AN ECONOMETRIC ANALYSIS OF BI-PROPORTIONAL PROPERTIES IN AN ECONOMETRIC INPUT-OUTPUT MODELING SYSTEM

by

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Abstract: Methods and models, such as bi-proportional adjustment procedure and econometric input-output model, to generate a series of input-output tables over time has been long studied and implemented. In this paper, temporal changes of input-output coefficients are examined in order to analyze their behavior. Within the Chicago Region Econometric Input-Output Model, a set of input-output relationships has been extracted analytically for the period 1975-2018. Using the empirical evidence for Chicago, this paper conducts econometric time series analysis to determine whether or not certain coefficients or sets of coefficients exhibit tendencies toward stability or predictable change while others require more extensive econometric estimation.

1. Introduction

Since its introduction in the *Programme for Growth* series (Cambridge University, Department of Applied Economics, 1963), the RAS or bi-proportional adjustment technique has become one of the most popular methods for adjusting input-output, social accounting, and demographic matrices. In the input-output literature, the technique has been used for two primary purposes: 1) the initial application, namely, the adjustment of a matrix observed at one time period to a new matrix for a subsequent time period, in which only row and column totals for intermediate demand and output are known; and 2) the adjustment of national input-output tables to represent regional tables in the regional input-output literature. The technique has been reviewed extensively; while critics have claimed that it is nothing more than a mechanical adjustment process, both Stone and Leontief, among others, have attempted to provide some theoretical basis to justify its application.

This paper's interest in the technique stems from entirely different premises. Since a series of input-output tables over time is available from the Chicago Region Econometric Input-Output Model (CREIM), the issue is not focused on the need to update tables. Rather, the interest lies in whether there exist any bi-proportional properties from this time series that would allow an analyst to efficiently update this series. Since the input-output tables were generated within a general equilibrium model (CREIM), a further issue centers on the degree to which the additional information attached by the non–input-output components produces tables that are not

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merely simple extrapolations of the earlier one. In part, initial stimulus for this approach was a paper of Lecomber (1969) in which he explored RAS projections when two or more matrices were known. In fact, the Chicago input-output tables were adjusted in the spirit that Lecomber proposed for cases where a large number of matrices were known.

In the next section, the RAS technique is presented and reviewed, as well as Lecomber's formulation and some extensions. Section 3 analyzes the estimates from the RAS procedure, focusing on its ability to capture structural changes. The fourth section introduces some experiments using Lecomber's and some other formulations conducted on the tables. Some evaluation and concluding comments complete this paper.

2. The RAS or Bi-Proportional Technique

One of the problems associated with input-output analysis is based on an assumption of constant production relationships (coefficients) over time, especially when this time horizon stretches over a period of more than a decade. At the regional level, the issues are further complicated by the potential for change in trading relationships and problems that may arise if input-output components are nested, linked, or integrated with other models, such as computable general equilibrium models and demo-economic models (Israilevich *et al.*, 1997). The costs of constructing survey-based regional input-output tables over time are prohibitive; as a result, the development of regional input-output tables has relied on two alternatives, non-survey and partial survey techniques. In the following subsections, one of the partial survey techniques, the *RAS* or *bi-proportional adjustment technique*, is presented and reviewed.

2-1. Overview of the Technique

The *RAS* or *bi-proportional* technique was developed by Stone (1961) and subsequently Stone and Brown (1962), Cambridge University, Department of Applied Economics (1963), and summarized by Bacharach (1970). These studies approached the problem of finding the most efficient way to update the U.K. input-output tables by adopting the following procedure:

$$A_{(t+1)} = \hat{r} A_{(t)} \hat{s} \tag{1}$$

where $\hat{r}$ and $\hat{s}$ may be considered as multipliers that implicitly transform a prior matrix to a new one. Equation (1) can be transformed to the following general expression:

$$A_{(t+1)} = f\left[A_{(t)}, u_{(t+1)}, v_{(t+1)}, X_{(t+1)}\right] \tag{2}$$

where $u_{(t+1)}$ is the vector of total intermediate outputs at time *t+1*, $v_{(t+1)}$ is the vector of total intermediate inputs at *t+1*, and $X_{(t+1)}$ is the vector of total outputs at *t+1*. Given these three sets of data at *t+1* and the input coefficient matrix at *t*, $A_{(t)}$, $\hat{r}$, and $\hat{s}$ can be estimated as follows.

First, an estimated vector of intermediate outputs is obtained using $A_{(t)}$ and known $X_{(t+1)}$:

$$u_1 = A_{(t)} X_{(t+1)} \tag{3}$$

This estimate, u_1, is adjusted to conform to the observed value, $u_{(t+1)}$, via adjustment of the matrix, $A_{(t)}$. A new matrix, A_1, will be produced:

$$A_1 = r_1 A_{(t)} \tag{4}$$

where $r_1 = \hat{u}_{(t+1)} \hat{u}_1^{-1}$, and $\hat{u}_{(t+1)}$ and $\hat{u}_1$ are the diagonal matrices of $u_{(t+1)}$ and u_1, respectively. This new matrix, A_1, is now used to obtain the estimate of intermediate input vector, v_1, and the matrix A_1 is further adjusted to a new matrix, A_2, to ensure equality with observed intermediate inputs, $v_{(t+1)}$:

$$v_1 = \hat{X}_{(t+1)} A_1^T i \tag{5}$$

$$A_2 = A_1 s_1 \tag{6}$$

where $s_1 = \hat{v}_{(t+1)} \hat{v}_1^{-1}$, and A_1^T is the transposed matrix of A_1, and i is a vector with all elements equal to unity. Then, the process returns to equation (3), where u_2 is now estimated as follows:

$$u_2 = A_2 X_{(t+1)} \tag{7}$$

and so on through equation (6). Equations (3) through (6) represent one complete iteration. In this first iteration, empirical evidence suggests that the process converges rapidly, usually within ten iterations (Hewings, 1985). After achieving the conversions of r_k and s_k at the kth iteration, the estimated r and s would be derived as follows:

$$r = r_n \cdots r_2 r_1;\; s = s_1 s_2 \cdots s_n \tag{8}$$

Because the adjustment process operates on the A matrices, the adjustment process is conservative, making only the minimally necessary adjustments to ensure agreement with the vectors $u_{(t+1)}$ and $v_{(t+1)}$.

Bacharach (1970), responding to the information theoretic approach for updating matrices by Uribe *et al.* (1965), showed that this RAS method achieves "closeness" and is equivalent to the following minimization problem:

$$\min \sum_{i,j} a_{ij}^{t+1} \log \frac{a_{ij}^{t+1}}{a_{ij}^{t}}; \tag{9}$$

$$\text{subject to} \quad A_{(t+1)} i = u_{(t+1)},$$

$$iA_{(t+1)} = v_{(t+1)}$$

where $A_{(t)} = \left\| a_{ij}^{t} \right\|$, and $A_{(t+1)} = \left\| a_{ij}^{t+1} \right\|$. The solution is the bi-porportional estimates, r and s. Reviewing several methods to achieve closeness between two matrices with row- and column-sum constraints, Hewings and Janson (1980) concluded that, in applications to input-output matrices, the degree to which $A_{(t+1)}$ may be claimed to be within $A_{(t)}$'s neighbor can be showed only with the empirical observation of $A_{(t+1)}$.

The economic interpretation of the $\hat{r}$ and $\hat{s}$ has proven to be contentious; Stone (1962) offered the interpretation that r_i is a measure of "substitution effects"--the extent to which, during the time interval, the input *i* has substituted for other inputs, or has been substituted by them--and that s_j is a measure of "fabrication effects" in the production of *j*--the extent to which the industry *j* has decreased (increased) its consumption of intermediate inputs per unit of gross output. Leontief (1941) also suggested this bi-proportional property in input-output tables. His interpretation differs slightly from Stone's: r_i is defined as a measure of the increased productivity of *i* in all uses, and s_j is regarded as a measure of the joint effect of increased productivity in industry j and of a decrease in its rate of investment.

In contrast to these economic interpretations of $\hat{r}$ and $\hat{s}$, however, many researchers discount this "oversimplified" view of the RAS procedure in which such change is distributed throughout an economy (Miller and Blair, 1985). The critics view the RAS technique as a purely computational procedure, which emerges as the solution to a constrained optimization problem subject to the row and column sums, as seen in (9).

2-2. Modifications of the RAS Technique

The original RAS procedure utilizes only one complete matrix, $A_{(t)}$, with some future data, $u_{(t+1)}$, $v_{(t+1)}$, and $X_{(t+1)}$, in order to estimate the future matrix. When more data, especially two or more input coefficient matrices, are available, Lecomber (1969, 1975) proposed modifications of the RAS method to utilize this additional data in the most efficient way, put forward by Johansen (1968).

Johansen's problem was to estimate $A_{(t)}$ by given $A_{(0)} = \left\| a_{ij}^0 \right\|$, $A_{(1)} = \left\| a_{ij}^1 \right\|$, and $X_{(1)}$. He argued that individual coefficients contain more information than the row and column sums and that this information should be taken into account. In other words, the bi-proportional hypothesis is not only an imperfect representation of the underlying movement of coefficients over time, but also the coefficients for any year would be subject to disturbances and errors in measurement. Then, the RAS method can be transformed as an error minimization problem:

$$\sum_{i,j} \varepsilon_{ij}{}^2 = \sum \left(a_{ij}^1 - a_{ij}^0 r_i s_j \right)^2 \tag{10}$$

Furthermore, the assumption that both $A_{(0)}$ and $A_{(1)}$ are subject to disturbances would appear to be more plausible. Consider the following model:

$$a_{ij}^t = \alpha_{ij} r_i^t s_j^t + \varepsilon_{ij}^t \tag{11}$$

where α_{ij} is the true (disturbance-free) coefficient for the (*i*, *j*) pair and ε_{ij}^t is the error term for (*i*, *j*) at time *t*. Moreover, the following can be easily assumed:

$$r_i^0 = s_j^0 = 1 \tag{12}$$

From the model (11), α_{ij}, r_i, and s_j can be estimated by minimizing $\sum_{i,j} \varepsilon_{ij}{}^2$. Further, this minimization procedure may be simplified by assuming a multiplicative error term; consequently, the model (11) can be transformed to the log-linear form:

$$\log a_{ij}^t = \log \alpha_{ij} + \log r_i^t + \log s_j^t + \log \varepsilon_{ij}^t \quad (13)$$

Then, $\log \alpha_{ij}$, $\log r_i^t$, and $\log s_j^t$ can be estimated by minimizing $\sum_{i,j} \left(\log \varepsilon_{ij}^t\right)^2$. Given data $A_{(0)} = \left\| a_{ij}^0 \right\|$ and $A_{(1)} = \left\| a_{ij}^1 \right\|$, the model (13) can be further transformed by eliminating α_{ij}:

$$\log a_{ij}^1 - \log a_{ij}^0 = \log r_i^1 + \log s_j^1 + \log\left(\frac{\varepsilon_{ij}^1}{\varepsilon_{ij}^0}\right) \quad (14)$$

The model (14) can be estimated by ordinary least squares regressions with dummy variables for each r_i and s_j. Unlike the original RAS technique in which only the row and column sums are known, r_i and s_j can be estimated by the minimization of the squared error term for each (*i*, *j*). However, this Johansen-Lecomber formulation has at least two drawbacks. First, the use of dummy variables for each r_i and s_j necessitates dropping one variable from each r_i and s_j set in order to avoid the singularity problem. In other words, one of r_i and one of s_j are set equal to unity--no change over the period. Secondly, since the regression model (14) minimizes $\sum_{i,j}\left(\log\left(\frac{\varepsilon_{ij}^1}{\varepsilon_{ij}^0}\right)\right)^2$, the estimated r_i and s_j may not minimize $\sum_{i,j}\left(\varepsilon_{ij}^0\right)^2$, nor $\sum_{i,j}\left(\varepsilon_{ij}^1\right)^2$.

