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A HYBRID INPUT-OUTPUT MODEL OF WATER

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0. Introduction

Changes to the context in which water policy is drawn up in advanced economies are increasingly highlighting the inadequacies of traditional responses to the water problem, which have centred on the management of the resource from the supply standpoint[i].

Indeed, a paradigm shift in water policy definition and strategy is now being talked about. Traditional solutions are gradually giving way to mixed solutions in which water management is increasingly approached from the demand side (where the aim is to manage existing resources).

In Spain, the implementation of new water policies based on management of the existing resources[ii] is hampered by the lack of knowledge about the relationship between water, the economy and society. Specifically, from an economic point of view, the necessary tools have not yet been fully developed for the estimation and evaluation of either the impact of water policies (such as water tariffs) on production sectors or the impact of sectorial economic policies (such as irrigation policies) on water resources.

Against this background, this paper seeks to increase the number of instruments available for analysing the relationships between the economy and water through two input-output models which, by establishing the relationships between the various production sectors and the water sector, allow an evaluation of the direct and indirect repercussions of changes in certain magnitudes on the rest.[iii]

The paper is structured into three sections:

The first section presents the defined input-output models. These are hybrid partitioned models which treat a part of the economy (the water sector) in terms of physical units (m^3) and the rest of the economy in monetary units.

The second section shows the data used to apply the model to the Spanish autonomous region of Andalusia: the *Tabla Input-Output de Andalucía 1990* (1990 Input-Output Table (IOT) of Andalusia*)* and an estimate of sectorial water consumption in the region included at the *Tabla Input-Output Medioambiental de Andalucía 1990* (1990 Environmental IOT of Andalusia).

The third section applies these models to the case of Andalusia and analyses the results.

The fourth section defines the price in water terms (P_w) of the various goods and services making up the Andalusian economy as a function of the water required for their production.

1. Input-Output models (I-O)

I-O models are an economic simulation and forecasting tool. Leontief's open model, by establishing the relationships between the different magnitudes in the economic system, allows estimating the effects of variations in certain variables on the others[iv].

Leontief's initial model (1951) is in physical quantities and nominal prices. However, the model has no empirical applications; applied work is done entirely with data measured in monetary units.

In contrast to the copious literature on I-O analysis and its applications, our approach is something of a novelty in that it is a hybrid partitioned model in which one part of the economy is dealt with in physical quantities and another part in monetary values[v].

1.1 The demand model

If x is the sectors output column vector, D the final demand by sectors and A the technical coefficient matrix, the demand model can be represented as follows:

$$[x] = [A][x] + [D] \quad (1) \qquad \text{or:} \qquad [x] = [I - A]^{-1}[D] \quad (2)$$

where x, the sectors output vector, is a function of final demand (D), and of $(I - A)^{-1}$, Leontief's inverse matrix.

In our application, based on (1), and following Stone (1961), a partitioned system is defined (3) which gives rise to equations (4) and (5) :

$$\begin{bmatrix} X_1 \\ \cdots \\ X_2 \end{bmatrix} = \begin{bmatrix} A_{11} & \vdots & A_{12} \\ \cdots & \cdots & \cdots \\ A_{21} & \vdots & A_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ \cdots \\ X_2 \end{bmatrix} + \begin{bmatrix} D_1 \\ \cdots \\ D_2 \end{bmatrix} \quad (3)$$

$$X_1 = A_{11}X_1 + A_{12}X_2 + D_1 \quad (4)$$

$$X_2 = A_{21}x_1 + A_{22}x_2 + D_2 \quad (5)$$

The first component (4) refers to the water sector and is expressed in physical units (m^3). It shows how total water output in m^3 is distributed between the water consumed by the sector itself, that used by the rest of sectors and that going to water final demand. Matrix A_{11} has m^3/m^3 dimension and its coefficient expresses the proportion of total water in m^3 that is consumed by the water sector itself. Matrix A_{12} has m^3/Ptas dimension and its coefficients express, for each sector, the quotient between its consumption of water in m^3 and the value of its output.

