

# Economies of Plant-Scale and Structural Change\*

Iwao Ozaki  
Masahiro Kuroda  
Koji Nomura

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## 1 Introduction

The purpose of the paper is to investigate statistically the effects of the expansion of plant-capacity,  $\Delta X_{0j}$ , on the economic changes, particularly on the structural changes in economic dynamic process. During the last half of 20th century, we experienced rapid increases of the capital coefficients in almost all of industrial sectors as well as in the aggregated level of the Japanese economy. During the same period, labor productivity improved with the high upheaval of the relative price of factor inputs, labor and capital. Simultaneously, we should stress that there existed the rapid expansion of the plant-capacity in the individual establishment along with the capital accumulation. In these drastic changes through the economic growth of the Japanese economy, we experienced various structural changes in the economy. In this paper, we would like to focus on the investigation of the implication of the expansion of the plant-capacity on the structural changes. We will propose a model, in which we can determine the scale of the plant endogenously in each establishment and induce the changes of the capital coefficients in the aggregated sectoral level consistently with the changes of the plant scale in the establishment level. We finally can make clear an implication of the expansion of the plant-scale quantitatively through the dynamic structural change. The term “structural change”, here, is defined as changes in structural parameters themselves which were caused by the change of endogenous variables in the dynamic model. Theoretically, the concept of “plant” is considered here as a compound commodity of durable goods which is utilized in the production process during a definite period. In this sense, each “plant” is a “lump of capital goods” in physical term that has its own capacity  $X_{0j}$ , measured by quantity of the produced final goods. The procedures of experiments in this paper are as follows;

(1) In order to investigate the effect of expansion of the plant-scale on economic system, we will begin with a statistical estimation of the parameters of production function in Section 3, after a brief explanation of our data in Section 2. In the estimation, we will use the data of “Census of Manufactures” (cross-section analysis), where individual establishment data as measured unit were used as a proxy of “plant” in the theoretical unit. We can confirm there the statistical stability of the factor limitational production function.

(2) In our theoretical concept, each “plant” corresponds to a specific technology. The concept of plant based technology is characterized by the plant scale and the productivity of labor and capital inputs. We call it the “Capital embodied technology” hypothesis. Then in order to determine the optimum level of the plant scale, we tried to introduce “unit cost minimization model” in Section 3.3. In the model, the optimum level of plant scale,  $X_{0j}^*$  is determined at the level of given factor prices, on the basis of the factor limitational production function estimated in the previous stage (1). The optimum scale of plant definitely corresponded to the productivity of labor and capital and the expansion of the scale dominated the improvement of the labor productivity along with the increases of the capital coefficients in each establishment.

(3) According to our estimation of the parameters of factor limitational production function, there were dominant the properties of the technology with the economy of scale, in which the scale of the plant capacity has been optimally chosen to enlarge along with the increases of relative factor price.

(4) Based on the results of the (1) and (2), the impacts of enlargement of the plant scale in the time-series on structural change were examined in the aggregated sectoral level in Section 4. From the

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estimated results in the sectoral time-series data, we can categorize the industrial sectors into six types of the technological properties. In the case that the technological properties are characterized by the factor limitational production function, theoretically, the changes of capital coefficients basically depend upon the changes of the plant capacity in the individual establishment and the distribution of changes of the plant scales consistently with the changes of relative factor prices.

(5) Especially, in the case which the technological properties are characterized by K(II) type; ( $\beta_{Kj} > 1$ , and  $\beta_{Kj} > \beta_{Lj}$ ), we can induce that the changes of the plant capacity in the individual establishment are endogenously determined through the unit cost minimization behavior. In this case the capital coefficient in the aggregated level definitely increases regardless the expansion of the market demand size.

(6) Consequently, we obtained the result that the change over time of the capital coefficient matrix,  $\Delta B^t$ , proposed in the Leontief's The "Dynamic Inverse" Model, is largely dependent on the enlargement of plant-scale,  $\Delta X_{0j}$ . Especially, it will be shown that the effect of economy of plant-scale in each establishment base on structural change is dominant in capital using technology sectors.

## 2 Data

Two sets of basic data were used to estimate the technology parameters. One is individual establishment data of "Census of Manufactures" published by MITI, Japan. The Census includes totally samples of about 300,000(1970) ~400,000(1985) establishments in each year of 1970,1971,1980 and 1985, in which all establishments are classified into about 600 sectors (industries) in 4-digit level and about 150 sectors in 3-digit level of JSIC (Japan standard Industries Classification). In the Census Data, annual gross output  $X_{0j}$ , required labor  $L_{0j}$ , and fixed capital stock at the end of the period,  $K_0$  can be available for the estimation of parameters in the individual establishment base. Here, the suffix "0" attached to each variable represent that  $X_{0j}, L_{0j}, K_{0j}$  in the establishment base are proxies of individual plant-base variables.

The other is the set of aggregated time-series data for sectoral gross output  $X_j$ , labor input  $L_j$ (man), and real capital stock  $K_j$ . Also, the deflators of gross output and labor input in the sectoral level,  $p_i$  and  $w$ , are available over the period 1951-1968, which are estimated by the Center for Economic Data Development and Research.<sup>1</sup>

## 3 Measurement I: Cross-section Approach

### 3.1 The estimation

Let us begin with the statistical estimation of the technology parameters of the production function in the establishment base, which is specified with the factor limitational type functions (1) and (2).

$$\begin{aligned} L_{0j} &= \alpha_{Lj} X_{0j}^{\beta_{Lj}} \\ \log L_{0j} &= \log \alpha_{Lj} + \beta_{Lj} \log X_{0j} \end{aligned} \quad (1)$$

$$\begin{aligned} K_{0j} &= \alpha_{Kj} X_{0j}^{\beta_{Kj}} \\ \log K_{0j} &= \log \alpha_{Kj} + \beta_{Kj} \log X_{0j} \end{aligned} \quad (2)$$

where

$X_{0j}$ : gross output for each establishment in the j-th industry(one-dollar worth),

$L_{0j}$ : annual labor input(man term),

$K_{0j}$ : value of fixed capital stock at the end of year.

Equation system (1) and (2) is usually referred to "factor limitational production function", which means labor-capital substitution could occur only through the expansion of the production scale.