In order to improve accuracy, additional modified versions of the RAS technique have been proposed. Allen and Lecomber (1975) introduced the *generalized version of RAS* in which some of the elements in the forecast matrix, $A_{(t+1)}$, are estimated from exogenous information and the remaining part of the matrix is adjusted by the RAS procedure. Barker (1975) and Snower (1990) have extended the RAS method to incorporate price information. Although they may improve the accuracy of the estimation, these modifications substantially increase the data requirement.

Similar to the utilization of the *two-stage least squares* approach to the estimation of input coefficients by Gerking (1976a, 1976b, and 1979), Toh (1998) interpreted r_i and s_j as iterative instrumental variable estimates and thus was able to derive the asymptotic standard errors. Toh's main idea is to consider the given row and column sum of the intermediate vectors, $u_{(t+1)}$ and $v_{(t+1)}$, as the instrumental variables for r_i and s_j. Toh further proposed the RAS as an iterative sectoral optimization model; he concluded, however, that the RAS technique may not be useful for projection but for the study of structural change, especially when the economy experiences rapid structural change. However, no test of the accuracy of projections using his method has been conducted.

Bi-porportionality in an input-output system has been explored and extended to investigate different aspects of temporal changes. Mesnard analyzed interindustry dynamics (1990) and coefficient variation between original Leontief demand-driven system and Ghosh's supply-driven system employing bi-proportional filter (1997) in French cases. Although, methodologically, Mesnard's methods are an extension to RAS technique, his results are informative: for any sector, both row and column coefficients are found to change simultaneously.

2-3. The RAS Technique and the Field of Influence

Sonis and Hewings (1992) presented the relationship between the RAS procedure and the *field of influence* concept. Consider the relative coefficient change, ε_{ij} :

$$e_{ij} = a_{ij}\varepsilon_{ij} \tag{15}$$

where e_{ij} is the incremental change. Then

$$Q(\varepsilon) = \frac{\det B}{\det B(\varepsilon)} = 1 - \sum_{i_1, j_1} b_{j_1 i_1} a_{i_1 j_1} \varepsilon_{i_1 j_1} + \sum_{k=2}^{n} (-1)^k \sum_{\substack{i_r \neq i_s \\ j_r \neq j_s}}{}' B_{or}\begin{pmatrix} j_1 j_2 \cdots j_k \\ i_1 i_2 \cdots i_k \end{pmatrix} a_{i_1 j_1} \cdots a_{i_k j_k} \varepsilon_{i_1 j_1} \cdots \varepsilon_{i_k j_k} \tag{16}$$

With this formulation , the following form of the Leontief inverse results:

$$B(\varepsilon) = B + \frac{1}{Q(\varepsilon)} \left[\sum_{k=1}^{n} \sum_{\substack{i_r \neq i_s \\ j_r \neq j_s}}{}' F\begin{pmatrix} i_1 \cdots i_k \\ j_1 \cdots j_k \end{pmatrix} a_{j_1 i_1} \cdots a_{j_k i_k} \varepsilon_{i_1 j_1} \cdots \varepsilon_{i_k j_k} \right] \tag{17}$$

This form of relative coefficient change can be shown to be a general form of the RAS procedure. The RAS procedure is usually in the form

$$a_{ij}^{t+1} = r_i a_{ij}^t s_j \tag{18}$$

Set

$$r_i = 1 + \delta_i \, ; \; s_j = 1 + \eta_j \tag{19}$$

With the relative change, equation (18) can be transformed:

$$a_{ij}^{t+1} = a_{ij}^t \left(1 + \varepsilon_{ij}\right) = r_i a_{ij}^t s_j = \left(1 + \delta_i\right) a_{ij}^t \left(1 + \eta_j\right) = a_{ij}^t \left(1 + \delta_i + \eta_j + \delta_i \eta_j\right) \tag{20}$$

Therefore:

$$\varepsilon_{ij} = \delta_i + \eta_j + \delta_i \eta_j \tag{21}$$

Furthermore, if relative changes in δ_i and η_j are small, the products $\delta_i \eta_j$ can be ignored:

$$\varepsilon_{ij} \approx \delta_i + \eta_j \tag{22}$$

Hence, the RAS procedure is equivalent to the problem of relative coefficient change.

3. Bi-Proportionality of Changes

As indicated earlier, the Chicago input-output tables are extracted from the Chicago Region Econometric Input-Output Model (CREIM). This system of 250 equations includes both exogenous and endogenous variables. Endogenous coefficient change serves as the mechanism to clear markets in the quantity-adjustment process (see Israilevich *et al.*, 1997, for more details). The input-output coefficient matrix is not observed directly; however, it is possible to derive analytically a Leontief inverse matrix and, through inversion, the estimated direct coefficient matrix. Thus, the interest lies in the way these exogenous changes are manifested in the input-output coefficients and the degree to which these input-output coefficients are predictable under the usual conditions associated with the RAS technique. Using the Chicago region input-output tables derived from the CREIM during the period of 1980-1997, the $\hat{r}$ and $\hat{s}$ vectors can be estimated by the repeated iteration procedure described in equations (3) through (6). An important assumption here is that the error terms in derived input-output coefficients from the CREIM are normally distributed, and are independent and identically distributed; thus, the coefficients can be not "real" observations but treated as such.

3-1. General Observations

Figures 1 and 2 reveal the trends in values of r_i and s_j from equation (1), ranked by the volume of output in 1980. By and large, smaller sectors (those with lower rank) tend to exhibit greater variance over time while the larger sectors tend to have more r_i values that are <1 than in the case with the values of s_j. Overall, there seems to be greater volatility in the r_i values than in the s_j entries. This may confirm the *hollowing-out* process in the Chicago economy reported by Hewings *et al.* (1998), in which the level of dependence on local purchases and sales is declining; the tendency of the sectors with larger output to have r_i <1 may be the evidence of substitution, not across sectors, but in the location of purchase; the smaller volatility in the s_j entries indicates that the fabrication effect (technological change) seems relatively insignificant. Casual inspection would suggest few pronounced trends in either of these entries.

<<insert Figures 1 and 2 here>>

Figures 3 through 5 provide the sample trends in individual coefficients (a_{ij}, b_{ij}, r_i, s_j); these trends vary among various (i, j) combinations. For example, the interaction between Sectors 18 (Fabricated Metals) and 19 (Industrial Machinery and Equipment) reveal that the trends in the direct and inverse coefficients are mainly associated with changes in s_j (in this case, Sector 19). The variations in the direct and inverse coefficients can be seen to mirror the changes in s_j. For the interaction between 10 (Paper and Allied Products) and 11 (Printing and Publishing), the r_i seems to generate the changes in the coefficients. For self-influenced changes (Sector 5, Food and Kindred Products), the row and column effects seem to offset each other, producing little change in the coefficients. Among the inverse-important coefficients, the trends in 17 of the top 25 field of influence coefficients are mainly associated with the row effects of r_i, while 2 and 6 of them are generated by the changes in s_j and the combined effects of r_i and s_j, respectively. This result also confirms the observations in Figure 1 and 2.

<<insert Figures 3 to 5 here>>

3-2. Analysis of Estimation Error

As mentioned in the previous section, the RAS procedure does not claim to minimize the sum of squared errors, only to find a matrix that is as close as possible to the prior one, subject to the row and column constraints. Thus, the analyses of estimation error were conducted in order to investigate how well the RAS procedure can capture the changes in the direct input matrices over time. Paelinck and Waelbroeck (1963) identify the errors in the RAS technique as derived from the following three possibilities: 1) aggregation errors in the industrial classification, which may arise in all input-output analysis; 2) error derived from variations in substitution effects over utilizing industries, in violation of the assumed uniformity of these effects; and 3) a false estimate of any one cell, which would force offsetting errors in other elements of its row and column--such errors would spread over a wide area of the matrix. In conjunction with the field of influence concept, the last notion of Paelinck and Waelbroeck indicates the system-wide impact of changes in each coefficient.

The error terms are derived as follows:

$$e_{ij}^t = a_{ij}^t - r_i^t a_{ij}^{t-1} s_j^t \qquad (23)$$

Using Jensen's (1980) definition, this e_{ij}^t can be considered as *partitive accuracy*, which measures the cell-by-cell accuracy of the estimation. *Holistic accuracy*, on the other hand, emphasizes a "mathematical portrait" interpretation of the estimation, relying on the accuracy of the estimated table as a whole. For this holistic accuracy, at first, the estimation errors of each coefficient in the estimated Leontief inverse are employed:

$$\tilde{e}_{ij}^t = b_{ij}^t - \bar{b}_{ij}^t \qquad (24)$$

where $\left\|\bar{b}_{ij}^t\right\| = \bar{B}_{(t)} = (I - \hat{r}_{(t)} A_{(t-1)} \hat{s}_{(t)})^{-1}$.

Table 1 shows the estimation performance of the RAS technique on an annual basis, as indicted in equation (2), using mean absolute deviation (MAD) and mean absolute percentage error (MAPE). For partitive accuracy, some small fluctuations over the years can be observed (except the 85–86 estimation); in general, however, the estimation errors are rather small (less than 5%). The estimation performances for holistic accuracy are notably improved--less than 1%. Practically, however, updating an input-output table annually is quite rare, and tables are usually updated over a 5 or 10 year period. Tables 2 and 3 exhibit the estimation performance over 5 and 10 years, respectively; these estimations are carried out by the following models:

$$A_{(t+5)} = f\left[A_{(t)}, u_{(t+5)}, v_{(t+5)}, X_{(t+5)}\right] \qquad (25)$$

$$A_{(t+10)} = f\left[A_{(t)}, u_{(t+10)}, v_{(t+10)}, X_{(t+10)}\right] \tag{26}$$

The results clearly indicate that, as the estimation period becomes longer, both the partitive accuracy and holistic accuracy deteriorate. Although the partitive accuracy decreases relatively rapidly, the holistic accuracy remains small (about 2% over 10 years). This tendency of increasing errors for longer estimation periods does not result from the exponential nature of RAS projection; rather, as Toh (1998) noted, the RAS technique may not be appropriate for estimation for longer time periods, and hence larger structural changes, since the RAS technique tries to derive the estimated matrix as close as possible to the base-year matrix.

