0.0001	0.0040	0.0019	0.0033	0.0126	0.3609	0.9997
	2	0.0162	0.0008	0.0254	0.0232	0.0002	0.0040	0.0045	0.0081	0.0379	0.3609	0.9997
	3	0.0162	0.0008	0.0263	0.0232	0.0002	0.0043	0.0045	0.0081	0.0379	0.3923	0.9999
1-5	1	0.0082	0.0004	0.0252	0.0097	0.0001	0.0040	0.0019	0.0033	0.0126	0.3609	0.9997
	2	0.0154	0.0008	0.0257	0.0214	0.0002	0.0041	0.0040	0.0071	0.0317	0.3609	0.9997
	3	0.0159	0.0008	0.0263	0.0228	0.0002	0.0043	0.0044	0.0084	0.0357	0.3923	0.9999
1-6	1	0.0082	0.0004	0.0083	0.0097	0.0001	0.0019	0.0019	0.0033	0.0126	0.3923	0.9999
	2	0.0094	0.0005	0.0083	0.0149	0.0001	0.0019	0.0029	0.0051	0.0173	0.4599	0.9999
	3	0.0094	0.0005	0.0263	0.0149	0.0001	0.0043	0.0029	0.0051	0.0173	0.4599	0.9999
1-7	1	0.0082	0.0004	0.0054	0.0097	0.0001	0.0017	0.0019	0.0033	0.0126	0.3923	0.9998
	2	0.0106	0.0006	0.0063	0.0207	0.0002	0.0017	0.0038	0.0071	0.0262	0.5085	0.9998
	3	0.0113	0.0006	0.0263	0.0211	0.0002	0.0043	0.0039	0.0072	0.0291	0.5137	0.9999
1-8	1	0.0029	0.0002	0.0024	0.0044	0.0000	0.0005	0.0005	0.0007	0.0057	0.0631	0.9999
	2	0.0057	0.0003	0.0076	0.0061	0.0000	0.0009	0.0015	0.0008	0.0069	0.0875	1.0000
	3	0.0063	0.0003	0.0103	0.0126	0.0001	0.0020	0.0015	0.0045	0.0245	0.2507	1.0000
1-9	1	0.0082	0.0004	0.0091	0.0097	0.0001	0.0017	0.0019	0.0033	0.0126	0.3826	0.9998
	2	0.0103	0.0005	0.0159	0.0149	0.0001	0.0026	0.0032	0.0044	0.0168	0.3923	0.9999
	3	0.0120	0.0006	0.0263	0.0201	0.0002	0.0043	0.0039	0.0070	0.0255	0.5075	0.9999

Table 4A
Estimation statistics. Case 2-1 to Case 2-8: row coefficients.
Position obtained by each of the methods used

CASE	POSITION	STE	MAD	MRD	TII	RMSE	RMSRE	WAD	IM	MXAD	MXRD	CC
2-1	1	G1	G1	G1	G1	G1	G1	G2	G1	G1	G1	G3
	2	G2	G2	G2	G2	G2	G2	G1	G2	G2	G2	G2
	3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G1
2-2	1	G1	G1	G1	G3	G3	G1	G3	G1	G1	G1	G2
	2	G3	G3	G2	G1	G1	G2	G1	G2	G3	G2	G3
	3	G2	G2	G3	G2	G2	G3	G2	G3	G2	G3	G1
2-3	1	G1	G1	G2	G1	G1	G1	G1	G1	G1	G1	G3
	2	G2	G2	G1	G2	G2	G2	G2	G2	G2	G2	G2
	3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G1
2-4	1	G1	G1	G1	G1	G1	G1	G3	G1	G2	G1	G3
	2	G2	G2	G2	G2	G2	G2	G2	G2	G1	G2	G2
	3	G3	G3	G3	G3	G3	G3	G1	G3	G3	G3	G1
2-5	1	G2	G2	G1	G2	G2	G1	G2	G2	G3	G1	G3
	2	G1	G1	G2	G1	G1	G2	G1	G1	G2	G2	G1
	3	G3	G3	G3	G3	G3	G3	G3	G3	G1	G3	G2
2-6	1	G1	G1	G2	G1	G1	G2	G1	G1	G3	G1	G3
	2	G2	G2	G1	G2	G2	G1	G2	G2	G1	G2	G2
	3	G3	G3	G3	G3	G3	G3	G3	G3	G2	G3	G1
2-7	1	G2	G2	G2	G2	G2	G2	G2	G2	G3	G1	G3
	2	G1	G1	G1	G1	G1	G1	G3	G1	G1	G2	G1
	3	G3	G3	G3	G3	G3	G3	G1	G3	G2	G3	G2
2-8	1	G1	G1	G1	G2	G2	G1	G1	G1	G3	G1	G3
	2	G2	G2	G2	G1	G1	G2	G2	G2	G2	G2	G1
	3	G3	G3	G3	G3	G3	G3	G3	G3	G1	G3	G2