Now, using the individual establishment base data of Census Manufactures, we can obtain the results of the estimates of  $\hat{\alpha}_{Lj}$ ,  $\hat{\beta}_{Lj}$ ,  $\hat{\alpha}_{Kj}$ , and  $\hat{\beta}_{Kj}$  in equation (1) and (2) in about 600 sectors classified into 4-digit industries. Since the results of all industries can not be described because of the limitation of the

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<sup>1</sup>The Center for Economic Data Development and Research was established in April 1970 as an independent organization.

space, we would like to focus on several industries; occidental paper industry, aluminum refining industry, copper smelting industry and zinc refining industry. Figure 1 and Figure 2 represent the distribution of samples in the occidental paper producing sector at 1980 and 1985, where  $X_{0j}$  and  $L_{0j}$  in both axes are plotted in the logarithmic measures respectively. From these observations, we can expect to obtain fairly stable estimations for above two functions. When we tries to estimate the parameters in the samples for the years 1980 and 1985, the parameter,  $\hat{\beta}_{Lj}$  is stable estimated in the range of 0.67 - 0.66 in this sector with the determination coefficients more than 0.90. Figure 3 represents the sample plots of  $X_{0j} - K_{0j}$  in the occidental paper industry at the year 1985. Estimated parameter,  $\hat{\beta}_{Kj}$  shows 1.14 with the determination coefficient of 0.87.

On the other hand, in the case that the parameters show  $\hat{\beta}_{Lj} < 1$ ,  $\hat{\beta}_{Kj} < 1$  and  $\hat{\beta}_{Kj} - \hat{\beta}_{Lj} > 0$ , the number of the establishments would be converge to the limited number by the oligopolistic competition in the market. Figure 4 through Figure 6 represent the sample plots of  $X_{0j} - L_{0j}$  in copper smelting industry, zinc refining industry and aluminum refining at the year 1985. In these industries number of establishments has been converged into the limited numbers because of the oligopolistic competition. In such industries, it is difficult to estimate parameters in the cross-section data. We will try to estimate parameters in these sectors by using time-series data in section 4. It is assumed that these oligopolistic competitions in such sectors would be realized by the properties of the parameters in the technology.

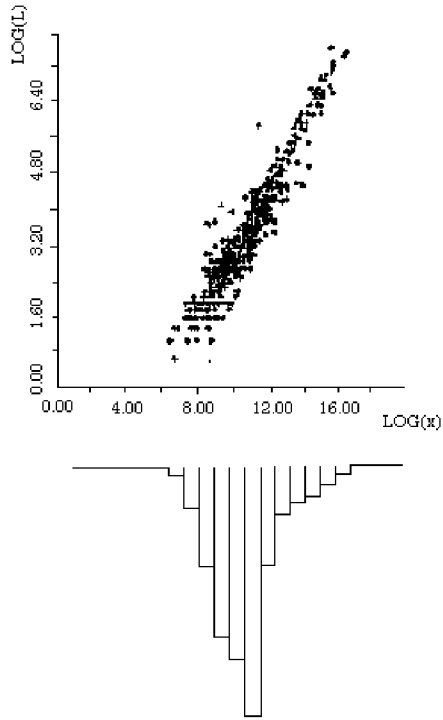


Figure 1: Distribution of Establishment 1980  
Occidental paper producing industry

Estimated Equation  
 $\log(L_{\theta}) = \log(\alpha_L) + \beta_L \log(X_{\theta})$

Occidental paper 1980 Code:1821  
 Sample size 519

|                  | Estimate | t-value |
|------------------|----------|---------|
| $\log(\alpha_L)$ | -3.723   | -35.468 |
| $\beta_L$        | 0.674    | 70.466  |
| R-square         | 0.905    |         |
| Corr Coef        | 0.951    |         |

|           | Mean   | Variance |
|-----------|--------|----------|
| $\log(X)$ | 10.823 | 3.314    |
| $\log(L)$ | 3.571  | 1.662    |

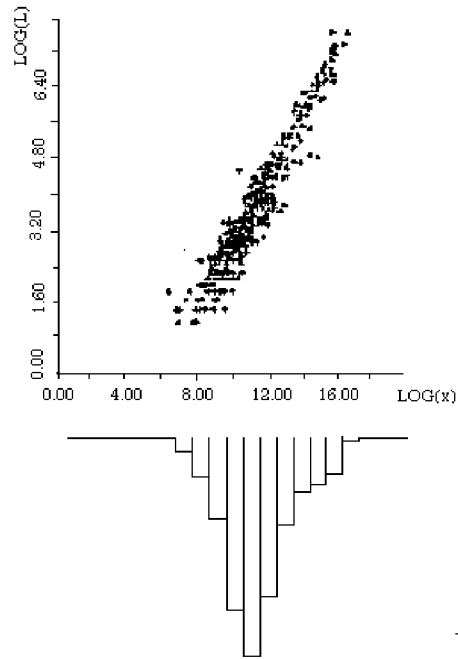


Figure 2: Distribution of Establishment 1985  
Occidental paper producing industry

Estimated Equation  
 $\log(L_{\theta}) = \log(\alpha_L) + \beta_L \log(X_{\theta})$

Occidental paper 1985 Code:1821  
 Sample size 400

|                  | Estimate | t-value |
|------------------|----------|---------|
| $\log(\alpha_L)$ | -3.693   | -33.090 |
| $\beta_L$        | 0.658    | 66.385  |
| R-square         | 0.917    |         |
| Corr Coef        | 0.957    |         |

|           | Mean   | Variance |
|-----------|--------|----------|
| $\log(X)$ | 11.074 | 3.798    |
| $\log(L)$ | 3.604  | 1.798    |

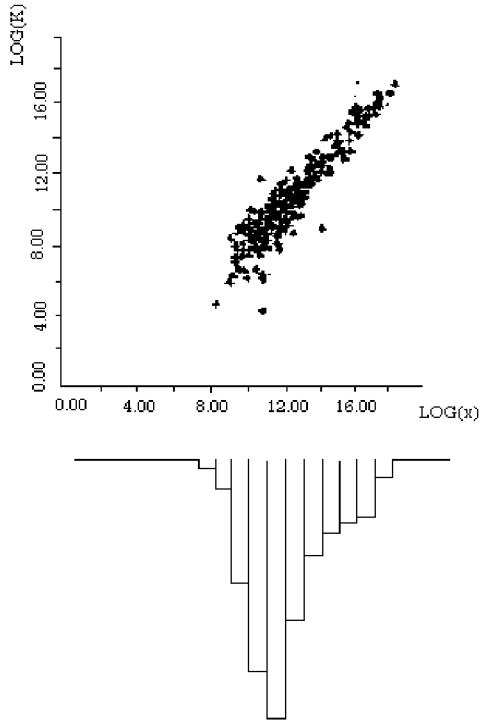


Figure 3: Distribution of Establishment 1985  
Occidental paper producing industry

Estimated Equation  
 $\log(K_\theta) = \log(\alpha_K) + \beta_K \log(X_\theta)$

Occidental paper 1985 Code:1821  
 Sample size 338

|                  | Estimate | t-value |
|------------------|----------|---------|
| $\log(\alpha_K)$ | -3.307   | -12.261 |
| $\beta_K$        | 1.140    | 49.256  |
| R-square         | 0.878    |         |
| Corr Coef        | 0.937    |         |

|        | Mean   | Variance |
|--------|--------|----------|
| Log(X) | 11.074 | 3.798    |
| Log(K) | 3.604  | 1.798    |

