

# **"Input-Output Structural Decomposition for Andalusia and Madrid (Spain)."**

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## **Abstract:**

The structural comparison of an economy, as represented, for example, by variations in the level of value added and of employment, raises questions about the sources of structural differences between two different regions through different production processes of individual industries. Input-output structural decomposition can ascribe the source of these differences to four kind of causes: production technological processes, domestic final demand, foreign trade and labor productivity. Through input-output analysis, this paper tries to cast light on structural differences between Andalusia, one of the less developed regions in the E.U., and Madrid, which is a good example of an agglomeration economy.

**KEYWORDS:** Input-Output analysis; Structural change; Regional economy.

## **1.- INTRODUCTION.**

The structural comparison of an economy, as represented, for example, by variations in the level of value added and of employment, raises questions about the sources of structural differences between two different regions. Input-output structural decomposition can ascribe the source of these differences to four kind of causes: production technological processes, domestic final demand, foreign trade and labor productivity.

According to Skolka (1989), structural change or inter-country comparisons are closely related to two main aspects. The first is about the sources of the differences in the position of individual industries. The input-output structural decomposition analysis approach reflects the logical structure of the input-output model, and relates the variations in the levels and composition of value added and employment to differences in production technological processes, domestic final demand, foreign trade and labor productivity.

The second is about the structural differences caused mainly by governments economic policy. This second aspect is not only more difficult to analyse but also far from our intentions in this paper. Thus, no attempt will be made to separate the effects of economic policies and of other types of influences.

The aim of this paper is not to analyse the structural changes of a region in a certain period of time. On the contrary, we are going to analyse two different production structures such as the Community of Madrid and Andalusia. Thus, from now on, it would be more precise to consider differences or variations instead of changes, which involve in some way periods of time.

On the one hand, we are considering a region with only one province (i.e. Madrid) and whose main city is actually one of the biggest and most open cities of Spain. Madrid is mainly an agglomeration economy around services industries, as well.

On the other hand, we are considering a region with eight provinces (i.e. Andalusia) and which is, nowadays, one of the less developed regions in the European Union.

In conclusion, the aim of this paper is to determine the causes and sources of the differences in the level and composition of value added and of employment between two different production structures related to a less developed economy and an agglomeration economy.

## **2.- INPUT-OUTPUT STRUCTURAL DECOMPOSITION ANALYSIS. METHODOLOGY.-**

According to Rose and Miernyk (1989) , structural decomposition analysis can be defined as a method of distinguishing major changes within an economy by means of comparative static changes in key sets of parameters (Skolka, 1989). Its origins date back to the work of Leontief (1941) on the structure of the United States economy (Skolka, 1989). A few years later, in the sixties, Chenery, Shishido and Watanabe (1963) and Carter (1960) extended in several ways this basic methodology. In this paper, we carry out the input-output decomposition analysis which Skolka offers in the Appendix of its work for the Austrian Institute for Economic Research (1989). This is based on a sophisticated set of equations to be used for empirical work, although it will be presented here as an inter-country comparison of different production structures.

### **2.1. Basic Considerations.-**

Differences in the structure of an economy between two regions can be shown on production data, value added data and employment data, both disaggregated by industries, as follows:

$$\Delta x_j = x_j^M - x_j^A \quad [1]$$

$$\Delta v_j = v_j^M - v_j^A \quad [2]$$

$$\Delta I_j = I_j^M - I_j^A \quad [3]$$

where  $x$  refers to domestic output,  $v$  refers to value added and  $I$  refers to the number of economically active persons, for each industry  $j$  and for each region (i.e. Andalusia and Madrid).

Moreover, we can define the shares of value added in the domestic output values for each industry as  $a_{v(j)}$  and the shares of the number of economically active persons in the total value added for each industry as  $a_{I(j)}$ . Then, together with the basic equation and solution of the input-output demand model, we have the following expressions:

$$\Delta x_j = x_j^M - x_j^A = [I - A^M]^{-1} Y^M - [I - A^A]^{-1} Y^A \quad [4]$$

$$\Delta v_j = v_j^M - v_j^A = a_{v(j)}^M x_j^M - a_{v(j)}^A x_j^A \quad [5]$$

$$\Delta I_j = I_j^M - I_j^A = a_{I(j)}^M v_j^M - a_{I(j)}^A v_j^A \quad [6]$$

with (i.e. Madrid):

$$(I - A^M) X^M = Y^M$$

$$(I - A^M)^{-1} Y^M = X^M$$

where  $X^M$  is the column vector of domestic output,  $Y^M$  is the column vector of final demand (excluding imports),  $A^M$  is the matrix of total input coefficients and  $I$  is the identity matrix.