Equation (5) refers to output in the rest of production sectors and is expressed in monetary units. It indicates that the total output of production sectors (excluding the water sector) is distributed between the water sector, the rest of sectors and final demand of each sector. Matrix A_{21} has Ptas/m^3 dimension and its coefficients represent, for each sector, the relationship between the value of its water consumption and the output of the water sector in m^3. Matrix A_{22} has Ptas/Ptas dimension and its coefficients are the technical coefficients of all the production sectors except the water sector.

Equations (6) and (7) are obtained by expanding equations (4) and (5) respectively:

$$[X_1] = [I - A_{11}]^{-1}[A_{12}X_2 + D_1] \quad (6) \qquad [X_2] = [I - A_{22}]^{-1}[A_{21}X_1 + D_2] \quad (7)$$

Model (6) allows the evaluation of the impact of a variation in output in the rest of sectors (in Ptas) or a variation in water final demand (in m^3) on output in the water sector, in m^3. Model (7) allows an evaluation of the impact that a variation in water output in m^3 or a variation in final demand in all sectors except water has on the output value of the rest of the sectors.

Following Stone (1961), we can calculate X_1 and X_2 as a function of D_1 and D_2. By substituting (7) into (6) and expanding, we obtain (8) and substituting (6) into (7) and expanding, we obtain (9);

$$[X_1] = \left[I - (I - A_{11})^{-1} A_{12}(I - A_{22})^{-1} A_{21}\right]^{-1} \left[(I - A_{11})^{-1} \left[A_{12}(I - A_{22})^{-1} D_2 + D_1\right]\right] \quad (8)$$

$$[X_2] = \left[I - [I - A_{22}]^{-1} A_{21} (I - A_{11})^{-1} A_{12}\right]^{-1} \left[[I - A_{22}]^{-1} \left[A_{21} (I - A_{11})^{-1} D_1 + D_2\right]\right] \quad (9)$$

Equation (8) defines X_1, the output of the water sector in m^3, as a function of D_1, final demand for water, and of $A_{12}(I - A_{22})^{-1} D_2$, demand for water derived from the economy's internal production process, and makes it possible to evaluate the impact of a variation of D_1 and/or D_2 on water output in physical units. Model (9) defines X_2, the output of the Andalusian economy, as a function of D_2, final demand in sector 2, and of $A_{21} (I - A_{11})^{-1} D_1$, demand for goods and services in the economy derived from the internal production process of the water sector.

1.2 The price model

If P is the sectors' unit price index vector, and V the sectors' added value per unit of output, the price model can be expressed as follows:

$$[P] = [A'][P] + [V] \quad (10) \qquad \text{or:} \qquad [P] = [I - A']^{-1} [V] \quad (11)$$

Model (11) represents Leontief's dual price system which can be used to evaluate the impact of a percentage price variation in any of the sectors on the price in the rest of the sectors.

In our case, based on (10) we define the partitioned system (12) from which the matrix relationships (13) and (14) are obtained:

$$\begin{bmatrix} P_1 \\ \cdots \\ P_2 \end{bmatrix} = \begin{bmatrix} A'_{11} & \vdots & A'_{21} \\ \cdots & \cdots & \cdots \\ A'_{12} & \vdots & A'_{22} \end{bmatrix} \begin{bmatrix} P_1 \\ \cdots \\ P_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ \cdots \\ v_2 \end{bmatrix} \quad (12)$$

$$P_1 = A'_{11}P_1 + A'_{21}P_2 + v_1 \quad (13)$$

$$P_2 = A'_{12}P_1 + A'_{22}P_2 + v_2 \quad (14)$$

Equation (13) refers to the nominal price per m^3 of water, which is a function of the price of water consumed by the sector itself, the unit price indices of the rest of sectors, and the water sector's added value per m^3. Equation (14) represents the unit price indices of the rest of sectors as a function of the price per m^3 of water, the unit price indices of the rest of sectors and the added value coefficients.

Equations (15) and (16) are obtained by expanding (13) and (14) respectively:

$$\left[P_1\right] = \left[I - A'_{11}\right]^{-1} \left[A'_{21}P_2 + v_1\right] \qquad (15) \qquad \left[P_2\right] = \left[I - A'_{22}\right]^{-1} \left[A'_{12}P_1 + v_2\right] \qquad (16)$$

Model (15) can be used to calculate the nominal price per m^3 of water and to evaluate the impact on this price of a percentage variation in the unit price index of the rest of sectors. Model (16) makes it possible to calculate the price variation in the rest of economic sectors due to variations in the nominal price per m^3 of water.