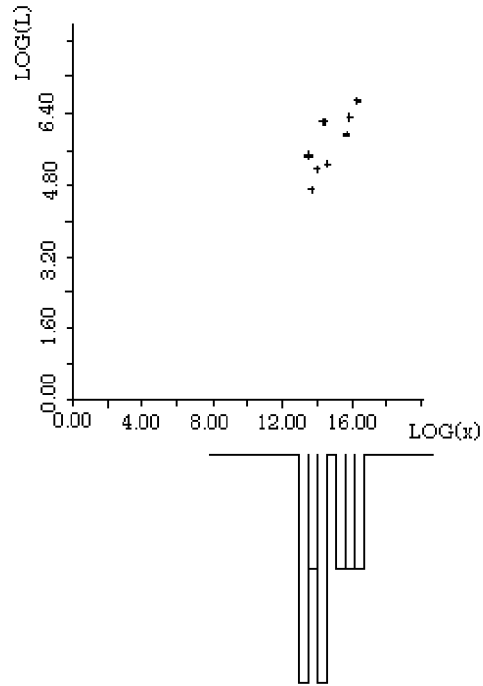


Figure 4: Distribution of Establishment 1985  
Copper smelting industry  
(oligopoly case)

Estimated Equation  
 $\log(L_\theta) = \log(\alpha_L) + \beta_L \log(X_\theta)$

Copper smelting 1985 Code:2711  
 Sample size 8

|                  | Estimate | t-value |
|------------------|----------|---------|
| $\log(\alpha_L)$ | -1.880   | -0.835  |
| $\beta_L$        | 0.521    | 3.387   |
| R-square         | 0.656    |         |
| Corr Coef        | 0.810    |         |

|        | Mean   | Variance |
|--------|--------|----------|
| Log(X) | 14.596 | 0.969    |
| Log(L) | 5.727  | 0.400    |

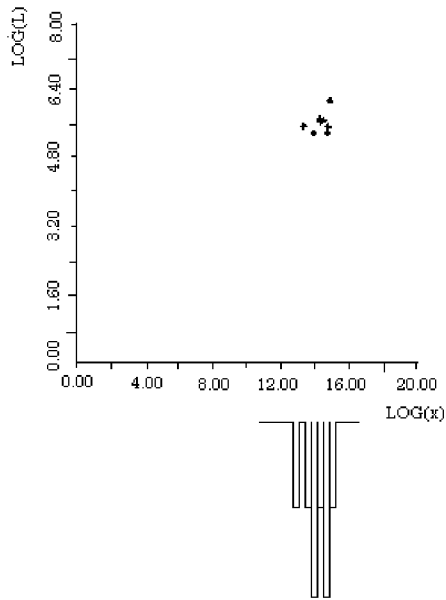


Figure 5: Distribution of Establishment 1985  
Zinc refining industry  
(oligopoly case)

Estimated Equation  
 $\log(L_{\theta}) = \log(\alpha_L) + \beta_L \log(X_{\theta})$

Zinc refining 1985      Code:2713  
 Sample size 8

|                   | Estimate | t-value |
|-------------------|----------|---------|
| Log( $\alpha_L$ ) | 2.898    | 1.152   |
| $\beta_L$         | 0.192    | 1.091   |
| R-square          | 0.165    |         |
| Corr Coef         | 0.407    |         |

|        | Mean   | Variance |
|--------|--------|----------|
| Log(X) | 14.268 | 0.234    |
| Log(L) | 5.642  | 0.052    |

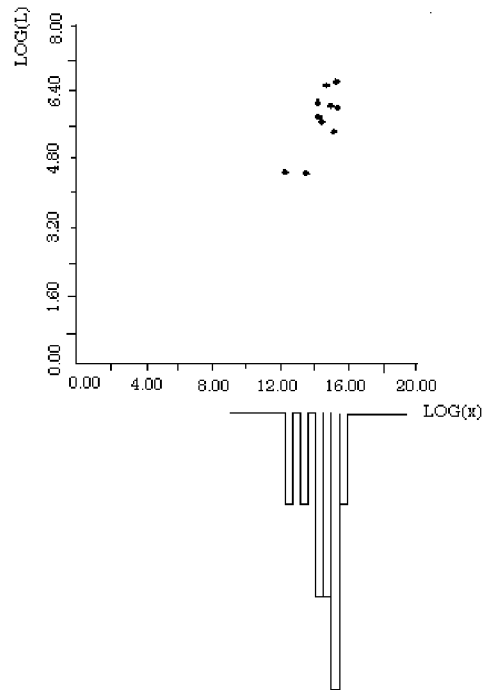


Figure 6: Distribution of Establishment 1985  
Aluminum refining industry  
(oligopoly case)

Estimated Equation  
 $\log(L_{\theta}) = \log(\alpha_L) + \beta_L \log(X_{\theta})$

Aluminum refining 1985      Code:2716  
 Sample size 10

|                   | Estimate | t-value |
|-------------------|----------|---------|
| Log( $\alpha_L$ ) | -2.966   | -1.202  |
| $\beta_L$         | 0.6      | 3.491   |
| R-square          | 0.603    |         |
| Corr Coef         | 0.777    |         |

|        | Mean   | Variance |
|--------|--------|----------|
| Log(X) | 14.321 | 0.845    |
| Log(L) | 5.632  | 0.505    |

### 3.2 Stability in the overtime of the technology parameters

As we mentioned in the previous section, the estimations of the technology parameters in the establishment base cross-section data show us the following stable findings:

1. In occidental paper industry, as shown in Figure 1-3, parameters of the production functions are dominantly estimated with the following properties.

