Methods to analyze structural change over time and space: a typological survey

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ABSTRACT. In structural analysis applied to matrix structures of production, as input-output analysis, to determine how the structure has changed over time an interesting question, or what are the differences between two structures, e.g., two countries for the same time. This can be performed by directed methods based on the computation of technical (or column) coefficients what removes the effect of differences between the two structures for their column margins; same thing can be done with row coefficients. This predertemines the direction of the economy, demand or supply driven, and both results are not comparable. However, the comparison of the two matrices can be performed by removing simultaneously the differences between the column margins and the row margins of the two matrices. The paper surveys all ways to perform this: the methods based on additive method (minimization of differences, minimization of square differences, etc.) and the methods based on multiplicative methods biproportional.

I. Introduction

Determining how structures of exchange have change is a necessity in many fields: in input-output analysis when one wants to determine the change of a structure of production (i.e., a structure of exchange between sectors of production) over time, but also in spatial and regional science when one wants to analyze how the exchanges between regions have been modified over time, or in finance theory when one is concerned with the evolution of cross-shareholding, or in sociological sciences when one is decided to see how the communications between individuals have varied, etc. Similarly, it can be asked for what are the differences over space between two different but comparable structures as two countries for the same year. Not all these applications are economic, but all have a common point: change in the structure of production must be measured.

The paper surveys some methods allowing to evaluate how an exchange structure has changed over time or what are the differences between two exchange structures over space. These two structures will be represented by two matrices denoted $\mathbf{Z}$ and $\mathbf{Z}^*$.

There are two major sets of methods: the *directed methods* where the economy is assumed to be either demand-driven either supply driven and the *non-directed methods* where such a hypothesis is not assumed.

II. Directed methods

By directed methods, I mean the methods of structural comparison that are based on the comparison of two matrices of coefficients, technical coefficients or allocation coefficients, or more generally, column coefficients and allocation coefficients.

A. Naive method: simple comparison of two matrices of technical coefficients or two matrices of allocation coefficients

1. The direct comparison of coefficients

To evaluate change in a structure of exchanges, you can compare two matrices of technical coefficients to remove the differences between column margins. A technical coefficient a_{ij} is the ratio of the flow z_{ij} from a sector i to a sector j, over the output x_j of sector j: it indicates how much a sector has to buy of commodity i to produce one unit of commodity j. For Leontief, these coefficients are assumed to be stable, what implies that the economy is demand driven, so two technical coefficient matrices $\mathbf{A}$ and $\mathbf{A}^*$ are compared, an initial matrix $\mathbf{A}$ and a final matrix $\mathbf{A}^*$, belonging of two different periods.

The same thing can be done with allocation coefficients: in the Ghosh perspective, the allocation coefficients are assumed to be stable, what imply that the economy is supply driven. An allocation coefficient b_{ij} is a coefficient that divides the flow z_{ij} by the output x_i of sector

i: it indicates how the output of a sector, i, will be allocated to other sectors, j. In this case, one compares two allocation coefficient matrices $\mathbf{B}$ and $\mathbf{B}^*$.

This direct comparison of technical or allocation seems simple, but it has to be sophisticated to be rigorous.

2. The comparison of normalized coefficients

With the demand-driven model, the matrix $\mathbf{A}$ deduced form $\mathbf{Z}$ is compared to the matrix $\mathbf{A}^*$ deduced from $\mathbf{Z}^*$. However, even many authors use this approach directly, this is not so simple because the two matrices have not the same column margins. If you compare directly two matrices of technical coefficients that have not the same column margins, you mix two effects: 1) the pure variation of the technical coefficients and 2) the variation of the rate of added value that perturbs the analysis. In one hand, in the view point of the Leontief production function, if a technical coefficient a_{ij} decreases, all coefficients being equal, then this has a technical signification; but, as a counterpart, the rate of added value $v_j = \frac{w_j}{x_j}$ (where w_j is the added value) has been increased mechanically: added value is endogenous, depending on the variation of technical coefficients. In the other hand, in a structural analysis viewpoint, the influence of j over i has decreased and the influence of j over the revenue has increased: the added-value is exogenous and it can vary by itself. In this last case, it is preferable to catch the pure variation of technical coefficients, for example when a comparative structural analysis (based on graph analysis) is done over time. Remember that among the two components of added value, the profit is traditionally a residue and it is endogenous, but the salaries are largely exogenous, that is to say, they are not less exogenous than technical coefficients.

So, either added value is included into the analysis and the matrices of technical coefficients are borded with a supplementary row, the row of added value: they have 1 as margin for all their columns but they are rectangular:

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ \dots & & \dots \\ a_{n1} & \dots & a_{nn} \\ v_1 & \dots & v_n \end{bmatrix}$$
$$1 \quad \dots \quad 1$$

either only the technical coefficient matrices are analyzed, but there is a need for an additional operation of normalization to allow a significant comparison of the two matrices. To perform it, a column normalization of $\mathbf{A}$ and $\mathbf{A}^*$ is done: $\mathbf{A}^{*M} - \mathbf{A}^M$ is computed, where $\mathbf{A}^M = \mathbf{A}\,\mathbf{M}_A^{-1}$ and $\mathbf{A}^{*M} = \mathbf{A}^* \,(\mathbf{M}_A^*)^{-1}$ are the matrices obtained by transforming A and $\mathbf{A}^*$ into Markovian matrices, with 1 as margin of all columns.

<u>Remark</u>. A margin is the column sum or a row sum. Here, the margin matrices $\mathbf{M}_A$ and $\mathbf{M}_A^*$ are the diagonal matrices whose diagonal elements are the sum of the column j of the matrices $\mathbf{A}$ and $\mathbf{A}^*$, excluding added value: $m_{j_A} = \sum_i a_{ij}$. ■

Additionally, as $\mathbf{A}^M$ and $\mathbf{A}^{*M}$ have the same column margins, it could be skilled to compute the Frobenius norm of the columns of the difference matrix to give the variability, an indicator of change of sector computed in percentage:

(1) $$\sigma_j = \sqrt{\sum_i \left(a_{ij}^{*M} - a_{ij}^M\right)^2}$$

that indicates what column sectors vary the more in terms of pure variation of technical coefficients. Note that this quantity is also the relative variability, computed in percentage, of the non-normalized technical coefficients, because σ_j can be written:

(2) $$\sigma_j = \sqrt{\sum_i \left(a_{ij}^{*M} - a_{ij}^M\right)^2} = \sqrt{\sum_i \left(\frac{a_{ij}^*}{m_j^*} - \frac{a_{ij}}{m_j}\right)^2} = \frac{1}{m_j^*} \sqrt{\sum_i \left(a_{ij}^* - \hat{a}_{ij}\right)^2}$$

where the terms $\hat{a}_{ij} = \frac{a_{ij}}{m_j} m_j^*$ are the transformation of $\mathbf{A}$ such as it has the same column margins than $\mathbf{A}^*$. Or it can be written also

(3) $$\sigma_j = \frac{1}{m_j} \sqrt{\sum_i \left(\hat{a}_{ij}^* - a_{ij}\right)^2}$$

where $\hat{a}_{ij}^* = \frac{a_{ij}^*}{m_j^*} m_j$ is the transformation of $\mathbf{A}^*$ so that it has the same margins than $\mathbf{A}$.

So, at least for the sector level, normalizing technical coefficients is the same thing than to do a comparison of $\hat{\mathbf{A}} = \mathbf{A}\,\mathbf{M}^{-1}\,\mathbf{M}^*$ to $\mathbf{A}^*$, then to compute relative variabilities: $\sigma_j = \frac{1}{m_j^*} \sqrt{\sum_i \left(a_{ij}^* - \hat{a}_{ij}\right)^2}$.

Reciprocally, for the supply-driven model, the same procedure is applied to the matrix $\mathbf{B}$, deduced from $\mathbf{Z}$, to compare it to the matrix $\mathbf{B}^*$, deduced from $\mathbf{Z}^*$, to give the relative variabilities $\sigma_i = \sqrt{\sum_j \left(b_{ij}^{*M} - b_{ij}^M\right)^2}$, that indicates what row sectors are the most changing.

B. The causative matrix method

The causative matrix method has been extended to input-output analysis by Jackson, Rogerson, Plane and O hUallachain (1990) [1]. They start from the inverse matrix $\mathbf{\Pi} = (\mathbf{I} - \mathbf{A})^{-1}$

[1] See also Rogerson and Plane (1984), Plane and Rogerson (1986), Jackson et al. (1990).

and they compute two transition matrices as Markovian matrices $\mathbf{\Pi}^M = \mathbf{\Pi}\ \mathbf{M}^{-1}$ and $\mathbf{\Pi}^{*M} = \mathbf{\Pi}^* \left(\mathbf{M}^*\right)^{-1}$. Note that the method would not have been changed, only the interpretation of the results would, if the direct matrices would have been used instead of the inverse matrices. Matrix $\mathbf{\Pi}^{*M}$ is assumed to be linked to the matrix $\mathbf{\Pi}^M$ by the formula:

(4) $$\mathbf{\Pi}^{*M} = \mathbf{C}\ \mathbf{\Pi}^M$$

Matrix **C** is the *left causative matrix* and it explains the change between $\mathbf{\Pi}^M$ and $\mathbf{\Pi}^{*M}$; it is found by inverting $\mathbf{\Pi}^M$:

(5) $$\mathbf{C} = \mathbf{\Pi}^{*M}\ (\mathbf{\Pi}^M)^{-1}$$

As matrix C is completely filled with n^2 terms, its interpretation is not easy. This is why matrix **C** is compared to the identity matrix: all diagonal elements are compared to 1, while all off-diagonal elements of each row are compared to 0, i.e., only $2\,n$ elements to analyze; in other terms, one hopes that $\pi_{ij}^{*M} = \pi_{ij}^M$ for all *i*, *j* and all gap is inscribed. However, as formula (4) implies that each coefficient π_{ij}^{*M} of $\mathbf{\Pi}^{*M}$ is not linked to the corresponding coefficient π_{ij}^M of $\mathbf{\Pi}^M$, but to all coefficients of one column of $\mathbf{\Pi}^M$: $\pi_{ij}^{*M} = \sum_k c_{ik}\ \pi_{kj}^M$, the interpretation of gaps remains difficult. For Jackson et al. (1990, p. 265-266), a large diagonal element c_{ii} indicates that "final demand impacts [of sector *i*], relative to others, are increasingly being internalized within the sector", while a large off-diagonal element c_{ij} indicates "an increasing proportionate importance of final demand deliveries from ... [sector *j*] in stimulating ... output [of sector *i*], either directly from final demand deliveries from ... [sector *j*], or relative to the impacts of final deliveries from all other sectors".