2. Data

The data used in the application of the models are Andalusia's 1990 I-O table divided into 4 sectors (table 1) and the results of an estimate of sectorial water consumption in the region used in the preparation of Andalusia's 1990 environmental IOT (table 2).

Table 1. Andalusia's 1990 Input-Output Table
(in millions of pesetas)

ANDALUSIA I-O 1990	Water	Agriculture and Fishing	Industry and Construction	Services	Intermed. Consumpt.	Internal consumpt.	Gross fixed capital	Exports	Final demand	TOTAL USAGE
Water	0	4,912	5,476	7,742	**18,130**	15,915	14	0	**15,929**	**34,059**
Agriculture and Fishing	0	65,873	379,324	59,249	**504,446**	253,670	23,179	257,374	**534,223**	**1,038,669**
Industry and Construction	8,088	167,290	1,641,907	660,545	**2,477,830**	1,917,546	1,407,056	1,689,625	**5,014,227**	**7,492,057**
Services	2,994	99,923	712,320	1,282,492	**2,097,729**	3,782,601	106,490	142,999	**4,032,090**	**6,129,819**
INTERMEDIATE CONSUMPTION	**11,082**	**337,998**	**2,739,027**	**2,010,028**	**5,098,135**	**5,969,732**	**1,536,739**	**2,089,998**	**9,596,469**	**14,694,604**
GAV market prices	**14,107**	**499,921**	**1,997,578**	**3,330,033**						
OUTPUT	**25,189**	**837,919**	**4,736,605**	**5,340,061**						
Total transfers	7,810	819	-11,175	2,546						
Distributed Output	32,999	838,738	4,725,430	5,342,607						
Imports	0	184,867	2,520,121	637,239						
VAT	1,060	15,064	246,506	149,973						
TOT. RESOURCES	**34,059**	**1,038,669**	**7,492,057**	**6,129,819**						

Table 2
Average water consumption and price in Andalusian sectors in 1990

Sector	Water consumption in m^3	% of total	Average price per m^3
Agriculture	3,042, 437,624	80.43	1.6
Industry	155,179,469	4.11	30.3
Services	155,643,736	4.12	45.0
Construction	17,103,310	0.45	45.0
Households consumption	410,587,424	10.88	38.8
TOTAL	**3,774,552,403**		**6.67**

3. Application of models and analysis of results

3.1 The demand model

With the Andalusian IOT for 1990 and the sectorial water consumption, the partitioned demand system (3) takes on the following values[vi]:

$$
\begin{pmatrix} 3{,}774{,}552{,}403 \\ \hline 837{,}919 \\ 4{,}736{,}605 \\ 5{,}340{,}061 \end{pmatrix} =
\begin{pmatrix} 0.0000000 & 3{,}623{,}308 & 36{,}373 & 29{,}146 \\ \hline 0.0000000 & 0.0786150 & 0.0800835 & 0.0110952 \\ 0.0000021 & 0.1996494 & 0.3466422 & 0.1236962 \\ 0.0000008 & 0.1192514 & 0.1503862 & 0.2401643 \end{pmatrix}
\begin{pmatrix} 3{,}774{,}552{,}403 \\ \hline 837{,}919 \\ 4{,}736{,}605 \\ 5{,}340{,}061 \end{pmatrix} +
\begin{pmatrix} 410{,}587{,}424 \\ \hline 333{,}473 \\ 2{,}258{,}775 \\ 3{,}242{,}332 \end{pmatrix}
$$

The first component refers to the distribution of the water sector's output in m^3 and the second to the distribution of the rest of the economy's output in monetary units.

3.1.1 Water output in m^3 as a function of the output of the rest of sectors.

Model 6 is used to evaluate the consequences of a variation in output of the rest of sectors on output in the water sector in m^3. From the results of table 3 (second column) it can be deduced that water output in the region (which, in this study, is equivalent to water consumption) is sensitive to variations in output of the agriculture sector (0.8 elasticity) and relatively insensitive to variations in the output of the rest of sectors (less than 0.1 elasticity).