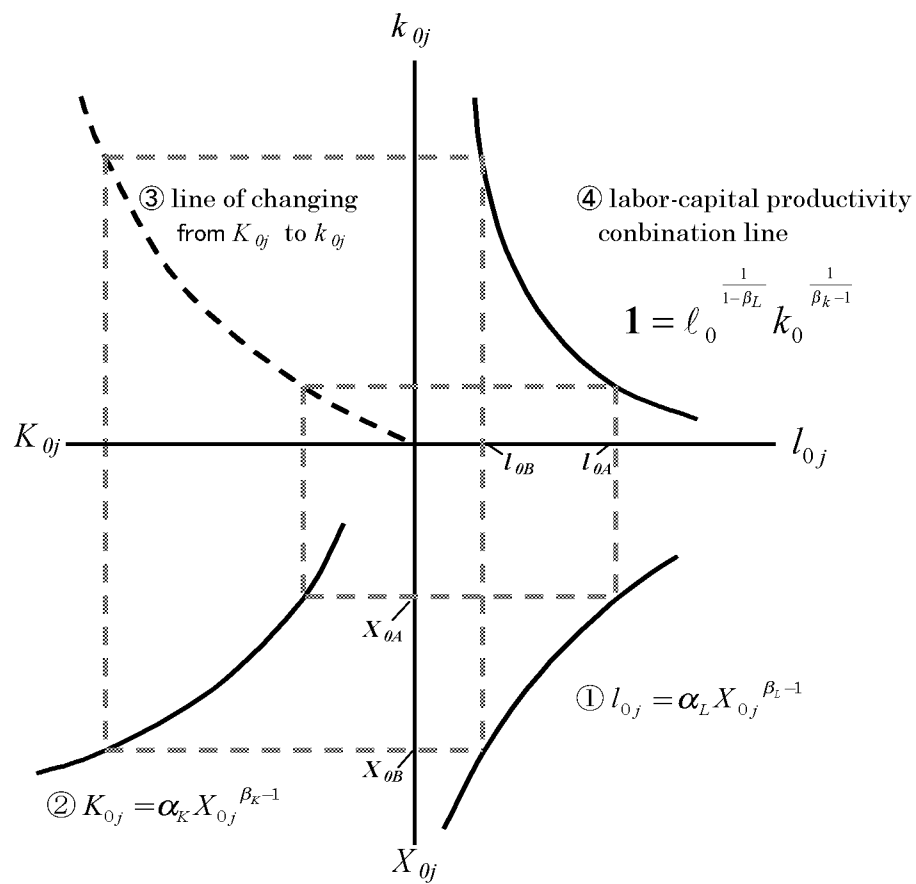
$$\hat{\beta}_{Lj} < 1, \hat{\beta}_{Kj} > 1 \text{ or } \hat{\beta}_{Kj} \simeq 1 \quad (3)$$

2. Table 1 shows the results of the estimated parameters in nineteen sectors with fairly large samples more than 140 establishments. The estimated value of the technology parameters,  $\hat{\beta}_{Lj}$  and  $\hat{\beta}_{Kj}$  were almost stable during the years 1970-85. In these sectors as well as occidental paper industry, there exists the large scale and medium scale establishments simultaneously at the certain period. The parameters in these sectors are also characterized in the same patterns as that in occidental paper industry.
3. we can verify the degree of stability of the estimated parameters in the overtime for each industry in Table 1. It can be seen that the estimated values of  $\hat{\beta}_{Lj}$  were highly stable for each industry and also the values of  $\hat{\beta}_{Kj}$  were fairly stable statistically during the years of 1970 through 1985.

When we focus on the properties of technology parameters typically shown in occidental paper industry, we can illustrate them in the technology system such as Figure 7. In Figure 7 we divided the chart in four quadrants. In the fourth quadrant we can represent the relationship between  $l_{0j}$  and  $X_{0j}$ ,  $l_{0j} = \alpha_{Lj} X_{0j}^{\beta_{Lj}-1}$ , where  $l_{0j}$  stands for the labor coefficient. In the third quadrant, we represent the relationship between  $K_{0j}$  and  $X_{0j}$ ,  $K_{0j} = \alpha_{Kj} X_{0j}^{\beta_{Kj}}$ . In the second quadrant the line represents the relationship between  $K_{0j}$  and  $k_{0j}$ , where  $k_{0j}$  stands for the capital coefficient. Finally, we can induce the relationship between capital and labor coefficients,  $k_{0j}$  and  $l_{0j}$  corresponding the certain scale of gross output,  $X_{0j}$ . The relationship in the first quadrant implies the combination line of labor-capital productivity corresponding to the certain level of gross output. This relationship is induced as follows:

$$1 = l_{0j}(X_{0j})^{\frac{1}{1-\beta_{Lj}}} k_{0j}(X_{0j})^{\frac{1}{\beta_{Kj}-1}} \quad (4)$$

This is the illustration of the capital embodied technology in the plant base. Next, in this system of the technology we have to show how to determine the level of the plant scale optimally.



$l_{0j}$  : required labor for producing unit of products.  
 $X_{0j}$  : plant capacity  
 $K_{0j}$  : capital stock in terms of commodity  
 $k_{0j}$  : capital coefficient

Figure 7: The Illustration of the Capital Embodied Technology in the Plant Base



Table 1: Stability over time of the Parameters by Cross-section Analysis

| Sector                             | Sample | $\beta_L$ |      |      |      | $\beta_K$ |      |      |      |
|------------------------------------|--------|-----------|------|------|------|-----------|------|------|------|
|                                    |        | 1970      | 1971 | 1980 | 1985 | 1970      | 1971 | 1980 | 1985 |
| Raw silk                           | 141    | 0.61      | 0.60 | 0.62 | 0.63 | 0.98      | 0.92 | 0.93 | 0.98 |
| Chemical fertilizer                | 201    | 0.77      | 0.76 | 0.59 | 0.53 | 1.30      | 1.31 | 1.05 | 1.12 |
| Industrial Inorganic chemicals     | 202    | 0.67      | 0.67 | 0.64 | 0.60 | 1.06      | 1.09 | 1.10 | 1.11 |
| Industrial organic chemicals       | 203    | 0.67      | 0.66 | 0.62 | 0.62 | 1.09      | 1.14 | 1.08 | 1.02 |
| Synthetic fiber                    | 204    | 0.69      | 0.69 | 0.76 | 0.78 | 0.84      | 0.92 | 1.17 | 1.18 |
| Industrial rubber products         | 233    | 0.72      | 0.72 | 0.66 | 0.65 | 0.96      | 1.00 | 0.93 | 0.98 |
| Grass products                     | 251    | 0.65      | 0.64 | 0.69 | 0.67 | 1.13      | 1.12 | 1.06 | 1.05 |
| Cement                             | 252    | 0.39      | 0.39 | 0.49 | 0.49 | 0.86      | 0.93 | 0.76 | 0.81 |
| Industrial machinery               | 301    | 0.64      | 0.65 | 0.62 | 0.58 | 0.90      | 0.90 | 0.86 | 0.86 |
| Household electric appliances      | 302    | 0.55      | 0.55 | 0.59 | 0.58 | 0.92      | 0.93 | 0.83 | 0.85 |
| Electric bulbs & lighting fixtures | 303    | 0.59      | 0.60 | 0.60 | 0.61 | 0.85      | 0.92 | 0.82 | 0.80 |
| Communication equipment            | 304    | 0.58      | 0.59 | 0.58 | 0.58 | 0.85      | 0.89 | 0.84 | 0.87 |
| Applied electronic equipment       | 306    | 0.64      | 0.67 | 0.62 | 0.59 | 0.99      | 1.03 | 0.86 | 0.84 |
| Electric measuring instruments     | 307    | 0.64      | 0.66 | 0.66 | 0.63 | 0.89      | 0.97 | 0.88 | 0.87 |
| Medical instruments                | 323    | 0.60      | 0.61 | 0.66 | 0.63 | 0.93      | 0.96 | 0.85 | 0.97 |
| Physical & chemical instruments    | 324    | 0.57      | 0.61 | 0.63 | 0.61 | 1.04      | 1.29 | 0.82 | 0.78 |
| Optical instruments                | 325    | 0.63      | 0.62 | 0.64 | 0.62 | 0.76      | 0.82 | 0.86 | 0.87 |
| Spectacles                         | 326    | 0.65      | 0.64 | 0.71 | 0.71 | 0.81      | 0.86 | 0.84 | 0.86 |
| Watches & clocks                   | 327    | 0.65      | 0.67 | 0.62 | 0.63 | 0.91      | 0.95 | 0.89 | 0.95 |

### 3.3 Unit cost minimization: the determination of optimum plant scale

Let us turn to evaluate the efficiency of the production of each plant with different scale of capacity,  $X_{0j}$ . In the following framework of the unit cost minimization behavior of the producer, producer intends to determine the optimum scale of the plant capacity and corresponding labor and capital coefficients at given factor prices.

