INPUT-OUTPUT AND ISOMORPHIC ANALYTICAL TOOLS IN SPATIAL ECONOMICS

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Summary.

In the course of spatial economic research, input-output specifications were regularly encountered; a number of them are reviewed, and recent extensions analysed. Moreover, isomorphic concepts can be derived; in particular location elasticities are taken up, with a resulting multiregional growth model.

Key-words : input-output analysis; location elasticity; mathematical programming; multiregional growth.

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Table of contents.

1.Introduction.

Input-output analysis is all-pervasive; in [67] (figures in brackets refer to the list of references, section 5), a study dedicated to Wassily Leontief, he was quoted as saying : *"Dependence and independence, hierarchy and circularity (or multiregional dependence) are the four basic concepts of structural analysis"*, and this could already be a sufficient argument.

In fact, focusing this review on spatial applications of input-output analysis, there are more reasons that could be involved, and the next sections will argue this from different angles of incidence.

Those sections are organised in two parts. A first part relates to - classical and less classical - applications of input-output analysis at the spatial - regional and urban - level; a second part treats some more distant applications, including isomorphic concepts that could be seen as complementary to input-output analysis proper, and in fact include its workings.

One will not dwell here on non-spatial studies in which the input-output framework was encountered ([9], [24], [26], [27], [28], [29], [30]), nor on general introductions to spatial input-output analysis ([e.g. in [42]) or on related but for the analysis to follow not immediately relevant studies ([8], [31], [36]); for the sake of completeness, a recent series of specialised studies ([11]) should not be omitted from mentioning.

2. Input-output analysis proper.

In what follows a certain number of applications of spatialised input-output analysis will be presented, and some of them will be given more recent extensions; for the sake of clarity, all parameters and coefficients are assumed to be strictly positive, unless otherwise stated..

2.1. Exploration, impact, simulation.

In the sixties, regional development problems attracted great attention; different types of regional inequalities were analysed (declining, mostly old industrial regions; traditionally stagnating regions; peripheral regions, i.a. border regions, [34]) and the problem was raised how to prepare them for a (new) take-off.

One possibility to solve that problem was to use input-output analysis to explore the impacts of certain - often new - lines of activity, "filières' as the french term for this concept is; references are [7], [32] and [49]. The technique used, a reverse one, was in fact relatively simple : it consisted in removing a sector from an input-output table, and replacing its deliveries by imports, all other things being equal (though refinements, like adapting local final demand, were also introduced). In the Liège region, an old industrial region in Belgium, metal working was directly and indirectly good for nearly 50% of total value added, which showed the one-sided structure of the region; to the contrary, chemicals contributed for only less than 8%. The main criticism raised against this type of exploration was its global character - one will come back to this point -, plus the hypothesis that supply of primary factors would be sufficiently elastic to adapt to a radically new situation, even in the longer run.

A variant on this method consisted in breaking down the impacts of a cluster of local activities - call it an industrial complex - on the global workings of the local economy ([33], [37], [68]). The derivation is as follows, *1* being the index of the complex, *2* that of all other activities, the vectors and matrices being the usual symbols of input-output analysis :

$$\begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \end{bmatrix} \qquad (1)$$

hence :

$$\mathbf{q}_1 = [\mathbf{I} - \mathbf{A}_{11} - \mathbf{A}_{12}(\mathbf{I} - \mathbf{A}_{22})^{-1}\mathbf{A}_{21}]^{-}1[\mathbf{f1} + \mathbf{A}_{12}(\mathbf{I} - \mathbf{A}_{22})^{-1}\mathbf{f}_2] \qquad (2)$$

The average decomposition of $\mathbf{q_1}$ is given by table 1 hereafter.

Tablr 1 : decomposition of impacts on an industrial complex

Entries	*%*
intraregional final deliveries	1.27
exports	82.62
impact of final demand for other sectors' products	3.00
effect of the complex's own multiplier matrix	9.00
effect of its insertion in the rest of the local economy	4.11

A marginal variant of the two methods just described analyses the effects of differential increases , e.g. in intermediate deliveries ([45], pp.199-208). The impact on the regional product of an increase in regional deliveries - at the expense, this time, of imports - of activity *i* to activity *j* has been computed as :

$$\partial r/\partial x_{ij} = \mathbf{v'C_{ij}f}\ [(\mathbf{b_j}+a_{ij}\mathbf{c_{ij,j}})'\mathbf{f}]^{-1} \qquad (3)$$

where :

* $\mathbf{C_{ij}} = \partial \mathbf{B}/\partial a_{ij}$;

* $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$;

* $\mathbf{b_j}'$ = *j*-th row of **B**;

* $\mathbf{c_{ij,j}}'$ = *j*-th row of $\mathbf{C_{ij}}$;

* **v** = value added coefficients;

* **f** = final demand.

The result of the analysis - applied to a Spanish province - was that the multipliers of intermediate demand can reach much higher values than those of final demand, for which the maximum is *1*; a multiplier as high as *13.5* was computed. Such multipliers can be obtained, either from the observed (partial) input coefficients**,** or from the maximum (technical coefficients proper) values.

Focusing now on the planning aspect of the regional development problem, input-output relations are taken into account in setting up a mathematical programming model for deriving a minimal investment (*I*) viable industrial complex, in the sense that all the activities present are minimally profitable ([39]).

The program is specified as follows :

$$\min I = \sum_i p^*_i (I_{io}/Q_{io}^{a_i})\, Q_i^{a_i}\, x_i \qquad (4)$$

s.t., $\forall i$,

$$p_i Q_i x_i - \sum_{j\neq i} p_j a_{ji} \exp(-a_{ji} Q_j x_j) Q_i x_i - w_i (L_{io}/ Q_{io}^{b_i}) Q_i^{b_i} x_i$$

$$- r_i p^*_i (I_{io}/ Q_{io}^{a_i}) Q_i^{a_i} x_i \geq \pi_i p^*_i (I_{io}/ Q_{io}^{a_i}) Q_i^{a_i} x_i \qquad (5)$$

$$Q_i x_i \geq Q_{io} \qquad (6)$$

$$x_i = x_i^2 \qquad (7)$$

$\sum_i x_i \geq 1$ (8)

with :

* Q_i : production level of plant *i*;

* Ii : investment in plant *i*;

* a_{ji} : input coefficient of product *j* into production of *i;*

* p_i : unit price of product *i*, net of intermediate products not considered for the industrial complex;

* p^*_i : unit price of investment in plant *i*;

* w_i : unit wage in production *i*;

* r_i : unit period depreciation rate for investment *i*;

* π_i : desired unit period profitability rate for activity *i*.