Table 3
Impact of variation in output and final demand on the water sector

	Variation in output	Variation in water output (m^3)	Variation in final demand	Variation in water output (m^3)
Agriculture	1%	0.804%	1%	0.362
Industry and construction	1%	0.046%	1%	0.361
Services	1%	0.041%	1%	0.169
All sectors	1%	0.891%	1%	0.891

3.1.2 Water sector output as a function of final demand in the rest of sectors

Model (10) can be used to evaluate the impact of variations in final demand in the water sector and in the rest of sectors on output in the water sector in m^3.

The fourth column in table 3 shows that output in the water sector in m^3 is sensitive to variations in final demand in agriculture and in industry and construction (0.362 and 0.361 elasticity, respectively) and relatively insensitive to variation in final demand in the services sector (0.168 elasticity).

Final demand in the water sector (D_1) is a function of the population in the region, and final demand in the rest of the economy (D_2) is a function of the GDP. The model (10) can be used to evaluate the percentage impact on output in the water sector of a 1% variation in population and in GDP, i.e. it gives an approximate value for population elasticity and income elasticity of water consumption in the region.

Population-elasticity of water consumption: in order to estimate this, a hypothetical fixed relationship is established between final demand for water and the 1990 population. If we assume that this relationship remains constant over time, a 1% increase in population implies a 0.109% increase in water output and consumption in the region. The population elasticity of water consumption is therefore 0.109.

Income-elasticity of water consumption: in order to calculate this, a hypothetical fixed relationship is established between D_2 and Andalusia's 1990 GDP. Assuming this relationship remains constant, a 1% increase in GDP implies a 0.891% increase in water sector output. That is to say, the income-elasticity of water consumption is 0.891.

3.1.3 Direct and indirect water needs for generation of final demand in the Andalusian economy

Vector $A_{12}(I - A_{22})^{-1}D_2$ of equation (10) refers to water demand deriving from the economy's internal production process. Vector $A_{12}(I - A_{22})^{-1}$ therefore represents the direct and indirect water needs per unit of final demand in each of the sectors. The main results for Andalusia are shown in table 4.

Table 4
Water needed to generate final demand in the Andalusian economy

	A_{12} (I-A_{22})-1 (m^3/million Ptas)	(D_2) (million Ptas)	A_{12} (I-A_{22})-1 D_2 (m^3)
Agriculture	4.08825	333,473	1,363,322
Industry and construction	0.60190	2,258,775	1,359,557
Services	0.19604	3,242,332	635,631
Total production sectors	4.88619	5,834,580	3,358,510

The agricultural sector's water needs per unit of final demand are considerably greater than those of the rest of sectors. However, in absolute terms, industry and construction have similar needs to the agricultural sector. As a whole, production processes linked to agriculture represent around 85% of the direct and indirect water needs which the Andalusian economy requires to generate its final demand.

Given the sizeable water requirements of the Andalusian agricultural and food sectors, it can be argued that the sector's development is only sustainable from the point of view of water management if it is accompanied by a substantial decrease in the coefficients of water consumption per unit of final demand, which necessarily requires structural reforms and modernisation of the region's irrigation sector.

3.2 The price model

3.2.1 The price per m^3 of water as a function of the price in the rest of sectors

Model (15) provides the average price per m^3 of water as a function of the economy's unit price index and the water sector's added value coefficient. Solving the model gives a price of 6.67 Ptas/m^3.

The model can calculate how this price is affected by percentage variations in prices in the rest of sectors. Thus, a 1% increase in Andalusian prices causes a 0.44% increase in the price of water per m^3.

4.2.2 Prices in the Andalusian economy as a function of water prices.

Model (16) defines the economy's unit price index as a function of the water price per m^3. Solving the system using the average water price per m^3 calculated by equation (15) should give unit price indices. However, non-unit P_2 values are obtained since the average water prices in each sector do not coincide with the average water price in the economy as a whole. In order to achieve unit price indices, and to maintain the assumption of price homogeneity in the model, an adjustment is made to the sectorial added value coefficients (v_2).