We assume the following factor limitational production function in the plant base.

$$\begin{aligned} L_{0j} &= \alpha_{Lj} X_{0j}^{\beta_{Lj}} \\ l_{0j} &= \alpha_{Lj} X_{0j}^{\beta_{Lj}-1} \end{aligned} \quad (5)$$

$$\begin{aligned} K_{0j} &= \alpha_{Kj} X_{0j}^{\beta_{Kj}} \\ k_{0j} &= \alpha_{Kj} X_{0j}^{\beta_{Kj}-1} \end{aligned} \quad (6)$$

Unit cost is defined in each production as follows:

$$UC_{0j} = \frac{C_{0j}}{X_{0j}} = w_j \alpha_{Lj} X_{0j}^{\beta_{Lj}-1} + r_j \alpha_{Kj} X_{0j}^{\beta_{Kj}-1} \quad (7)$$

where  $w$  and  $r$  stand for factor prices of labor and capital.

A necessary condition for a minimum unit cost is  $(\partial UC_{0j} / \partial X_{0j}) = 0$ ,

$$\frac{\partial UC_{0j}}{\partial X_{0j}} = (\beta_{Lj} - 1) w_j \alpha_{Lj} X_{0j}^{\beta_{Lj}-2} + (\beta_{Kj} - 1) r_j \alpha_{Kj} X_{0j}^{\beta_{Kj}-2} = 0 \quad (8)$$

As a result, we obtained optimum solution,  $X_{0j}^*$

$$\begin{aligned} X_{0j}^* &= \left( \frac{(1 - \beta_{Lj}) \alpha_{Lj}}{(\beta_{Kj} - 1) \alpha_{Kj}} \right)^{\frac{1}{\beta_{Kj} - \beta_{Lj}}} \left( \frac{w_j}{r_j} \right)^{\frac{1}{\beta_{Kj} - \beta_{Lj}}} \\ &= A_j \left( \frac{w_j}{r_j} \right)^{\frac{1}{\beta_{Kj} - \beta_{Lj}}} \end{aligned} \quad (9)$$

Substituting equation (9) in equation (7) gives the following minimum unit cost ( $UC_{0j}^*$ ):

$$\begin{aligned} UC_{0j}^* &= w \alpha_{Lj} A_j^{\beta_{Lj}-1} \left( \frac{w_j}{r_j} \right)^{\frac{\beta_{Lj}-1}{\beta_{Kj}-\beta_{Lj}}} + r \alpha_{Kj} A_j^{\beta_{Kj}-1} \left( \frac{w_j}{r_j} \right)^{\frac{\beta_{Kj}-1}{\beta_{Kj}-\beta_{Lj}}} \\ &= \left( \alpha_{Lj} A_j^{\beta_{Lj}-1} + \alpha_{Kj} A_j^{\beta_{Kj}-1} \right) r_j \left( \frac{1 - \beta_{Lj}}{\beta_{Kj} - \beta_{Lj}} \right) \left( \frac{\beta_{Kj} - 1}{\beta_{Kj} - \beta_{Lj}} \right) w_j \end{aligned} \quad (10)$$

We can show examples of the unit cost minimization procedure by specific sets of parameters in Figure 8 and 9.