The expressions $(I_{io}/ Q_{io}^{ai}) Q_i^{ai}$ and $(L_{io}/ Q_{io}^{bi}) Q_i^{bi}$, $0 \leq ai, bi \leq 1$, are derived from technological production functions with scale economies; the exponential function reflects local externalities.

As such the problem boils down to a mixed integer-continuous geometric program, a solution method being presented in the article quoted. The method can be used, either to build up an industrial complex from scratch, or to complement an already - loosely integrated - "pseudo" industrial complex as referred to above.

It should finally be noted that if public authorities start launching several of those programs, problems of consistency will creep up ([12], [13], [38[, [41]); we will come back to this point further down.

In order to gain more insight in the interlinkages of the input-output economy, the following method was developed recently ([63], [64]). One first defines a "club" as a series of activities with high degrees of interlinkages, e.g. of the input-output type; the general idea then is to extract from an input-output table a series of clubs with each time maximal total interlinkages.

The mathematical program can be written as :

$$\max_{\mathbf{x}} \varphi \overset{\Delta}{=} \mathbf{vec(A')'x} \quad (9)$$

subject to :

$$\mathbf{Jx \leq i} \quad (10)$$

$$\hat{\mathbf{x}}\mathbf{x} = \mathbf{x} \quad (11)$$

with :

* **vec(A')** : the vectorialisation of the transpose of **A**, diagonal elements omitted, of order $n(n\text{-}1)_* 1$, *n* being the order of **A**;

* **x** : a vector, equally of dimension $n (n\text{-}1)_* 1$, of binary variables [conditions (11), where $\hat{\mathbf{x}}$ is the diagonal matrix constructed from **x**];

* conditions (10) : weakened assignment conditions, matrix **J** beingt binary and of order $2n_* n(n\text{-}1)$; if the weak inequalities were to be replaced by equalities, exactly *n directed* relations would be selected, each sector appearing

twice; relaxation so allows of generating *incomplete* clubs.

Once the solution to the mathematical program has been obtained, one has generated a first club with maximal internal cohesion, here total mutual deliveries; one then cancels the corresponding entries of **A**, leading up to a matrix **A*** to be treated in the same manner, and so on until all the entries have been exhausted..

The method was applied to a *10** *10* input-output matrix for the United Kingdom in 1950 (source : [66], p.33); the results were as follows.

Only the first three rounds produced at least one club, the very first round a complete set of three clubs; those rounds together accounted for respectively 40.60%, 18.34% and 11.62% of the total flows. Here are the clubs generated :

* round 1 : a. agriculture, forestry, fishing => food, drink, tobacco => agriculture, forestry, fishing;
b. mining, quarrying => gas, electricity,water => chemicals, allied trades => textiles, leather, clothing => other manufacturing => building , contracting => mining, quarrying;
c. metals, engineering, vehicles => other production, trade => metals, engineering, vehicles.

* round 2 : agriculture, forestry, fishing => textiles, leather, clothing => mining, quarrying => chemicals, allied trades => agriculture, forestry, fishing.
* round 3 : mining, quarrying => textiles, leather, clothing => chemicals, allied trades => metals, engineering, vehicles => mining, quarrying.

Round *1,a* obviously reproduces the agricultural food cycle, round 1,*b* evidently centers around chemical production, while round 1,*c* pictures the metalworking complex. Rounds *2* and *3* refer to more involved technologies, but the level of aggregation is to high to allow to disentangle them correctly. Anyhow, the above mentioned "filières" are clearly present.

2.2. Supply effects.

In the previous section local externalities have been mentioned; in Weberian location theory they are held - partly - responsible for the locational decisions of potential investors. It is well-known that input-output anlysis proper is demand-oriented, but in spatial applications supply effects should not be absent from modelling; a possible specification could be the following one ([40], [44]).

Start from the input-output ("pull")-model for all sectors :

$$\mathbf{q} = \mathbf{A}\,\mathbf{q} + \mathbf{f} + \mathbf{e} \qquad (12)$$

where **f** is regional final demand, and **e** net exports; a "push" equation can now bespecified as :

$$\mathbf{q} = \hat{\mathbf{b}}^{-1}\,\tilde{\mathbf{A}}'\,\mathbf{q} \qquad (13)$$

where $\tilde{\mathbf{A}}$ is the matrix of intermediate allocation coefficients.

(12) and (13) can be combined linearly by a diagonal weighing-matrix $\hat{\lambda}$ to be applied to (13); a reasonable assumption is that :

$$\lambda = \tilde{\mathbf{A}}\,\mathbf{i} \qquad (14)$$

hence, defining :

$$\hat{\mu} \overset{\Delta}{=} \mathbf{I} - \hat{\lambda} \qquad (15)$$

there comes :

$$\mathbf{q} = \hat{\mu}\,\mathbf{A}\,\mathbf{q} + \hat{\mu}\,\mathbf{f} + \hat{\mu}\,\mathbf{e} + \hat{\lambda}\,\hat{\mathbf{b}}^{-1}\,\tilde{\mathbf{A}}'\,\mathbf{q} \qquad (16)$$

Consider now **f**; in part it will be demand-pulled :

$$\mathbf{f} = \alpha\,\mathbf{v}'\,\mathbf{q} \qquad (17)$$

where α is a vector of consumption propensities and, as already defined above, $\mathbf{v}' = \mathbf{i}'(\mathbf{I}-\mathbf{A})$. As to the push-effect, it can be specified as :

$$\mathbf{f} = \hat{\varphi}\,\mathbf{q} \qquad (18)$$

where $\hat{\varphi}$ is the diagonal matrix of the interior final demand allocation coefficients. If we combine (17) and (18) linearly, the weights for (18) could be reasonbly put equal to φ, those for (17) to $\mathbf{i} - \varphi$. A same argument applies to **e** with weighing-vector ψ.

Inserting these developments in (16) allows of writing finally that :

$$\mathbf{q} = \mathbf{Q}\,\mathbf{q} \qquad (19)$$

a system of homogeneous equations in **q**.