Formulae (4) or (5) allow to describe this approach as "multiplicative", by contrast to the above comparison of technical or allocation coefficients that could be referred to as "additive": transposed to the inverse matrices, it could be written $\mathbf{D} = \mathbf{\Pi}^{*M} - \mathbf{\Pi}^M$, what should be compared to the null matrix. In this case, the interpretation would be easier because there is no mix of coefficients from $\mathbf{\Pi}^M$ to $\mathbf{\Pi}^{*M}$ and each π_{ij}^{*M} is linked to the corresponding coefficient π_{ij}^M by the simple formula $\pi_{ij}^{*M} = d_{ij} + \pi_{ij}^M$ that gives a more simple interpretation of the gaps.

Remark. A reverse comparison can be done by posing $\mathbf{\Pi}^M = \widetilde{\mathbf{C}}\ \mathbf{\Pi}^{*M}$, where $\widetilde{\mathbf{C}}$ is the causative matrix for this reverse analysis. $\widetilde{\mathbf{C}}$ is the inverse of **C**:

(6) $$\widetilde{\mathbf{C}} = \mathbf{\Pi}^M\ (\mathbf{\Pi}^{*M})^{-1} = \mathbf{C}^{-1}$$

However, this is a right causative matrix a defined by Jackson et al. (1990): $\mathbf{\Pi}^{*M} = \mathbf{\Pi}^{M}\,\mathbf{R} \Rightarrow \mathbf{R} = \left(\mathbf{\Pi}^{M}\right)^{-1}\,\mathbf{\Pi}^{*M}$. ■

C. Discussion

The difficulty with directed methods is that the results obtained for technical coefficients will be not comparable to the results obtained for allocation coefficients. If technical coefficients are assumed to be stable, allocation coefficients cannot be stable: assuming that technical coefficients are stable, i.e., $\mathbf{A}^* = \mathbf{A}$, then $\mathbf{B}^* = \hat{\mathbf{x}}^{*\ -1}\,\mathbf{A}\,\hat{\mathbf{x}}^* \neq \mathbf{B}$ and if allocation coefficients are stable, i.e., $\mathbf{B}^* = \mathbf{B}$, then $\mathbf{A}^* = \hat{\mathbf{x}}^*\,\hat{\mathbf{x}}^{-1}\,\mathbf{A}\,\hat{\mathbf{x}}\,\hat{\mathbf{x}}^{*-1}$. This works except in the case of absolute join stability (Chen and Rose, 1986 and 1991): if $\mathbf{A}^* = \mathbf{A}$ and $\mathbf{x}^* = k\,\mathbf{x}$, then $\mathbf{B}^* = \hat{\mathbf{x}}^{*\ -1}\,\mathbf{A}\,\hat{\mathbf{x}}^* = \grave{\mathbf{B}}$.

There is a large literature about what model can be considered as the more attractive: Bon, 1986; Oosterhaven, 1988, 1989, 1996; Miller, 1989; Gruver, 1989; Rose and Allison, 1989; Dietzenbacher, 1997. Generally, the demand driven model is considered as the more plausible.

However, the true stability over time of one type of coefficient or the other is doubtful: in Mesnard (1997), it is shown that the temporal stability of column coefficients is not higher to the temporal stability of row coefficients. If technical coefficients are not stable, the model can be declared as not demand driven but the reciprocal of this proposition is false: if technical coefficients are stable, the model is not necessarily demand driven while if allocation coefficients are not stable, the model cannot be declared as supply driven but the reciprocal is false again. So, one may want to compare technical coefficients over time by assuming the demand driven hypothesis and the normal stability of technical coefficients, but one will have no information about allocation coefficients. Or, alternately, one may want to compare allocation coefficients over time by assuming the supply driven hypothesis by assuming the normal stability of allocation coefficients, but one will have no information about technical coefficients.

III. Not directed methods

In this group of methods, no hypotheses are made about the direction of the economy, demand or supply driven. Both hypotheses are incompatible and the results in the first case are not comparable with the results in the second case. However, even if the demand-driven hypothesis could seem more plausible, in an epistemological view it is always preferable not to pose a hypothesis when it can be avoided, provided that the hypothesis could be falsified later. So, while for directed methods, either column coefficients were compared after a column normalization either row coefficients after a row normalization, for non-directed methods the idea consists into comparing coefficients but without a column or row normalization that would imply a demand driven model or a row driven model.

A. The methods that generalize the comparison of technical and allocation coefficients

The idea is to generalize the comparison of technical coefficients and of allocation coefficients but by becoming free from the direction of the economy -- demand-driven or supply-driven hypothesis -- . The *ex ante* stability of technical and allocation coefficients will not be posed and their stability will be measured eventually *ex post*, what could help to dismiss one of the alternative hypothesis or both eventually.

Flow matrices can be compared directly, without passing by the technical and allocation coefficient matrices. Remember that, instead of to compare two technical coefficients (respectively two allocation coefficients) a_{ij} and a_{ij}^*, one is able to compare two absolute values $z_{ij}\frac{x_j^*}{x_j}$ and z_{ij}^*: $a_{ij} \leftrightarrow a_{ij}^* \Leftrightarrow \frac{z_{ij}}{x_j} \leftrightarrow \frac{z_{ij}^*}{x_j^*} \Leftrightarrow z_{ij}\frac{x_j^*}{x_j} \leftrightarrow z_{ij}^*$, where the symbol "$\leftrightarrow$" signifies "compared to".

In a general way, the most simple principle of this type of non-directed methods will consist into projecting one matrix to give her the margins of another matrix, what is close to do a normalization of both column and rows. Nevertheless, while for directed methods it was equivalent to compare $\hat{\mathbf{A}} = \mathbf{A}\,\mathbf{M}^{-1}\,\mathbf{M}^*$ with $\mathbf{A}^*$ or $\mathbf{A}^M$ with $\mathbf{A}^{*M}$, now it will not be the case: this will generate some variants in the procedure.

Starting from an initial flow matrix $\mathbf{Z}$ and a final flow matrix $\mathbf{Z}^*$, the principle consists into computing a matrix the closer as possible to $\mathbf{Z}$ but with the row and column margins of $\mathbf{Z}^*$; see figure 1.

Figure 1 here

However, there are many tools to perform the projection of a matrix and the problem is to choose one of these tools, or, in other words, there are an infinite number of matrices that can have the same margins and the problem is to choose one of these matrices. The resulting matrix may vary depending on the tool chosen to perform the projection, and consequently the results of the methods may vary, again generating a typology. Moreover, to evaluate the variation from the projections, some methods compute the difference between the projection and the target, some other compute the ratio between the projection and the target: this will add two more branches to the typology.

1. The methods that minimize a distance between the projected matrix and the final matrix

To find the matrix $\hat{\mathbf{Z}}$ that is the nearest to a matrix $\mathbf{Z}$ and that respects the margins of another matrix $\mathbf{Z}^*$ (i.e. under constraints of margins: $\sum_i \hat{z}_{ij} = \sum_i z_{ij}^*$ and $\sum_j \hat{z}_{ij} = \sum_j z_{ij}^*$), it is possible to use one of the methods that minimize a distance between the projected matrix $\hat{\mathbf{Z}}$ and the initial matrix $\mathbf{Z}$. However, this may create negative terms in the projection $\hat{\mathbf{Z}}$ because of their additive form. For example, the orthogonal projection, i.e., the minimization of the least squares, that is $\min \Sigma_i \Sigma_j (\hat{z}_{ij} - z_{ij})^2$ gives $\hat{\mathbf{Z}} = \mathbf{P} + \mathbf{Z} + \mathbf{Q}$, where $\mathbf{P}$ and $\mathbf{Q}$ are diagonal matrices. The non negativity of terms of the projected matrix is not guaranteed (Mesnard, 1990a). Remember that negative terms are impossible to explain in an input-output context: if $\mathbf{Z}$ has no negative terms, how to justify in an economic view point, the existence of some negative terms inside the projected matrix $\hat{\mathbf{Z}}$?

Among the possible methods, can be found:

- The above minimization of the quadratic deviation (the square of the Frobenius norm of the difference matrix): min $\sum_i \sum_j (\hat{z}_{ij} - z_{ij})^2$.
- The minimization of the absolute differences: min $\sum_i \sum_j |\hat{z}_{ij} - z_{ij}|$, what is not continuously derivable.
- The minimization of the Hölder norm at the power p: min $\sum_i \sum_j |\hat{z}_{ij} - z_{ij}|^p$, knowing that the Hölder norm (Rotella and Borne, p. 78) is $\left\|\hat{\mathbf{Z}} - \mathbf{Z}\right\|_p = \left[\sum_i \sum_j |\hat{z}_{ij} - z_{ij}|^p\right]^{1/p}$, what is a generalization of two preceding,
- Pearson's χ^2: min $\sum_i \sum_j \frac{(\hat{z}_{ij} - z_{ij})^2}{z_{ij}}$,
- Neyman's χ^2: min $\sum_i \sum_j \frac{(\hat{z}_{ij} - z_{ij})^2}{\hat{z}_{ij}}$.

Besides the negative terms, these methods often lead to various problems as some non-linearities or non-differentiabilities that can be found in the system of equation (Neyman, absolute differences).

2. The methods based on a biproportion

In (Mesnard, 1990a, 1990b, 1996, 1977), a biproportional filter was proposed to analyze structural change. Remember that the result $\hat{\mathbf{Z}}$ of a biproportion that gives to $\mathbf{Z}$ the same margins than to $\mathbf{Z}^*$, $\hat{\mathbf{Z}} = K(\mathbf{Z},\ \mathbf{Z}^*)$, is equal to $\mathbf{P}\,\mathbf{Z}\,\mathbf{Q}$, where $\mathbf{P}$ and $\mathbf{Q}$ are diagonal matrices that allow to respect two conditions:

1) $\hat{\mathbf{Z}}$ must have the same row and column margins than $\mathbf{Z}^*$:

(7) $$\left\{ \begin{array}{l} \sum_j \hat{z}_{ij} = \sum_j z^*_{ij} \text{ for all } i \\ \sum_i \hat{z}_{ij} = \sum_i z^*_{ij} \text{ for all } j \end{array} \right\}$$

2) $\hat{\mathbf{Z}}$ is the matrix the nearest to $\mathbf{Z}$ following a certain criterion. This criterion can be, among others:

- The maximization of entropy (Wilson, 1970): max $-\sum_i \sum_j \hat{z}_{ij} \log \hat{z}_{ij}$, under the constraint $C = \sum_i \sum_j \hat{z}_{ij}\, c_{ij}$ where C is the total cost and c_{ij} is a cost, that can be considered as representative of $\mathbf{Z}^0$.
- Kullback and Liebler's minimization of information (Kullback and Liebler, 1951), (Kullback, 1959), (Snickars and Weibull, 1977): min $\sum_i \sum_j \hat{z}_{ij} \log \frac{\hat{z}_{ij}}{z_{ij}}$.
- The minimization of interactions of Watanabe (1969) and Guiasu (1979). Etc.

