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Analysis of the calculation methodologies of National Accounts in the I-O framework: consistency, constraints and independent estimation.

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1. Introduction

National accounts figures are obtained by using a model based on a complex system of statistical surveys on enterprises, institutions, household, etc. These data sources could be biased by several factors: missing responses (partial or total), sample errors, non sample errors, problems related to the timeliness of data collection, etc[1]. Furthermore, the same data sources could be treated by using different methodologies in order to estimate different aggregates. Once accounted for that, the initial estimates of each branch of economic activity could not satisfy the balance constraint imposed by the account of resources and uses and, in many cases, a discrepancy between the supply side and the demand side figures may be found. This paper demonstrates that these discrepancies do not represent just a problem to avoid, but they include an information set very important for national account statisticians.

In order to eliminate these discrepancies, the Italian National Account Department uses a balancing method based on a symmetric input-output (hereinafter I-O) scheme[2], that allows to switch from an initial estimation system free from accounting constraints to a balanced system. The redistribution of differences between aggregates was made on the basis of a priori information on the degree of relative reliability attributed in terms of quality, exhaustiveness of statistical sources and accuracy of the calculation techniques adopted in the estimation. The balancing process implied the correction of entries considered as less reliable, mostly by redistributing accounting residuals.

In this work we use an I-O scheme mostly as a tool to analyse and validate the estimation process of national accounts figures. In particular, the validation procedure based on the comparison of different estimates of the same aggregates is analysed. Such methodology is currently applied to validate each item of national accounts, but in this paper we try to generalise such an approach in order to consider the account of the resources and uses as a whole.

On the basis of balance constraint, that imposes the equality of each row total with each column total of the I-O matrix, we can consider the total of resources and the total of uses as the same aggregate. These totals are the result of an estimation process of information on both the demand and the supply side. Then, the validation process is based on the comparison of data sources and methodology used to set out resources and uses.

According to this definition, in order to use the I-O scheme to validate the national account figure it is necessary to verify the actual independence of supply and use estimates. Such independence will be checked both on data sources and on statistical methodologies.

In the next paragraph we describe the general framework of the I-O scheme used, whilst the following two sections are devoted to the formalisation of supply and demand side estimation procedures. In paragraph 5 we discuss the proprieties of the balance constraint and the last section reports some concluding remarks.

2. The general framework

The Italian I-O table is based on the homogeneous production units classification of economic activities. This is a relevant concept which allows for the reconciliation between the supply side (manly based on enterprise data) and the demand side (mainly based on products) estimates.[3]

In this paper the attention is focused on the method to calculate the supply aggregates based on the enterprise surveys[4] and we compare this method with two different methodologies followed to calculate the demand side aggregates[5]: commodity flow[6] and price times quantity approaches.

[1] See Puggioni (2000).
[2] For the specific algorithm used see: Stone, Champernowne, Meade (1942), Nicolardi (1998), Mantegazza and Mastrantonio (2000).
[3] Eurostat (1996).
[4] Further information on supply side data sources and methods are in Pisani (2000).
[5] Further information on supply side data sources and methods are in ISTAT (2000)
[6] See ISTAT (2000a).

A general overview of the terminology is reported in table 1, where a schematic presentation of the I-O table at market prices is shown and where all acronyms used are illustrated.

Table 1 – Schematic presentation of a symmetric input-output table at market prices

branches	1	…	**j**	…	n	Intermediate consumption	Household consumption	Other final consumption	Gross fixed capital formation	Changes in stocks	Export	Total uses
1												
…												
i			**IC_{ij}**			**TII_i**	**THC_i**	**OFC_i**	**$TGCF_i$**	**DST_i**	**EXP_i**	**TUP_i**
…												
n												
Intermediate costs			**TIC_j**									
Value added			**TVA_j**									
Taxes			**TAX^t_j**									
Vat			**VAT^t_j**									
Transfer			**TRA_j**									
Output by product			**DOU_j**									
Imports by product			**IMP^t_j**									
Trade and transport margins			**TTM^t_j**									
Supply by product			**TRP_j**									

n= 101 branches of economic activities; i= row index, i= 1, 2, …, n; j= column index, j= 1, 2, …, n;

3. The supply side estimates

In the first step we analyse the procedure to estimate the intermediate costs used as input in the generic branch j (TIC_j). Such an estimate is based on the data provided by the enterprises classified in the branch j (ICI_j)[7]. Survey data were processed to obtain estimation of per-capita values and they were expanded to the target population of national accounts using the full time equivalent units (ULA_j). In formal terms:

$$TIC_j = \frac{ICI_j}{WI_j} \bullet ULA_j \qquad (3.1)$$

Taking into account the proprieties of the full time equivalent units[8], the transformation of (3.1) allows to achieve two important targets:

- the unit of analysis changes, that is the data classified by enterprise shift into data classified by Local Kind of Activity;
- a relevant share of the hidden economy is considered.