Table 4A
Estimation statistics. Case 2-1 to Case 2-8: row coefficients.
Values obtained by each of the methods used

CASE	POSITION	STE	MAD	MRD	TII	RMSE	RMSRE	WAD	IM	MXAD	MXRD	CC
2-1	1	0.2154	0.0359	0.1309	0.3641	0.0106	0.0367	0.0956	1.6976	0.5820	2.2735	0.8659
	2	0.2154	0.0359	0.1309	0.3641	0.0106	0.0367	0.0956	1.6976	0.5820	2.2735	0.9084
	3	0.3559	0.0593	4.4630	0.4235	0.0124	2.2880	0.1227	8.1548	0.6416	199.9887	0.9084
2-2	1	0.4800	0.0800	0.3968	0.4975	0.0145	0.0599	0.1582	3.3607	0.5233	2.2578	0.7457
	2	0.4940	0.0823	0.6369	0.4989	0.0146	0.1078	0.2073	5.9560	0.5243	4.7134	0.8061
	3	0.6438	0.1073	7.9078	0.6507	0.0190	2.7349	0.2748	13.1568	0.5896	199.9887	0.8382
2-3	1	0.2128	0.0355	0.1309	0.3555	0.0104	0.0364	0.0941	1.6201	0.5575	2.2735	0.8659
	2	0.2154	0.0359	0.1325	0.3641	0.0106	0.0367	0.0956	1.6976	0.5820	2.2735	0.9084
	3	0.3559	0.0593	4.4630	0.4235	0.0124	2.2880	0.1227	8.1548	0.6416	199.9887	0.9123
2-4	1	0.4758	0.0793	0.4866	0.4814	0.0141	0.0785	0.1582	3.3761	0.5183	3.8840	0.8061
	2	0.4923	0.0820	0.8090	0.4826	0.0141	0.1420	0.1895	3.8005	0.5185	5.6369	0.8408
	3	0.4940	0.0823	7.9078	0.4975	0.0145	2.7349	0.1928	13.1568	0.5243	199.9887	0.8451
2-5	1	0.2228	0.0371	0.1647	0.3319	0.0097	0.0381	0.0965	1.4638	0.4262	2.2734	0.9059
	2	0.2228	0.0371	0.1647	0.3319	0.0097	0.0381	0.0965	1.4638	0.4741	2.2734	0.9224
	3	0.3091	0.0515	3.0034	0.3565	0.0104	2.1210	0.1078	4.2802	0.4741	220.4765	0.9224
2-6	1	0.2164	0.0361	0.1647	0.3125	0.0091	0.0381	0.0925	1.3418	0.4262	2.2698	0.9059
	2	0.2228	0.0371	0.1737	0.3318	0.0097	0.0406	0.0965	1.4633	0.4421	2.2734	0.9225
	3	0.3091	0.0515	3.0034	0.3565	0.0104	2.1210	0.1078	4.2802	0.4740	220.4765	0.9308
2-7	1	0.2228	0.0371	0.1647	0.3318	0.0097	0.0381	0.0965	1.4635	0.4262	2.2663	0.9059
	2	0.2606	0.0434	0.2118	0.3333	0.0097	0.0420	0.1078	1.5120	0.4415	2.2734	0.9210
	3	0.3091	0.0515	3.0034	0.3565	0.0104	2.1210	0.1111	4.2802	0.4740	220.4765	0.9225
2-8	1	0.2109	0.0351	0.1468	0.3319	0.0097	0.0369	0.0928	1.4591	0.4262	2.2651	0.9059
	2	0.2228	0.0371	0.1647	0.3344	0.0098	0.0381	0.0965	1.4637	0.4741	2.2734	0.9218
	3	0.3091	0.0515	3.0034	0.3565	0.0104	2.1210	0.1078	4.2802	0.4914	220.4765	0.9225

Table 5A
Estimation statistics. Case 2-1 to Case 2-8: column coefficients.
Position obtained by each of the methods used