Several algorithms respect these two conditions. Among them, there is RAS. For example, the terms $\mathbf{P}$ and $\mathbf{Q}$ can be of the following form:

$$p_i = \frac{z^*_{i\bullet}}{\sum_{j=1}^{m} q_j\, z_{ij}} \text{, for all } i \text{, and } q_j = \frac{z^*_{\bullet j}}{\sum_{i=1}^{n} p_i\, z_{ij}} \text{, for all } j$$

It is demonstrated (Mesnard, 1994) that all algorithms respecting the two conditions of a biproportion provide to the same results. These algorithms have a unique solution as demonstrated for RAS by Bacharach (1970). One of the properties that cause difficulties with these algorithms is that they have to be solved iteratively, but generally, the convergence speed is good even if it depends on the choice made for the algorithm (Bachem and Korte, 1979). Also, note that if all terms $p_i^{(k)}$ are positive, all terms $q_j^{(k)}$ will be also positive, as soon as all terms of $\mathbf{Z}$ are positive: this guarantees to avoid negative terms, always hard to interpret.

However, there are more than one manner to complete the job.

i. The subtractive biproportional methods

a. Projection of a flow matrix on a flow matrix

1) The basic method

With the *biproportional ordinary filter*, the flow matrix $\mathbf{Z}$ is projected such that it obtains the same margins than the flow matrix $\mathbf{Z}^*$: $\hat{\mathbf{Z}} = K(\mathbf{Z},\ \mathbf{Z}^*)$. Then the result $\hat{\mathbf{Z}}$ is compared to $\mathbf{Z}^*$ by calculating the difference matrix $\mathbf{Z}^* - K(\mathbf{Z},\ \mathbf{Z}^*)$. Relative variations are then computed:

$$\sigma_j^C = \frac{\sqrt{\sum_i \left[z^*_{ij} - K(\mathbf{Z},\ \mathbf{Z}^*)_{ij} \right]^2}}{\sum_i z^*_{ij}} \text{, for column } j \text{ and, } \sigma_i^R = \frac{\sqrt{\sum_j \left[z^*_{ij} - K(\mathbf{Z},\ \mathbf{Z}^*)_{ij} \right]^2}}{\sum_j z^*_{ij}} \text{, for row } i$$

Remark. Also, a χ^2 is able to be computed, for example:

$$\sqrt{\sum_i \frac{\left[z_{ij}^* - K(\mathbf{Z},\ \mathbf{Z}^*)_{ij}\right]^2}{z_{ij}^*}} \quad \text{or} \quad \sqrt{\sum_i \frac{\left[z_{ij}^* - K(\mathbf{Z},\ \mathbf{Z}^*)_{ij}\right]^2}{z_{ij}}}\ \blacksquare$$

Remark. As $K(\mathbf{Z},\ \mathbf{Z}^*)$ has the same row and column margins as $\mathbf{Z}^*$, both matrices are compared without the differential growth effect of sectors: this generalizes the "shift-and-share method". ■

A difficulty is that it is also possible to project $\mathbf{Z}^*$ on the margins of **Z** to compare the result to **Z**, so we have two ways to do the computation, the direct computation from **Z** to $\mathbf{Z}^*$ and the reverse computation from $\mathbf{Z}^*$ to **Z**, what multiplies all computations by two and what gives two sets of different results without a criterion to declare the superiority of one over the other.

2) *Avoiding the double projections, direct and reverse*

Understanding that the basic idea of biproportional filtering consists into giving to the flow matrices **Z** and $\mathbf{Z}^*$ the same margins, to remove these difficulties, one can try to find a third matrix $\mathbf{Z}^B$ to provide these margins. If $\mathbf{Z}^B$ has the same margins than **Z**, or is equal to **Z**, then $K(\mathbf{Z},\ \mathbf{Z}^B) = \mathbf{Z}$ and $K(\mathbf{Z}^*,\ \mathbf{Z}^B) = K(\mathbf{Z}^*,\ \mathbf{Z})$, that is the reverse projection of the ordinary biproportional projector; if $\mathbf{Z}^B$ has the same margins than $\mathbf{Z}^*$, then $K(\mathbf{Z}^*,\ \mathbf{Z}^B) = \mathbf{Z}^*$ and $K(\mathbf{Z},\ \mathbf{Z}^B) = K(\mathbf{Z},\ \mathbf{Z}^*)$, that is the direct projection of the ordinary biproportional projector. For all positions between these two "polar" matrices, one can obtain a wide range of results. A good idea could consist into choosing $\mathbf{Z}^B$ such a manner that the variance would be minimized:

$$\min\ \left\|K(\mathbf{Z},\ \mathbf{Z}^B) - K(\mathbf{Z}^*,\ \mathbf{Z}^B)\right\|_F^2$$

Unfortunately, this expression is not linear regarding to the terms of **Z**, $\mathbf{Z}^*$ and to the margins of $\mathbf{Z}^B$ and it has no analytical solution because biproportion is a transcendent operator. Such a problem can be solved only by a succession of computations, what is too much heavy even for small matrices.

However, one type of matrix is a good candidate to play the role of $\mathbf{Z}^B$: a function of **Z** and $\mathbf{Z}^*$; for example, $\bar{\mathbf{Z}}$, the mean of **Z** and $\mathbf{Z}^*$, with $\bar{\mathbf{Z}} = \frac{1}{2}(\mathbf{Z} + \mathbf{Z}^*)$. I call this the *biproportional mean filter* (Mesnard, 1998).

Remark$\mathbf{Z}^B$ could be also a third matrix of an intermediary year, 1988, if **Z** is 1980 and $\mathbf{Z}^*$ is 1996, but remember that only the margins of this matrix are important. ■

In the *biproportional mean filter*, each matrix $\mathbf{Z}$ and $\mathbf{Z}^*$ is projected to the margins of $\bar{\mathbf{Z}}$, the mean of $\mathbf{Z}$ and $\mathbf{Z}^*$, to give $K\left(\mathbf{Z},\ \bar{\mathbf{Z}}\right)$ and $K\left(\mathbf{Z}^*,\ \bar{\mathbf{Z}}\right)$; then, $K\left(\mathbf{Z},\ \bar{\mathbf{Z}}\right)$ is compared to $K\left(\mathbf{Z}^*,\ \bar{\mathbf{Z}}\right)$ by calculating the Frobenius norm of the difference matrix $K\left(\mathbf{Z}^*,\ \bar{\mathbf{Z}}\right) - K\left(\mathbf{Z},\ \bar{\mathbf{Z}}\right)$ as it is done in the ordinary biproportional filter, except that there is only one set of computations and not two:

$$\sigma_i^R = \frac{\sqrt{\sum_j \left(K\left(\mathbf{Z}^*,\ \bar{\mathbf{Z}}\right)_{ij} - K\left(\mathbf{Z},\ \bar{\mathbf{Z}}\right)_{ij} \right)^2}}{\bar{z}_{i\bullet}}\text{, for row } i$$

$$\text{and } \sigma_j^C = \frac{\sqrt{\sum_i \left(K\left(\mathbf{Z}^*,\ \bar{\mathbf{Z}}\right)_{ij} - K\left(\mathbf{Z},\ \bar{\mathbf{Z}}\right)_{ij} \right)^2}}{\bar{z}_{\bullet j}}\text{, for column } j.$$

This allows to remove the effects of differential growth of sectors, but not the effect of differences in the size of sectors.

A figure based on an Edgeworth box will illustrate the method (see figure 2). Consider the matrices:

$$\mathbf{Z} = \begin{bmatrix} 5 & 5 \\ 4 & 1 \end{bmatrix} \begin{matrix} 10 \\ 5 \end{matrix} \quad \text{and} \quad \mathbf{Z}^* = \begin{bmatrix} 3 & 1 \\ 6 & 5 \end{bmatrix} \begin{matrix} 4 \\ 11 \end{matrix}$$
$$\begin{matrix} 9 & 6 \end{matrix} \qquad\qquad\qquad \begin{matrix} 9 & 6 \end{matrix}$$

so, $K(\mathbf{Z},\ \mathbf{Z}^*) = \begin{bmatrix} 1.42 & 2.58 \\ 7.58 & 3.42 \end{bmatrix}$ and $K(\mathbf{Z}^*,\ \mathbf{Z}) = \begin{bmatrix} 6.74 & 3.26 \\ 2.26 & 2.74 \end{bmatrix}$

This matrix is represented by the following Edgeworth box, where the sides of the box correspond to the column constraints of matrix $\mathbf{Z}$, the line *AB* corresponds to the row constraints of $\mathbf{Z}$ and the point *z* corresponds to $\mathbf{Z}$. With matrix $\mathbf{Z}^*$, column constraints are the same and row constraints become the line *CD*, while $\mathbf{Z}^*$ is represented by point z^*. The length of segment $\{K(z,\ z^*),\ z^*\}$, which corresponds to the variation found by the direct projection, is closed to the length of segment $\{K(z^*,\ z),\ z\}$, which corresponds to the variation by the reverse projection. Consider another matrix $\mathbf{Z}_1$ with the same margins than $\mathbf{Z}$:

$$\mathbf{Z}_1 = \begin{bmatrix} 8 & 2 \\ 1 & 4 \end{bmatrix} \begin{matrix} 10 \\ 5 \end{matrix}$$
$$\begin{matrix} 9 & 6 \end{matrix}$$

so, $$K(\mathbf{Z}_1,\ \mathbf{Z}^*) = \begin{bmatrix} 3.74 & 0.26 \\ 5.26 & 5.74 \end{bmatrix}$$

As $\mathbf{Z}$ and $\mathbf{Z}_1$ have the same margins, $K(z^*,\ z_1)$ is confused with $K(z^*,\ z)$. The segment $\{K(z_1,\ z^*),\ z^*\}$ is clearly shorter than the segment $\{K(z^*,\ z_1),\ z_1\}$. This is because the projection of z_1 is near the limit of the box: the orthogonal projection of z_1, found by an additive method, is even outside the limit of the box (it corresponds to negative terms in the projected matrix) and the ordinary biproportional projection corrects it.