The appropriate interpretation for this adjustment vector is as follows: since water is a homogeneous product, in the sense of the Leontief model assumption and Walrasian equilibrium, its price should be the same in all markets in which it is involved. In this case, we have not a monopolistic supplier who attempts to take advantage of the price discrimination to maximise profits, but a public supplier who, for socio-political reasons, establishes separate segments in the market; the result of this segmentation should be interpreted as an implicit system of cross levies and subsidies. The differences between the observed v_2 vector and the adjusted v_2' vector can therefore be interpreted as an indicator of water tariff differences (table 5).

Table 5
Differences between original and adjusted coefficients for added value

	Initial v_2s	Adjusted v_2s	Difference (v_2-v_2')
Agriculture	0.59662211	0.57830456	+0.01831755
Industry and construction	0.42173202	0.4226454	-0.00091337
Services	0.62359456	0.62484985	-0.00125529

The positive differences (agricultural sector) indicate that the average water price in the sector is lower than the average water price in the economy as a whole, while the negative differences (rest of sectors) indicate that the average water price is higher in these sectors than the average water price in the region. According to this interpretation, there is an implicit water subsidy in agriculture which represents approximately 3% (difference/ adjusted v_2) of the added value of this activity.

The sectorial differences in the price of water are not only a result of this implicit system of cross levies and subsidies. They are also the result of the different costs of water in its different uses. Water itself has no price. The amount paid for it, its price, refers to the cost generated by its management and distribution. And the cost of these processes is not the same for all the different uses of water. Water for domestic use, tertiary activities and industries requires a complex process of regulation, transport, purification and distribution, whose cost is significantly higher than that of the water used in agriculture, which does not normally require prior treatment and which is generally consumed near the collection and regulation points. The differences in cost are therefore reflected in prices and explain, at least in part, the sectorial differences in water prices.

Model (16) with the adjusted added-value coefficients affords an analysis of the impact of variations in the average price per m^3 of water on unit prices in the different sectors. Thus, a 1% increase in the average water price per m^3 causes a 0.02% increase in the average price in the agricultural sector, a 0.004% increase in average prices in industry and construction and a 0.001% increase in prices in services.

4. Water content as a guide to the price of goods and services

This section defines prices in water terms (P_w) in the Andalusian economy, and their evolution in relative terms, when water becomes the production factor which determines the value of human output.

Basically, it is an attempt to establish an analogy with Marx's labour-value theory. Marx considered that the scarce and truly limiting factor in production processes was labour and he maintained that, as a result, the value of goods and services and, logically, their price, should be directly proportional to the amount of labour required to produce them (Marx, 1974).

Broadly speaking, in Marx's labour-value concept, if the production of *X* either directly or indirectly requires two hours of work and the production of *Y* just one hour, then *X* 's labour value would be double *Y*'s. If we adopt the water-value concept, the reasoning would be as follows: if the production of *X* directly or indirectly needs 1 m^3 of water and *Y* needs 3 m^3 of water, then *Y*'s water value should be three times that of *X*.

In the world of Walrasian general static equilibrium, equilibrium prices or cost/prices reflect all production costs obtained in a situation of market competition. This section aims to calculate the prices in water terms (P_w) of the different products with a view to comparing them with the cost/prices. It is worth noting that, since this is a value model, the cost/prices are implicit unit value prices; P_w values are therefore interpreted as price indices linked to the unit cost/prices, in other words, as deviations from these cost/prices.

In order to calculate the P_w we use vector $\left[A_{12}(I - A_{22})^{-1}\right]$ from equation (8) which shows the direct and indirect water needs per unit of final demand in the Andalusian economy, the values of which are shown in the first column of table 6. Dividing the economy's final demand value between the litres necessary to generate this demand we can see that, on average, 0.575 m^3 is required to produce one million pesetas in final demand. With these average requirements, it is possible to define the P_w of the Andalusian production sectors, which can be seen in the second column of table 6.

Table 6
Water value in Andalusian production sectors

	m^3/million Ptas final demand	Water value	% of average water value
Agriculture	4.08	7.10	610%
Industry and construction	0.60	1.05	5.5%
Services	0.19	0.34	-66.7%
Total economy	0.575	1	0%

If the average price in water terms of the Andalusian economy were 1, the P_w in the agriculture sector would be 7.1 (610% higher than the average), in the industrial and construction sector it would be 1.04 (4% higher) and in services it would be 0.34 (66% lower than the average P_w).