Splitting up **q** into $\mathbf{q}_1$ and $\mathbf{q}_2$, and **Q** comformably into $\mathbf{Q}_{11}$, $\mathbf{Q}_{12}$, $\mathbf{Q}_{21}$ and $\mathbf{Q}_{22}$, one can express the impacting effects symmetrically as :

$$\mathbf{q}_1 = (\mathbf{I} - \mathbf{Q}_{11})^{-1}\mathbf{Q}_{12}\mathbf{q}_2^* \qquad (20)$$

and :

$$\mathbf{q}_2 = (\mathbf{I} - \mathbf{Q}_{22})^{-1}\mathbf{Q}_{21}\mathbf{q}_1^* \qquad (21)$$

where $\mathbf{q}_2^*$ and $\mathbf{q}_1^*$ are taken alternatively as exogenous, so mutual push-and-pull impacts can also be computed. The relevant partial coefficients are of course those of the matrices $(\mathbf{I} - \mathbf{Q}_{11})^{-1}\mathbf{Q}_{12}$ and $(\mathbf{I} - \mathbf{Q}_{22})^{-1}\mathbf{Q}_{21}$.

It should be noted that all the parameters can be derived from a standard input-output table; possible applications are the computation of impacting effects of transportation sectors ([47], [48], [65]).

2.3. Input-output and spatial econometrics.

As input-output analysis has been combined with macro-economic modelling, so has it been combined with spatial econometrics.

Let it be recalled ([2], [13]) that the latter discipline is characterised by five features or principles :

* spatial interdependence of endogenous variables;

* spatial heterogeneity and asymmetry;

* "allotopy", or the presence at a distance of exogenous explanatory variables;

* ex-ante non-linearity with possible ex-post linearity;

* presence of topological variables : distances already mentioned, coordinates, densities,...

In [1] the spatial econometrics of an interregional model has been presented, the so-called European FLEUR-model

(Factors of Location in EURope). To quote an important passage from that study, pp.231-232 : " Regions cannot be regarded as closed systems, nor is sectoral growth an isolated phenomenon; spatial interaction is not an intra-regional mechanism. Indeed, both input and output markets of most modern industrial sectors are spatially dispersed far beyond the frontiers of the region where the industries are settled, and industrial technologies are also interdependent beyond regional borders. Industries may settle in a given region not because that region itself is so attractive, but because it is next to an important market area for their products that is too congested to admit new firms. Or a region may be chosen as a spatial compromise between various contiguous regions of which one offers an output market, the second offers access to primary inputs, the third offers ancillary services, and so on. Obviously, such interregional effects are very important elements to include, at least implicitly, in the model."

On the basis of those elements, an estimable dynamic function was specified, input-output playing an essential part in estimating the demand and supply variables that would act as locational factors. Applying the third principle mentioned above, those estimates were "potentialised", in other words they were "discounted' over space with the help of an appropriate spatial decay function. Here follow both specifications.

The dynamic sectoral function was specified with a double error correction :

$$\Delta' y_t = \alpha(y^o_t - y_t) + \beta(\Delta' y^o_t - \Delta' y_t) \qquad (22)$$

where y_t is the natural logarithm of a non-linear transform of the activity indicator, y^o_t a log-linear function of the locational factors - the equilibrium values -, all variables at time *t*. The function y^o_t includes potentialised values of relevant demand and supply variables, the flexible decay function being ;

$$f_{rs}(d_{rs}) = \exp(1-\gamma^*)[\ln(1+\gamma\, d_{rs}) + \gamma^*](\gamma d_{rs} + 1)^{-1} \qquad (23)$$

with :

$$\gamma^* \overset{\Delta}{=} \gamma\,(1+\gamma)^{-1} \qquad (24)$$

$$\gamma \geq 0 \qquad (25)$$

where d_{rs} is an appropriate distance measure between regions *r* and *s*.

Apart from the usual projection exercises, the estimated equations were also put to two other applications which deserve to be mentioned.

The first one was a classification of European regions in two sets : less and more developed regions; the model rested indeed on a preliminary discriminant specification ([46], p.169).

The second was a complementation of the econometric model, in a simplified version, with a mathematical programming exercise. The general idea was to devise a minimal investment program (recall the industrial complex programming model of section 2.1) for the less developed regions, such as to guarantee all of them to reach at least a critical "threshold level", in terms of their endowment with locational factors, a level in fact which separated those less developed regions from the more developed ones. To that effect, investment funds could - over two long term periods - be channeled to five types of policy instruments : improving infrastructure, adding to the industrial base, extend the availability of services, use financial stimuli, or enhance urban externalities.

Some interesting findings were the following ([46], pp.176-177). Improvement of accessibility, through bettering of the outbound infrastructure, in concordance with its long term character and relative expensiveness, was generally applied , and that throughout the two periods. Increasing local supply of industrial products and services was already more selective, between regions and periods. Financial stimuli had to be applied at rather low levels, and showed a complementary or accompanying character. Finally urban externalities should only be stimulated in the second period.

A remarkable hierarchy so presented itself from that first exploratory exercise in multiregional consistent planning, already hinted at in section 2.1. In fact, there is some spatial logic behind it : accessibility to demand - both intermediate and final - and to supply is an important factor in the workings of spatial markets, and so is the local activity base in terms of goods and services produced. Once those elements are sufficiently strong, urban externalities - favouring hi-tec developments, information and communication technoilogies, and the like - come into play; looking jointly at space and time is an essential requirement for designing consistent, possibly optimal, multiregional policies.

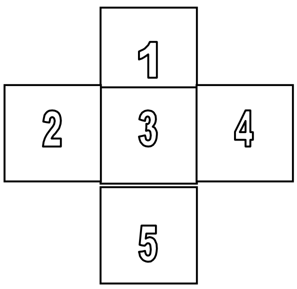
2.4. General spatial economic equilibrium.

General spatial economic equilibrium will be approached here from the angle of Tinbergen-Bos Systems; those models were developed in the sixties by the two scholars named, and designed to operationalise Lösch's general spatial economic equilibrium model, in particular to derive propositions on economic "landscapes" in terms of clusters of activities ("centres"), a specific combination of clusters being called a "system".