<<insert Tables 1 to 3 here>>

Tables 4 through 6 are similar analyses of the estimation errors but only for the coefficients with the top 25 direct fields of influence--the most important coefficients. The results reveal the same tendencies as in Tables 1 through 3, in which the estimation error increases as the estimation time becomes longer. However, overall, the partitive and holistic accuracy are considerably better than the results for all coefficients. These results might imply that the inverse important coefficients are stable over time. Recall that most of the top 25 cells with the largest fields of influence are concentrated across rows.

<<insert Tables 4 to 6 here>>

Although the findings in this sub-section are consistent with the previous studies (Szyrmer, 1989, and Miller and Blair, 1985, among others), the results here would raise an interesting question: If the trends of r_i and s_j are relatively easily determined over time using some econometric method, can the RAS technique replace more complex models for the estimation of input-output coefficients? In the following sub-section, the trends of r_i and s_j are investigated.

3-3. Tests for the Trends of r_i and s_j

As presented in the previous section, the RAS technique can trace the changes in direct input coefficients, a_{ij}, relatively well, especially for a short period of time and in terms of holistic accuracy. The question arises whether $\hat{r}$ and $\hat{s}$ themselves are predictable so that the future input coefficient matrices can be estimated solely by the predicted $\hat{r}$ and $\hat{s}$. The following tests investigate the behaviors of r_i and s_j over time.

Table 7 provides a summary of a *runs test* that was applied to the time series of r_i and s_j in order to test the randomness of the trends. For the majority of sectors, r_i and s_j indicate random trends (no particular monotonic or cyclical trends). An interesting observation is that the sectors with non-random trends vary between r_i and s_j; this also confirms the different trends in

r_i and s_j shown in the Figures 1 and 2. By and large, the trends of r_i and s_j are random. Since the *run test* is for non-parametric analysis, more extensive tests were adopted to explore the statistical properties of r_i and s_j, assuming that r_i and s_j can be considered random variables.

<<insert Table 7 here>>

If r_i and s_j can be considered as univariate time series models, in which dependence is based only on the past (autoregressive) trends, the models applied to these data need to be examined in terms of their integration (non-stationary) process in order to specify the models. Investigation focused on models of $I(1)$, integrated of order one, referred to as a unit root process. The importance of the tests is that if the process is a *random walk*,

$$y_t = y_{t-1} + \varepsilon_t \quad (27)$$

the current observation is the simple sum of random disturbance terms, which possibly may be explained by exogenous variables. Furthermore, if a variable is truly $I(1)$, then shocks to it will have permanent effects. Augmented Dicky-Fuller (ADF) tests, with lagged differences for auto-regressive specification were employed to test the processes of r_i and s_j with the following three model specifications, since no *a priori* specification is known:

$$y_t = y_{t-1} + \varepsilon_t \text{ (random walk model)} \quad (27')$$

$$y_t = \mu + y_{t-1} + \varepsilon_t \text{ (random walk with drift)} \quad (28)$$

$$y_t = \mu + \beta t + y_{t-1} + \varepsilon_t \text{ (random walk with drift and trend)} \quad (29)$$

where y_t is either r_i and s_j at time *t*, and *t*=1981,...,1997.

Tables 8 and 9 show the results of ADF tests. Allowing maximum lags up to *t-5*, in a pure random walk specification (27') for all r_i and all s_j except Sector 3 (Mining), the null hypothesis of a unit root is not rejected at 5% level. In a random walk with drift specification, the results were more varied: Sectors 1, 3, and 31 for r_i and Sectors 4, 5, 13, 19, 20, and 22 for s_j reject the null hypothesis. Model (29), the random walk with drift and trend model, resulted in most sectors not rejecting the null hypothesis; the exceptions were Sectors 1, 25, 28, 31, and 33 for r_i and Sectors 1, 3, 4, 5, 16, 17, and 34 for s_j. In either model, the majority of sectors did not reject the null hypothesis of unit root.

<<insert Tables 8 and 9 here>>

Since the entire sample consists of only 17 observations (1980 to 1997), ADF tests with lagged differences consume observations so that with five lagged differences the sample size was decreased to only 11 observations. In order to address this problem and to analyze the sensitivity of the lagged differences, ADF tests with lesser lags (maximum of three) were conducted, and the results are shown in Tables 8 and 9. In the majority of cases, no changes occurred in the behavior of a sector in terms of whether the null hypothesis was accepted or rejected. However, there were some variations; with a maximum lag of three, a smaller number of sectors rejected the null hypothesis across the models. Overall, one might claim that the majority of the sectors do not reject the null, indicating unit root behavior, and hence non-stationary process. The results suggest the option of constructing *autoregressive integrated moving average* [ARIMA(p,d,q)] models for each r_i and s_j; however, the ARIMA model requires larger sample sizes (Harvey, 1989 and 1990). Furthermore, since r_i and s_j can capture the sudden changes of economic structure and technological advancement, the model for these using constant coefficients might perform poorly, either for forecasting or for analyzing the effect of policy change (Maddala and Kim, 1998). Likewise, if there is a break (sudden trend shift) in the deterministic trend, then unit root tests will lead to a misleading conclusion (Perron, 1989). Again, given the size of samples, the results here need to be considered carefully.

4. Experiments

In order to further investigate the bi-proportional properties of input-output systems, the following experiments were implemented and analyzed. First, utilizing the Johansen-Lecomber formulation, r_i and s_j are estimated by regression models for each year. Second, systems of equations approaches are examined, and the *vector autoregression* (VAR) model is employed to analyze the estimates of the RAS technique.

4-1. Lecomber Revisited

As presented in Section 2-2, the Johansen-Lecomber formulation of the RAS model is based on the regression model, (14). While it preserves the bi-proportional structure of the estimation procedure, this formulation has the advantage of minimizing the error terms. The comparison of the estimation error between the RAS technique and the Johansen-Lecomber formulation may elucidate the properties of the estimation procedures. In order to estimate the Lecomber model, r_i and s_j for Sector 6 (Tobacco Products) are set to unity (zero in the model) in order to avoid the singularity problem in regression estimation as noted in 2-2. Sector 6 was chosen because it has the smallest output throughout the period of 1980–1997.

Tables 10 through 13 show the results from the Johansen-Lecomber estimation. Table 10 displays the partitive and holistic accuracy of annual estimation, corresponding to Table 1 for the RAS procedure. Seemingly, the MAPEs for partitive accuracy in the Lecomber model are comparable to the ones in the RAS procedure; however, for holistic accuracy, the Lecomber model exhibits larger errors throughout the estimates. Furthermore, these larger MAPEs fluctuate across the estimates in a wider range than for the RAS estimates. Since the Johansen-

Lecomber formulation is basically a regression model, the estimation procedure provides statistical outputs, such as significance of the coefficients. Table 12 shows the estimation errors of the Lecomber model without insignificant coefficients--set to one (Lecomber model is in log-linear form: =1 when log =0) for corresponding r_i and s_j. The results indicate that both partitive and holistic accuracy are less than in Table 10; omitting insignificant coefficients has some, relatively small (1–2% increase in MAPE), impact on the estimation. Tables 12 and 13 exhibit the estimation accuracy for 5-year and 10-year spans. As found in Tables 2 and 3, the estimation accuracy deteriorates as the estimation span becomes longer, for both partitive and holistic accuracy. While the partitive accuracy for the Lecomber model is comparable to the RAS results, better accuracy in several cases, the holistic accuracy is, again, less accurate than the RAS counterparts.

<<*insert Tables 10 to 13 here*>>

Estimation errors were further investigated for the inverse important coefficients, the top 25 coefficients with the largest direct fields of influence. Tables 14 through 17 present the results. As in the RAS results, the estimation accuracy, for both partitive and holistic, improves for the annual estimation (Table 14). However, the MAPEs for partitive and holistic accuracy are much larger than in the RAS results, and partitive and holistic accuracy have similar values unlike in the RAS procedure, where the MAPEs for holistic accuracy are smaller than the ones for partitive accuracy. This tendency continues in the case without the inclusion of insignificant coefficients (Table 15). The most striking finding is that the MAPEs in Table 15, for both partitive and holistic accuracy, are larger than the ones in Table 14 with all the estimated r_i and s_j (some r_i and s_j for inverse important coefficients are statistically insignificant). As the estimation period becomes longer, the tendency found in Tables 14 and 15 continues. Although the increase of the MAPEs with longer periods is smaller than that in the RAS results, the estimation accuracy for partitive and especially for holistic accuracy is lower than the ones shown in Tables 5 and 6.

<<*insert Tables 14 to 17 here*>>

Although the Lecomber estimates may vary by setting r_i and s_j to unity in a different sector, the findings illustrate that the estimation by the RAS technique is quite accurate, especially in terms of holistic accuracy, compared to the Lecomber regression model. This may imply that, although both models possess a similar bi-proportional structure (taking into account row and column effects), adjusting coefficients by row and column sums will provide better estimation than the estimation by each coefficient—in this case, less information, the better results. This seems contradictory to received theory; however, after all, regression allows error terms (while minimizing them), whereas the RAS procedure adjusts a matrix with a pair of constraints (row and column sums) to derive the closest matrix to the base matrix. These row and column constraints may provide better results, especially for the inverted matrix, indicating the existence of row and column structures in the matrices. One aspect of the Lecomber model may contribute to this difference in estimation capability. As indicated in 2-2, the Lecomber model does not directly minimize the estimation errors; rather, it minimizes the sum of the squared logarithm of error ratios, $\sum_{i,j}\left(\log\left(\frac{\varepsilon_{ij}^{1}}{\varepsilon_{ij}^{0}}\right)\right)^2$.

4-2. Systems of Equations Approaches

The Johansen-Lecomber formulation can be considered as a regression model of bi-proportional estimates for a pair of time periods, as in the RAS procedure. If, however, a time series model can be formed over the observation period, it can utilize more information on the structural changes of input-output tables than by the Lecomber model. There may be two candidates to formulate such a model: the *seemingly unrelated regressions* (SUR) model and the *vector autoregression* (VAR) model.

Since an input-output table can be considered as a system with bi-proportional influence row- and column-wise, the univariate time series model for each coefficient may be improved upon by generating the estimates jointly. The *seemingly unrelated regressions* (SUR) model may be employed to form such a system of equations. However, since there are no exogenous variables on the right-hand side and each equation can only be a univariate time series model, the SUR model in this case is equivalent to a vector autoregression model.

