

Industrial Specialisation, Trade, and Labour Market  
Dynamics  
in a Multisectoral Model of Technological Progress

Robert Stehrer

The Vienna Institute for International  
Economic Studies (WIIW), Vienna, Austria.  
Oppolzergasse 6, A-1010 Vienna.  
e-mail: stehrer@wsr.ac.at

Vienna, October 20, 2000

Paper prepared as part of the project:  
*Technology, Productivity, and Employment in the Accession Countries*  
financed by the OENB, Research project number 8001.

- PRELIMINARY VERSION -

## Abstract

The issue of the impact of trade on specialisation structures and the effects of trade liberalisation on employment and labour markets has been intensively discussed in the recent literature on trade liberalisation and globalisation. In Europe this debate has gained new momentum in the discussion on the effects of the catching-up processes of the transforming economies in Eastern European Countries. But the bulk of the existing literature in this area employs almost without exception a static Heckscher-Ohlin framework based on factor-endowment differences and thus seems not to be a suitable tool for analysing dynamic issues of technology catching-up and dynamic adjustment processes.

In this paper I present a model to explore the issue of productivity catching-up, international specialisation and labour market effects in a dynamic multi-sectoral framework with heterogeneous labour. The model is basically an input-output model, but also has some Schumpeterian features. These Schumpeterian features are the impact of monopoly rents, emerging from (labour) productivity-enhancing technological progress or catching-up processes, upon the price-, wage- and quantity system of the trading economies. Relative productivity and relative wage rate dynamics across sectors determine comparative cost advantages and trade specialisation. The second part of the paper presents some simulation studies of the evolution of prices, output, employment and wage structures, where various stylized types of technological progress and industrial catching-up processes are modelled. In the third part of the paper the equilibrium solutions of the model are derived in a general way.

*JEL-Classification:*

*Keywords:*

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# INDUSTRIAL SPECIALISATION, TRADE, AND LABOUR MARKET DYNAMICS IN A MULTISECTORAL MODEL OF TECHNOLOGICAL PROGRESS

Robert Stehrer <sup>1</sup>

## 1 Introduction

This paper presents a dynamic multisectoral model to study the effects of technological progress, catching-up and trade liberalisation on the labour market performance of different skill-types of workers in advanced and catching-up economies. The issue of the impact of trade on labour markets in the more advanced economies was widely discussed in the beginning of the 1990's, when the free trade agreement (especially the NAFTA between the US, Canada and Mexico) came into being. The debate focused mainly on the impact of developing countries and exporters of low-skill intensive goods on the relative wages of skilled to unskilled workers in the more advanced countries, especially the US. In this debate the empirical evidence of a large widening of the wage differential between skilled and unskilled workers in the US in the 1990's was the starting point. On the one hand, especially Leamer (1994, 1996) and Wood (1995) argued that trade liberalisation was the main reason for the worsened labour market position of the unskilled workers. This explanation was, on the other hand, criticised e.g. by Lawrence and Slaughter (1993) and especially Berman et al. (1994). The latter pointed to skill-biased technological progress as the main explanation for the labour market positions of low skilled workers.

In the debate on the effects of trade on labour markets mainly the static framework of the Heckscher-Ohlin model was used. In the case that the advanced country is relatively better endowed with skilled (relative to unskilled) labour than the less-developed country, skill-intensive goods are relatively cheaper in the advanced country. In a free trade regime, this country should then specialise in skill-intensive goods, which then raises the demand for the skilled workers, and thus relative wages of skilled workers are increasing. The opposite is expected for the less-developed country, which is relatively better endowed with unskilled workers.