Furthermore, expressions [4], [5] and [6] can be depicted as follows:

$$\Delta x_j = x_j^M - x_j^A = \left[ (I - A^M)^{-1} - (I - A^A)^{-1} \right] Y^A + (I - A^M)^{-1} (Y^M - Y^A) \quad [7]$$

$$\Delta v_j = v_j^M - v_j^A = \left[ a_{v(j)}^M - a_{v(j)}^A \right] x_j^A + a_{v(j)}^M \left[ x_j^M - x_j^A \right] \quad [8]$$

$$\Delta I_j = I_j^M - I_j^A = \left[ a_{I(j)}^M - a_{I(j)}^A \right] v_j^A + a_{I(j)}^M \left[ v_j^M - v_j^A \right] \quad [9]$$

In these three equations, differences in output values, value added and employment can be shown as a weighted sum of those variations caused by technological reasons in terms of production, employment and value added; and others caused by final demand, output values and value added, respectively.

Certainly, each of the weights obtained in these three expressions is a mere result of the mathematical method of differences calculus applied here. It is evident that in structural decomposition analysis which involves two different periods of time, period 1 is after period 0 and so we study the structural change from period 0 to 1. Nonetheless, it is not clear enough who differs from who between two different regions or countries in the same period of time. Thus, in this paper, an a priori hypothesis is considered to analyse structural differences between Madrid and Andalusia. That is to say, structural differences will be analysed from Madrid with respect to Andalusia and so we obtain the weights shown in [7], [8] y [9].

## 2.2- Sources of structural differences.-

Differences in output values (see equation [7]) will be in both cases explained only by means of final demand and the matrix of total input coefficients. In contrast, differences in value added levels (see equation [8]) will be caused by variations in output values ([7]) and in value added shares. Finally, differences in employment (see equation [9]) will be explained by variations in value added levels ([8]) and in labor productivity.

Nevertheless, it is evident that these elements presented above will not be the only ones which determine the differences in employment, value added and output values between two different production structures. In this paper, following Skolka (1989), final demand will be split into several components (i.e. differences in production technological processes, in the commodity composition of final demand components and exports, in the weight of final demand components, etc.) in order to account separately the effects of each one on the differences in output values (see equation [7]).

The structural decomposition analysis is based on the original specifications of the Leontief input-output demand model (Skolka, 1989). That is to say (i.e. Madrid),

$$\begin{aligned} (I - A^M) X^M &= Y^M \\ (I - A^M)^{-1} Y^M &= X^M \end{aligned}$$

Therefore, output values for each region and industry can accordingly be expressed as:

$$\left[ I - A^{M(d)} \right]^{-1} Y_{(d)}^M = X^M \quad [10]$$

$$\left[ I - A^{A(d)} \right]^{-1} Y_{(d)}^A = X^A \quad [11]$$

where  $(d)$  refers to regional output values, final demand and the matrix of input coefficients.

Hence, Andalusian regional final demand can also be expressed as a function of two different groups of elements (or sources of deviations): exports and domestic demand which is split into consumption and gross capital formation ( $k=2$ ):

$$Y_{(d)i}^A = Y_{(d)ik}^A + Y_{(d)ie}^A \quad [12]$$

where andalusian regional domestic demand of industry  $i$  with respect to final demand component  $k$  as well as regional exports from industry  $i$  is obtained.

If we add and subtract the following expression to [12]:

$$Y_{(d)ik}^A \left( \frac{E_A}{D_A} \right)$$

where  $E_A$  y  $D_A$  stands for exports and andalusian total final demand respectively, [13] is obtained:

$$Y_{(d)i}^A = Y_{(d)ik}^A + Y_{(d)ie}^A + Y_{(d)ik}^A \left( \frac{E_A}{D_A} \right) - Y_{(d)ik}^A \left( \frac{E_A}{D_A} \right) \quad [13]$$

and with a convenient transformation:

$$Y_{(d)i}^A = Y_{(d)ie}^A + \frac{Y_{(d)ik}^A}{D_A} (D_A + E_A) - \frac{Y_{(d)ik}^A}{D_A} (E_A) \quad [14]$$