A *vector autoregression* (VAR) model can be utilized to estimate the future vector (or, vectorized matrix) using the past trend of the vector. Typically, the VAR model can be formulated as follows:

$$\mathbf{y}_t = \boldsymbol{\mu} + \Gamma_1 \mathbf{y}_{t-1} + \cdots + \Gamma_p \mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t \qquad (29)$$

where $\mathbf{y}_t$ and $\boldsymbol{\varepsilon}_t$ are $n \times 1$ vectors, $\boldsymbol{\mu}$ is the $n \times 1$ mean vector, and $\Gamma_1, \cdots, \Gamma_p$ are $n \times n$ parameter matrices. The elements of $\Gamma_1, \cdots, \Gamma_p$ may be estimated by multivariate least squares, which is exactly the same as applying ordinary least squares to each equation. If input-output tables reflect this kind of autoregressive process over time, this type of VAR model can be applied to forecast the future vectors (and hence matrices), and can be employed to analyze the autoregressive properties of input-output tables. However, serious drawbacks may prevent the use of such a model. First, if an input-output table is large enough, such as 36×36 in this study, the dimension of $\mathbf{y}_t$, a vectorized matrix of 36×36, becomes 1296×1; hence, the dimension of $\Gamma_1, \cdots, \Gamma_p$ can be 1296×1296. Even for a single equation in the system, at least 36 unknown variables must be estimated while, in this study, there are only 18 observations (without lagged variables; a smaller number with lags). Secondly, the essential condition for the VAR estimators is that the series in $\mathbf{y}_t$ should be jointly stationary. This is rarely true for time series data, and attempting to tackle the problem by taking differences is rarely satisfactory (Harvey, 1990).

Consider the following VAR model:

$$\begin{pmatrix} \log a_{ij}^{t} \\ \log r_i^{t+1} \\ \log s_j^{t+1} \end{pmatrix} = \Gamma_1 \begin{pmatrix} \log a_{ij}^{t-1} \\ \log r_i^{t} \\ \log s_j^{t} \end{pmatrix} + \cdots + \Gamma_p \begin{pmatrix} \log a_{ij}^{t-p} \\ \log r_i^{t-p+1} \\ \log s_j^{t-p+1} \end{pmatrix} + \boldsymbol{\varepsilon}_t \qquad (30)$$

This formulation is equivalent to

$$\begin{aligned} \log a_{ij}^{t} &= \varphi_{11}^{1} \log a_{ij}^{t-1} + \varphi_{12}^{1} \log r_i^{t} + \varphi_{13}^{1} \log s_j^{t} + \cdots \\ \log r_i^{t+1} &= \varphi_{21}^{1} \log a_{ij}^{t-1} + \varphi_{22}^{1} \log r_i^{t} + \varphi_{23}^{1} \log s_j^{t} + \cdots \\ \log s_j^{t+1} &= \varphi_{31}^{1} \log a_{ij}^{t-1} + \varphi_{32}^{1} \log r_i^{t} + \varphi_{33}^{1} \log s_j^{t} + \cdots \\ &\quad + \varphi_{11}^{p} \log a_{ij}^{t-p} + \varphi_{12}^{p} \log r_i^{t-p+1} + \varphi_{13}^{p} \log s_j^{t-p+1} + \varepsilon_1^{t} \\ &\quad + \varphi_{21}^{p} \log a_{ij}^{t-p} + \varphi_{22}^{p} \log r_i^{t-p+1} + \varphi_{23}^{p} \log s_j^{t-p+1} + \varepsilon_2^{t} \\ &\quad + \varphi_{31}^{p} \log a_{ij}^{t-p} + \varphi_{32}^{p} \log r_i^{t-p+1} + \varphi_{33}^{p} \log s_j^{t-p+1} + \varepsilon_3^{t} \end{aligned} \qquad (31)$$

The first equation in (31) is the autoregressive form of the Lecomber model. The second and third equations are of lesser interest. The model for each element (*i*, *j* pair) is estimated using the value of a_{ij}, and the estimated values of r_i and s_j from the RAS procedure. The *Schwarz (Bayesian) information criterion* (SIC, or BIC) is employed to determine the number of lags (maximum lag set to be three) for each model (30).

Table 18 presents the results for the selected elements with the top 10 largest fields of influence; the table shows only the results for the first equation in (31). Overall, all top 10 elements have a high R-squared. These results imply that r_i and s_j have statistically explanatory power on a_{ij}. For the autoregressive structure, only the 6th (34, 4) and the 9th (28, 30) pairs exhibit longer lags (=3); however, the coefficients for the second and third lags are insignificant. Otherwise, the lag structures for other elements are only one. In the 3rd, 4th, 8th, and 10th elements, the coefficient for s_j is insignificant, indicating a smaller influence for a_{ij}. By and large, for the most important coefficients, the results reinforce the Lecomber's formulation.

<<insert Table 18 here>>

Two things must be noted: first, since the estimated values of r_i and s_j by the RAS procedure are used as data, the coefficients for r_i and s_j may indicate the accuracy of the estimates; if r_i and s_j can perfectly estimate a_{ij} based on its lagged value, the coefficients for them should be unity. Second, most of the coefficients for a_{ij} are very close to one, implying possible unit root behavior of a_{ij}. The results from the augmented Dicky-Fuller tests for each a_{ij} suggest that only 22%, 10%, and 15% of a_{ij} are stationary processes (random walk, random walk and drift, and random walk with drift and trend models, respectively; see Figures 6 through 8 for the distributions of a_{ij} with stationary processes). Given the results of the unit root tests for r_i and s_j in 3-3, the results from the VAR models above must be viewed with caution. It may be possible that some models with non-stationary variables can generate spurious regressions. However, given the simplicity of the RAS procedure, further investigation of the VAR specifications are infeasible and beyond the scope of this study.

5. Evaluation and Summary

In this section, the major findings in this paper are evaluated as an adjustment procedure and as a tool for structural analysis. Further extensions and concluding comments are also provided.

5-1. Evaluation

Overall, the results seem to indicate that the RAS technique is a reasonable estimation procedure for a short time period, or for a period with few structural changes, as compared to some regression models. For a longer period and/or rapid structural changes, the RAS procedure tends to generate larger errors. In addition, there seems to be significant sectoral variation; no doubt, the level of aggregation would also be important.

Barker (1985) made a strong case for the use of other macro variables in any adjustment process; in fact, the economic interpretations of r_i and s_j are still ambiguous. In this regard, Lecomber's (1969) suggestion that even if other variables are introduced there are still advantages in assuming that such effects act uniformly across rows and down columns seems unnecessarily restrictive. However, from a more practical perspective, the RAS procedure may provide minimum data requirement for updating an input-output table for a relatively short time interval.

As Lecomber (1975) and Allen and Lecomber (1975) suggested, the generalized RAS approach--in which key elements are estimated by exogenous variables, say the top 25 to 50 fields of influence, and the remaining elements by the RAS procedure--might improve the estimation accuracy without creating the need for a large estimation model. Nevertheless, the RAS procedure can only provide the evidence of structural changes, not the mechanisms and interpretation of the changes.

5-2. Conclusions

There are several issues that need to be further explored. First, can a taxonomy of tables be developed in a way that enables the analyst to classify the tables themselves or the changes in terms of certain tendencies? If this were the case, analysis could move in the direction suggested by Jensen *et al.* (1988); certain coefficients or sets of coefficients might exhibit tendencies toward stability or predictable change while others might require more extensive econometric estimation. Second, new approaches can be employed for evaluating sets of tables. One alternative would be to adopt a nonlinear redistributive dynamic approach. The following models can be used to trace the movement of coefficients over time:

$$y_t = ay_{t-1}(1 - y_{t-1}) + \varepsilon_t \quad (32)$$

$$y_t = ay_{t-1}(1 - y_{t-2}) + \varepsilon_t \quad (33)$$

These logistic forms, with different lag functions, provide an alternative way to handle annual changes with a greater focus on changes over a longer time period. Another alternative might be to exploit the input-output time series drawing on Markov properties:

$$A_{t+1} = M_L A_t \quad (34)$$

$$A_{t+1} = A_t M_R \quad (35)$$

The statistical evaluation of the matrix multipliers, M_L and M_R, requires the estimation of a linear system of equations; this formulation differs from the VAR model with vectorized matrices, since the unknown variables are $36 \times 36 = 1296$. Equations (34) and (35) provide for two alternatives, based on row and column properties; however, the respective *M* matrices are derived from the time series of interactions between matrices over each of the two time periods. All of these approaches share similar perspectives with the notion of temporal changes in input-output systems introduced in Sonis and Hewings (1998).

Since the publication of Lecomber's (1969) paper, nonlinear regression and the application of maximum likelihood estimators offer new opportunities to explore the nature of change in the time series of input-output matrices. However, recall the findings of Feldman *et al.* (1987) and Sonis *et al.* (1996) that in an evaluation of sources of structural change, changes in final demand rather than in input-output coefficients were frequently more important components of change in output in a time series evaluation of U.S. input-output tables.

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Appendix

Sectoring Scheme in the CREIM Model

Sector	Title	SIC
1	Livestock, Livestock Products, and Agricultural Products	01, 02
2	Agriculture, Forestry, and Fisheries	07, 08, 09
3	Mining	10, 12, 13, 14
4	Construction	15, 16, 17
5	Food and Kindred Products	20
6	Tobacco	21
7	Apparel and Textile Products	22, 23
8	Lumber and Wood Products	24
9	Furniture and Fixtures	25
10	Paper and Allied Products	26
11	Printing and Publishing	27
12	Chemicals and Allied Products	28
13	Petroleum and Coal Products	29
14	Rubber and Misc. Plastics Products	30
15	Leather and Leather Products	31
16	Stone, Clay, and Glass Products	32
17	Primary Metals Industries	33
18	Fabricated Metal Products	34
19	Industrial Machinery and Equipment	35
20	Electronic and Electric Equipment	36
21	Transportation Equipment	37
22	Instruments and Related Products	38
23	Miscellaneous Manufacturing Industries	39
24	Railroad Transportation and Transportation Services	40-47
25	Communications	48
26	Electric, Gas, and Sanitary Services	49
27	Wholesale and Retail Trade	50-57, 59
28	Finance and Insurance	60-64, 66, 67
29	Real Estate	65
30	Lodging, Business, Engineering, Management, and Legal Services	70, 73, 81, 87, 89
31	Eating and Drinking Places	58
32	Auto Repair, Services, and Parking	75
33	Motion Pictures, and Amusement and Recreation Services	78, 79
34	Other Services (Health, Education, Social, etc.)	
35	Federal Government Enterprises	
36	State and Local Government Enterprises	