In order to construct an I-O table it is necessary to focus the attention on each single cluster that represents the intermediate consumption purchased by the branch j from the branch i (IC_{ij}). This estimate is performed by decomposing (3.1) using a specific parameter (q) derived from a specific survey on enterprises. The procedure can be written as follows:

[7] The intermediate costs are equal to the purchases of goods and services to be used in the production process, corrected by the changes in stocks of the same kind of goods.

[8] See ISTAT (1993)

$$IC_{i,j} = \frac{ICI_j}{WI_j} \bullet ULA_j \bullet q_{ij} \qquad (3.2)$$

On the basis of (3.2) we can rewrite (3.1) in the form:

$$TIC_j = \sum_{i=1}^{j-1} \frac{ICI_j}{WI_j} \bullet ULA_j \bullet q_{ij} + \frac{ICI_j}{WI_j} \bullet ULA_j \bullet q_{jj} + \sum_{i=j+1}^{n} \frac{ICI_j}{WI_j} \bullet ULA_j \bullet q_{ij} \qquad (3.3)$$

(3.3) allows to highlight the diagonal element (j,j) that is common to the supply and demand side estimates.

The value added at factor costs follows the same procedure of intermediate costs. Also in this case we expand the per capita value provided by enterprises using the full time equivalent units. This takes the form:

$$TVA_j = \frac{TOI_j^t + ORI_j - TIC_j + OST_j}{WI_j} \bullet ULA_j + GSU_j \qquad (3.4)$$

where:
TOI^t = turnover of the specific product sold by enterprises;
ORI = other revenues
OST = change in stocks by enterprise
GSU = subsides on output and products

For the specific target of this paper, it is necessary to give special attention to the turnover. In particular, for a better comparison with the demand side estimates, it is useful to revolve TOI^t into its economic uses. In order to be coherent with the aggregates that will be described in the next paragraph, we will take into account six categories of uses (even if some of them could be equal to zero):

1. intermediate costs;
2. household consumption;
3. other final consumption;
4. gross fixed capital formation;
5. change in stocks;
6. exports.

Where t is referred to the total turnover.

By arranging (3.3) and (3.4) we can write the output by product (DOU_j) as:

$$DOU_j = TIC_j + TVA_j + TAX_j^t - GSU_j + VAT_j^t + TRA_j \qquad (3.5)$$

where:
TAX^t = taxes on output and products;
VAT^t = tax on value added;
TRA = transfers.

Also for TAX and VAT we use the index t as well as for TOI, as it is important to split their values into the six components of uses.

It is now useful to look more in depth at the TRA aggregate in (3.5). It is possible, from this aggregate, to transform the data classified by local KAU into data classified by homogenous production units. By definition TRA could be positive or negative for each branch of economic activity and the sum is equal to zero for the whole economy.

Finally, the supply by product estimation (TRP_j) is given by:

$TRP_j = DOU_j + TTM^t_c + IMP^t_j$ (3.6)

where:
TTM^t = internal trade and transport margins;
IMP^t = imports.

As mentioned above for TAX^t and VAT^t, also TTM^t e IMP^t are theoretically decomposable in the six categories of uses. Furthermore, for TTM, the index c is used for the purpose of underlining that such estimates are based on the information derived from the internal trade and transport branches.

4. The demand side estimates

Following the same approach of the previous paragraph, the intermediate consumption (TII) is considered as the first aggregate. In this case we have to sum by row the data previously summed by column. Therefore, we can rewrite (3.3) simply changing the summation index j in i, that is:

$$TII_i = \sum_{j=1}^{i-1} \frac{ICI_j}{WI_j} \bullet ULA_j \bullet q_{ij} + \frac{ICI_j}{WI_j} \bullet ULA_j \bullet q_{jj} + \sum_{j=i+1}^{n} \frac{ICI_j}{WI_j} \bullet ULA_j \bullet q_{ij} \qquad (4.1)$$

Two important aspects emerge from the comparison between (3.3) and (4.1). On the one hand, it is clear that the total of TII is equal to the total of TIC by definition, that means that the two aggregates are not independent in the total. On the other hand, since this equality is not verified for each branch of economic activity, we can assert that a certain degree of independence at the desegregate level does exist. More particularly, for each branch of economic activity the dependence between the demand side and the supply side estimates is ensured by the diagonal term j,j in (3.3) and i,i in (4.1).