Indeed, if in theory the system of equations characterising a Lösch equilibrium allows of computing locations of firms, quantities produced and their unit factory prices, market areas and boundaries, no specific "landscapes" can *theoretically* be derived; moreover the problem itself is in principle of a non-linear mixed continuous-discrete nature, so rather hard to solve. The fundamental idea of Tinbergen-Bos Systems was to simplify the Lösch model to a linear one without prices, locations and market areas, much along the lines Leontief operationalised Walras' non-spatial equilibrium model.

The original model was implicitly based on a discrete (*0-1*) metric; Kuiper, Paelinck and Rosing (1990) introduced a metric on a Manhattan network within what was termed a "Manhattan square" of a given radius r (see figure 1 for an example with $r=1$), and in this way Tinbergen-Bos Systems became a special case of a location-allocation problem ([57]). For more details one is referred to [17], [18], [19], [20], [21], [22], [23],[52], [53], [55], [56] ,[57], [60] and [61].

Figure 1 : Manhattan circle of radius 1



The equilibrium is computed by minimising, for given unit prices, an objective function derived from consumption and production coefficients; the composition and location of centres - the "economic landscape" or "system" - are indeed determined by the minimisation of global transport cost (though this may correspond to profit maximisation behaviour by individual firms : see [59]), i.e. of a function :

$$\varphi = \mathbf{w}'\mathbf{x} \qquad (26)$$

where $\mathbf{x}$ is a vector of the exports of goods and services, and $\mathbf{w}$ a vector depending on the distances d_{ij} between all the possible locations i and j (whatever the metric used; in figure 1, $i,j=1,...,5$; the locations are situated at the center of each "elementary square", and linked together by a Manhattan network), on unit transportation costs, and on relative quantities shipped, the latter depending on consumption propensities and technical coefficients in the following way.

Define :

a_k : propensity to consume product k;

y^* : total production value (or value added in the absence of interindustry relations) of the system (exogenous);

$a_k y^*$: production of sector k.

So, in the absence of interindustry relations, the value transported between sectors *k* and *l* equals $a_k a_l y^*$, each firm of sector *l* demanding $a_k a_l y^*/n^*_l$ from sector *k*, n^*_l being the number of plants of type *l*. The total weight for deliveries between *k* and *l* becomes $(t_k+t_l)a_k a_l y^*$, so the *relative* weights (excluding distances) relating to the above mentioned flows become :

$$w_{kl} = (t_k+t_l)a_k a_l n_l^{-1} \qquad (27)$$

the t_k, t_l being the unit transportation costs.

In the same vein equation (27) can be generalised to the presence of input-output relations, already studied by Bos, but fully integrated in [16]; if a_{kl} is the input coefficient of *k* in sector *l*, and m_k the value added - production multiplier of sector *k*, then for deliveries between sectors *k* and *l* total transportation costs become :

$$T_{kl} = [t_k(a_k+a_{kl}m_l)a_l + t_l(a_l+a_{lk}m_k)a_k]y^* \qquad (28)$$

How complex the interactions can be is shown by figure 2, which pictures the potential flows between agricultute (sector *0*) and two industrial sectors, sectors *1* and *2* (indices of the flows; the indices inside the squares refer to their spatial characteristics : agricultural centers *0*, centers producing goods *1* or *2,* and centers *3* including both types of production).

Figure 2 :Spatial delivery relations

The next figure (source : [22]) shows how varying elements of function (26) can affect the economic landscape; the relative weights, w_3/w_1 refer to deliveries between industries *1* and *2*, on the one hand, activities *1* and *0* on the other. As equation (27) shows, the causes of those changes may be multiple : changes in transportation costs, in consumption propensities, in technological data. Crosses refer to activity *1*, circlets to activity *2*.

Figure 3 :Different economic landscapes on a Manhattan circle of radius 1

3. Extensions.

In what follows, some isomorphic extensions to input-output analysis will be treated; they center arount the concept of location elasticity.

3.1. Location elasticities.

Location elasticities were first presented in early eighties ([14]) and subsequently nurtured a number of analyses (i.a. [15], [58]).

The central notion of a location elasticity is that of a relative reaction of a spatialised (regionalised) variable to the rate of growth of that same variable measured over a reference area (e.g. a country); formally :

$$E_{ir} \overset{\Delta}{=} \partial \ln x_{ir} / \partial \ln x_{i.} \qquad (29)$$

where index *i* stands for an activity, say, index *r* for a spatial unit (a region, possibly), *x* being a given variable (production, or value added, for instance). The "."-notation has been carried over from statistics, meaning the aggregation of the corresponding variable over all the values of the index having been replaced.

Generalising, one can define a spatialised transversal or cross-location elasticity as :

$$E_{ijrr} \overset{\Delta}{=} \partial \ln x_{ir} / \partial \ln x_{jr} \qquad (30)$$

and a matrix $\mathbf{E_{rr}}$ of those elasticities for region *r*; a generalised model can now be set up as :

$$\boldsymbol{\rho_r} = \mathbf{E_{rr}}\, \boldsymbol{\rho_r} + \boldsymbol{\rho^*_r} \qquad (31)$$

where $\boldsymbol{\rho_r}$ is the column-vector of the ρ_{ir}'s, the regional growth rates of activity *i*, and $\boldsymbol{\rho^*_r}$ a column-vector of locally autonomous ρ^*_{ir}'s, to be expanded further down.

Expression (31) leads to :

$$\boldsymbol{\rho_r} = \mathbf{M_{rr}}\, \boldsymbol{\rho^*_r} \qquad (32)$$

where $\mathbf{M_{rr}}$ is now an elasticity matrix multiplier applicable to the elements of $\boldsymbol{\rho^*_r}$; the latter can be linked to national activity growth rates by :

$$\boldsymbol{\rho^*_r} = \hat{\mathbf{e}}_\mathbf{r}\, \boldsymbol{\rho_i} \qquad (33)$$

One has further that :

$$\rho_{ir} = E_{ir}\, E_{i.}\, \rho_{..} \qquad (34)$$

so diagonal matrix $\hat{\mathbf{e}}_\mathbf{r}$ comprises in a sense "residual" elements E_{ir}.