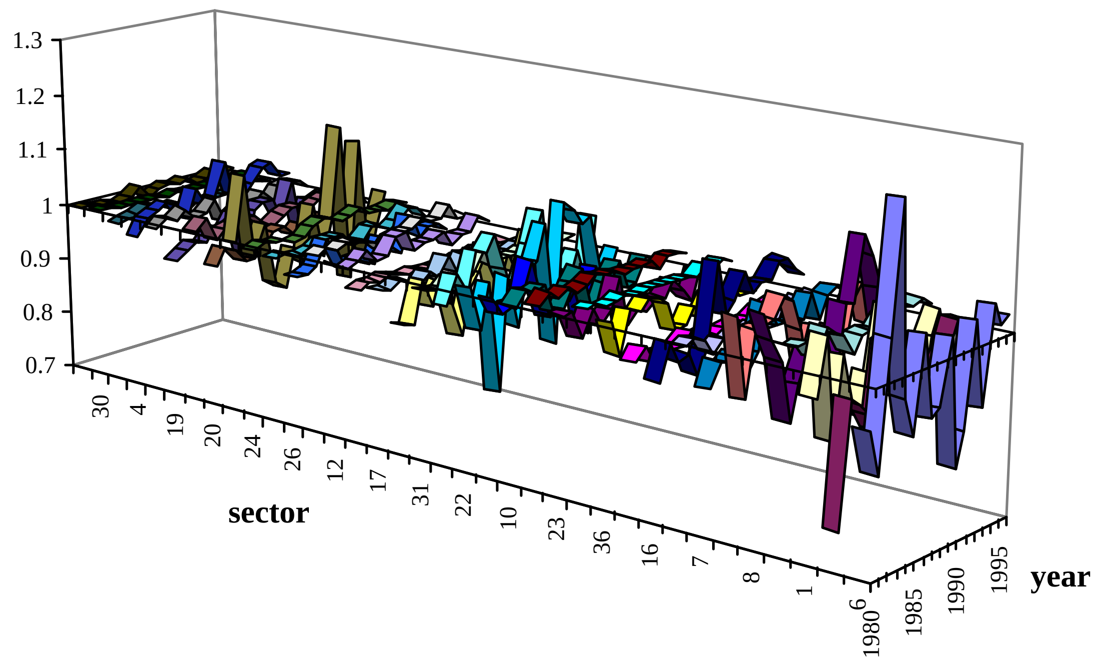

Figure 1. Trends of r_i

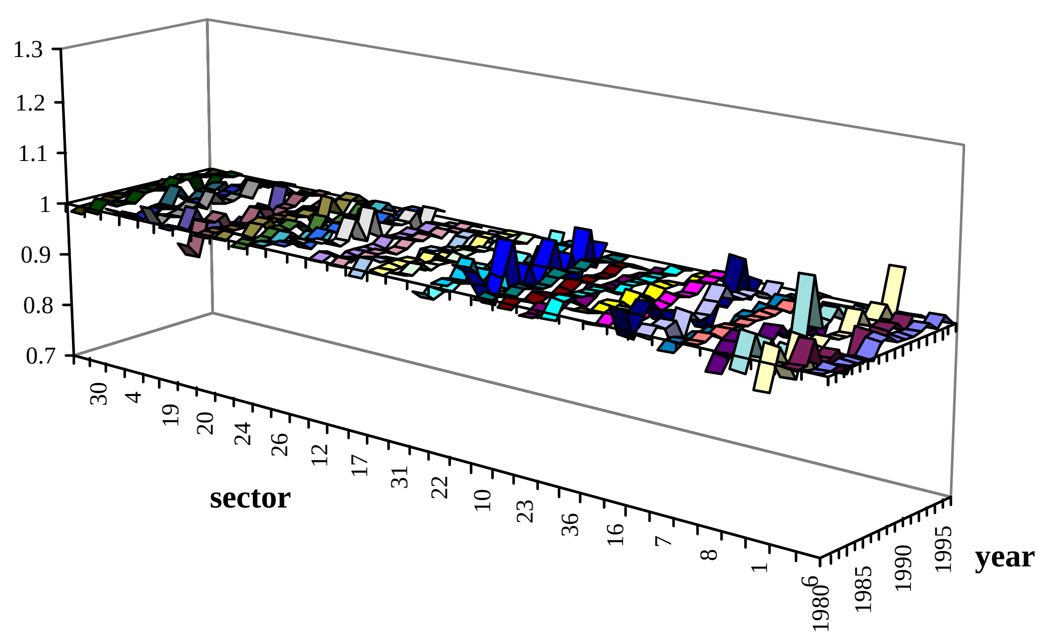

Figure 2. Trends of s_j

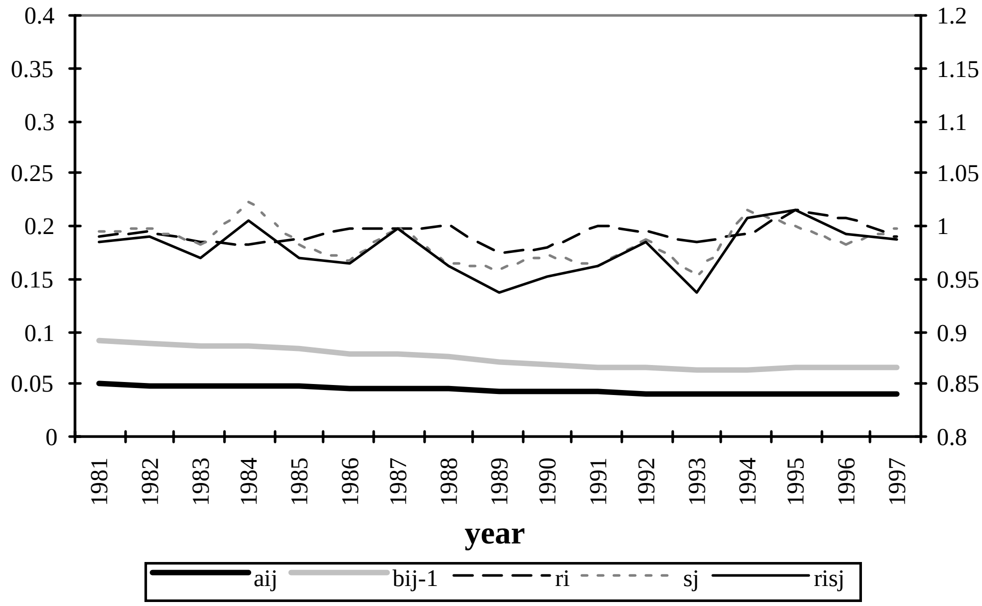

Figure 3. Trends in a_{ij}, b_{ij}, r_i, and s_j

(i=18: fabricated metals; j=19: industrial machinery and equipment)

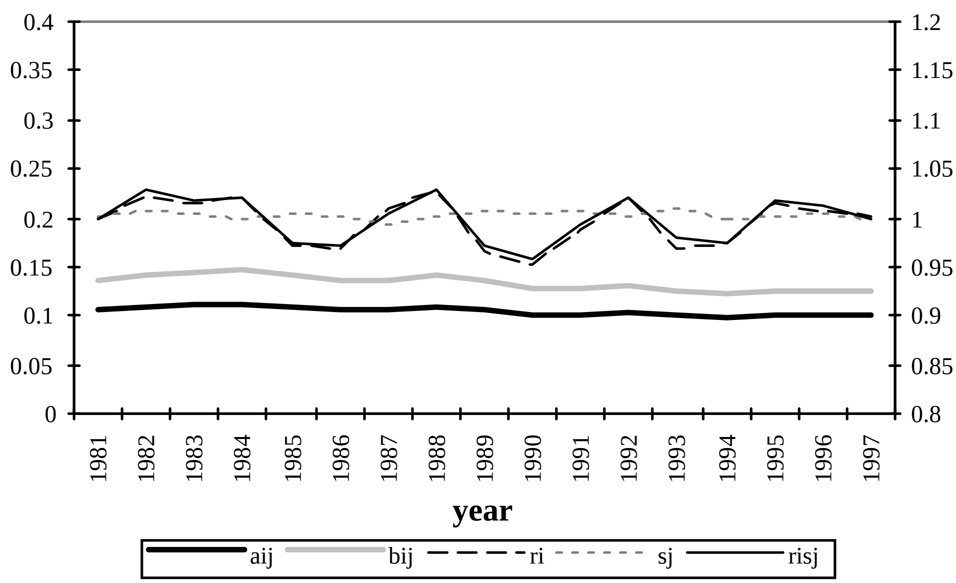

Figure 4. Trends in a_{ij}, b_{ij}, r_i, and s_j

(i=10: paper and allied products; j=11: printing and publishing)

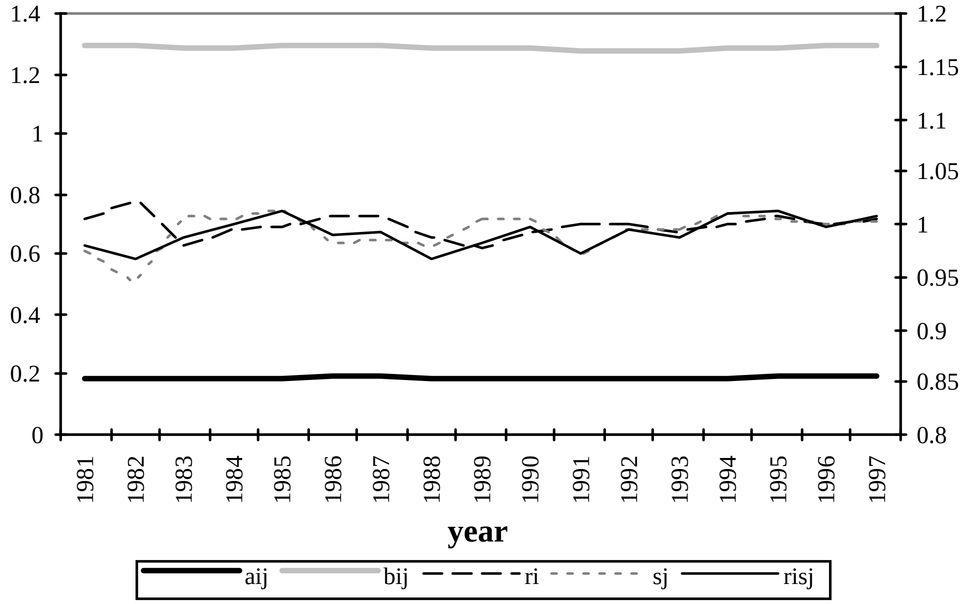

Figure 5. Trends in a_{ij}, b_{ij}, r_i, and s_j
(i=5; j=5: food and kindred products)

Table 1. Estimation Accuracy of the RAS Procedure (Annual Estimation)

		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
years base	target	MAD	MAPE %	MAD	MAPE %
80	81	0.00017	4.03	0.00022	0.41
81	82	0.00015	2.57	0.00019	0.36
82	83	0.00012	2.07	0.00016	0.29
83	84	0.00015	2.88	0.00019	0.36
84	85	0.00012	4.41	0.00016	0.39
85	86	0.00016	20.37	0.00019	0.47
86	87	0.00015	3.27	0.00018	0.37
87	88	0.00016	3.27	0.00020	0.45
88	89	0.00017	3.44	0.00020	0.43
89	90	0.00013	2.33	0.00015	0.32
90	91	0.00015	3.60	0.00019	0.46
91	92	0.00011	4.44	0.00013	0.32
92	93	0.00014	2.95	0.00016	0.34
93	94	0.00011	3.83	0.00013	0.29
94	95	0.00012	2.33	0.00015	0.34
95	96	0.00011	2.36	0.00013	0.27
96	97	0.00009	1.46	0.00011	0.21

Remark: 1986 table (*A* matrix) contains a very small coefficient.