Theoretical and empirical studies then focused on the relative impact of trade versus technological progress. In most studies technological progress was found to have the most important impact on the labour market performance of the lower-skilled versus the higher skilled workers (measured either in relative wages or relative unemployment rates). Using factor content analysis, Wood (1995) and others confirmed this view for the advanced countries. Lawrence and Slaughter (1993) criticised this view, as in the Heckscher-Ohlin model relative prices of goods must change, causing changes in relative factor prices (Stolper-Samuelson effect). But they could not find empirical evidence for such a change in relative prices. On the technology side, Berman et al. (1994) examined the impact of technological progress on skilled relative to unskilled workers. Finding that relative demand for skilled workers had risen in each industry (intraindustry versus interindustry shifts in relative demand), they concluded that skill-biased technological progress was much more important than the effects of trade liberalisation. Further Feenstra and Hanson (1996) showed that relative wages of the skilled workers in Mexico (the

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<sup>1</sup>I acknowledge support from the *Jubiläumsfonds* of the Austrian National Bank in the context of the project 'Technology, Productivity and Employment in the Accession Countries'.

country relatively better endowed with unskilled workers) has also risen (contrary the expected effect of the Heckscher-Ohlin model). In explaining this fact, they used a Heckscher-Ohlin model, where outsourcing activities may lead to increases in relative wages of the skilled workers in both countries.

From the viewpoint of this paper there are several drawbacks in analysing the issue of trade and labour markets with the models mentioned above. The main criticism of the Heckscher-Ohlin framework can be summarized in two items: First, its genuine static nature and, second, the assumption of equal technologies in the countries.<sup>2</sup> In this paper we shall start from the evidence that countries use different technologies (expressed here as labour productivity levels) but are able to catch-up to the more advanced countries. The period from the start of the catching-up process to reaching the technology frontier is by itself interesting and worth studying, but the shape of the transition and the positioning within this period can have long-term effects. Landesmann and Stehrer (2000b) find that the half-time of catching-up in labour productivity levels in different manufacturing sectors ranged from 10-30 years, a time period which should not be neglected in analytical research. Further the catching-up process is different across sectors. From this viewpoint a model where comparative *dynamic* analysis can be made, rather than purely comparative *static* analysis, seems to be justified. Further, the Heckscher-Ohlin framework relies only on differences in factor endowments. Although this assumption seems to be relevant (at least partly) for trade between advanced and developing countries, it seems less satisfactory when studying effects of trade liberalisation between Eastern European countries and the EU, as here the differences in factor endowments seems not be that large (or even reversed, if one compares with EU-Southern countries only).<sup>3</sup>

In the model presented below, a Ricardian framework with catching-up in labour productivity levels is used, where also differences of payments on factors of production can be introduced.

In Europe this issue is now debated with respect to the integration of the Eastern European countries and the impact on the labour markets on the EU and the Eastern European countries. Although there are also large differences in (labour) productivity, some countries manage to catch-up quite rapidly to the level of the Western European countries, whereas some others are staying or even falling behind.

In two recent papers (Stehrer and Landesmann, 1999; Landesmann and Stehrer, 2000b) we focused on the technological performance of a rather large sample of catching-up economies (European Southern countries, Eastern Asian countries, etc.) at a disaggregated manufacturing level. The main findings were that, first, there are huge differences in labour productivity levels between the developing and the developed countries, and, second, these differences are closed quite rapidly by catching-up processes (at least for some countries or country groups). Further the relative speed of catching-up in levels of labour productivity was found to be quite different if one compares different manufacturing (e.g. high-to-medium tech and lower tech) sectors.

This paper presents a dynamic, multisectoral model of catching-up, where these issues can be discussed in an integrated framework. Further the model allows to discuss labour market effects on different skill-types of workers. The main focus of this model is to analyse the impact of 'shocks' (either technology or trade liberalisation) and it thus to deals mainly with non-steady-state and non-balanced growth and fluctuations. Thus it is not the aim of this paper to study long-term dynamics or even traverses from one long-term equilibrium to another long-term

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<sup>2</sup>This means that the countries are on the same isoquant, although they are on different points on this isoquant due to different factor endowments.

<sup>3</sup>This statement has to be checked further with empirical studies.

equilibrium. Further some limitations of this approach (and especially restrictions on parameter values) are discussed at the end of the paper.