CASE	POSITION	STE	MAD	MRD	TII	RMSE	RMSRE	WAD	IM	MXAD	MXRD	CC
2-1	1	G1	G1	G1	G1	G1	G1	G1	G1	G1	G1	G3
	2	G2	G2	G2	G2	G2	G2	G2	G2	G2	G2	G1
	3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G2
2-2	1	G2	G2	G1	G2	G1	G1	G2	G2	G1	G1	G3
	2	G1	G1	G2	G1	G2	G2	G1	G1	G2	G2	G2
	3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G1
2-3	1	G1	G1	G1	G1	G1	G1	G1	G1	G1	G1	G3
	2	G2	G2	G2	G2	G2	G2	G2	G2	G2	G2	G2
	3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G1
2-4	1	G2	G2	G2	G1	G1	G1	G2	G2	G1	G1	G3
	2	G1	G1	G1	G2	G2	G2	G1	G1	G2	G2	G1
	3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G2
2-5	1	G1	G1	G2	G2	G2	G2	G1	G2	G1	G1	G3
	2	G2	G2	G1	G1	G1	G1	G2	G1	G2	G2	G1
	3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G2
2-6	1	G2	G2	G1	G2	G2	G1	G2	G1	G1	G1	G3
	2	G1	G1	G2	G1	G1	G2	G1	G2	G2	G2	G1
	3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G2
2-7	1	G1	G1	G1	G1	G1	G1	G1	G1	G1	G1	G3
	2	G2	G2	G2	G2	G2	G2	G2	G2	G2	G2	G2
	3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G1
2-8	1	G2	G2	G2	G2	G2	G2	G2	G2	G1	G1	G3
	2	G1	G1	G1	G1	G1	G1	G1	G1	G2	G2	G1
	3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G3	G2

Table 5B
Estimation statistics. Case 2-1 to Case 2-8: column coefficients.
Values obtained by each of the methods used

CASE	POSITION	STE	MAD	MRD	TII	RMSE	RMSRE	WAD	IM	MXAD	MXRD	CC
2-1	1	0.2135	0.0119	0.3028	0.1991	0.0029	0.0616	0.0387	0.3793	0.2379	3.7207	0.9367
	2	0.2135	0.0119	0.3028	0.1991	0.0029	0.0616	0.0387	0.3793	0.2379	3.7207	0.9771
	3	0.2874	0.0160	0.3856	0.3472	0.0050	0.1188	0.0741	0.7824	0.3920	11.3085	0.9771
2-2	1	0.1268	0.0070	0.1640	0.1800	0.0026	0.0449	0.0216	0.2464	0.2379	3.1142	0.9684
	2	0.1268	0.0070	0.1640	0.1800	0.0026	0.0449	0.0216	0.2464	0.2379	3.1142	0.9810
	3	0.2112	0.0117	0.3398	0.2319	0.0033	0.1168	0.0405	0.4479	0.2379	11.3085	0.9810
2-3	1	0.2086	0.0116	0.2879	0.1973	0.0028	0.0558	0.0379	0.3523	0.2379	3.1142	0.9367
	2	0.2135	0.0119	0.3028	0.1991	0.0029	0.0616	0.0387	0.3793	0.2379	3.7207	0.9771
	3	0.2874	0.0160	0.3856	0.3472	0.0050	0.1188	0.0741	0.7824	0.3920	11.3085	0.9776
2-4	1	0.1268	0.0070	0.1640	0.1800	0.0026	0.0449	0.0216	0.2464	0.2379	3.1142	0.9684
	2	0.1268	0.0070	0.1640	0.1800	0.0026	0.0449	0.0216	0.2464	0.2379	3.1142	0.9810
	3	0.2112	0.0117	0.3398	0.2319	0.0033	0.1168	0.0405	0.4479	0.2379	11.3085	0.9810
2-5	1	0.1530	0.0085	0.2175	0.1825	0.0026	0.0484	0.0251	0.2761	0.2379	3.1142	0.8670
	2	0.1530	0.0085	0.2175	0.1825	0.0026	0.0484	0.0251	0.2761	0.2379	3.1142	0.9805
	3	0.3801	0.0211	0.5610	0.4777	0.0069	0.2058	0.0755	1.9311	0.4687	19.3607	0.9805
2-6	1	0.1530	0.0085	0.2134	0.1825	0.0026	0.0478	0.0251	0.2733	0.2379	3.1142	0.8670
	2	0.1560	0.0087	0.2175	0.1827	0.0026	0.0484	0.0270	0.2761	0.2379	3.1142	0.9805
	3	0.3801	0.0211	0.5610	0.4777	0.0069	0.2058	0.0755	1.9311	0.4687	19.3607	0.9805
2-7	1	0.1361	0.0076	0.1826	0.1806	0.0026	0.0465	0.0226	0.2564	0.2379	3.1142	0.8670
	2	0.1530	0.0085	0.2175	0.1825	0.0026	0.0484	0.0251	0.2761	0.2379	3.1142	0.9805
	3	0.3801	0.0211	0.5610	0.4777	0.0069	0.2058	0.0755	1.9311	0.4687	19.3607	0.9809
2-8	1	0.1530	0.0085	0.2175	0.1825	0.0026	0.0484	0.0251	0.2761	0.2379	3.1142	0.8670
	2	0.1823	0.0101	0.2449	0.1878	0.0027	0.0499	0.0298	0.3075	0.2379	3.1142	0.9794
	3	0.3801	0.0211	0.5610	0.4777	0.0069	0.2058	0.0755	1.9311	0.4687	19.3607	0.9805

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