Consider the matrix $\bar{\mathbf{Z}}$ represented by the segment EF:

$$\bar{\mathbf{Z}} = \frac{1}{2}(\mathbf{Z} + \mathbf{Z}^*) = \begin{bmatrix} 4 & 3 \\ 5 & 3 \end{bmatrix} \begin{matrix} 7 \\ 8 \end{matrix}$$
$$\begin{matrix} 9 & 6 \end{matrix}$$

Then,

$$K\left(\mathbf{Z},\ \bar{\mathbf{Z}}\right) = \begin{bmatrix} 3.00 & 4.00 \\ 6.00 & 2.00 \end{bmatrix},\ K\left(\mathbf{Z}^*,\ \bar{\mathbf{Z}}\right) = \begin{bmatrix} 5.00 & 2.00 \\ 4.00 & 4.00 \end{bmatrix},\ K\left(\mathbf{Z}_1,\ \bar{\mathbf{Z}}\right) = \begin{bmatrix} 6.25 & 0.75 \\ 2.75 & 5.25 \end{bmatrix}$$

Figure 2 here

The second variant is a generalization of the normalization of the technical or allocation coefficients: for this reason, it constitutes a special category.

b. Binormalization of flow matrices

In the *biproportional bimarkovian filter*, both flow matrices are binormalized, that is to say normalized by columns and by rows simultaneously: each matrix $\mathbf{Z}$ and $\mathbf{Z}^*$ is transformed into a bimarkovian matrix, $\mathbf{Z}^M$ and $\mathbf{Z}^{*M}$.

A bimarkovian or binormalized matrix is a matrix of which all margins of both sides, column and row, are equal to 1: this is exactly possible only for square matrices. Any other number can be chosen; the important thing is that all margins of the same side would be equal. For rectangular matrices of dimension $(n,\ m)$, the margins of one side -- say the side of dimension n -- are equal to μ, and the margins of the other side are equal to λ: $\mathbf{1}^M\,\mathbf{s} = \mu\,\mathbf{s}$ and $\mathbf{s}'\,\mathbf{1}^M = \lambda\,\mathbf{s}'$. For example, one can take:

(8) $$\mathbf{1}^M = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} \begin{matrix} 1 \\ \dots \\ 1 \end{matrix}$$
$$\frac{n}{m} \;\dots\; \frac{n}{m}$$

or,

(9) $$\mathbf{1}^M = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} \begin{matrix} m \\ \dots \\ m \end{matrix}$$
$$n \;\dots\; n$$

Two biproportional projections are done: $\mathbf{Z}^M = K(\mathbf{Z},\ \mathbf{1}^M)$ and $\mathbf{Z}^{*M} = K(\mathbf{Z}^*,\ \mathbf{1}^M)$. For example, $K(\mathbf{Z},\ \mathbf{1}^M) = \mathbf{P}\ \mathbf{Z}\ \mathbf{Q}$ and with matrix $\mathbf{1}^M$ defined in (8):

(10) $$p_i = \frac{1}{\sum_{j=1}^{m} q_j\, z_{ij}} \text{ , for all } i \text{ , and } q_j = \frac{n}{m}\,\frac{1}{\sum_{i=1}^{n} p_i\, z_{ij}} \text{ , for all } j$$

or with the matrix $\mathbf{1}^M$ defined in (9):

(11) $$p_i = \frac{m}{\sum_{j=1}^{m} q_j\, z_{ij}} \text{ , for all } i \text{ , and } q_j = \frac{n}{\sum_{i=1}^{n} p_i\, z_{ij}} \text{ , for all } j$$

The same reasoning holds for $\mathbf{Z}^*$. Then, $\mathbf{Z}^M$ is compared to $\mathbf{Z}^{*M}$, by calculating the Frobenius norm of the difference matrix $\mathbf{Z}^{*M} - \mathbf{Z}^M$. The rest of the method is similar to the biproportional mean filter replacing matrix $\bar{\mathbf{Z}}$ by the bimarkovian matrix $\mathbf{1}^M$. With a bimarkovian matrix as $\mathbf{1}^M$ in (9), one divide by m for columns and by n for rows when calculating relative variations: $\sigma_j = \frac{1}{n}\sqrt{\sum_i \left(z_{ij}^{*M} - z_{ij}^{M}\right)^2}$ and $\sigma_i = \frac{1}{m}\sqrt{\sum_j \left(z_{ij}^{*M} - z_{ij}^{M}\right)^2}$.

> Remark. The choice between forms (8) or (9) for matrices $\mathbf{1}^M$ is neutral because the above matrices (8) and (9) are equivalent; a separable modification of $\mathbf{Z}$ (or $\mathbf{Z}^*$) is ineffective (Mesnard, 1994, 1997), i.e., if $\mathbf{Z}$ is replaced by $\tilde{\mathbf{Z}} = \mathbf{\Psi}\ \mathbf{Z}\ \mathbf{\Omega}$, then $K\left(\tilde{\mathbf{Z}},\ \mathbf{1}^M\right) = K(\mathbf{Z},\ \mathbf{1}^M)$. ■

This method could seem to distort strongly the data but it is only a generalization of the normalization procedure applied to technical or allocation coefficients as seen in the beginning of this paper. In the other hand, not only the effect of the differential growth of sectors is removed without predetermining if the economy is demand or supply - driven as in the ordinary biproportional filter, not only the problem of the double result is removed as in the biproportional mean filter, but the effect of differential size of sectors is removed: after projection all sectors in column will have the same margin, i.e. the same size, and all sectors in column will have the same margin.

ii. The divisive biproportional methods

Van der Linden and Dietzenbacher (1995) have had an idea similar to those exposed above: to compare two matrices, they project the first on the margins of the second. To perform this, they compute $\hat{\mathbf{A}} = K(\mathbf{A},\ \mathbf{A}^*)$. There are only two differences with the biproportional ordinary filter exposed above.

- They do not compute variablities of rows or columns, but only variabilities of single cells of the matrices: they compute d_{ij} by dividing $\hat{a}_{ij}$ by a^*_{ij}:

$$d_{ij} = \frac{\hat{a}_{ij}}{a^*_{ij}} \text{ for all } i, j$$

- They work on technical coefficient matrices, when flow matrices are used in the biproportional filters above. Note that under biproportion, it is not the same thing to work on flow matrices or on technical coefficient matrices even if **A** is derived from **Z** by a diagonal matrix multiplication (Mesnard, 1994):

$$K(\mathbf{A},\ \mathbf{A}^*) = K\left(\mathbf{A},\ \mathbf{Z}^* \langle \mathbf{x}^* \rangle^{-1}\right) = K(\mathbf{A},\ \mathbf{Z}^*) \quad \text{but} \quad K(\mathbf{A},\ \mathbf{A}^*) = K\left(\mathbf{Z} \langle \mathbf{x} \rangle^{-1},\ \mathbf{A}^*\right) \neq K(\mathbf{Z},\ \mathbf{A}^*).$$

  Even if these authors remain on technical coefficient matrices, both margins are projected, so the method is relevant to the category of non-directed methods: it could be applied as well as to flow matrices. There is a certain contradiction to start from a directed model (demand-driven) and then to apply a non-directed method: it could seem curious to project on both margins, while only the column margin has a signification when technical coefficient matrices are used.

B. The bicausative method

The principle of the bicausative method (Mesnard, 2000) consists into keeping the idea of causative matrices, but in abandoning the directed character of the causative method. The bicausative method starts from the *double causative* method proposed by Jackson et al. (1990, p. 268): $\mathbf{\Pi}^{*M} = \mathbf{C}_L\ \mathbf{\Pi}^M\ \mathbf{C}_R$; the double causative method could be qualified as non-directed after replacing Markovian matrices by flow matrices, but it does not allow to compute **C** matrices and it obliges to estimate a too large number of parameters, $2\,n^2$ -- . In the bicausative method, **two diagonal matrices**, **L** and **R**, are replacing the matrices $\mathbf{C}_L$ and $\mathbf{C}_R$: the number of parameters falls to $2\,n$. Change between flow matrices is assumed to be of the form $\mathbf{L\,Z\,R}$ but this form is *not* biproportional because **L** and **R** are diagonal matrices. As $\mathbf{L\,Z\,R}$ is generally not equal to $\mathbf{Z}^*$, the choice is made to find the matrices $\mathbf{L}(n,\ n)$ and $\mathbf{R}(m,\ m)$ by minimizing the sum of squares of the differences between z^*_{ij} and $l_i\ z_{ij}\ r_j$:

(12)

$$\min SS\ ; SS = \sum_{i=1}^{n}\sum_{j=1}^{m}\left[z_{ij}^{*} - l_i\, z_{ij}\, r_j\right]^2$$

what gives:

(13)

$$\left\{\begin{array}{l} l_i = \dfrac{\sum_{j=1}^{m} z_{ij}^{*}\, z_{ij}\, r_j}{\sum_{j=1}^{m}\left(z_{ij}^{0}\right)^2\left(r_j^{0}\right)^2}, \text{ for all } i \\ r_j = \dfrac{\sum_{i=1}^{n} z_{ij}^{*}\, z_{ij}\, l_i}{\sum_{i=1}^{n}(z_{ij})^2\,(l_i)^2}, \text{ for all } j \end{array}\right\}$$

This is solved iteratively and is denoted $LS(\mathbf{Z},\ \mathbf{Z}^{*}) = \mathbf{L}\,\mathbf{Z}\,\mathbf{R}$. The diagonal matrix **R** affects equally all terms of a column and the diagonal matrix **L** affects equally all terms of a row.

Remark. In reverse form, from $\mathbf{Z}^{*}$ to **Z**, one obtains:

(14) $$LS(\mathbf{Z}^{*},\ \mathbf{Z}) = \tilde{\mathbf{L}}^{*}\ \mathbf{Z}^{*}\ \tilde{\mathbf{R}}^{*}$$

with,

(15) $$\left\{\begin{array}{l} \tilde{l}_i^{*} = \dfrac{\sum_{j=1}^{m} z_{ij}^{*}\, z_{ij}\, \tilde{r}_j^{*}}{\sum_{j=1}^{m}\left(z_{ij}^{*}\right)^2\left(\tilde{r}_j^{*}\right)^2}, \text{ for all } i \\ \tilde{r}_j^{*} = \dfrac{\sum_{i=1}^{n} z_{ij}^{*}\, z_{ij}\, \tilde{l}_i^{*}}{\sum_{i=1}^{n}\left(z_{ij}^{*}\right)^2\left(\tilde{l}_i^{*}\right)^2}, \text{ for all } j \end{array}\right\}$$

However, unlike the case for causative matrices, the direct and inverse results will not be the same. To compare the results of the direct and reverse methods, one should compute $LS\left(\mathbf{Z},\ \mathbf{Z}^{*}\right) = \mathbf{L}\,\mathbf{Z}\,\mathbf{R}$ and $\left(\tilde{\mathbf{L}}^{*}\right)^{-1} LS\left(\mathbf{Z}^{*},\ \mathbf{Z}\right)\left(\tilde{\mathbf{R}}^{*}\right)^{-1} = \mathbf{Z}^{*}$ that are both on the space of year 1 (while $LS\left(\mathbf{Z}^{*},\ \mathbf{Z}\right) = \tilde{\mathbf{L}}^{*}\ \mathbf{Z}^{*}\ \tilde{\mathbf{R}}^{*}$ is on the space of year 0) so matrix **R** should be compared to matrix $\left(\tilde{\mathbf{R}}^{*}\right)^{-1}$ and matrix $\left(\tilde{\mathbf{L}}^{*}\right)^{-1}$ should be compared to matrix **L**. ■