$\boldsymbol{\rho_i}$ in turn can be expressed as :

$$\boldsymbol{\rho_i} = \mathbf{E_i}\, \boldsymbol{\rho_i} + \boldsymbol{\rho^*_i} \qquad (35)$$

hence :

$$\boldsymbol{\rho_i} = \mathbf{M_i}\, \boldsymbol{\rho^*_i} \qquad (36)$$

and proceeding as for (33), one obtains :

$$\boldsymbol{\rho_r} = \mathbf{M_{rr}}\, \hat{\mathbf{e}}_\mathbf{r}\, \mathbf{M_i}\, \mathbf{e_i}\, \rho_{..} \qquad (37)$$

and from (37) :

$$\rho_{ir} = (\mathbf{m'_{ir}}\, \hat{\mathbf{e}}_\mathbf{r})\, (\mathbf{M_i}\, \mathbf{e_i})\, \rho_{..} \qquad (38)$$

which should be compared with (34) : in fact the latter expression is a simplification of the multiplier-weighted expressions in the parentheses of (38), and E_{ir} and $E_{i.}$ should be considered as already incorporating multiplier operations. This also suggests an immediate generalisation of (34) to :

$$\rho_{ir} = \textstyle\sum_j E_{jr}\, E_{j.}\, \rho_{..} \qquad (39)$$

A multiregional extension of (37) can be envisaged and leads to :

$$\boldsymbol{\rho} = \mathbf{M_{rs}}\, \hat{\mathbf{e}}_s\, \mathbf{M_i}^{\sim}\, \mathbf{e_i}^{\sim}\, \rho_{..} \qquad (40)$$

where $\mathbf{M_i}^{\sim}\,\mathbf{e_i}^{\sim}$ is a repetitive vector of $\mathbf{M_i}\,\mathbf{e_i}$ from (37) or (38). The vector of total regional growth rates, $\boldsymbol{\rho}^{>}$, is finally given by :

$$\boldsymbol{\rho}^{>} = \mathbf{L}\, \mathbf{M_{rs}}\, \hat{\mathbf{e}}_s\, \hat{\mathbf{w}}_i^{\sim}\, \mathbf{M_i}^{\sim}\, \mathbf{e_i}^{\sim} \qquad (41)$$

where $\mathbf{L}$ is a matrix displaying the regional location quotients as row-vectors in suitable positions, and $\mathbf{w_i}^{\sim}$ a (repetitive) diagonal matrix of national activity shares.

The formal, isomorphic, relatedness of the analysis just presented, can easily be seen from the equations. As a first example take equations (31) which obviously represent an open system; equations (40), on the other hand, can be transformed into a closed system by expressing $\rho_{..}$ as :

$$\rho_{..} = \mathbf{w}'\boldsymbol{\rho} \qquad (42)$$

where $\mathbf{w}$ is a suitable weighting vector. Rewriting (40) as :

$$\boldsymbol{\rho} = \mathbf{m}\, \rho_{..} \qquad (43)$$

one obtains the closed system :

$$(\mathbf{I} - \mathbf{m}\,\mathbf{w}')\, \boldsymbol{\rho} = \mathbf{0} \qquad (44)$$

This purely formal aspect should not let one forget that the whole analysis is based on technological links between spatialised activities, as has already been said in section 2.3.

The preceding analysis can again be used as a basis for setting up a mathematical program like in section 2.1; this was done in [58], were the objective function was the national rate of growth of employment, ψ , the instruments being regional policy, *p*, sectoral policy, *s*, and macro-economic policy, *m*. Table 2 reports the results of the maximising exercise (source : [58]).

Table 2 : optimal solutions

Variable	Value(s)
p	1.25
s	1.00 or 0
m	4.1875 or 4.6875
ψ	.0685705

One notices the equivalence of two substitutable solutions in *s* and *m*, a fact which should get more attention in applied policy design; the overall result would have been an increase of nearly seven percentage points in the growth rate of employment.

3.2. Dynamics and growth.

Starting from the notion of location elasticity, a number of dynamic models has been developed ([10], [50], [51]).

From the FLEUR-model, presented in section 2.3, a remarkable expression can be derived for the equilibrium (shown by a circlet) share location elasticity for a sector and a region (the sectoral index *i* will be omitted here for reasons of simplicity; only the regional index *r* will be used) :

$$E^{o}_{ar} = 1 - b^{-1} a^{o}_{r} \qquad (45)$$

where a^o_r is the regional equilibrium share of region *r* (for activity *i*, say), and where *b* is defined as :

$$b \stackrel{\Delta}{=} \sum_s a^{o\,2}_{s}, \; s = 1,...,R \qquad (46)$$

From (45) and (46), with for the observed a_r an error correction specification, one can set up the following dynamic model, ρ being equivalent to ρ.. of equations (37) and following above, and η the error correction parameter, $0 < \eta < 1$:

$$\dot{a}^{o}_{r} = \rho\, a^{o}_{r} (1 - b^{-1} a^{o}_{r}) \qquad (47)$$

$$\dot{a}_{r} = \eta\, (a^{o}_{r} - a_{r}) \qquad (48)$$

$$\dot{b} = 2\, \rho\, b\, (1 - b^{-2} \sum_s a^{o\,3}_{s}) \qquad (49)$$

The model has a singular point $a^o_r = b, a_r = a^o_r, b = +\sqrt{\sum a^{o\,3}_s}$; the model converges to that point for $\rho > 0$; for $\rho = 0$ the shares converge to their initial equilibrium values, and for $\rho < 0$ the region (or regions) with the highest initial equilibrium share(s) see their share(s) converge to *1* (or n^{-1}, *n* being the multiplicity hinted at).

All this is true for constant locational factors, and even for changing ones, though in the latter case movement would be sustained; adding a differential correction factor to (48) - as was the case in (22) - would also not impair the conclusions. Reflecting anew on model (47) through (49), if to the right hand side of (48) one adds a term like $-\theta (1 - b^{-1} a^o_r)$, which for small a^o_r would be negative, and for relatively - to η - large θ would dominate the first term; for $\rho > 0$, the region concerned would decline first, to grow again in the long term, hence short term concerns for that type of region. The new term represents in fact preference for spatial externalities present in larger and/or denser areas, better equipped with locational factors.

So complex but realistic multiregional dynamics can be encompassed in a simple model based on location elasticities; extension to a multisectoral one is immediate.