The paper goes as follows: In the first part, the dynamic model used afterwards in the simulation studies is presented. This is done step by step: First we introduce the basic model with one autarkic country and homogeneous labour. Then the model is extended to the case with heterogeneous labour and finally to a trade model with interacting countries. In the appendix the long-term dynamic properties of the model are discussed. This is useful as the 'behaviour' of the model and some specific assumptions in the non-equilibrium transition phase become clearer, if the long-term properties are accounted for. In the second part of the paper three simulation studies are presented. The first one shows the impact of sector specific technological progress in a closed economy with homogenous labour. This also helps to interpret the dynamics of the system in more complicated environments (e.g. with more skill-types of workers, more countries, etc). The second simulation run then shows the model with two types of labour (skilled and low-skilled workers). Finally, the third simulation discusses a model with two trading economies, where one of them catches up in terms of productivity levels with the more advanced economy. Here especially the impacts on the labour markets of both countries are discussed. The contribution of this paper is mainly to introduce this kind of dynamic structural modeling in a very simple way, i.e. using quite simple specifications which can be improved in further research. Extensions of the model including sensitivity analysis and applications to specific topics are to be analysed in future research.

## 2 The model

### 2.1 The basic N-sectoral model

In this section we now discuss the detailed structure of the model, which is used afterwards in simulation studies. To be more explicit on the equations in this section we do not use matrix notation. The model is based on a paper by Landesmann and Stehrer (2000a) and is an extension of the model presented therein with some respect.

#### 2.1.1 Technology

We start with a simple matrix of technical input coefficients, denoted by

$$\mathbf{A} = \begin{pmatrix} a_{11} & \dots & a_{N2} \\ \vdots & \ddots & \vdots \\ a_{1N} & \dots & a_{NN} \end{pmatrix}$$

which is assumed to be stable over time. Labour productivity is given by a vector of labour input coefficients

$$\mathbf{a}_l = (a_{l1}, \dots, a_{lN})$$

Labour is used in fixed proportions (meaning that there is no substitution between labour and (flow) capital or (as discussed below) between different types of labour when relative factor prices are changing. These labour input coefficients  $a_{li}$  may decline over time at an exogenous rate  $g_{a_{li}} \leq 0$  to a predetermined level  $\bar{a}_{li}$ :

$$\dot{a}_{li} = g_{a_{li}} (a_{li} - \bar{a}_{li}) \tag{2.1}$$

This formulation implies that  $a_{li} > 0$  for all  $t$  and thus, that labour is seen as a necessary input for the production of each good. Further the growth rate of labour productivity is going to 0,  $\frac{\dot{a}_{li}}{a_{li}} \rightarrow 0$  when  $t \rightarrow \infty$ .

### 2.1.2 Prices and rents

**Prices** Prices are modeled as adjustment to unit costs

$$\dot{p}_i = \delta_{p_i} [(1 + \pi)c_i - p_i] \quad (2.2)$$

where

$$c_i = \sum_j p_j a_{ji} + \omega_i$$

are the costs of production and

$$\omega_i = w_i a_{li}$$

denote the unit labour costs (for the moment we assume only one skill-type of workers). We assume that wage rates  $w_i$  need not be equalised across sectors, although it is possible in general that wage rates equalise in the long run as we shall see below. The parameter  $0 \leq \delta_{p_i} \leq 1$  gives the speed of adjustment of prices to (equilibrium) unit labour costs. There exists a long run mark up on prices  $\pi$  which is equal across all sectors (this leads to equal real per unit profits in each sector).