Some drawbacks of the bicausative-matrices method are indicated in (Mesnard, 2000). The estimators **L** and **R** are not identified because they are specified at a coefficient of proportionality. The initialization by $r_j(0) = \lambda$, for all j, gives $l_i^{*} = \frac{1}{\lambda}\,\bar{l}_i^{*}$, for all i, and $r_j^{*} = \lambda\,\bar{r}_j^{*}$, for all j, where $\bar{l}_i^{*}$ and $\bar{r}_j^{*}$ denote the values obtained after an initialization by

$r_j(0)=1$, for all *j*, and the product $l_i^* \, x_{ij} \, r_j^*$ remains unchanged: $l_i^* \, x_{ij} \, r_j^* = \bar{l}_i^* \, x_{ij} \, \bar{r}$, for all *i* and *j*. It is similar if one initializes by $l_i(0)=\lambda$ instead of $l_i(0)=1$ for all i. When the computation is initialized by any set of values, i.e., by $\exists\, j_1,\, j_2 \,/\, r_{j_1}(0) \neq r_{j_2}(0)$, for all *j*, then the result is not predictable. The problem is embarrassing because non identification concerns the coefficients **L** and **R** that are searched, while it is not the case with other methods as the biproportional filter where only the identified products $\mathbf{P\,Z\,Q}$ are searched. Problems of convergence of the iterative algorithm were found, with local equilibria.

Moreover, interpretation of the bias between $\mathbf{Z}^*$ and $LS\,(\mathbf{Z},\ \mathbf{Z}^*)$ is problematic: $\mathbf{Z}^* - LS\,(\mathbf{Z},\ \mathbf{Z}^*)$ can be approximately different to **0** but the quality of the analysis depends on the size of this bias. As it is said in (Mesnard, 2000), this manner of calculating the bias mixes two phenomenona, the bias caused by the differences in the sector size and the bias caused by the true structural effect because both matrices **Z** and $\mathbf{Z}^*$ do not have the same margins. Thus, one could give to both matrices the same margins to eliminate the size effect of differential growth of margins, for example by doing a biproportion:

(16)

$$K\left[LS\,(\mathbf{Z},\ \mathbf{Z}^*),\ \mathbf{Z}^*\right]$$

However, the bias of the bicausative method is similar to the difference matrix of the biproportional filtering method, then, if one computes the structural bias of the bicausative method, one is forced to compute the structural change as found by the biproportional filtering method. It is simple to prove: as **P** and **Q** are diagonal in the expression $LS(\mathbf{Z},\mathbf{Z}^*) = \mathbf{L\ Z\ R}$, one has (Mesnard, 1994):

(17)

$$K\left[LS\,(\mathbf{Z},\ \mathbf{Z}^*),\ \mathbf{Z}^*\right] \ = \ K\,(\mathbf{L\ Z\ R},\ \mathbf{Z}^*) = K\,(\mathbf{Z},\ \mathbf{Z}^*)$$

and the bias is:

(18)

$$\mathbf{Z}^* - K\left[LS\,(\mathbf{Z},\ \mathbf{Z}^*),\ \mathbf{Z}^*\right] \ = \ \mathbf{Z}^* - K\,(\mathbf{Z},\ \mathbf{Z}^*)$$

that is what it is computed with the ordinary biproportional filter.

Remark. In the biproportional filtering method, the difference $\mathbf{Y} - K(\mathbf{X},\ \mathbf{Y})$ is analyzed: it is not a bias, it is the subject of the analysis itself. ■

IV. Application for France

All filters will be compared to the ordinary biproportional filter by an application based on data for France, for the period 1980-1997. These two tables, the definitive table for 1980, the temporary table for 1997 (INSEE, various years), are used in their original form but aggregated into 9 sectors. Data are given in the base of 1980; the price deflation used is those made by the INSEE itself: all tables are at the prices of 1980, so price effects are removed. Only the intermediate block of tables is used. To obtain a square table that can be analyzed by the causative method, sectors T25 (*Trade*) and T38 (*Non Marketable Services*) have been removed because they have not a row in the French accounting system.

To be allowed to compute technical coefficients and allocation coefficients, I have computed final demand and added-value so that the account of each sector is in equilibrium (total of column equal to total of corresponding row). So, the final demand is simply the difference between the grand total of each sector and its intermediate sales and the added-value is the difference between the grand total of each sector and its intermediate buyings. Imports, customs duty, commercial margins and VAT are included into the added-value and the gross formation of fixed capital, stock variations and exportations into the final demand; but, for this simple illustrative application, it is not a problem (and it is difficult to avoid the difficulty...). These data are presented by tables 1 and 2. This includes the imports, customs duty, commercial margins and VAT into the added-value and the gross formation of fixed capital, stock variations and exportations into the final demand, but, for this simple illustrative application, it is not a problem (and it is difficult to avoid the difficulty...).

Table 1 here

Table 2 here

All these results are not always comparable. For example, the causative method is not comparable to other methods: they are not relative variations in percentage. For the causative method, to allow a minimal degree of comparability, the direct matrices $\mathbf{A}$ and $\mathbf{A}^*$ are used along with the inverse matrices $\mathbf{\Pi}$ and $\mathbf{\Pi}^*$: the results are denoted in table 11 as "direct" when the ordinary causative method based on inverse matrices is denoted "inverse". The first column for the causative method indicates the value of diagonal term of the left causative matrix, the second column the sum of the off-diagonal terms of each row. The results of the bicausative method are not provided because they are not significant, as seen above.

For other methods, the percentages of variation obtained with all methods can be compared only in a first approximation, especially for the methods that compare technical or allocation coefficients and for the other methods. Also remember that with directed methods the results for column sectors are not comparable to the results for row sectors, while they are with non-directed methods. There are two ways of projection in the ordinary biproportional filter: one is obliged to synthesize these results, direct and reverse, in a completely empirical way, by computing the average of these two. The causative matrix and various matrices obtained by a biproportion are shown in tables 3 to 9. As the multiplicative biproportional filter does not give results for columns and rows, and as it is similar to the ordinary biproportional filter except for presenting the results, it will not be published.

Table 3 here

Table 4 here

Table 5 here

Table 6 here

Table 7 here

Table 8 here

Table 9 here

Table 10 here

In tables 11 and 12, the results for the comparison of technical or allocation coefficients will be presented in a first column, a second column will contain the results of the causative method, a third and fourth column will present the results for the ordinary biproportional filter for direct and reverse computations while a fourth column will indicate the average of column three and four; the sixth column contains the results of the biproportional mean filter and the last column gives the results of the biproportional bimarkovian filter.

Table 11 here

Table 12 here

With all methods, the main result is the overwhelming dynamism of *Financial Services*, for both column and row vectors. This is caused by the strong development of exchanges between financial institutions, what can appear partially artificial because all financial movements between banks, positive and negative, are measured in the French system, while only the balances are really exchanged each month. This is why in the future reform of the French national accounting system, only these balances will be taken into account. However a discussion remains concerning these phenomenons. For other sectors, the results are the following with the biproportional bimarkovian filter:

- For columns: *Buildings*, *Energy* and *Trade* (but this last one does not appear in the list of the most changing sectors with the bimarkovian filter), are the most changing. Remember that change in columns reflects change in the production function.
- For rows: *Buildings*, *Trade*, *Transport and Telecommunications*, *Services* (again, *Trade* and *Transport and Telecommunications* do not appear as the most changing with the bimarkovian filter). Remember that change in rows reflects change in the distribution function.

The biproportional mean filter, the average of the ordinary biproportional filter and the bimarkovian filter provide very similar results for columns. For rows, the two firsts give similar results also but those of the bimarkovian filter diverge for sectors *Energy*, *Trade*, *Transport and Telecommunications* and *Services*.

Some large differences between direct and reverse projections can be noted with the ordinary biproportional filter. For technical coefficients or allocation coefficients, the magnitude of indicators diverges to the biproportional analysis but the ordering of sectors remains similar.

With the causative method based on inverse matrices, only diagonal elements are found to exceed 1, *Financial Services* and *Transport and Telecommunications* and *Minerals*; the smaller diagonal elements are for *Buildings* and *Energy*. For off-diagonal elements, *Financial Services* and *Services* are the only positive, when *Energy* has the larger negative one. With the causative method based on direct matrices, *Financial Services, Transport and Telecommunications*, *Services* and *Manufacturing* have a diagonal element that exceeds one, while the same coefficient for *Buildings* is near zero. The off-diagonal element of *Financial Services* is highly positive, those of *Agriculture*, *Services*, *Transport and Telecommunications* are strongly negative.

V. Conclusion

In this paper, I have presented some methods to compare two input-output matrices, and more generally two flow matrices, at two different dates. The following graph will summarize the typology of these methods. Bicausative method is crossed because it suffers theoretical problems that prevent its use. The dotted line indicates that the original version of the multiplicative biproportional method works on technical coefficient matrices (even if it could be functional on flow matrices).

Figure 3 here

The question that remains is: what is the best method? It is hard to answer but, what is certain, is that it is always preferable to avoid to choose a method that obliges to pose a hypothesis when this hypothesis cannot be verified with the method. In other terms, it is preferable to choose a more general method that allows not to pose this hypothesis especially if the method allows to verify the hypothesis. The directed methods fall into the first category of method that oblige to pose an unverifiable hypothesis (the demand-driven or the supply-driven hypothesis). The non-directed methods fall into the second category: they do not force to pose the demand-driven or the supply-driven hypothesis but they give a tool to choose between them, even if the results are not always decisive (Mesnard, 1997).

VI. Bibliographical references

AÏT-SAHALIA Yacine, Michel L. BALINSKI and Gabrielle DEMANGE, 1988. "Le redressement des tables de contingence: deux nouvelles approches", in *Mélanges Economiques, Essais en l'honneur de Edmond Malinvaud*, Economica - Editions EHESS.

BACHARACH, Michael, 1970. *Biproportional Matrices and Input-Output Change*. Cambridge University Press, Cambridge.

BACHEM, Achim and Bernhard KORTE, 1979. "On the RAS-Algorithm," *Computing* 23, 189-198.

BALINSKI, Michel L. and Gabrielle DEMANGE, 1989. "An axiomatic approach to proportionality between matrices", *Mathematics of Operations Research*, Vol. 14, No 4, pp. 700-719.

BALINSKI Michel L. and H. P. YOUNG, 1994. "Apportionment", in S.M. POLLOCK and al., Eds., Handbooks in OR & MS, Vol. 6, Elsevier Science B.V.