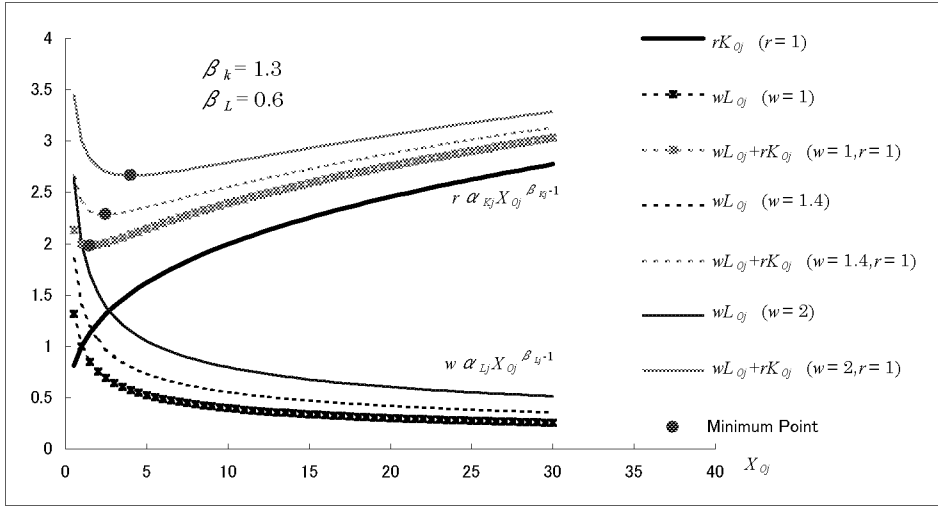


Figure 8: Example of the unit cost minimization I

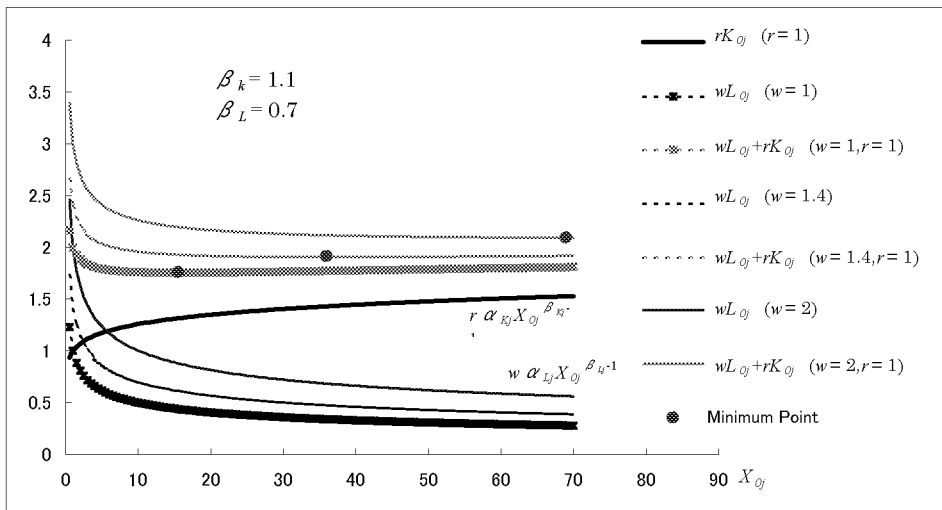


Figure 9: Example of the unit cost minimization II

In Figure 8 and 9 we try to show the unit cost minimization procedure with the typical sets of technological parameters as examples. In these case the changes of relative factor prices could induce the changes of the optimum scale of plant capacity. It is because if the relative price of factor cost would increase, the optimum scale of plant capacity definitely increases. Furthermore, we can show that although the unit cost at the point of the optimum scale of plant capacity increases because of the increase of the relative factor price, the level of the unit cost after changes definitely decreases in comparison with the level of the unit cost at the previous level of plant capacity attached with the relative factor price. It implies that because of the effect of the economy of scale in the factor limitational production function, producers intend to expand the scale of plant capacity in order to pursue the economy of scale in our sense.

## 4 Measurement II:Time-series Analysis

In the earlier papers by Ozaki [15],[16], the following factor limitational type of production function in the  $j$ -th sector was statistically estimated by using time-series data. In the paper we tried to use the aggregated data in each sector as time-series. The variables used in the time-series analysis for the estimation of technology parameters are the plant aggregated sectoral one, which is represented by the following equations:

$$X_j = \sum_0 X_{0j} \quad (11)$$

$$L_j = \sum_0 L_{0j} \quad (12)$$

$$K_j = \sum_0 K_{0j} \quad (13)$$

where

$$L_{0j} = \alpha_{Lj} X_{0j}^{\beta_{Lj}} \quad (14)$$

$$K_{0j} = \alpha_{Kj} X_{0j}^{\beta_{Kj}} \quad (15)$$

Then plant base labor coefficient,  $l_{0j}$  and capital coefficient,  $k_{0j}$  are defined in the properties of the technology in each plant, where the technology in each plant is characterized by the factor limitational production function, as we mentioned above. The aggregated output and inputs are defined by the aggregated measures in the sector in the  $j$ -th sector.

We can formulate the aggregated relationships for each sector as follows:

$$L_j = \alpha_{Lj} X_j^{\beta_{Lj}} \quad (16)$$

$$K_j = \alpha_{Kj} X_j^{\beta_{Kj}} \quad (17)$$

where  $L_j$  and  $K_j$  represent, respectively, labor and capital stock required for the production level  $X_j$  in the  $j$ -th sector, which are sectoral plant aggregated data.

When we try to estimate above functions by the aggregated time-series data, we can identify the technology type for each sector statistically with the estimated parameters of production function in (16) and (17). It is summarized in Table (2). All Sectors were grouped into the following six technology types.<sup>2</sup>

- (i) Type K (I-B) technology; large quantity processing technology
- (ii) Type K (I-M) technology; large-scale assembly production technology
- (iii) Type K (II) technology; capital using technology
- (iv) Type L(I) technology; labor-using technology
- (v) Type L(II) technology; labor-small scale technology
- (vi) Type (L-K) technology; traditional type technology

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<sup>2</sup>This classifications of technology types are slightly different from the classification in [?],[?]. It is because we just focus on the values of estimated parameters instead of the capital intensity in this time.

As shown in Table 2, industries categorized in Type K(I-B) are characterized by their properties, where both of  $\hat{\beta}_{Lj}$  and  $\hat{\beta}_{Kj}$  are less than unity with the highest capital-labor ratio. Although industries categorized in Type K(I-M) also have the same properties in terms of the estimated parameters, values of the capital-labor ratio are relatively capital intensive, but not so high. In Table (2), we observed that K(I-B) type technology sectors include 1. Electric power supply, 2. Gas & water supply, 3. Petroleum refining products, 4. Basic organic chemicals, 5. Artificial fiber materials, 6. Iron and steel, 7. Nonferrous primary products, and K(I-M) type technology sectors include, 8. Ships & ship repairing, 9. Motor vehicles, 10. Machinery, 11. Electrical machinery, 12. Precision instruments, 13. Fiber spinning, 14. Beverages & alcoholic drinks. According to the properties of technology in these industries, we can not deduce the optimal scale of the plant by above unit cost minimization procedure. Although the determination of the plant scale is depending upon the scale of demand in the market, the technology is dominantly characterized by the properties of the economies of plant scale.

On the other hand, industries categorized in Type K(II) are characterized by their technology properties, where  $\hat{\beta}_{Lj}$  is less than unity and  $\hat{\beta}_{Kj}$  is more than unity. Industries categorized in this type should be distinguished from industries in Type L(I), Type L(II) and Type (L-K) because of their capital using technology. According to the technology properties in Type K(II), we can determine the optimum scale of the plant in the above unit cost minimization. Given the changes of factor prices, producers intend to determine the optimum scale of the plant. When the changes of labor input price were relatively high in comparison with the changes of capital input price, the optimum scale of plant would be enlarged in order to pursue the economic efficiency. Industries categorized in above three types, K(I-B), K(I-M) and K(II) are characterized by the properties of the technology with high capital intensity and the benefit of the economies of scale in the plant level.

Next two types L(I) and L(II) are characterized by the small scale plant and the low capital intensity. Also, Type K-L includes traditional sectors such as agriculture and mining.

In this paper, we are focussing on the properties of technology included in Type K, especially Type K(II). In these industries, the values of parameters  $\beta_K$ 's are more than unity, while the values of  $\beta_L$ 's are all less than unity.

$$\begin{aligned} \beta_{Lj} &< 1 \\ &\text{and} \\ \beta_{Kj} &> 1 \end{aligned}$$

We should note that in the developing process of the Japanese economy, the industries categorized in Type K, especially in Type K(II) made important roles as one of leading sectors.