4. Conclusions.

Two points should be mentioned to conclude the above scanning of a number of input-output related spatial analyses.

The first is that the input-output framework - and, more generally, its insertion in a spatialised social accounting matrix, or a consistent series of them - allows of operationalising approaches to spatial development; such as is the case of the theory of growth pole sustained development, as exposed in [6], [25], and [35].

The second point is the fact that the overview presented shows a great absentee : a price system. As far as Tinbergen-Bos Systems are concerned (section 2.4) a first attempt to introduce it has been developed in [62]; an empirical counterpart would be a spatialised computable general equilibrium model, as presented - but still in a non-spatialised version - in [3], [4], [5] and [54]. Location elasticities too could be readily generalised, as the following developments show.

Let x_r be a regional quantity variable, p_r its unit price, and the same for the referential totals *x* and *p*. The "nominal location elasticity" could then be expressed as :

$$E^{*}_{r} = d \ln p_r x_r / d \ln px = d \ln p_r + d \ln x_r / d \ln p + d \ln x \qquad (50)$$

whence :

$$d \ln x_r = E^*_r (d \ln p + d \ln x) - d \ln p_r \quad (51)$$

But as $d \ln x_r = E_r d \ln x$ there comes :

$$E_r d \ln x = E^*_r (d \ln p + d \ln x) - d \ln p_r \quad (52)$$

or :

$$E_r (1 + d \ln p_r / d \ln x_r) = E^*_r (1 + d \ln p / d \ln x) \quad (53)$$

so finally :

$$E^*_r = E_r (1 + \varepsilon_r^{-1})(1 + \varepsilon^{-1})^{-1} \quad (54)$$

wher ε_r and ε are respectively the local and referential price elasticities, to be analysed by an appropriate model.

5. References

[1] Ancot, J.-P. and Paelinck, J.H.P., 1983, The Spatial Econometrics of the European FLEUR-Model, in D.A.Griffith and A.C. Lea (eds*), Evolving Geographical Structures*, The Hague, Martinus Nijhoff Publishers, pp.229-246.

[2] Ancot, J.-P., Kuiper, J.H. and Paelinck, J.H.P., 1990, Five principles of spatial economnetrics illustrated, in M Chatterji and R.E. Kuenne (eds), *Dynamics and Conflict in Regionals Structural Change*, London, Macmillan, pp.141-155.

[3] Burg, H.G. and Paelinck, J.H.P., 1991, Politiques économiques et contrôle des prix de la santé : un modèle, in A.S. Bailly and M. Périat (eds), *L'Etat et la Santé*, Paris, Economica, pp.55-66.

[4] Burg, H.G. and Paelinck, J.H.P., 1993, Structure et simulation d'un modèle d'équilibre général calculable axé sur les activités médicales (MEGICAL), in J.-P. Gallet (ed.), *Actes des premières journées françaises de médicométrie*, Chinon, A.E.S.V.L., pp.85-92.

[5] Burg, H.G. and Paelinck, J.H.P., 1998, Médecine et alcool : relations systémiques, *Health and System Science*, Vol.1, Nos3-4, pp.307-320.

[6] Caevel, J. de, Degueldre, J. and Paelinck, J.H.P., 1963, Análisis cuantitativo de ciertos fenómenos del desarrollo regional polarizado, *Cuadernos de la Sociedad Venezolana de Planificación*, vol. II, no. 5-6, pp.45-73.

[7] Caevel, J. de, Degueldre, J. and Paelinck, J.H.P., 1965, Analyse quantitative de certains phénomènes du développement régional polarisé, Essai de simulation statique d'itinéraires de propagation*, Collection de l'Institut de Science Economique de l'Université de Liège*, No 8, Paris et Genève; (reproduced in J.Boudeville (ed.), 1968*, L'Univers rural et la planification,* Paris, Presses Universitaires de France).

[8] Caevel, J. de, Degueldre, J. Loriaux, M. and Paelinck, J.H.P., 1965, *Régions et structures industrielles: le cas de la Belgique*, Paris, Genin.

[9] Castermans, M. and Paelinck, J.H.P., 1965, Deux problèmes dans l'analyse d'entrée et de sortie, *Revue belge de statistique et de recherche opérationnelle*, Vol.4, No1, pp.3-14.

[10] Girardi, R. and Paelinck, J.H.P., 1994, A Regional Equilibrium Growth Model and its Disequilibrium Dynamics : A One Sector Approach , *Regional Studies*, vol. 28, No 3, pp.305-317; reproduced in K.E. Haynes e.a. (eds), *Regional Dynamics*, Cheltenham, Edw. Elgar Publishing, 1996.

[11] Hewings, G.J.D., Sonis, M., Madden, M. and Kimura, Y., 1999, *Understanding and Interpreting Economic Structure*, Berlin-Heidelberg, Springer-Verlag.

[12] Klaassen, L.H. and Paelinck, J.H.P., 1972, Desarrollo regional, medio socio-cultural y ambiente natural : algunos modelos integrados, *Boletin de Estudios Economicos*, No 86, pp.335-364.

[13] Klaassen L.H. and Paelinck, J.H.P., 1974, *Integration of Socio-Economic and Physical Planning*, Rotterdam, University Press.

[14] Kuiper, J.H. and Paelinck, J.H.P., 1981, Macro, Sectoral, Regional Policies and Regional Growth, *Revue d'Economie Régionale et Urbaine*, No 4, pp.517-534.

[15] Kuiper, J.H. and Paelinck, J.H.P., 1983, Implémentation empirique d'élasticités de localisation, *Revue d'Economie Régionale et Urbaine*, No 2, pp.235-247.

[16] Kuiper, J.H. and Paelinck, J.H.P., 1984, Tinbergen-Bos Systems Revisited, in J.M. Pillu and R. Guesnerie (eds), *Modèles Economiques de la Localisation et des Transports*, Paris, E.N.P.C., pp.117-140.

[17] Kuiper, J.H., Paelinck, J.H.P. and Rosing K.E., 1990a, Transport Flows in Tinbergen-Bos Systems, in K. Peschel (ed.), *Infrastructure and the Space-Economy,* Heidelberg-Bonn, Springer-Verlag, pp.29-52.