BALINSKI Michel and Victoriano R. GONZALEZ, 1996. "Mexican Electoral Law", roneo, to appear in *Electoral Studies*.

BON, Ranko 1986. "Comparative Stability Analysis of Demand-Side and Supply-side Input-Output Models," *International Journal of Forecasting* 2, 231-235.

CHEN, Chia-Yon and Adam ROSE 1986. "The Joint Stability of Input-Output Production and Allocation Coefficients," *Modeling and Simulation* 17, 251-255,

______ 1991. "The absolute and relative Joint Stability of Input-Output Production and Allocation Coefficients," in A.W.A. Peterson (Ed.) *Advances in Input-Output Analysis*. Oxford University Press, New-York, pp. 25-36.

DIETZENBACHER, Erik. 1997. "In vindication of the Ghosh model: a reinterpretation of a price model", *Journal of Regional Science* 37, 629-651.

GRUVER, Gene W. 1989. "A Comment on the Plausibility of Supply-Driven Input-Output Models," *Journal of Regional Science* 29, 441-450.

GUIASU, S., 1979. "An entropic measure of connection and interdependence between the subsystems of a given large system", *Proceedings the 3d formation symposium on mathematical methods for the analysis of large-scale systems (May 16-19, 1978)*, Academia, Prague, pp. 113-124.

INSEE, various years. "Comptes et Indicateurs Economiques, Rapport sur les comptes de la nation", *INSEE Résultats*, Séries générales.

JACKSON, Randall W., P. ROGERSON, D. PLANE and B. O hUALLACHAIN, 1990, "A Causative Matrix Approach to Interpreting Structural Change", *Economic Systems Research,* Vol. 2, No 3, pp. 259-269.

KULLBACK, S., and R.A. LIEBLER, 1951. "On information and sufficiency", *Annals of Mathematics and Statistics*, Vol. 22, pp. 79-86.

KULLBACK S., 1959. *Information, Theory and Statistics*, Wiley.

van der LINDEN, J. and E. DIETZENBACHER, 1995. "The nature of changes in the EU cost structure of production 1965-85: an RAS approach", in H.W. ARMSTRONG and R.W. VICKERMAN, Eds, *Convergence and divergence among European Regions,* Pion Limited, London, pp. 124-139.

de MESNARD, Louis 1990*a*. *Dynamique de la structure industrielle française*. Economica, Paris.

______ 1990*b*. "Biproportional Method for Analyzing Interindustry Dynamics: the Case of France", *Economic Systems Research*, 2, 271-293.

______ 1994. "Unicity of Biproportion", *SIAM Journal on Matrix Analysis and Applications* 2, 15, 490-495.

______ 1996. "Biproportional Methods And Interindustry Dynamics : The Case of Energy in France", in *Models for Energy Policy*, Jean-Baptiste LESOURD, Jacques PERCEBOIS et François VALETTE Eds., ROUTLEDGE, London and New-York.

______ 1997. "A biproportional filter to compare technical and allocation coefficient variations", *Journal of Regional Science*, 37, 4, pp. 541-564.

______ 1998. "Analyzing structural change: the biproportional mean filter and the biproportional bimarkovian filter", *Twelfth International Conference on Input-Output Techniques,* New-York City, 18-22 mai 1998.

______ 2000. "Bicausative matrices to measure structural change: are they a good tool?", to appear in *The Annals of Regional Science*.

MILLER, Ronald E., 1989. "Stability of Supply Coefficients and Consistency of Supply-Driven and Demand-Driven Input-Output Models: a Comment," *Environment and Planning A* 21, 1113-1120.

MILLER Ronald E. and Peter D. BLAIR, 1985. *Input-output analysis, Foundations and extensions*, Prentice-Hall, Englewood Cliffs.

OOSTERHAVEN, Jan 1988. "On the Plausibility of the Supply-Driven Input-Output Model," *Journal of Regional Science* 28, 203-217.

______ 1989. "The Supply-Driven Input-Output Model: A New Interpretation but Still Implausible," *Journal of Regional Science* 29, 459-465.

______ 1996. "Leontief Versus Ghoshian Price And Quantity Models", *Southern Economic Journal* 62-3, pp. 750-759.

PLANE David, and P. ROGERSON 1986. "Dynamic Flow Modeling with Interregional Dependency Effects", *Demography*, 23, pp. 91-104.

ROGERSON, Peter and D. PLANE, 1984. "Modeling temporal change in flow matrices", *Papers of the Regional Science Association*, 54, pp. 147-164.

Rose, Adam and Tim ALLISON 1989. "On the Plausibility of the Supply-Driven Input-Output Model: Empirical Evidence on Joint Stability," *Journal of Regional Science* 29, 451-458.

ROTELLA, Frédéric and Pierre BORNE, 1995. *Théorie et pratique du calcul matriciel*, Editions Technip.

SNICKARS, F. and J.W. WEIBULL, 1977. "A Minimum Information Principle. Theory and Practice.", *Regional Science and Urban Economics,* Vol. 7, pp. 137-168.

SNOWER, Dennis J., 1990, "New methods of updating input-output matrices", *Economic Systems Research*, 2, 1, pp. 27-37.

STONE, R. and J.A.C. BROWN, 1962. "A long-term growth model for the British economy", in: R.C. GEARY (ed.), *Europe's future in figures*, North-Holland, Amsterdam.

TOH, Mun-Heng, 1998. "The RAS Approach in Updating Input-Output Matrices: An Instrumental Variable Interpretation and Analysis of Structural Change", *Economic Systems Research* 10, 63-78.

WATANABE, S., 1969. *Knowing and guessing. A quantitative study of inference and information*. Wiley, New-York.

WILSON, A.G., 1970. *Entropy in Urban and Regional Modeling*, Pion.

VII. Tables and figures

1980	Buildings	Energy	Minerals	Manu-facturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services	Final Demand	Output
Agriculture...	270 732	196	63	24 955	0	25 520	233	2 305	0	468 699	**792 703**
Energy	18 603	167 784	23 722	48 846	8 091	6 285	28 118	7 129	877	221 557	**531 012**
Minerals	1 962	2 303	83 346	72 775	60 063	1 880	493	810	0	71 271	**294 903**
Manufacturing	50 722	13 485	10 610	439 871	74 100	11 480	13 867	59 304	3 437	1 136 942	**1 813 818**
Buildings	1 033	6 042	381	2 050	231	406	627	2 917	5 891	431 123	**450 701**
Trade	831	263	1 401	2 627	813	3 524	1 866	8 703	823	136 133	**156 984**
Transport and Telecomm.	5 632	5 985	10 125	36 106	13 034	4 026	24 126	21 715	4 407	143 731	**268 887**
Services	18 792	12 857	9 866	83 142	48 570	12 646	15 907	103 334	12 802	476 609	**794 525**
Financial Services	1 038	568	829	5 826	5 940	790	636	1 796	3 812	115 447	**136 682**
Added-value	423 358	321 529	154 560	1 097 620	239 859	90 427	183 014	586 512	104 633	3 201 512	**5 240 215**
Output	**792 703**	**531 012**	**294 903**	**1 813 818**	**450 701**	**156 984**	**268 887**	**794 525**	**136 682**	**5 240 215**	

Table 1. Input-output table for 1980

1997	Agri-culture...	Energy	Minerals	Manu-facturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services	Final Demand	Output
Agriculture...	322 195	82	18	26 579	0	29 155	262	3 793	0	652 127	**1 034 211**
Energy	21 967	131 572	17 340	57 330	9 039	7 886	37 493	11 455	1 511	278 729	**574 322**
Minerals	1 897	13 704	73 056	75 138	52 009	2 019	294	1 147	0	86 226	**305 490**
Manufacturing	65 350	13 689	9 949	643 225	77 183	14 998	22 418	110 662	3 360	1 876 975	**2 837 809**
Buildings	1 308	7 462	311	2 567	205	450	779	5 147	11 980	435 214	**465 423**
Trade	902	283	908	2 756	595	3 834	2 524	12 399	420	168 423	**193 044**
Transport and Telecomm.	8 304	7 026	9 786	66 975	15 001	7 352	53 145	59 148	8 055	253 161	**487 953**
Services	34 278	26 771	13 246	160 772	65 040	21 598	25 851	224 065	34 205	838 362	**1 444 188**
Financial Services	3 168	1 791	1 616	18 459	12 291	1 341	2 107	5 507	987 446	145 990	**1 179 716**
Added-value	574 842	371 942	179 260	1 784 008	234 060	104 411	343 080	1 010 865	132 739	4 735 207	**8 522 156**
Output	**1 034 211**	**574 322**	**305 490**	**2 837 809**	**465 423**	**193 044**	**487 953**	**1 444 188**	**1 179 716**	**8522156**	

Table 2. Input-output table for 1997

C based on inverse matrices	**Agri-culture...**	**Energy**	**Minerals**	**Manu-facturing**	**Buildings**	**Trade**	**Transport and Telecom.**	**Services**	**Financial Services**
Agriculture...	0.978020	-0.000188	-0.000588	-0.007009	-0.000213	-0.032388	-0.000909	-0.001059	-0.002325
Energy	-0.004692	0.928007	-0.035239	-0.011832	-0.005093	-0.003089	-0.039038	-0.000944	-0.013546
Minerals	-0.001302	0.023335	1.004467	-0.022213	-0.053228	-0.003719	-0.003390	-0.000441	-0.006258
Manufacturing	-0.002428	-0.000301	-0.004832	0.988599	-0.017293	0.001931	-0.007881	-0.000600	-0.045208
Buildings	0.000198	0.001970	-0.000297	0.000086	0.914776	0.000280	-0.000781	0.000314	-0.027662
Trade	-0.000040	0.000308	-0.001498	-0.000352	-0.000421	0.952228	-0.001363	-0.002084	-0.006364
Transport and Telecommunications	0.001252	0.002198	-0.002823	0.003449	0.000187	0.013095	1.033929	0.013584	-0.033719
Services	0.012133	0.026504	0.012507	0.014496	0.031391	0.036846	-0.003537	0.971099	-0.074636
Financial Services	0.016858	0.018167	0.028304	0.034777	0.129894	0.034817	0.022970	0.020131	1.209719