**Rents** As there is a constant mark-up on prices  $\pi$  there are long run per unit profits  $r_i$  defined as

$$r_i = \pi c_i$$

As prices do not adjust immediately to unit costs plus mark-up there arises transitory profits depending on the speed of technological progress  $g_{a_{li}}$  and the price adjustment parameter  $\delta_{p_i}$  and the dynamics of wages as we shall see below:

$$s_i = p_i - (1 + \pi)c_i = p_i - c_i - \pi c_i = p_i - c_i - r_i$$

**Price index** Further for consumers the price index is an important indicator. The consumer price index is defined as

$$P^C = \sum_i \alpha_i p_i \quad \text{with} \quad \sum_i \alpha_i = 1$$

which is defined as a weighted sum of prices (weights are the nominal shares of consumption  $\alpha_i$  which are introduced more specifically below).

### 2.1.3 Labour market

**Wage rates** Nominal wages are also growing or falling as (especially transitory) rents are partly distributed to workers and because of excess supply (demand) of workers in the labour market:

$$\dot{w}_i = \kappa_{s_i} \frac{s_i}{a_{li}} + \kappa_u u w_i + \kappa_w \frac{w_i - \bar{w}}{w_i} \quad (2.3)$$

$0 \leq \kappa_{s_i} \leq 1$  is the proportion of per unit (transitory) rents  $s_i$  paid to workers. Rents are distributed only to workers in the respective sector where the rents arises. (A more general formulation would also allow that wages of workers in other sectors are rising due to profits in a particular sector.)

The second term on the rhs of the wage dynamics equation reflects the impact of unemployment on the dynamics of the wages ( $\kappa_u \leq 0$ ), where unemployment is defined as

$$u = \frac{L^S - \sum_i a_{li} q_i}{L^S} = \frac{L^S - L^D}{L^S}$$

Third, there is an impact on the wage dynamics if wages (for the same type of worker) differ across sectors. This reflects the common assumption (e.g. in the standard Ricardian trade model), that wages get equalised across sectors because of labour mobility. The (weighted) average wage  $\bar{w}$  is defined as

$$\bar{w} = \frac{\sum_i L_i^D w_i}{\sum_i L_i^D}$$

If the average wage  $\bar{w}$  is higher than the sectorial wage  $w_i$  the wage dynamics is positive ( $\dot{w}_i > 0$  for  $\kappa_w < 0$ ), in the other case negative. This term works symmetrically across sectors.

Thus in the formulation used in the simulations, there is a sector specific term and two economy wide terms having an influence on wage rates in each sector. Further there can occur wage differentiation across sectors in the short run, wages are equalised, however, in the long run.

**Labour supply** Labour supply  $L^S$  is assumed to adjust to labour demand according to

$$\dot{L}^S = \delta_{L^S} (L^D - L^S) \quad (2.4)$$

where  $L^D = \sum_i a_{li} q_i$  and

$$\begin{aligned} \delta_{L^S} &= 0 & \text{for} & \quad L^S > L^D \\ \delta_{L^S} &\geq 0 & \text{for} & \quad L^S \leq L^D \end{aligned}$$

This formulation implies that labour supply is adjusting to labour demand if there is excess demand of labour, but there is no adjustment in the other direction; i.e. that workers leave the labour market in case of unemployment (excess supply of labour).<sup>4</sup> In the simulations below it was actually assumed, that there can be no excess demand for labour as labour is supplied with

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<sup>4</sup>A less strong assumption would be that the parameters differ for the two situations, so that high unemployment leads to a falling participation rate.

infinite elasticity and adjusts immediately to labour demand.<sup>5</sup> This assumption can be justified for two reasons: First, there is some evidence that shortage of labour has been no constraint in the long run growth of economies (e.g. McCombie and Thirlwall, 1994). Further with application to actual catching-up processes of some countries a shortage of labour was never discussed as limiting factor, as maybe the human resources has been build up before the growth process starts or are available from other sectors (as for example in the model by Lewis, 1954). or are accumulated in the pre catching-up phase (e.g. human capital). Second, from a modelling point of view, a constraint of labour supply would imply a further specific assumption on the distribution of labour across the sectors, which is especially a difficult problem when assuming more than one skill-type of workers.<sup>6</sup> On the other hand there are two facts which would justify to allow for (short term) excess *demand* for labour. First, if firms are not fully employing there labour force, a higher demand could result in higher productivity levels of labour, which are not captured in the long-term trend of the labour input coefficients  $a_{li}$  in this model. This implies that fluctuations of labour demand are absorbed at the firm level. Second, if labour markets are also include some unemployment because of labour mobility, an adjustment of this frictional unemployment rate could help to deal with situations of excess labour demand, which implies an absorption at the market level.