Table 2: Production Function Parametric Characteristics of Various Industries by Technology Type (1)

| Sector  | Type of Technology | $\beta_L$ | $\beta_K$ | $\frac{K}{L}$ | $\frac{1}{\beta_K - \beta_L}$ | $\frac{\beta_K - 1}{\beta_K - \beta_L}$ |
|---|--------------------|-----------|-----------|---------------|-------------------------------|---|
| <b>Large-quantity processing technology</b>       |                    |           |           |               |                               |   |
| Electric power supply                             | K(I-B)             | 0.12      | 0.80      | 17.43         | 1.46                          | -0.30                                   |
| Petroleum refining products                       | K(I-B)             | 0.27      | 0.65      | 14.76         | 2.58                          | -0.89                                   |
| Basic organic chemicals                           | K(I-B)             | 0.33      | 0.72      | 5.70          | 2.60                          | -0.74                                   |
| Artificial fiber materials                        | K(I-B)             | 0.10      | 0.84      | 3.89          | 1.35                          | -0.21                                   |
| Iron & steel                                      | K(I-B)             | 0.30      | 0.80      | 3.86          | 2.01                          | -0.41                                   |
| Nonferrous metal products                         | K(I-B)             | 0.38      | 0.73      | 3.84          | 2.85                          | -0.78                                   |
| <b>Large-scale assembly processing technology</b> |                    |           |           |               |                               |   |
| Ships & ship repairing                            | K(I-M)             | 0.07      | 0.80      | 1.19          | 1.38                          | -0.28                                   |
| Motor vehicles                                    | K(I-M)             | 0.46      | 0.70      | 2.12          | 4.20                          | -1.26                                   |
| Machinery   | K(I-M)             | 0.52      | 0.88      | 0.62          | 2.74                          | -0.32                                   |
| Electrical machinery                              | K(I-M)             | 0.55      | 0.91      | 1.00          | 2.76                          | -0.24                                   |
| Precision instruments                             | K(I-M)             | 0.53      | 0.97      | 0.59          | 2.25                          | -0.06                                   |
| Fiber spinning                                    | K(I-M)             | 0.26      | 0.59      | 2.07          | 2.99                          | -1.23                                   |
| Beverages & alcoholic drinks                      | K(I-M)             | 0.33      | 0.79      | 2.26          | 2.19                          | -0.47                                   |
| <b>Capital using technology</b>                   |                    |           |           |               |                               |   |
| Paper   | K(II)              | 0.13      | 1.03      | 3.07          | 1.11                          | 0.03                                    |
| Pulp  | K(II)              | -0.29     | 1.23      | 3.94          | 0.66                          | 0.15                                    |
| Cement  | K(II)              | 0.08      | 1.03      | 9.07          | 1.06                          | 0.03                                    |
| Basic inorganic chemicals                         | K(II)              | 0.04      | 1.01      | 2.71          | 1.03                          | 0.01                                    |
| Chemical manure                                   | K(II)              | -0.71     | 1.71      | 4.97          | 0.41                          | 0.29                                    |
| Miscellaneous coal products                       | K(II)              | -0.09     | 1.67      | 1.50          | 0.57                          | 0.38                                    |
| Building & construction                           | K(II)              | 0.53      | 1.32      | 0.25          | 1.27                          | 0.40                                    |
| Seafood, preserved                                | K(II)              | 0.46      | 1.13      | 0.59          | 1.50                          | 0.20                                    |
| Paints  | K(II)              | 0.35      | 1.09      | 1.51          | 1.36                          | 0.12                                    |
| Other transport equipment                         | K(II)              | 0.28      | 1.05      | 1.01          | 1.31                          | 0.06                                    |
| Metal products                                    | K(II)              | 0.51      | 1.01      | 0.49          | 2.02                          | 0.02                                    |
| Structural clay products                          | K(II)              | -0.04     | 1.10      | 0.57          | 0.87                          | 0.09                                    |
| Printing & publishing                             | K(II)              | 0.47      | 1.15      | 0.57          | 1.48                          | 0.22                                    |
| Other food, prepared                              | K(II)              | 0.42      | 1.36      | 0.65          | 1.06                          | 0.38                                    |
| Finance & insurance                               | K(II)              | 0.45      | 1.22      | 0.70          | 1.29                          | 0.29                                    |
| Communication services                            | K(II)              | 0.25      | 1.46      | 0.17          | 0.83                          | 0.38                                    |

Table 2: Production Function Parametric Characteristics of Various Industries by Technology Type (2)

| Sector                             | Type of Technology | $\beta_L$ | $\beta_K$ | $\frac{\bar{K}}{L}$ | $\frac{1}{\beta_K - \beta_L}$ | $\frac{\beta_K - 1}{\beta_K - \beta_L}$ |
|------------------------------------|--------------------|-----------|-----------|---------------------|-------------------------------|---|
| <b>Small scale technology</b>      |                    |           |           |                     |                               |   |
| Meat                               | L(I)               | 1.15      | 0.80      | 1.52                | -2.85                         | 0.56                                    |
| Transport services                 | L(I)               | 0.47      | 0.99      | 1.04                | 1.92                          | -0.01                                   |
| Rubber products                    | L(I)               | 0.47      | 0.84      | 0.99                | 2.74                          | -0.45                                   |
| Grass products                     | L(I)               | 0.31      | 0.97      | 1.46                | 1.51                          | -0.05                                   |
| Miscellaneous industrial products  | L(I)               | 0.52      | 0.61      | 0.78                | 11.67                         | -4.60                                   |
| Leather products                   | L(I)               | 0.46      | 0.63      | 0.40                | 5.75                          | -2.10                                   |
| Furniture & fixtures               | L(I)               | 0.41      | 0.57      | 0.40                | 6.15                          | -2.63                                   |
| Other wood products                | L(II)              | 0.28      | 0.50      | 0.26                | 4.47                          | -2.21                                   |
| Paper articles                     | L(II)              | 0.45      | 0.75      | 0.72                | 3.32                          | -0.84                                   |
| Pottery , china, & earthenware     | L(II)              | 0.37      | 0.89      | 0.51                | 1.89                          | -0.20                                   |
| Other nonmetallic mineral products | L(II)              | 0.46      | 0.67      | 1.15                | 4.74                          | -1.55                                   |
| Medicine                           | L(II)              | 0.24      | 0.87      | 1.25                | 1.60                          | -0.21                                   |
| Weaving & other fiber products     | L(II)              | 0.28      | 0.77      | 0.79                | 2.03                          | -0.46                                   |
| Footwear & wearing apparel         | L(II)              | 0.42      | 0.60      | 0.31                | 5.68                          | -2.28                                   |
| Trading                            | L(II)              | 0.33      | 0.40      | 0.65                | 14.45                         | -8.62                                   |
| <b>Traditional technology</b>      |                    |           |           |                     |                               |   |
| Agriculture, forestry, & fisheries | L-K                | -1.09     | 1.82      | 0.46                | 0.34                          | 0.28                                    |
| Coal & I ignite                    | L-K                | -1.68     | 2.66      | 0.90                | 0.23                          | 0.38                                    |
| Mining                             | L-K                | 0.04      | 1.53      | 0.56                | 0.67                          | 0.35                                    |
| Silk reeling & spinning            | L-K                | -2.12     | 2.06      | 0.59                | 0.24                          | 0.25                                    |
| Vegetable & animal oil & fat       | L-K                | -0.06     | 1.39      | 1.91                | 0.69                          | 0.27                                    |
| Wood milling                       | L-K                | 0.36      | 1.17      | 0.68                | 1.23                          | 0.21                                    |

## 5 Enlargement of the plant scale and structural change

Results in above two sections estimated in cross section data and time series data are summarized consistently from the viewpoints of the technology. In the cross section data in establishment base we can estimate the properties of technology in the plant level. As we mentioned in the previous section, their properties of the technology in the plant level could be connected with the properties of technology realized in the aggregated sectoral level by the distribution of the plants in each period. Changes of the factor prices in time series could be induced changes of the distribution of the plant scale endogenously as the changes of the optimum scale of plants. Even if the distribution of the optimum plant scale in the time series would be changed by the changes of factor prices, in the aggregated level we could observe a stable technological property as parameters of the factor limitational production functions. As shown in Figure 1-3, the distributions of the optimum scales of plants were fairly stable in the development process in the economy. Therefore, in the aggregated level, changes in the capital coefficients in the Leontief Dynamic Model could be endogenously determined by the changes of the optimum scales of plants, which are determined by the unit cost minimization in each plant level. Finally the changes of capital coefficient in the aggregated level could be dominated by endogenously determinate optimal scales of plants and their distribution.