Table 3. Causative matrix 1980/1997, based on inverse matrices

C based on direct matrices	**Agri-culture...**	**Energy**	**Minerals**	**Manu-facturing**	**Buildings**	**Trade**	**Buildings**	**Services**	**Financial Services**
Agriculture...	0.958951	-0.011314	0.009073	-0.022090	0.210353	-0.906155	0.047631	0.077316	-0.421638
Energy	0.009650	0.826050	-0.002153	-0.003934	-0.384087	0.158514	-0.052877	-0.000919	0.449191
Minerals	0.003914	0.042692	0.979570	-0.033269	1.035944	0.176445	-0.046811	0.007051	-1.588412
Manufacturing	0.010407	0.018109	0.009308	1.006449	-0.246185	0.134790	-0.003001	-0.063338	-0.317095
Buildings	0.000078	0.041925	-0.004515	-0.000760	0.109691	-0.018828	-0.048532	0.032454	-0.131094
Trade	0.000586	0.006263	-0.002530	0.000722	-0.145419	0.839969	-0.001829	-0.010946	0.083538
Transport and Telecommunications	-0.001645	0.023506	-0.030048	0.006167	-0.714651	0.202697	1.281932	0.010606	-0.403282
Services	0.017805	0.180426	0.023596	0.042680	-2.619437	0.599590	-0.311563	1.077434	0.855066
Financial Services	0.000253	-0.127658	0.017698	0.004034	3.753791	-0.187022	0.135049	-0.129658	2.473725

Table 4. Causative matrix 1980/1997, based on direct matrices

***K*(1980, 1997)**	**Agri-culture...**	**Energy**	**Minerals**	**Manu-facturing**	**Buildings**	**Trade**	**Transport and Telecom.**	**Services**	**Financial Services**
Agriculture...	318 681.59	210.33	50.96	30 271.13	0.00	28 527.08	350.48	3 992.43	0.00
Energy	18 057.72	148 475.97	15 822.34	48 861.03	4 855.38	5 793.55	34 878.55	10 182.58	8 665.87
Minerals	2 429.58	2 599.87	70 918.02	92 868.44	45 981.21	2 210.80	780.14	1 475.93	0.00
Manufacturing	67 661.55	16 399.18	9 725.24	604 678.58	61 108.96	14 542.76	23 638.62	116 406.95	46 672.16
Buildings	418.84	2 233.33	106.15	856.55	57.90	156.33	324.87	1 740.33	24 314.69
Trade	636.23	183.57	737.04	2 072.65	384.81	2 562.17	1 825.65	9 804.63	6 414.25
Transport and Telecommunications	7 565.83	7 329.65	9 346.05	49 983.50	10 824.61	5 136.02	41 416.47	42 924.26	60 265.60
Services	24 341.50	15 182.38	8 781.22	110 980.98	38 894.09	15 555.60	26 330.36	196 954.98	168 804.89
Financial Services	19 576.16	9 765.72	10 742.97	113 228.13	69 256.05	14 148.70	15 327.85	49 840.90	731 839.53

Table 5. $K(\mathbf{Z},\ \mathbf{Z}^*)$

K(1997, 1980)	Agri-culture...	Energy	Minerals	Manu-facturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services
Agriculture...	273 889.90	84.50	21.83	22 082.10	0.00	25 267.38	190.00	2 468.30	0.00
Energy	20 982.61	152 345.43	23 633.24	53 519.88	11 860.75	7 679.54	30 552.36	8 376.08	505.10
Minerals	1 566.47	13 717.60	86 078.54	60 639.78	58 997.72	1 699.73	207.11	725.06	0.00
Manufacturing	46 507.00	11 809.23	10 102.70	447 383.68	75 456.67	10 881.68	13 610.55	60 287.65	836.83
Buildings	1 121.01	7 752.34	380.32	2 150.16	241.36	393.19	569.57	3 376.85	3 593.21
Trade	864.63	328.84	1 241.92	2 581.94	783.51	3 746.83	2 064.04	9 098.41	140.90
Transport and Telecommunications	4 772.20	4 894.60	8 024.58	37 617.41	11 842.82	4 307.52	26 055.54	26 021.29	1 620.03
Services	19 471.11	18 433.89	10 736.07	89 254.48	50 752.67	12 507.73	12 527.33	97 433.03	6 799.70
Financial Services	170.09	116.56	123.80	968.58	906.51	73.40	96.51	226.34	18 553.23

Table 6. $K(\mathbf{Z}^*,\ \mathbf{Z})$

K(1997, 1980)	Agri-culture...	Energy	Minerals	Manu-facturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services
Agriculture...	443 611.27	252.94	69.75	34 575.44	0.00	31 133.74	372.38	4 125.98	0.00
Energy	22 559.34	160 249.48	19 436.02	50 086.37	6 054.57	5 674.61	33 257.97	9 444.19	6 744.94
Minerals	2 691.08	2 487.85	77 236.96	84 402.84	50 836.16	1 919.88	659.54	1 213.68	0.00
Manufacturing	69 824.67	14 620.63	9 868.26	512 017.82	62 946.10	11 766.36	18 619.28	89 184.53	30 007.34
Buildings	520.68	2 398.60	129.75	873.72	71.85	152.37	308.25	1 606.21	18 832.06
Trade	791.24	197.23	901.28	2 115.02	477.68	2 498.22	1 732.96	9 052.53	4 969.85
Transport and Telecommunications	7 566.51	6 332.86	9 190.53	41 016.57	10 805.59	4 027.12	31 614.47	31 870.26	37 550.11
Services	24 458.55	13 179.53	8 675.84	91 500.90	39 008.92	12 254.58	20 193.61	146 924.40	105 674.67
Financial Services	14 414.66	6 212.38	7 778.11	68 410.81	50 901.62	8 168.12	8 614.54	27 246.23	335 734.03

Table 7. $K(\mathbf{Z},\ \text{mean})$

K(1997, mean)	Agri-culture...	Energy	Minerals	Manu-facturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services
Agriculture...	447 695.56	102.47	24.61	30 584.01	0.00	31 405.51	284.76	4 044.59	0.00
Energy	26 607.69	143 317.39	20 668.93	57 505.61	10 530.94	7 404.95	35 521.77	10 647.78	1 302.44
Minerals	2 089.59	13 575.02	79 192.21	68 540.24	55 103.96	1 724.09	253.31	969.58	0.00
Manufacturing	65 996.65	12 432.17	9 887.54	537 936.50	74 973.55	11 741.90	17 708.47	85 763.47	2 414.75
Buildings	1 352.14	6 936.93	316.38	2 197.51	203.83	360.62	629.88	4 083.15	8 813.06
Trade	1 042.86	294.24	1 033.09	2 638.69	661.68	3 436.36	2 282.52	11 001.00	345.56
Transport and Telecommunications	7 762.20	5 906.15	9 001.92	51 844.42	13 487.39	5 327.60	38 856.87	42 429.23	5 358.21
Services	31 642.52	22 223.84	12 032.97	122 901.65	57 749.23	15 456.03	18 665.55	158 729.30	22 469.93
Financial Services	2 248.79	1 143.30	1 128.85	10 850.86	8 391.93	737.94	1 169.87	2 999.90	498 809.05

Table 8. $K(\mathbf{Z}^*,\ \text{mean})$

K(1997, mean)	Agri-culture...	Energy	Minerals	Manu-facturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services
Agriculture...	0.716452	0.000834	0.000347	0.036431	0.000000	0.236817	0.001885	0.007235	0.000000
Energy	0.037608	0.545333	0.099677	0.054474	0.022610	0.044555	0.173760	0.017095	0.004888
Minerals	0.006306	0.011901	0.556792	0.129035	0.266846	0.021189	0.004844	0.003088	0.000000
Manufacturing	0.084256	0.036014	0.036632	0.403081	0.170143	0.066870	0.070413	0.116849	0.015742
Buildings	0.028669	0.269591	0.021978	0.031386	0.008862	0.039512	0.053192	0.096026	0.450786
Trade	0.022224	0.011308	0.077876	0.038756	0.030054	0.330478	0.152544	0.276074	0.060686
Transport and Telecommunications	0.028159	0.048109	0.105219	0.099586	0.090079	0.070586	0.368727	0.128781	0.060753
Services	0.044341	0.048773	0.048386	0.108222	0.158413	0.104634	0.114732	0.289210	0.083287
Financial Services	0.031984	0.028138	0.053093	0.099030	0.252994	0.085359	0.059904	0.065641	0.323858

Table 9. 1980 bimarkovized: $K(\mathbf{Z},\ \mathbf{1}^M)$

K(1997, 1980)	Agri-culture...	Energy	Minerals	Manu-facturing	Buildings	Trade	Transport and Telecom.	Services	Financial Services
Agriculture...	0.720838	0.000270	0.000134	0.034507	0.000000	0.235614	0.001456	0.007180	0.000000
Energy	0.047732	0.421356	0.125007	0.072290	0.047219	0.061896	0.202379	0.021060	0.001060
Minerals	0.004291	0.045684	0.548240	0.098624	0.282818	0.016496	0.001652	0.002195	0.000000
Manufacturing	0.074098	0.022876	0.037427	0.423232	0.210399	0.061428	0.063144	0.106166	0.001230
Buildings	0.048258	0.405756	0.038069	0.054960	0.018184	0.059972		0.160674	0.142732
Trade	0.023670	0.010945	0.079055	0.041969	0.037538	0.363426	0.164535	0.275302	0.003559
Transport and Telecommunications	0.024620	0.030702	0.096263	0.115233	0.106928	0.078738	0.391423	0.148380	0.007712
Services	0.048265	0.055555	0.061880	0.131365	0.220170	0.109850	0.090421	0.266942	0.015553
Financial Services	0.008228	0.006855	0.013925	0.027820	0.076744	0.012580	0.013594	0.012101	0.828153

Table 10. 1997 bimarkovized: $K(\mathbf{Z}^*,\ \mathbf{1}^M)$

Sectors	**Norm. tech. coeff.**	**Causat. diag. inverse**	**Causat. off inverse**	**Causat. diag direct**	**Causat. off direct**	Biprop. direct	Biprop. reverse	**Average direct + reverse**	**Biprop. mean**	**Bimark.**
Agriculture...	4.025	0.978	-0.045	0.959	-1.017	4.368	1.614	2.991	2.690	3.474
Energy	17.647	0.928	-0.113	0.826	0.173	12.517	9.629	11.073	11.313	18.981
Minerals	5.025	1.004	-0.067	0.980	-0.402	8.325	2.611	5.468	5.856	5.265
Manufacturing	5.300	0.989	-0.077	1.006	-0.457	11.082	2.407	6.745	8.313	8.983
Buildings	8.449	0.915	-0.026	0.110	-0.129	28.205	3.310	15.758	21.918	19.439
Trade	8.081	0.952	-0.012	0.840	-0.070	16.434	2.629	9.532	10.873	8.514
Transport and Telecomm.	11.076	1.034	-0.003	1.282	-0.907	12.383	5.406	8.894	9.365	6.788
Services	5.090	0.971	0.056	1.077	-1.212	12.675	3.685	8.180	9.155	8.976
Financial Services	93.409	1.210	0.306	2.474	3.466	28.381	51.625	40.003	34.906	60.008

Table 11. Comparison of methods for France, column vectors, in %
(the results of the causative method are not at all comparable with those of other methods)