In a similar way, the P_w can be interpreted as deviations from the current cost/prices (implicit unit price index) and any rise in the relative prices of agricultural goods and any decrease in the relative prices of services would be favourable to water efficiency.

Performing the above calculations on a more desegregated level (65 sectors) leads to the P_w values of each of the Andalusian production areas. Table 7 presents the areas of activity with the highest P_w values.

Table 7
Areas of activity with highest P_w values

Nº	Area of activity	P_w	% difference over average
3	Citrus fruit.	32.848	3,184.8
1	Cereals and legumes.	20.451	1,945.1
36	Milling and baking.	12.738	1,173.8
2	Fruit and vegetables	7.650	665.0
6	Other agricultural products	7.383	638.3
5	Olive production	6.960	596.0
39	Other food industries	6.043	504.3
4	Industrial plants	4.506	350.6
7	Livestock	4.185	318.5
31	Oils and fats	4.086	308.6

The areas with the highest P_w are concentrated in the agricultural sector and in the food industry and those with the lowest P_w are in the services sector.

5. Summary and conclusions

The models

From a methodological point of view, the input-output models defined here present a novelty in relation to the copious literature on I-O analysis and its applications: they are "hybrid" partitioned models in which part of the economy (in this case the water sector) is dealt with in physical units (m^3) while the rest of the economy is dealt with in monetary units.

Application of the models

The application of the models to the case of Andalusia and the analysis of the results lead to the following conclusions:

- The agricultural sector is the main water consumer in the region, both in absolute terms (80% of total consumption) and relative terms (it consumes 3,623 m^3 of water per million pesetas of output, whereas the industry and services sectors consume 36 m^3 and 19 m^3, respectively, per million pesetas of output).
- Water consumption in the region is sensitive to variations in agricultural output and to variations in final demand in the agriculture, industry and construction sectors.
- The population-elasticity of water consumption in the region is low, whereas the income-elasticity is relatively high.
- As a whole, the production processes linked to agriculture are responsible for around 85% of the Andalusian economy's direct and indirect water needs for the generation of final demand.
- The sustainability of growth in the agriculture sector from the point of view of water management requires a substantial reduction in its production processes' water needs, which will necessarily imply structural reform and modernisation of the region's irrigation sector.

- The average water price in the region is 6.67 Ptas per m^3. There are considerable differences between water prices in the different production sectors, which must be attributed to differences in the cost of services and to an implicit system of cross levies and subsidies.

Prices in water terms (P_w)

The P_w calculated above refer to the prices of goods and services in the Andalusian economy and their deviation in relative terms from current prices, when water becomes the production factor which determines the value and price of human output. A water management policy that attempted to relate the price of goods to the absolute scarcity of water would have to include a pricing policy which would move the relative prices of goods and services in this direction.

According to the calculations, the P_w in the agricultural sector are 610% above the current prices, those of industry and construction are 4% over current prices and those of services are 66% lower than current prices.

It is clear that water is not currently so scarce as to reach a situation in which P_w took precedence; but it is also clear that, in the very long term and in a context of progressive development towards the optimisation of water use, it would be advisable for the relative prices of goods and services to better reflect future scarcity and to move in the direction of P_w. In other words, from the water economy standpoint, it would be positive if, over the next few decades, the relative prices of the more water-intensive products maintained an upward trend in relation to the relative prices of the rest of goods and services.

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[i] This paper is based on the Doctoral Thesis which I wrote under the direction of Professor Emilio Fontela and which I defended on June 24, 1999 in the Department of Applied Economics of the Universidad Autónoma de Madrid, Spain.

[ii] In contrast to traditional policies, which focus on increasing available resources through the construction of dams and other hydrological works.

[iii] Similar models can be seen in Bouhia (1998).

[iv] See Pulido and Fontela (1993)

[v] Other hybrid I-O models can be seen in Miller and Blair, 1985.

[vi] To define final demand in sector 2 the value of transfers, imports and their VAT is deducted from the values of final demand of Andalusia's 1990 IOT.

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