Growth rate of optimum scale of plant capacity in each establishment under the unit-cost minimum can formulate as follows:

$$\frac{\dot{X}_{0j}}{X_{0j}} = \frac{1}{\beta_{Kj} - \beta_{Lj}} \left( \frac{\dot{w}_{0j}}{r_{0j}} \right) \quad (18)$$

where relative factor price in each establishment,  $\frac{w_{0j}}{r_{0j}}$  is given exogenously at the certain period. The change of factor price in over-time might have changes of the optimum scale of plant capacity under the unit cost minimization behavior. Equation (18) represents the growth rate of the optimum scale of plant capacity consistently with changes of relative factor price in each individual establishment.

On the other hand, we can obtain the growth rate of sectoral aggregated plant capacity consistently with the changes of scale of plant capacity in each individual establishment as follows.

$$\frac{\dot{X}_j}{X_j} = \frac{1}{\beta_{Kj} - \beta_{Lj}} \sum_0 \frac{X_{0j}}{X_j} \left( \frac{\dot{w}_{0j}}{r_{0j}} \right) \quad (19)$$

We can define sectoral aggregated capital coefficient as follows.

$$\begin{aligned} B_j &= \frac{K_j}{X_j} \\ &= \frac{\sum_0 K_{0j}}{\sum_0 X_{0j}} = \frac{\sum_0 \alpha_{Kj} X_{0j}^{\beta_{Kj}}}{\sum_0 X_{0j}} \end{aligned} \quad (20)$$

Consequently, when we introduce our factor limitational production function as technological properties, we can induce the growth rate of sectoral aggregated capital coefficient as a function of the optimum scale of plant capacity in each establishment as follows.

$$\begin{aligned} \frac{\dot{B}_j}{B_j} &= \sum_0 \left( \beta_{Kj} \frac{K_{0j}}{K_j} - \frac{X_{0j}}{X_j} \right) \frac{\dot{X}_{0j}}{X_{0j}} \\ &= \sum_0 \left( \beta_{Kj} \frac{X_{0j}^{\beta_{Kj}}}{\sum_0 X_{0j}^{\beta_{Kj}}} - \frac{X_{0j}}{\sum_0 X_{0j}} \right) \frac{\dot{X}_{0j}}{X_{0j}} \end{aligned} \quad (21)$$

It implies that the changes of the capital coefficient in the sectoral aggregated level depend upon the technological parameters of the production function and the distribution of the optimum scale of



plant capacity in establishments consistently with given changes of factor price in overtime. if changes of relative factor price for individual plant is given at the constant level, we can assume,

$$\frac{\left(\frac{\dot{w}_{0j}}{r_{0j}}\right)}{\left(\frac{w_{0j}}{r_{0j}}\right)} = \frac{\left(\frac{\dot{w}_j}{r_j}\right)}{\left(\frac{w_j}{r_j}\right)}, \text{ for all } 0 \quad (22)$$

Substituting equation (22) into equation (19), and rearranging equations (19) and (22), we can obtain the following two equations.

$$\frac{\dot{X}_j}{X_j} = \frac{1}{\beta_{Kj} - \beta_{Lj}} \frac{\left(\frac{\dot{w}_j}{r_j}\right)}{\left(\frac{w_j}{r_j}\right)} \quad (23)$$

$$\frac{\dot{B}_j}{B_j} = \frac{\beta_{Kj} - 1}{\beta_{Kj} - \beta_{Lj}} \frac{\left(\frac{\dot{w}_j}{r_j}\right)}{\left(\frac{w_j}{r_j}\right)} \quad (24)$$

Finally, from (23) and (24), the change of the capital coefficient in the sectoral aggregated level can represent as the following equation.

$$\frac{\dot{B}_j}{B_j} = (\beta_{Kj} - 1) \frac{\dot{X}_j}{X_j} \quad (25)$$

Finally the changes of capital coefficients in the aggregated level in equation (25) depend upon the parameter,  $\beta_{Kj}$  as technological property and the growth rate of the plant capacity in the sectoral aggregated level. Needless to say, the growth rate of the plant capacity in the sectoral aggregated level is a function of the optimum scale of plant capacity in each individual establishment. As we mentioned before, in the case which the technological properties are represented as  $\beta_{Kj} > 1$  and  $\beta_{Kj} > \beta_{Lj}$  typically, the plant capacity in the sectoral aggregated level increases along with the expansion of the optimum plant scale in each establishment at given increases of relative factor price. At the same time, the capital coefficient in the sectoral aggregated level is also increasing. It implies that the capital coefficients in the Leontief Dynamic Model could be endogenously determined, consistently with the properties of technological relationships in the plant base.

## 6 Concluding Remarks

The results obtained in this study may be summarized as follows;

1. We can show the usefulness and effectiveness of the factor limitational type of the production function in the analysis of structural changes, where dynamic changes of the parameters in the Leontief's dynamic models are endogenously determined.
2. In order to determine the optimum plant-scale,  $X_0^*$ , two hypotheses were set up. One is the "capital embodied technology" hypothesis and the other is "unit cost minimization" principle. In this model, the concept of "economies of plant-scale" was newly defined in terms of unit cost.
3. Based on the above results, it was shown that there is a tendency toward the expansion of plant-scale,  $\Delta X_0$ , in the process of economic growth accompanied by changing of wage cost and other prices.
4. It was shown that changes over time of the sectoral aggregated capital coefficient  $B_j$  is dependent on the expansion of optimum plant-scale,  $\Delta X_0$ . The changes over time of the structural capital coefficient would be the most important factor which cause the structural change over time in the Leontief Dynamic Model.
5. Finally we can conclude that in the following Leontief Dynamic Model,

$$A^t X(t) + B^{t+1} \Delta X^t + C^t = X^t, \quad (26)$$

Capital coefficients,  $B_j$  would be endogenously induced by the changes of the optimum scale of plant. In our model, Leontief Dynamic Model with structural changes should be formulated as follows;

$$A^t X^t + B^{t+1}(X_{0j}) \Delta X^t + C^t = X^t, \quad (27)$$

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