The final demand estimate is based on a wider set of data sources and methods than the supply estimate. For the sake of simplicity, in this paper we refer only to the two most important methods:

1. the commodity flow, that calculates the quantity of goods and services available for the final use;
2. the price times quantity approach, that estimates, with selected quantity and price indicators, the amount of goods and services actually consumed for the final use.

The household consumption is one of the most relevant aggregates from the quantitative point of view. Both the above methods are largely used to estimate the household consumption. The commodity flow approach is quite similar to the supply method; also in this case, per-capita values are expanded to the target population of national account using the full time equivalent units. The starting point is the turnover of the specific product sold (TOI^2) by the enterprise that produces final consumption goods and/or services. As mentioned above, TOI^2 is a share of the total turnover TOI^t included in the value added formula (3.4).

Thus, we divide TOI^2 by the labour input of the same enterprise and then we gross up the per-capita values by multiply for full time equivalent units. As well as in the supply side calculation, the transformation performed using the ULA allows to include a relevant share of the hidden economy in the National Account figures. Furthermore it is necessary to make another adjustment in order to convert the data classified by enterprises in data classified by product. For this purpose, a specific source, that splits up the output of each enterprise in the actual goods produced, is used. On the basis of this source, a parameter is derived, in order to determine the quantity that reassigns the turnover by product (RHC). To complete the evaluation, all other demand components are considered as follows:

$$THC_i = \left(\frac{TOI_j^2}{WI_j} \bullet ULA_j \right) + RHC_i + TAX_j^2 + TTM_c^2 - EXP_i^2 + IMP_j^2 - DSIT_i^2 + VAT_j^2 \quad (4.2a)$$

where:
EXP^2 = household consumption exports;
$DSIT^2$ = household consumption change in stocks of the internal trade enterprises;
VAT^2 = valued tax on household consumption.
j = i
The index c is used in order to underline that such estimates are based on the information derived from the internal trade and transport branches.

If the price times quantity approach is used, (4.2a) becomes:

$$THC_i = PC_i \bullet QC_i \qquad (4.2b)$$

where PC_i e QC_i are consumer prices and consumed quantities respectively.

The other final consumption is estimated by using different sources and methods and, for the specific target of this paper, its estimation procedure is not reported and it will be indicated as OFC.

When the gross fixed capital formation estimate follows the commodity flow approach, its calculation is similar to that of household consumption. Then, we can write:

$$TGCF_i = \left(\frac{TOI_j^4}{WI_j} \bullet ULA_j \right) + RGF_i + TAX_j^4 + TTM_c^4 - EXP_i^4 + IMP_j^4 - DSIT_i^4 + VAT_i^4 \quad (4.3a)$$

where:
RGF = transfers of investments goods;
EXP^3 = exports of investments goods;
$DSIT^3$ = change in stocks of investments goods of internal trade enterprises;
VAT^3 = valued tax on investments goods.

In the case of price times quantity approach, (4.3a) becomes:

$$TGCF_i = PI_i \bullet QI_i \qquad (4.3b)$$

where PC_i e QC_i are investment prices and invested quantities respectively.

Regarding the change in stocks, it is necessary to split up this aggregate in the following two components:

1. change in stocks of the enterprise belonging to the branch j=i;
2. change in stocks of the internal trade branches.

In formal terms:

$$DST_i = \frac{OST_i}{WI_i} \bullet ULA_i + \frac{DSIT_c^t}{WI_c} \bullet ULA_c \qquad (4.4)$$

where index c is referred only to the internal trade branches that sold goods produced by the i branch.

Finally, if we indicate exports of good and services with the acronym EXP^t, we can write the total uses (TUP) formula as:

$$TUP_i = TII_i + THC_i + TOC_i + TGCF_i + DST_i^t + EXP_i^t \qquad (4.5)$$

5. The balance constraint

The proposed methodology is based on the analysis of the principal symmetric I-O balance constraint that imposes the identity between resources (supply) and uses (demand). That can be written as:

$$TRP_j = TUP_i \quad \text{con j = i} \qquad (5.1)$$

Substituting in (5.1) the formulas showed in paragraphs 3 and 4, it is possible to calculate the degree of dependence between the supply side and demand side estimates of each branch of economic activity. On the basis of the arguments showed above, such a degree of dependence will be different if the demand is calculated using the commodity flow or the price times quantity approach.

In the second case, it is easy to verify that the only element of dependence between demand and supply of each branch is the diagonal element of intermediate costs underlined in both (3.3) and (4.1). Hence, we can assert that, in this case, the two estimates are independent and that the national account balancing procedure is a successful validation instrument.