[18] Kuiper, J.H., Paelinck, J.H.P. and Rosing K.E. 1990b, Flux de transport dans un système de Tinbergen-Bos métrisé, *Revue d'Economie Régionale et Urbaine*, No 2, pp.281-287

[19] Kuiper, J.H., Paelinck, J.H.P. and Rosing, K.E., 1991, Systèmes de Tinbergen-Bos métrisés à deux industries avec flux de transports, *Les Cahiers Scientifiques de la Revue " Transports"*, No 23, pp.89-110.

[20] Kuiper. J.H. and Paelinck, J.H.P., 1992a, Optimal Tinbergen-Bos Location Patterns of Two Industrial Sectors on a Manhattan Circle (R = 1), in H. Birg et H.J. Schalk (eds), *Regionale und sektorale Stukturpolitik*, Münster, Institut für Siedlungs- und Wohnungswesen, pp.29-40.

[21] Kuiper, J.H. and Paelinck, J.H.P., 1992b, Optimale locatiepatronen volgens de principes van Tinbergen en Bos, in D.-J. Kamann et P. Rietveld (eds), *Nieuwe Ideeën in Nederlands Ruimtelijk Onderzoek*, Stichting RSA Nederland, pp.245-263.

[22] Kuiper, F.J., Kuiper, J.H. and Paelinck, J.H.P., 1993, Tinbergen-Bos Metricised Systems : Some Further Results, *Urban Studies*, vol. 30, No 10, pp.1745-1761.

[23] Kuiper, J.H., Mares N. and Paelinck, J.H.P., 1994, Alternative specifications for metricised Tinbergen-Bos systems with two activities, *Sistemi Urbani*, 1992-1/2/3, pp.99-107.

[24] Markey, P. and Paelinck, J.H.P., 1963, Impact des dépenses d'énergie et de main-d'oeuvre sur l'économie belge, Annexe au *Bulletin Mensuel de la Direction Générale des Etudes et de la Documentation*, Bruxelles, Ministère des Affaires Economiques et de l'Energie, No 2, 47 pages.

[25] Paelinck, J.H.P., 1963, La teoría del desarrollo regional polarizado, *Revista Latino-americana de Economía,* No. 9, pp. 175-229; reproduced in *Cahiers de l'ISEA*, Série L (Economies Régionales), 1965, pp. 5-47, as "La théorie du développement régional polarisé", and in italian as "La teoria dello sviluppo regionale polarizzato", in A. Testi (ed.), *Sviluppo e pianificazione regionale*, Torino, Giulio Einaudi, 1977.

[26] Paelinck, J.H.P. and Waelbroeck, J., 1963a, *Programmation économique et modèles économétriques de croissance: analyse des relations sous-jacentes au premier programme d'expansion 1962-1965 du Bureau de Programmation Economique*, Liège, Bibliothèque de l'Institut de Science Economique de l'Université, No3.

[27] Paelinck, J.H.P. and Waelbroeck, J. 1963b, Etude empirique sur l'évolution de coefficients input-output, *Revue belge de statistique et de recherche opérationnelle*, No 1, pp.3-12.

[28] Paelinck, J.H.P. and Waelbroeck, J. 1963c, Essai d'application de la procédure RAS de Cambridge au tableau interindustriel belge, *Economie Appliquée*, No 1, pp.81-111.

[29] Paelinck, J.H.P., 1965a, Recherches récentes en matière de modèles d'exploration, *Cahiers du Séminaire d'Econométrie*, No 8, pp.35-70.

[30] Paelinck, J.H.P., 1965b, Le modèle économétrique d'exploration utilisé par le Bureau de Programmation Economique Belge, Structure et Problèmes d'Estimation, in G. Parenti (ed.), *Modelli Econometrici per la Programmazione*, Firenze, Scuola di Statistica dell'Università.

[31] Paelinck, J.H.P. and Pirard, J., 1966, *Croissance et contribution à l'analyse matricielle,* Liège, Séminaire de Science Economique de l'Université, No 3.

[32] Paelinck, J.H.P., 1968a, Analyse quantitative de certains phénomènes du développement régional polarisé, Essai de simulation statique d'itinéraires de propagation, in J.R. Boudeville (ed.), *L'espace rural et la planification,* Paris, Presses Universitaires de France, pp.123-169.

[33] Paelinck, J.H.P., 1968b, Programmation de localisations conjointement efficientes, *Cahiers de l'ISEA*, vol. II, No6, pp.1311-1316.

[34] Paelinck, J.H.P., 1968c, Eléments pour l'étude fonctionnelle de régions frontalières, in *Economie Régionale sans Frontières*, Paris, Genin, pp.203-216.

[35] Paelinck, J.H.P., 1968d, Systématisation de la théorie du développement régional polarisé, in J.R. Boudeville (ed.), *L'espace et les pôles de croissance*, Paris, Presses Universitaires de France, pp.895-1000.

[36] Paelinck, J.H.P. (ed.), 1969, *Problèmes d'économie multirégionale en Belgique*, Namur, Publications de l'Institut de Science Economique Régionale.

[37] Paelinck, J.H.P., 1970, De quelques aspects opératoires dans l'usage de techniques d'entrée et de sortie au niveau régional et interrégional, *Revue juridique et économique du sud-ouest,* Série économique, No 1, pp.1-11.

[38] Paelinck, J.H.P., 1971, Techniques of Regional Plan Formulation: Problems of Interregional Consistency, in J.M. Dunham and J.G.M. Hilhorst (eds), *Issues in Regional Planning*, Paris and The Hague, Mouton, pp.184-194.

[39] Paelinck, J.H.P., 1972, Selecting a Minimum Investment Viable Industrial Complex, in N. Hansen (ed.), *Growth Centers in Regional Policy*, New York and London, Collier-McMillan, pp.139-159.

[40] Paelinck, J.H.P., 1973, Modèles de politique économique multirégionale basés sur l'analyse d'attraction, *L'Actualité Economique*, No 4, pp.559-564.

[41] Paelinck, J.H.P. and Rompuy, P. van, 1973, Regionaal en sectoraal subsidiebeleid : economische theorie en modellen, *Tijdschrift voor Economie*, vol. XVIII, No 1, pp. 39-55.

[42] Paelinck, J.H.P. and Nijkamp, P., 1975, *Operational Theory and Method in Regional Economics*, Farnborough, Saxon House.