Table 2. Estimation Accuracy of the RAS Procedure (Estimation Over 5 Years)

		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
years base	target	MAD	MAPE %	MAD	MAPE %
80	85	0.00039	8.00	0.00049	1.07
81	86	0.00039	48.27	0.00047	1.13
82	87	0.00042	8.66	0.00052	1.14
83	88	0.00046	8.24	0.00056	1.27
84	89	0.00047	10.71	0.00057	1.29
85	90	0.00051	10.36	0.00063	1.30
86	91	0.00055	7.97	0.00068	1.43
87	92	0.00052	8.18	0.00066	1.40
88	93	0.00050	8.22	0.00062	1.34
89	94	0.00043	10.85	0.00053	1.17
90	95	0.00036	7.69	0.00045	1.00
91	96	0.00033	7.42	0.00040	0.88
92	97	0.00034	5.53	0.00041	0.86

Remark: 1986 table (*A* matrix) contains a very small coefficient.

Table 3. Estimation Accuracy of the RAS Procedure (Estimation Over 10 Years)

		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
years base	target	MAD	MAPE %	MAD	MAPE %
80	90	0.00076	14.89	0.00092	2.03
81	91	0.00080	11.32	0.00097	2.10
82	92	0.00078	16.52	0.00096	2.03
83	93	0.00080	11.64	0.00099	2.12
84	94	0.00081	21.48	0.00101	2.12
85	95	0.00081	11.16	0.00102	2.10
86	96	0.00081	15.13	0.00100	2.09
87	97	0.00076	10.99	0.00095	1.94

Table 4. Estimation Accuracy of the RAS Procedure (Annual Estimation)
Top 25 Fields of Influence

years		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
base	target	MAD	MAPE %	MAD	MAPE %
80	81	0.00013	0.61	0.00065	0.19
81	82	0.00012	0.49	0.00046	0.16
82	83	0.00008	0.44	0.00040	0.13
83	84	0.00006	0.32	0.00047	0.16
84	85	0.00005	0.35	0.00047	0.18
85	86	0.00008	0.69	0.00052	0.16
86	87	0.00007	0.39	0.00058	0.18
87	88	0.00008	0.50	0.00050	0.19
88	89	0.00008	0.50	0.00066	0.23
89	90	0.00011	0.71	0.00113	0.33
90	91	0.00007	0.37	0.00027	0.09
91	92	0.00009	0.71	0.00036	0.12
92	93	0.00010	0.57	0.00055	0.18
93	94	0.00007	0.54	0.00070	0.20
94	95	0.00009	0.47	0.00063	0.16
95	96	0.00009	0.42	0.00042	0.11
96	97	0.00005	0.21	0.00029	0.07

Table 5. Estimation Accuracy of the RAS Procedure (Estimation Over 5 Years) Top 25 Fields of Influence

		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
years base	target	MAD	MAPE %	MAD	MAPE %
80	85	0.00033	1.33	0.00106	0.39
81	86	0.00030	1.94	0.00100	0.37
82	87	0.00026	1.46	0.00143	0.49
83	88	0.00023	1.27	0.00130	0.46
84	89	0.00024	1.87	0.00119	0.40
85	90	0.00026	1.86	0.00204	0.61
86	91	0.00029	1.73	0.00234	0.68
87	92	0.00032	1.77	0.00226	0.70
88	93	0.00026	1.62	0.00236	0.72
89	94	0.00015	1.34	0.00240	0.69
90	95	0.00024	1.07	0.00177	0.47
91	96	0.00027	1.36	0.00191	0.49
92	97	0.00026	1.13	0.00223	0.56

Table 6. Estimation Accuracy of the RAS Procedure (Estimation Over 10 Years) Top 25 Fields of Influence

		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
years base	target	MAD	MAPE %	MAD	MAPE %
80	90	0.00055	3.82	0.00245	0.75
81	91	0.00047	3.67	0.00272	0.80
82	92	0.00049	3.42	0.00266	0.77
83	93	0.00039	2.58	0.00278	0.81
84	94	0.00032	1.88	0.00325	0.94
85	95	0.00043	1.72	0.00364	0.98
86	96	0.00041	2.18	0.00397	1.02
87	97	0.00037	1.92	0.00378	0.96

Table 7. Summary of Run Test for r_i and s_j

sector	r_i	s_j
1	12	10
2	6	9
3	8	**13**
4	8	8
5	9	6
6	10	7
7	6	8
8	6	6
9	8	7
10	8	8
11	6	11
12	9	**5**
13	9	11
14	11	9
15	**4**	10
16	**4**	9
17	10	7
18	9	8
19	10	11
20	10	7
21	10	6
22	8	7
23	9	10
24	6	8
25	8	7
26	6	10
27	9	8
28	6	10
29	**4**	7
30	7	7
31	6	10
32	7	7
33	8	12
34	8	10
35	6	9
36	8	8

Remark: reject null hypothesis of randomness, if r<=5 or r>=13 (=0.063)

Table 8. Summary of Augmented Dickey-Fuller Test on r_i

sector	Random Walk	Random Walk	RW with drift	RW with drift	RW with drift and trend	RW with drift and trend
	(max lag=5)	(max lag =3)	(max lag=5)	(max lag =3)	(max lag=5)	(max lag =3)
1	-0.2139	-0.3942	**-4.0650**	**-4.1518**	**-3.6393**	**-3.9919**
2	-0.2077	0.18626	-2.5243	-2.2476	-2.1417	-2.1417
3	-0.1566	-0.1566	**-3.9600**	-2.0050	-2.9216	-2.1266
4	1.1971	0.4837	-2.3431	-2.3431	-2.9464	**-4.1731**
5	0.3425	0.1247	-1.5213	-1.5213	-1.3486	-1.3486
6	0.4699	0.5639	-1.6063	-1.6063	0.3166	-1.2226
7	0.5458	1.0737	-1.4969	-1.4969	-3.2385	-2.5444
8	0.1202	0.1202	-2.3855	-2.3855	-1.6577	-2.6834
9	0.8715	1.0488	-1.2351	-1.2351	-1.8117	-1.8117
10	0.3706	-0.5146	-2.6489	-2.6489	1.4334	-1.1895
11	1.0023	0.0976	-2.5271	-0.6418	-2.0485	-1.4121
12	0.1695	0.0695	-1.4248	-2.7506	-1.5668	-2.8168
13	-0.0399	-0.2878	-2.2438	-2.8177	-1.9078	-2.9652
14	0.2484	0.0402	-2.3900	-2.3900	-1.7539	-2.3414
15	0.4967	0.4570	-0.5757	-2.7502	-1.7437	-2.4010
16	1.5377	0.7866	-2.5252	-1.3693	-1.4167	-1.5832
17	0.0788	0.0788	-2.8892	-2.8892	-1.6841	-2.7896
18	0.9371	0.9410	-0.6204	-0.6204	-1.2458	-1.2458
19	0.6182	0.6182	-1.9892	-1.9892	-2.3269	-2.3269
20	1.1233	2.9622	-0.2819	-0.6857	-1.6102	-1.7122
21	0.2686	0.2686	-2.0416	-2.0416	-2.8445	-2.8625
22	0.4138	0.4138	-1.5684	-1.5684	-2.0125	-2.0125
23	0.1070	0.0258	-2.4146	-2.4146	-2.3079	-2.3079
24	0.3864	0.3864	-1.8863	-1.8863	-2.2050	-2.2050
25	-1.2103	-0.2980	-2.7318	-2.7319	**-5.2477**	-3.2150
26	-0.1570	-0.2891	-2.0248	-2.0278	-1.9607	-2.5233
27	1.1964	0.4229	-2.8212	-2.2439	-1.6514	-3.5428
28	-0.0116	-0.0116	-2.3572	-2.7581	**-3.6373**	-2.7860
29	-0.3390	-0.3390	-1.7085	-2.9566	-2.4285	-2.4285
30	0.4621	0.3445	-1.0067	-2.2860	-1.1727	-1.1727
31	1.0684	0.7551	**-5.2281**	-1.7226	**-10.5722**	-1.8439
32	0.7919	0.7919	-1.5332	-1.5332	-1.8695	-1.8695
33	0.5028	0.5028	-1.9240	-1.0482	**-3.7596**	**-3.7596**
34	0.2100	-0.0347	-2.8053	-1.3888	-2.4233	-1.4316
35	-0.2068	-0.2068	-1.7956	-1.7956	-2.0083	-2.0083
36	0.5871	-0.5871	-2.0206	-2.0206	-2.4661	-2.6095

Critical Values (=0.05): Random Walk= -1.95; Random Walk with Drift= -3.00; Random Walk with Drift and Trend= -3.60