Sectors	Buildings	Biprop. direct	Biprop. reverse	Average direct + reverse	Biprop. mean	Bimark.
Agriculture...	1.134	1.346	1.321	1.334	1.113	0.499
Energy	11.340	7.229	5.500	6.364	6.528	13.486
Minerals	7.465	9.984	7.568	8.776	9.000	4.931
Manufacturing	4.597	6.303	1.380	3.842	4.900	5.206
Buildings	11.860	46.245	14.837	30.541	45.759	34.591
Trade	9.812	27.363	4.137	15.750	22.753	6.755
Transport and Telecommunications	10.421	24.975	5.063	15.019	20.238	6.858
Services	6.726	24.652	3.938	14.295	20.028	10.146
Financial Services	86.703	27.410	77.517	52.463	34.245	55.073

Table 12. Comparison of methods for France, row vectors, in %
(no results for row coefficients with the causative method)

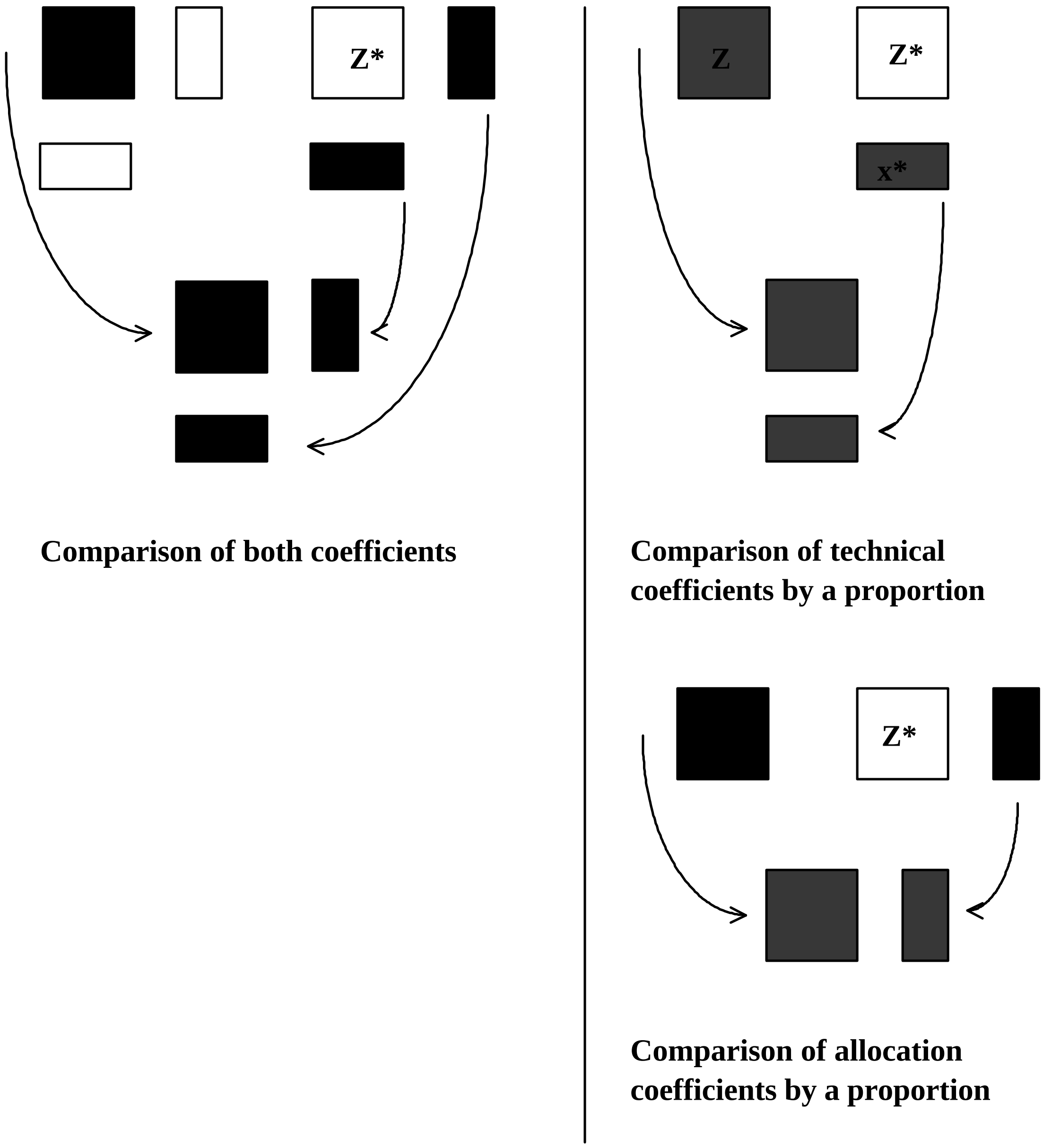

Figure 1. Principle of matrix comparisons over time.

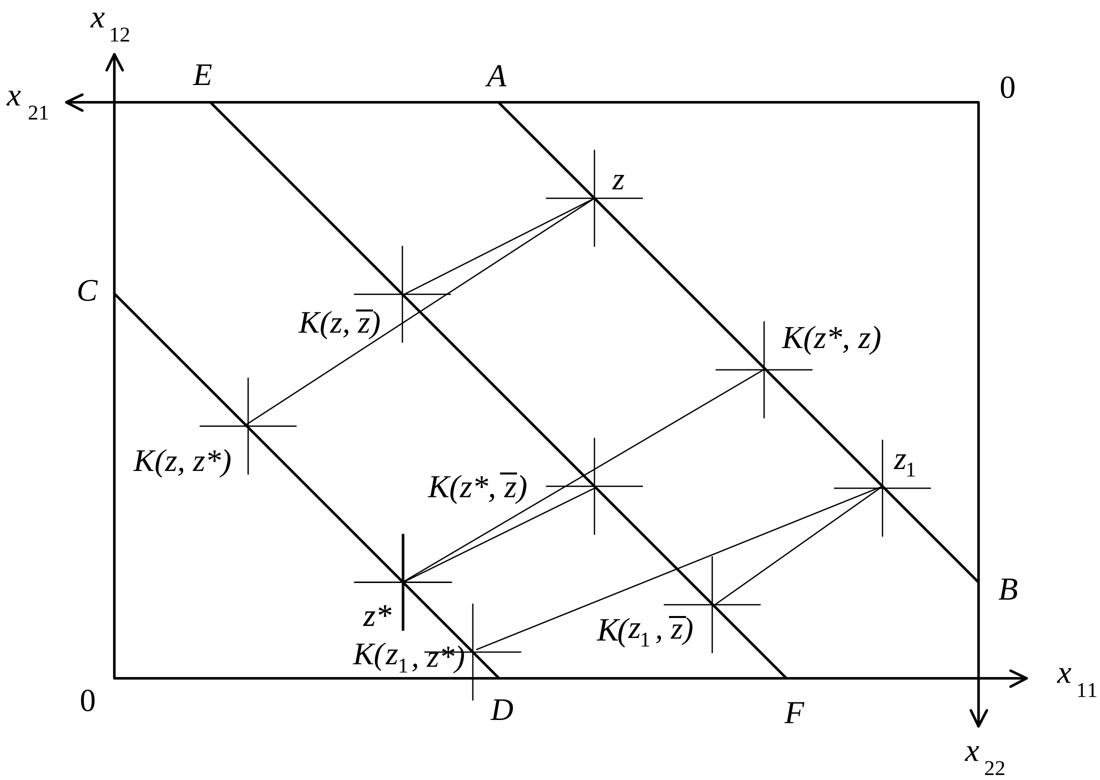

Figure 2. Edgeworth box for the ordinary biproportional projector
and the mean biproportional projector

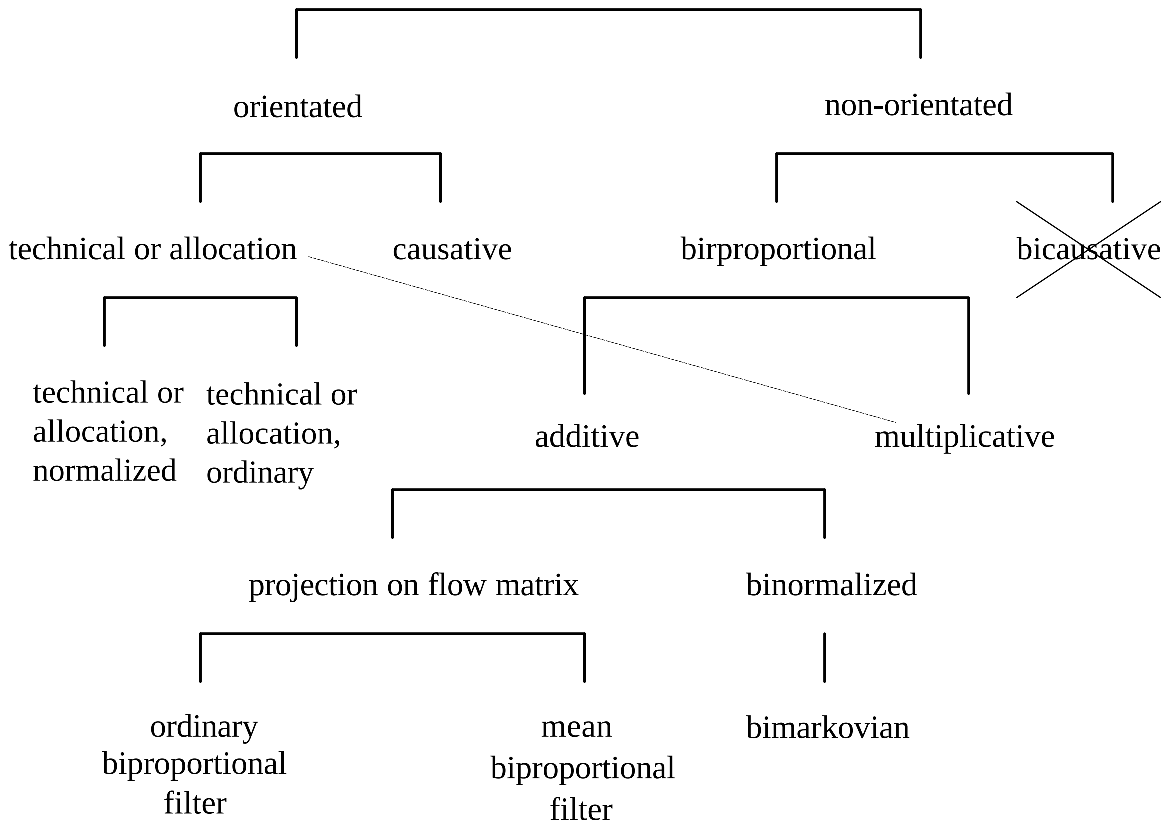

Figure 3. Typology of methods