[43] Paelinck, J.H.P. and Klaassen, L.H., 1979, *Spatial Econometrics*, Farnborough, Saxon House; published in polish as *Ekonometria przestrzenna*, Warszawa, Państwowe Wydawnictwo Naukowe, 1983.

[44] Paelinck, J.H.P. and Wagenaar, Sj., 1981, Supply Effects in Regional Modelling, *Canadian Journal of Regional Science*, vol.IV, No2, pp.145-168.

[45] Paelinck, J.H.P. (with the assistance of J.-P. Ancot and J.H. Kuiper), 1983a, *Formal Spatial Economic Analysis*, Aldershot, Gower Press.

[46] Paelinck, J.H.P., 1983b, Investment and the Development of Backward Regions, in A. Heertje (ed.), *Investing in Europe's Future*, London, Basil Blackwell, pp.152-187.

[47] Paelinck, J.H.P., 1985a, Bijdrage van de transportsector aan een nationale economie, *Tijdschrift voor Economie en Management*, pp.295-306.

[48] Paelinck, J.H.P., 1985b, Contribution des activités de transport au produit national : une méthode d'estimation, *Les Cahiers Scientifiques de la Revue "Transports"*, No11-12, pp.269-282.

[49] Paelinck, J.H.P., 1986a, La planification du développement local : une approche opérationelle, in W.-J. Coffey and R. Runte (eds), *Le développement local*, Nouvelle Ecosse, Presses de l'Université de Ste Anne, pp.113-126; published in English : Planning Local Development: An Operational Approach, in W.-J. Coffey and R. Runte (eds), *Local Development,* Nouvelle Ecosse, Presses de l'Université de Ste Anne, pp.111-122.

[50] Paelinck, J.H.P., 1986b, A consistent model for sectoral regional dynamics, in J.H.P. Paelinck (ed.), *Human Behaviour in Geographical Space,* Aldershot, Gower Press, pp.91-110.

[51] Paelinck, J.H.P., 1987, Some Multisectoral Multiregional Dynamics, in B. Guesnier and J.H.P. Paelinck (eds), *Modélisation spatiale: théorie et applications*, Dijon, Collection de l'Institut de Mathématiques Economiques, Série d'Econométrie Appliquée, pp.87-94.

[52] Paelinck, J.H.P., 1988, L'équilibre général d'une économie spatiale, in Cl. Ponsard (ed.), *Analyse Economique Spatiale*, Paris, Presses Universitaires de France, ch. 7, pp.277-319; published in polish as : Równowaga ogólna w ekonomii przestrzennej, in B. Gruchman (ed.), *Ekonomiczna Analiza Przestrzenna,* Poznań, Wydanictwo Akademii Ekonomicznej, 1992, pp.223-258.

[53] Paelinck, J.H.P., 1993, Metricised Tinbergen-Bos Systems with 2 Industries, in W. Welfe (ed.), Proceedings of the XXth Conference on Macromodels, Łódź, Institute of Econometrics and Statistics.

[54] Paelinck, J.H.P., and Burg, H.G. van der, 1994, De la santé avec l'Etat à celle sans Etat : un modèle, *Revue d'Economie Régionale et Urbaine*, No1, pp.15-22.

[55] Paelinck, J.H.P., 1997, Two Studies in Tinbergen-Bos Systems : (a) An Application of Tinbergen-Bos Analysis to the Case of the Japanese Prefectures; (b) Frequency Distribution of Distances over a Manhattan Circle; *Oikonomika*, vol. 34, No 1, pp.1-22.

[56] Paelinck, J.H.P., 1998, Recent Results in Tinbergen-Bos Analysis : On Two Problems in the Analysis of Tinbergen-Bos Systems, in D.A. Griffith, C. Amrhein and J.-M. Huriot (eds), *Econometric Advances in Spatial Modeling and Methodology*, Dordrecht, Kluwer, Advanced Studies in Theoretical and Applied Econometrics, vol.35, 1998, pp.9-14.

[57] Paelinck, J.H.P. and Kulkarni, R., 1999, Location-Allocation Aspects of Tinbergen-Bos Systems, *The Annals of Regional Science*, vol.33, No4, pp.573-580.

[58] Paelinck, J.H.P., 1999, Location Elasticities Revisited, or Employment and Competition in Regional Growth, in *Emprego e Desenvolvimento Regional*, Vol.1, APDR, Coimbra, pp.15-26.

[59] Paelinck, J.H.P., 2000a, Quasi-dynamique de systèmes de Tinbergen-Bos, accepted for publication in *Région*

et Développement.

[60] Paelinck, J.H.P., 2000b, Tinbergen-Bos Systems : A Compendium of Recent Results, accepted for publication in *Revue d'Economie Régionale et Urbaine.*

[61] Paelinck, J.H.P., 2000c, Regional Competition in the Framework of Quasi-Dynamic Tinbergen-Bos Systems, accepted for publication in J.R. Roy and W. Schulz (eds), *Theory of Regional Competition*, Baden-Baden, Nomos-Verlag

[62] Paelinck, J.H.P., 2000d, Prices in Tinbergen-Bos Systems : A first Exploration, paper presented at the annual WRSA cogress, Poipu, Hawai'i, February 2000.

[63] Paelinck, J.H.P., 2000f, Modélisation réticulaire par affectation quadratique, paper prepared for the annual ASFDLF congress, Crans-Montana, september 2000.

[64] Paelinck, J.H.P., 2000g, Médicométrie relationnelle : application aux Pays-Bas, paper prepared for the annual ASFDLF congress, Crans-Montana, september 2000

[65] Paelinck, J.H.P. and Stough, R.R., 2000e, CRATON : transportation impact analysis, a first theoretical exploration, paper prepared for the RSAI congress, Lugano, May 2000.

[66] Stone, R. and Croft-Murray, G., 1959, *Social Accounting and Economic Models*, London, Bowes and Bowes.

[67] Varii Auctores, 1966, *Etude comparée des tableaux d'entrées et de sorties des Communautés Européennes*, Namur, Travaux de la Faculté des Sciences Economiques et Sociales, Collection "Econométrie Européenne", No 1.

[68] Varii Auctores, 1967, *La Basse-Sambre, Essai de formulation d'un programme de développement sélectif*, Namur, Travaux de la Faculté des Sciences Economiques et Sociales, Collection "Economie Régionale", No2.