Table 9. Summary of Augmented Dickey-Fuller Test on s_j

	Random Walk	Random Walk	RW with drift	RW with drift	RW with drift and trend	RW with drift and trend
	(max lag=5)	(max lag =3)	(max lag=5)	(max lag =3)	(max lag=5)	(max lag =3)
1	1.8713	1.5329	0.2291	0.2126	**-3.7331**	-2.7372
2	0.0611	-0.1814	-1.9064	-1.9064	-1.8809	-1.8809
3	**-2.1923**	-0.1448	-0.2833	**-4.4583**	**-4.6058**	**-7.1006**
4	-0.3918	-0.3918	**-3.9674**	-1.7784	**-3.7325**	-1.6746
5	0.7013	0.4528	**-3.6977**	-1.8093	**-4.3563**	-2.4600
6	0.2984	0.2984	-1.8741	-1.8741	-2.3452	-2.3452
7	0.6743	0.3802	-0.9867	-0.9837	-2.5110	-1.7876
8	-0.3876	-0.2945	-1.8017	-1.8017	-0.2563	-1.2477
9	-0.1347	-0.1347	-1.4912	-1.4912	-1.6959	-1.8913
10	-0.2422	0.1811	-1.9555	-1.6304	-2.2561	-1.1756
11	-0.1448	-0.1325	-1.5965	-1.5965	-3.3036	-1.6046
12	-0.4745	-0.0268	-1.7127	-1.7127	-1.9356	-1.9356
13	0.2243	0.3675	**-3.3069**	**-3.3069**	-0.9254	-3.3712
14	-0.4919	-0.3381	-1.9796	-1.9796	-1.7189	-1.1789
15	-0.1373	-0.1373	-1.7319	-1.7319	-1.8036	-1.8036
16	0.1969	-0.2555	-1.3338	-1.3338	**-7.8214**	-0.9257
17	-0.3657	-0.3657	-2.5295	-2.5295	**-3.7150**	-2.3376
18	-0.1828	-0.3771	-2.0770	-2.0770	-1.9962	-1.9962
19	-0.1411	-0.3372	**-3.0025**	-1.1819	-2.6012	-0.9481
20	-0.6116	-0.7526	**-4.3505**	-2.7113	-3.3411	-2.5515
21	-0.4454	-0.4454	-2.1427	-2.1427	-2.0491	-2.0491
22	-0.0918	-0.5013	**-3.5399**	-1.6281	-1.4232	-2.3653
23	-0.3144	-0.0321	-2.2325	-2.5835	-2.1019	-2.5548
24	0.1542	0.3069	-1.7900	-1.7900	-1.6963	-1.6963
25	0.3854	0.3854	-2.0704	-2.0704	-2.0990	-2.0990
26	0.2276	0.2276	-2.2253	-2.2253	-2.0989	-2.0989
27	0.6048	0.3247	-2.1876	-2.1876	-0.9004	-1.3075
28	-0.2833	-0.3957	-1.8896	-1.8896	-2.5608	-2.1733
29	0.0866	0.0866	-2.9819	-2.9819	-2.8252	-2.8252
30	-0.0485	-0.0485	-2.0826	-2.0826	-2.1934	-2.1934
31	-0.9457	-0.5521	-1.5636	-1.5636	-1.3410	-3.3695
32	-1.1246	-1.1246	-3.3666	-1.3072	-3.3798	-1.5936
33	-0.1928	-0.2366	-1.9038	-1.9038	-1.7948	-1.7948
34	0.9492	0.2368	-1.2282	**-4.1009**	**-3.9280**	**-4.9244**
35	-0.3937	-0.2691	-0.9530	-0.9530	-1.0398	-1.0398
36	-1.1011	-0.7675	-2.2941	-2.2941	-3.1671	-3.2110

Critical Values (=0.05): Random Walk= -1.95; Random Walk with Drift= -3.00; Random Walk with Drift and Trend= -3.60

Table 10. Estimation Accuracy of Lecomber Model (Annual Estimation)

		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
years base	target	MAD	MAPE %	MAD	MAPE %
80	81	0.00043	4.63	0.00140	3.30
81	82	0.00030	3.36	0.00102	2.45
82	83	0.00022	2.65	0.00085	1.91
83	84	0.00033	4.13	0.00115	2.96
84	85	0.00044	5.80	0.00129	3.34
85	86	0.00068	16.84	0.00206	5.38
86	87	0.00047	4.75	0.00145	3.44
87	88	0.00037	3.71	0.00112	2.87
88	89	0.00030	3.71	0.00089	2.13
89	90	0.00024	2.80	0.00066	1.58
90	91	0.00038	4.20	0.00112	2.85
91	92	0.00031	5.06	0.00094	3.53
92	93	0.00032	3.89	0.00123	3.59
93	94	0.00026	4.47	0.00101	2.88
94	95	0.00037	3.79	0.00129	3.12
95	96	0.00020	2.75	0.00068	1.47
96	97	0.00019	2.06	0.00060	1.55

Remark: 1986 table (*A* matrix) contains a very small coefficient.

Table 11. Estimation Accuracy of Lecomber Model (Annual Estimation) without insignificant estimates

		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
years base	target	MAD	MAPE %	MAD	MAPE %
80	81	0.00063	6.05	0.00234	4.67
81	82	0.00041	3.97	0.00156	3.39
82	83	0.00040	3.50	0.00138	2.80
83	84	0.00033	4.13	0.00110	2.88
84	85	0.00055	7.34	0.00218	5.38
85	86	0.00080	18.57	0.00372	10.30
86	87	0.00061	6.03	0.00191	4.36
87	88	0.00045	4.22	0.00136	3.54
88	89	0.00045	4.43	0.00212	4.27
89	90	0.00030	3.29	0.00095	2.25
90	91	0.00040	4.31	0.00140	3.26
91	92	0.00056	6.11	0.00235	5.44
92	93	0.00041	4.91	0.00127	4.09
93	94	0.00073	6.46	0.00413	7.69
94	95	0.00051	5.00	0.00177	4.15
95	96	0.00029	3.57	0.00101	2.14
96	97	0.00025	2.61	0.00080	2.04

Remark: 1986 table (*A* matrix) contains a very small coefficient.

Table 12. Estimation Accuracy of Lecomber Model (Estimation Over 5 Years)

		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
years base	target	MAD	MAPE %	MAD	MAPE %
80	85	0.00078	7.68	0.00219	5.05
81	86	0.00081	30.34	0.00230	5.65
82	87	0.00069	9.92	0.00192	5.03
83	88	0.00075	8.48	0.00200	4.84
84	89	0.00078	10.51	0.00242	5.70
85	90	0.00092	11.85	0.00293	6.61
86	91	0.00098	8.39	0.00278	7.14
87	92	0.00086	8.51	0.00235	7.41
88	93	0.00080	8.06	0.00209	5.58
89	94	0.00081	10.38	0.00224	6.45
90	95	0.00064	8.33	0.00183	5.06
91	96	0.00059	8.19	0.00209	4.51
92	97	0.00063	6.83	0.00247	5.20

Remark: 1986 table (*A* matrix) contains a very small coefficient.

Table 13. Estimation Accuracy of Lecomber Model (Estimation Over 10 Years)

		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
years base	target	MAD	MAPE %	MAD	MAPE %
80	90	0.00112	13.36	0.00306	6.91
81	91	0.00110	10.65	0.00299	6.66
82	92	0.00111	13.96	0.00291	7.07
83	93	0.00112	10.78	0.00306	7.06
84	94	0.00122	16.96	0.00341	8.96
85	95	0.00122	12.46	0.00386	8.87
86	96	0.00142	15.87	0.00460	11.16
87	97	0.00113	10.76	0.00383	8.22

Table 14. Estimation Accuracy of Lecomber Model (Annual Estimation)
Top 25 Fields of Influence

years		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
base	target	MAD	MAPE %	MAD	MAPE %
80	81	0.00042	1.57	0.00528	1.85
81	82	0.00014	1.63	0.00368	1.32
82	83	0.00015	1.38	0.00297	1.04
83	84	0.00013	1.26	0.00253	0.86
84	85	0.00036	3.10	0.00446	1.65
85	86	0.00026	3.00	0.01277	4.25
86	87	0.00019	2.46	0.01256	3.93
87	88	0.00031	1.41	0.00292	1.11
88	89	0.00014	0.61	0.00310	1.15
89	90	0.00020	0.95	0.00253	0.83
90	91	0.00044	2.27	0.00447	1.54
91	92	0.00022	3.29	0.00304	1.02
92	93	0.00015	1.96	0.00313	1.10
93	94	0.00012	1.20	0.00433	1.38
94	95	0.00039	1.39	0.00628	2.00
95	96	0.00022	1.04	0.00437	1.35
96	97	0.00012	0.96	0.00288	0.85

Table 15. Estimation Accuracy of Lecomber Model (Annual Estimation)
Top 25 Fields of Influence; without insignificant estimates

		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
years base	target	MAD	MAPE %	MAD	MAPE %
80	81	0.00059	2.52	0.00578	2.03
81	82	0.00016	2.15	0.00468	1.64
82	83	0.00023	1.73	0.00364	1.27
83	84	0.00013	1.26	0.00228	0.78
84	85	0.00029	3.92	0.00672	2.43
85	86	0.00032	3.79	0.01558	5.20
86	87	0.00035	2.92	0.01546	4.98
87	88	0.00046	2.11	0.00364	1.38
88	89	0.00031	1.52	0.00828	3.03
89	90	0.00026	1.67	0.00426	1.47
90	91	0.00062	3.21	0.00675	2.37
91	92	0.00047	3.90	0.01429	4.85
92	93	0.00034	2.69	0.00335	1.16
93	94	0.00069	3.58	0.02661	8.89
94	95	0.00046	2.07	0.00877	2.78
95	96	0.00030	1.88	0.00583	1.79
96	97	0.00016	1.23	0.00404	1.23

Table 16. Estimation Accuracy of Lecomber Model (Estimation Over 5 Years)
Top 25 Fields of Influence

		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
years base	target	MAD	MAPE %	MAD	MAPE %
80	85	0.00063	2.73	0.00605	2.16
81	86	0.00044	4.25	0.01309	4.43
82	87	0.00047	3.17	0.00608	2.21
83	88	0.00062	3.01	0.00461	1.77
84	89	0.00059	3.29	0.00549	2.12
85	90	0.00058	3.00	0.01040	3.66
86	91	0.00060	6.67	0.01501	4.70
87	92	0.00075	5.35	0.00672	2.22
88	93	0.00059	3.12	0.00651	2.22
89	94	0.00071	4.31	0.00931	3.03
90	95	0.00067	3.69	0.00847	2.59
91	96	0.00061	3.10	0.01465	4.35
92	97	0.00069	4.81	0.01718	5.10

Table 17. Estimation Accuracy of Lecomber Model (Estimation Over 10 Years)
Top 25 Fields of Influence

		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
years base	target	MAD	MAPE %	MAD	MAPE %
80	90	0.00070	3.96	0.00720	2.50
81	91	0.00049	3.64	0.00780	2.74
82	92	0.00065	4.37	0.00637	2.17
83	93	0.00074	4.10	0.00698	2.37
84	94	0.00086	3.52	0.00927	3.03
85	95	0.00080	4.52	0.01586	4.89
86	96	0.00108	8.25	0.02821	8.13
87	97	0.00095	3.73	0.02042	5.88

Table 18. Summary of VAR Estimation (for a_{ij} equation: Top 10 Fields of Influence)

F of I Rank	Sector i	Sector j	Lag	R^2	SIC	Granger Test F-stat.	Coefficients (Standard Error) a_{ij}	r_i	s_j
1	28	27		0.9993	-32.76	390.2*			
			1				0.999* (0.0001)	0.887* (0.033)	0.904* (0.094)
2	28	4		0.9964	-30.31	77.87*			
			1				0.999* (0.0004)	0.840* (0.090)	1.306* (0.202)
3	2	27		0.9969	-29.35	67.27*			
			1				0.999* (0.0003)	0.939* (0.083)	0.695 (0.327)
4	2	4		0.9957	-28.69	45.11*			
			1				0.999* (0.0004)	0.920* (0.114)	0.678 (0.354)
5	34	27		0.9952	-32.26	238.56*			
			1				0.999* (0.0002)	0.942* (0.043)	1.641* (0.148)
6	34	4		0.9925	-30.37	38.37*			
			1				1.193* (0.471)	0.909* (0.090)	2.046* (0.335)
			2				-0.605 (0.643)	-0.114 (0.399)	-0.458 (1.064)
			3				0.412 (0.381)	0.510 (0.396)	0.586 (0.703)
7	36	27		0.9995	-31.60	1396.20*			
			1				0.999* (0.0001)	1.012* (0.021)	0.476* (0.129)
8	36	4		0.9950	-28.35	135.81*			
			1				1.000* (0.0002)	0.912* (0.056)	0.369 (0.269)
9	28	30		0.9997	-29.21	68.28*			
			1				0.907* (0.408)	1.058* (0.076)	0.902* (0.065)
			2				0.104 (0.508)	-0.053 (0.454)	0.149 (0.377)
			3				-0.011 (0.342)	0.032 (0.454)	-0.033 (0.327)
10	35	27		0.9996	-29.74	3845.27*			
			1				0.999* (0.0001)	1.006* (0.012)	0.311 (0.156)

Remark: * indicates rejecting the null hypothesis (no Granger causality; or insignificant coefficient) at 5%.