The case of commodity flow approach is quite different: besides a certain degree of dependence in data sources, there is independence with respect to the method.

In order to verify the independence, it is necessary to rearrange (5.1) using (3.3) – (3.6) and (4.1) – (4.5). Simplifying the common terms, it is possible to rewrite the balance constraint (5.1) of the generic column j and of the corresponding row i, as:

$$\frac{(TOI_j^1 + TOI_j^3 + TOI_j^5 + TOI_j^6) + ORI_j}{WI_j} \bullet ULA_j + TAX_j^1 + TAX_j^3 + TAX_j^6 + VAT_j^2 + VAT_j^3 +$$

$$+ TRA_j + TTM_c^1 + TTM_c^3 + TTM_c^6 + IMP_j^1 + IMP_j^3 + IMP_j^5 =$$

$$= \sum_{j=1}^{i-1} \frac{ICI_j}{WI_j} \bullet ULA_j \bullet q_{ij} + \frac{ICI_j}{WI_j} \bullet ULA_j \bullet q_{ij} + \sum_{j=i+1}^{n} \frac{ICI_j}{WI_j} \bullet ULA_j \bullet q_{ij} + RHC_i + TOC_i + RGF_i +$$

$$+ \frac{DSIT_i^1 + DSIT_i^3 + DSIT_i^6}{WI_c} \bullet ULA_c + EXP_i^1 + EXP_i^2 \qquad (5.2)$$

By definition, in (5.2) all supply and demand components that are included in the commodity flows calculation are annulled. Thus, the ratio between the annulled components and the total of resources/uses gives the measure of the dependence between supply and demand.

Concerning the independent components, on the supply side, in addition to the turnover (TOI) and to imports (IMP), we can find the trade and transportation margins (TTM), the aggregates related to tax (TAX e VAT) that are implicitly included in the corresponding items of intermediate uses, other final consumption and exports.

On the demand side, instead, the rule of intermediate costs (ICI), other final consumption (TOC), change in stocks of the internal trade (DSIT) and exports (EXP) is shown.

However, (5.2) highlights that, in order to avoid the discrepancy between resources and uses, it is necessary to balance the supply side components devoted to intermediate uses and to other final uses with the other final consumption, the change in stocks of the internal trade enterprises (different by consumer and investment goods) and exports.

Furthermore, a very important role in determining the independence between resources and uses is played by the correction adopted to transform the supply data according to the homogeneous production units classification and the demand data according to product. In order to point out this aspect we can assume that all the use components are estimated by using the commodity flow approach. In this case (5.2) becomes:

$$\frac{ORI_j}{WI_j} \bullet ULA_j + TRA = RII_i + RHC_i + ROC_i + RGF_i + RDSIT_i \qquad (5.3)$$

In (5.3) the discrepancy is only due to the correction items and to the other revenues (that are mainly referred to the kind of activities different from that specific of branch j).

5. Conclusions

The point of view of the statistician that builds the I-O table can be quite different from that of the economic analyst. Actually, on the one hand, the identity between resources and uses is an unavoidable necessity for the analyst, whilst, on the other hand, the statistician interest is mainly focused on the discrepancy between these aggregates.

The information described by the discrepancy between demand and supply is very important for the data maker since its existence is a guarantee that the estimates are independent but, at the some time, an high discrepancy shows an imbalance in the data production process.

This paper represents a first effort to formalise the data production process of the resources-uses account. The strengths of such a formalisation are:

1. it makes more transparent the data production process, underlining the interactions between the demand and supply methodologies and data sources;
2. it provides further information to the users, permitting the evaluation of the properties of the data generating model;
3. it is useful for the data maker, in particular if relatively reliable parameters (variances) have to be assigned to the different items of the resources-uses account.

Point three highlights the strong link existing between this paper and the balance procedure used in the Italian National Accounts.

Actually, as mentioned in the introduction, in order to consider the balancing process as a data validation procedure, it is necessary that a certain degree of independence exists in the initial estimates.

The proposed formalisation allows both to assign more precisely the variances to the various aggregates and to verify the degree of dependence between supply and demand.

If a complete independence holds, then, ignoring the generating function of the discrepancy, the balancing procedure should redistribute accounting residuals only on the basis of the assigned variances. If a certain degree of dependence instead exists, the formalisation proposed in this paper permits to split up the data production process so as it is possible to verify *ex ante* whether the dependent items are balanced and to reallocate the discrepancy only among the independent items.

This paper is only the first step of a more general project that should lead to apply this methodology in the National Account data generating process. In order to really implement this procedure, it is necessary to extend the formalisation and to consider all methods used in National Account estimates, without restricting to the most important as done in the present paper.

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