#### 2.1.4 Quantities

After this discussion of the price system the quantity system must be specified. Demand for goods consists of three different components which can be summarized in the following general equation:

$$\begin{aligned} q_i^D &= \sum_j a_{ij} q_j + \beta_i \sum_j \frac{(1 - \kappa_{sj}) s_j + r_j}{p_i} q_j + \alpha_i \sum_j \frac{w_j}{p_i} a_{lj} q_j \\ &= \sum_j a_{ij} q_j + q_i^I + q_i^C \end{aligned} \quad (2.5)$$

with  $\sum_i \alpha_i = \sum_i \beta_i = 1$ . The first term is demand for intermediate goods used in production, the second term is demand out of profits (which are entirely used for investment) and the third term reflects demand out of workers income (used for consumption). (The terms  $q_i^I$  and  $q_i^C$  will be discussed below in detail). Thus  $q_i^I + q_i^D$  is the final demand for good  $i$ . Specifically we assume further that workers do not save their income (or spend all money on consumption goods), whereas profits are entirely used for investment.<sup>7</sup> Further it is assumed that investments cannot be negative (see below).

**Consumption demand** For consumer demand we assume that the nominal shares of consumption  $\alpha_i$  are constant and  $\sum_i \alpha_i = 1$ . Or, stated differently, consumers maximize a Cobb-Douglas utility function,  $U = \prod_i x_i^{\alpha_i}$ , from which this kind of consumer behaviour results. Of

<sup>5</sup>Thus the production is not constrained by shortages in the supply of labour, although in the simulations we allow for a pressure on wage rates due to excess demand of labour via the unemployment term. But as labour supply adjusts rapidly to demand ( $\delta_{L^S} = 1$  for  $L^S \leq L^D$ ) this effect is not very large. In fact, this is mainly a numerical problem in the solution of the systems of equations due to the formulation of the equations.

<sup>6</sup>The limitations of (sectoral) growth due to a shortage of factors would of course be an interesting topic in itself, but is not a topic in this paper. Thus, research in this area has to be postponed.

<sup>7</sup>This assumption is not necessary but simplifies discussion and presentation of the model. E.g. if part of the profits is used for consumption with different structure as workers consumption, another term would have to be introduced in the equation above.

course, each other demand system - e.g. Stone-Geary, CES utility functions, AIDS demand system, or Dixit-Stiglitz type - which gives nominal shares for given income and prices - and thus would allow non-linear Engel curves, various price elasticities, and so on - could be used instead of the simple Cobb-Douglas system.<sup>8</sup>

**Investment demand** Demand for investments (which are then investments in the particular sector) are similarly formulated with a nominal share  $\beta_i$ . This specification describes only the aggregate outcome of investment decisions. For example, one could formulate sector specific investment behaviour which in the aggregate could then also be described by the formulation used in this paper.<sup>9</sup> As over time the structure of the economy changes (due to changes in relative prices, real income levels, and thus changes in consumption patterns) the structure of investments also has to change. As in this paper we are not particularly interested in the investment behaviour, we assume that the investment structure adjusts to the optimal structure over time. The specific formulation for the nominal shares of investment in the simulation model is

$$\dot{\beta}_i = \delta_{\beta_i^*} (\beta_i - \beta_i^*)$$

where

$$\beta_i^* = \frac{p_i q_i}{\sum_i p_i q_i}$$

This formulation assumes a myopic decision behaviour. For given quantities and prices at time  $t$  one can calculate the optimal (i.e. growth maximising) investment structure,  $\beta_i^*$ . The actual investment structure then adjusts to this optimal one gradually.<sup>10</sup> Investments in one sector are then given by

$$q_i^I = \max \left( 0, \beta_i \sum_j \frac{(1 - \kappa_{s_j}) s_j + r_j}{p_i} q_j \right)$$

where we assume that investment must be non-negative. The growth rate of the economy depends on this investments as we discuss in the next step.