Therefore, if andalusian total final demand can be shown as a sum of exports ( $E_A$ ) and total domestic demand ( $D_A$ ), we can define:

$$Y_{(t)}^A = D_A + E_A \quad [15]$$

and substituting [15] into [14] yields:

$$Y_{(d)i}^A = Y_{(d)ie}^A + \frac{Y_{(d)ik}^A}{D_A} Y_{(t)}^A - \frac{Y_{(d)ik}^A}{D_A} (E_A) \quad [16]$$

Finally, if we multiply and divide [16] by several convenient expressions as follows:

$$Y_{(d)i}^A = Y_{(d)ie}^A \left( \frac{E_A Y_{(t)}^A}{E_A Y_{(t)}^A} \right) + \frac{Y_{(d)ik}^A}{D_A} Y_{(t)}^A \left( \frac{Y_{(t)k}^A}{Y_{(t)k}^A} \right) - \frac{Y_{(d)ik}^A}{D_A} (E_A) \left( \frac{Y_{(t)}^A Y_{(t)k}^A}{Y_{(t)}^A Y_{(t)k}^A} \right) \quad [17]$$

[17] is obtained, where  $Y_{(t)k}$  refers to the total value of andalusian domestic final demand of component  $k$ .

As a result, let us now define the following matrices and vectors:

$$B_{(d)}^D = \begin{bmatrix} \frac{Y_{(d)11}^A}{Y_{(t)1}^A} & \dots & \frac{Y_{(d)1k}^A}{Y_{(t)k}^A} \\ \vdots & \ddots & \vdots \\ \frac{Y_{(d)n1}^A}{Y_{(t)1}^A} & \dots & \frac{Y_{(d)nk}^A}{Y_{(t)k}^A} \end{bmatrix}_{(nxk)}$$

$$B_{(d)}^E = \begin{bmatrix} \frac{Y_{(d)1e}^A}{E_A} \\ \vdots \\ \frac{Y_{(d)ne}^A}{E_A} \end{bmatrix}_{(nx1)}$$

$$C_{(d)} = \begin{bmatrix} \frac{Y_{(t)1}^A}{D_A} \\ \vdots \\ \frac{Y_{(t)k}^A}{D_A} \end{bmatrix}_{(kx1)}$$

Moreover,

$$C_{(d)}^E = \frac{E_A}{Y_{(t)}^A}$$

And if we now define from [17] the following expressions for Andalusia and consequently, for Madrid, [18] and [19] are obtained:

$$\left[ I - A^{M(d)} \right]^{-1} \left[ B_{(d)}^D C_{(d)} \left( 1 - C_{(d)}^E \right) + B_{(d)}^E C_{(d)}^E \right] Y_{(t)}^M = X^M \quad [18]$$

$$\left[ I - A^{A(d)} \right]^{-1} \left[ B_{(d)}^D C_{(d)} \left( 1 - C_{(d)}^E \right) + B_{(d)}^E C_{(d)}^E \right] Y_{(t)}^A = X^A \quad [19]$$

From [18] and [19], input-output structural decomposition analysis was carried out as a step-by-step transition from matrices, vectors and scalars of the andalusian input-output table for 1995 to those of the Madrid input-output tables for the year 1996 (see Appendix in Skolka, 1989). This approach is not the unique solution to the structural decomposition problem, which has not yet been derived. At least, its advantages are the fine level of disaggregation and the additivity of partial effects (Skolka, 1989).

### **3.- INPUT-OUTPUT STRUCTURAL DECOMPOSITION ANALYSIS BETWEEN ANDALUSIA (1995) AND MADRID (1996).**

A few previous adjustments had to be done before starting to compare two different production structures such as Andalusia and Madrid. Firstly, it should not be a problem to deal with two input-output tables from different years since we are analysing structural features which are not expected to be changed in so short periods of time. Secondly, there are actually in Spain only these two input-output tables according to the new European Accounting System (1995) and hence, comparables. Thirdly, both sets of different industries from each input-output table had to be rearranged into more homogeneous groups in order to obtain comparable results. And finally, in this paper we are considering the following sources of structural deviations:

- (a) Differences in domestic final demand.
- (b) Differences in labor productivity.
- (c) Differences in technological processes by means of intermediate inputs structure and value added shares.
- (d) Differences in domestic final demand by means of commodity composition and different weights of final demand components.
- (e) Differences in exports by means of its contribution to variations in total final demand, its commodity composition and its shares in total final demand.
- (f) Differences in import dependence by means of intermediate imports and direct imports for final demand.