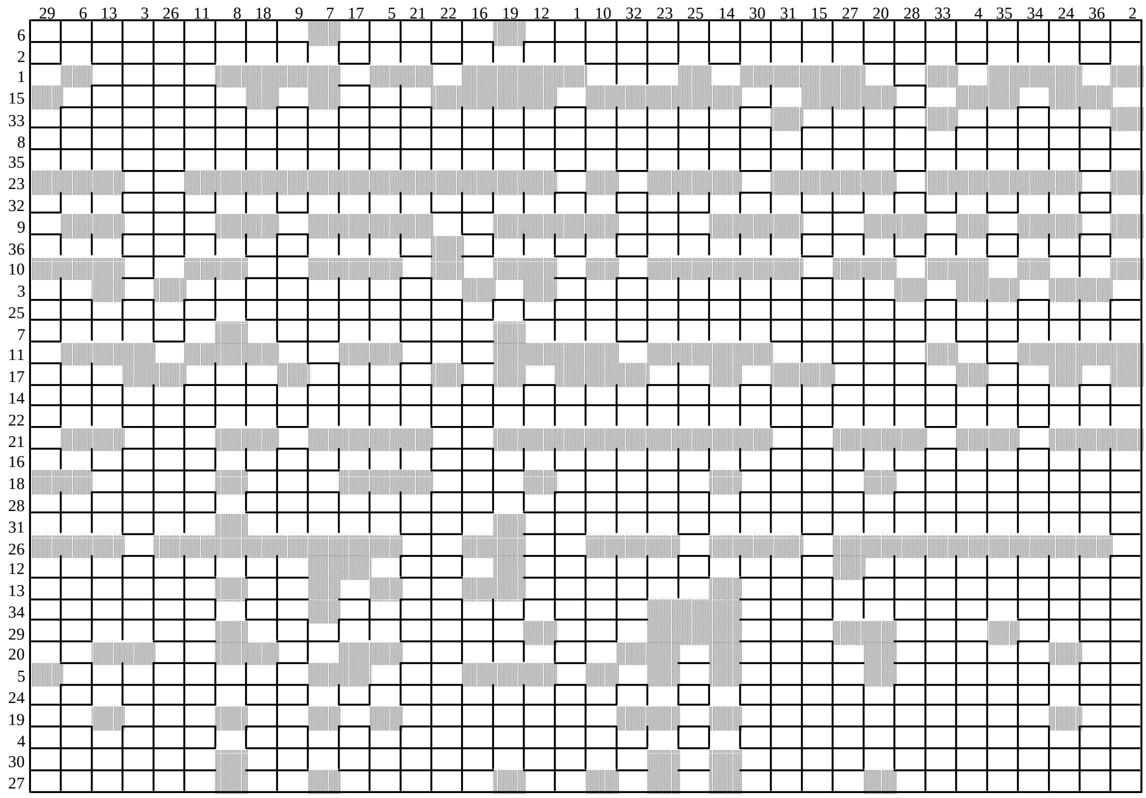

Figure 6. Distribution of a_{ij} with Stationary Process

(pure random walk; sorted by 1980 MPM hierarchy--lower right corner is the highest)

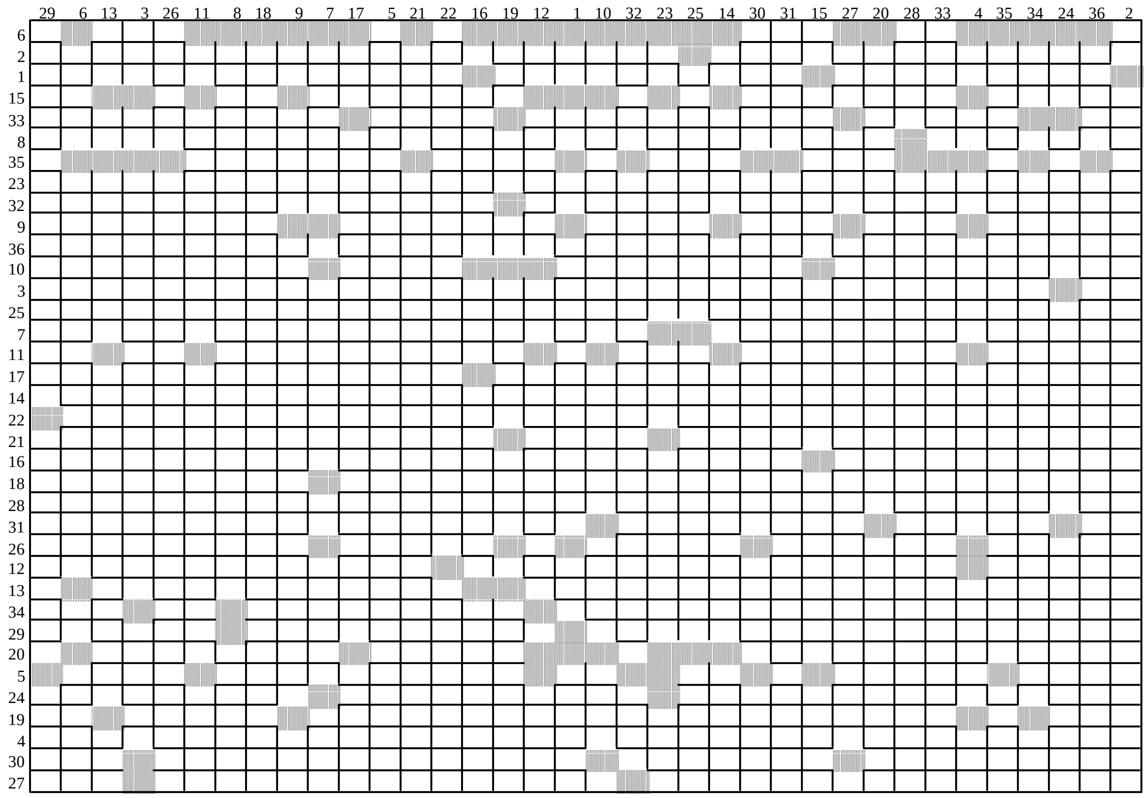

Figure 7. Distribution of a_{ij} with Stationary Process

(random walk with drift; sorted by 1980 MPM hierarchy--lower right corner is the highest)

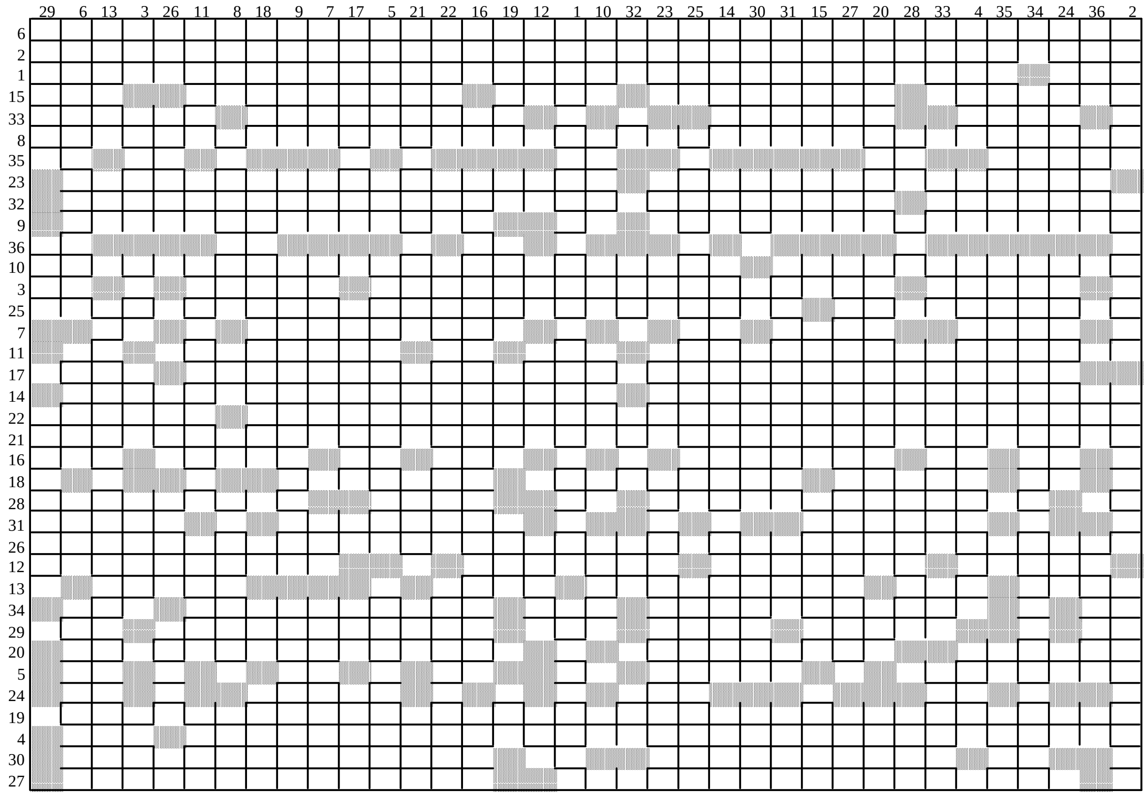

Figure 8. Distribution of a_{ij} with Stationary Process

(random walk with drift and trend; sorted by 1980 MPM hierarchy--lower right corner is the highest)