#### **4.- RESULTS OF THE CALCULATIONS AND CONCLUSIONS.**

As Table 1 shows, the difference in the level of value added between Madrid and Andalusia amounts to 2,3 billions of pesetas. With respect to employment, the number of economically active persons in Madrid is only slightly lower than those in Andalusia, though the latter is quite more extensive than the former. In particular, the difference in employment amounts to 93.000 economically active persons.

Moreover, Table 1 shows the main results of the input-output structural decomposition analysis of the differences in the levels of value added and employment, both split into several components.

The technological component caused a difference in value added of 0,386 billions of pesetas (approx. 2.320 millions of euros). The negative effect of variations in intermediate inputs (minus 0,463 billions of pesetas value added) was smaller than the positive effect of value added shares (plus 0,849 billions of pesetas).

Deviations in domestic final demand caused a slightly effect in value added which amounted to 0,076 billions of pesetas (approx. 456,8 millions of euros). In particular, commodities composition has a major importance than the consumption and gross capital formation shares of domestic final demand.

Foreign trade is the key factor in the differences in value added level between Madrid and Andalusia. With respect to exports, their contribution to domestic final demand differences is the most important influence in the value added level (i.e. 2,247 billions of pesetas or approx. 13.506 millions of euros). With rather less importance, the weights of exports in total final demand and the different commodities composition of exports caused variations of 0,788 and 0,376 billions of pesetas, respectively.

In terms of import dependence, intermediate inputs as well as final demand commodities (mainly for private consumption and for gross fixed investments) caused a negative effect in value added level of minus 0,653 billions of pesetas (approx. 3.925 millions of euros).

In conclusion, differences in value added levels between Madrid and Andalusia are mainly caused by foreign trade, and particularly, by exports. Total effects of domestic factors are slightly less important.

**TABLE 1: STRUCTURAL DECOMPOSITION OF THE DIFFERENCES IN VALUE ADDED AND EMPLOYMENT: RESULTS.**

	<b>VALUE ADDED EMPLOYMENT</b>	
	<b>(billions of pesetas)</b>	<b>(thousand of persons)</b>
<b>Andalusia</b>	9,301	1.828,386
<b>Madrid</b>	11,642	1.735,500
<b>Difference</b>	2,341	-92,886
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<b>Technological difference (1+2)</b>		
Intermediate inputs (1)	-0,463	-95,690
Value added shares (2)	0,849	-95,065
<b>Domestic final demand difference (3+4)</b>		
Commodity composition (3)	0,051	-37,751
Weights of components (4)	0,025	2,875
<b>Exports difference (5+6+7)</b>		
Major deviations in exports (5)	2,247	445,744
Commodity composition (6)	0,376	-168,955
Weights of exports (7)	0,788	63,002
<b>Import dependence difference (8+9)</b>		
Intermediate imports (8)	-0,275	-174,106
Final demand (9)	-0,378	-139,220
<b>Major deviations in domestic final demand (10)</b>	-0,879	-130,341
<b>Impact of labor productivity (11)</b>		236,621
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<b>A. Total effect of domestic factors (1+2+3+4+10+11)</b>	-0,417	-119,351
<b>B. Total effect of foreign trade (5+6+7+8+9)</b>	2,758	26,465
<hr/>		
<b>Total Difference (A+B)</b>	2,341	-92,886

Furthermore, let us note that exports and imports are referred to domestic terms in such a way that, for example, deliveries from Madrid to Barcelona would be considered as exports in the Madrid input-output table and those from the Canary Islands to Seville would be considered as imports in the andalusian input-output table.

From Table 1 it is of interest to notice that differences in terms of employment were caused mainly by domestic demand factors instead of foreign trade (in particular,

exports) or labor productivity. Finally, it is to be noted that differences in employment in large part depends on deviations in intermediate and final demand imports dependence.

This paper suggests a new approach to analyse the production structural dissimilarities among different regions or countries on the basis of deviations in the levels of employment and value added. It is to be noted that input-output analysis always involves several kind of methodological limitations, as well. In particular, exports can be overestimated because the fact that Madrid is the capital city of Spain and that most of the more important companies are centralized there.

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<sup>1</sup> J. Skolka (1989) mentions different papers referred to inter-country comparisons which were carried out by Balassa (1979) and Fay and Fink (1976).

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