**Supply of goods** The supply of goods is then modeled as an adjustment process where supply adjusts to demand in a growing economy with:

$$\dot{q}_i = \delta_{q_i} \left[ (1 + g) \sum_j \tilde{a}_{ij} (q_j^I + q_j^C) - q_i \right] \quad (2.6)$$

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<sup>8</sup>A specification of a utility function would also allow for analysis of welfare implications.

<sup>9</sup>One could also introduce sector specific parameters  $\beta_i$ . But for simplicity of notation we stick to the more simple formulation.

<sup>10</sup>In discrete time one could calculate an optimal path by an iterative process: Given prices  $\mathbf{p}_t$  and quantities  $\mathbf{q}_t^{(1)}$  one calculates in the first step (1) the parameters  $\beta_i^{(1)}$ . Inserting these into the quantity system yields new quantity values  $\mathbf{q}_t^{(2)}$ , which again leads to new parameters  $\beta_i^{(2)}$ , and so on. This iterative process at each point in (discrete) time then leads to the optimal investment structure  $\beta_i^*$  which maximises the growth rate (see below and the discussion in Appendix A).

where  $\tilde{a}_{ij}$  denotes a typical element of the Leontief matrix  $[\mathbf{I} - \mathbf{A}]^{-1}$  and  $\delta_{q_i}$  is an adjustment parameter. The overall growth rate of the economy is determined as

$$g = \min_i \left( \frac{q_i^I}{q_i} \right)$$

The rationale for this specification is as follows: At each point in time there exists a final demand vector,  $q_i^I + q_i^C$ . For the system to be able to produce these quantities the intermediate demand for the production of each good must be taken into account, which is done by the Leontief inverse. Further the economy is able to grow only if there are positive investments in each sector, which amount - in this model with circulating capital only - to a growing stock of intermediate inputs. Due to the linearity of the production system the overall growth rate is bounded by the sector with the lowest investment rate (i.e. the ratio of  $\frac{q_i^I}{q_i}$ ). In the Appendix A we show that in equilibrium (i.e. steady state balanced growth) the investment structure with nominal shares  $\beta_i^*$  as defined above guarantees that the system grows with  $g^* = \frac{\pi}{1+\pi}$ .

## 2.2 Extension I: S skill-types of workers

A simple but very interesting extension of the basic model above is the assumption that there are more than one type of worker (e.g. high- and low-skilled workers). In fact the model can in this respect generalised quite easily. Starting with the price equation one only has to change the expression for unit labour costs. The industry specific unit labour costs are now the sum of the products of the skill specific wages and input coefficients:

$$\omega_i = \sum_z w_i^z a_{li}^z \quad (2.7)$$

where  $z$  denotes the skill types of workers,  $z = 1, \dots, S$ . If wages by skill groups are equalised across sectors the unit labour costs by sectors are

$$\omega_i = \sum_z w^z a_{li}^z$$

and thus the differences in sectoral unit labour costs depends only on differences in the productivity levels and the structure of labour inputs. Further the technological progress can be different across skill groups, thus the differential equation for the labour input coefficients is

$$\dot{a}_{li}^z = g_{a_{li}^z} (a_{li}^z - \bar{a}_{li}^z)$$

The effects of transitory rents and unemployment on wage rates are now becoming skill-specific which is one of the most interesting characteristic of the extension of the model to more than one skill group. As the unemployment rate is different across skill groups and has equal impact on wage rates in the various sectors the wage rate dynamics differs not only from sector specific rents but also because of different skill-intensity and differing unemployment rates. In the simulations below we assume the following specification:

$$\dot{w}_i^z = \kappa_{s_i^z} \frac{s_i}{\sum_z a_{li}^z} + \kappa_{u^z} u^z w_i^z + \kappa_{w^z} \frac{w_i^z - \bar{w}^z}{w_i^z} \quad \text{with} \quad \kappa_{s_i^z} = \kappa_{s_i} \frac{w_i^z}{\sum_z w_i^z} \quad (2.8)$$

where

$$\bar{w}^z = \frac{\sum_i L_i^D w_i^z}{\sum_i L_i^D}$$

The specification of the first term on the rhs of equation 2.8 implies that wage rates of different skill types of workers within an industry are growing at equal percentage rates. This means that wage rates can (temporally) be different across sectors and skill-groups. The second term gives the impact of the skill-specific unemployment rates  $u^z$  on sectoral wage rates by skill group. The third term again rests on the assumption that wage rates for the same skill-type of workers equalise in the long run as discussed above. The unemployment rate  $u^z$  has to be defined skill specific too:

$$u^z = \frac{L^{S^z} - \sum_i a_{li}^z q_i}{L^{S^z}} = \frac{L^{S^z} - L^{D^z}}{L^{S^z}}$$

Again we assume as above that labour supply adjusts to demand according to

$$\dot{L}^{S^z} = \delta_{L^{S^z}} (L^{D^z} - L^{S^z}) \quad (2.9)$$

where  $L^{D^z} = \sum_{i,z} a_{li}^z q_i$  and

$$\begin{aligned} \delta_{L^{S^z}} &= 0 & \text{for} & \quad L^{S^z} > L^{D^z} \\ \delta_{L^{S^z}} &\geq 0 & \text{for} & \quad L^{S^z} \leq L^{D^z} \end{aligned}$$

The reasons for this assumption are the same as above. But here one has to notice that the assumption is even stronger for skilled workers (which in fact has to be educated) than for low- or unskilled workers.

Finally the demand component of workers has to be changed in the demand equations to

$$\alpha_i \sum_j \sum_z \frac{w_j^z}{p_i} a_{lj}^z q_j$$

This specification assumes that the nominal consumption shares are equal for all skill groups. In an even more general setting the nominal shares may depend on skill- and sector-specific wages and prices (e.g. when using demand systems with non-linear Engel curves).

### 2.3 Extension II: Trading economies

The next step is to introduce more countries and especially international relationships between these countries. First of all, all the variables have to be indicated for the different countries. In this paper we restrict to a two country model and denote the variables with  $L$  for the leader and  $r$  for the other country, respectively. If there are general relationships between the two countries we denote them by  $r$  and  $s$ , respectively. Further we assume that the exchange rate between the trading economies is set to  $X^{rs} = 1$  and there are no changes over time. (Effects of trade imbalances on exchange rates shall be introduced later.)

Then the various economic relationships between the countries have to be specified. Three different ways of economic linkages are specified in this paper: imports and exports, investment flows, and international learning processes. (Especially we do not discuss migration as a fourth linkage.)

### 2.3.1 Exports and imports for consumption

For consumer demand we make a specification which is similar to the specification in the closed economy case. Consumer demand in country  $r$  now depends also on wages of skill-group  $z$  in the particular industry  $i$  in country  $s$ ,  $w_i^{z,s}$ , multiplied by the input coefficient  $a_{li}^{z,s}$  and sectoral output in country  $s$ ,  $q_i^s$ . Summing up over all skill groups gives the nominal wage income  $\sum_z w_i^{z,s} a_{li}^{z,s} q_i^s$  in industry  $i$  of country  $s$ .

For simplicity we assume that a constant nominal share of wage income in country  $s$  is spent on goods from country  $r$ . This nominal share is denoted by  $\mu_i^{sr}$ . The nominal share of income in economy  $s$  spent on goods in economy  $r$  can then be written as

$$\alpha_i^{sr} = \alpha_i^s \mu_i^{sr}$$

where  $\sum_r \sum_i \alpha_i^{sr} = 1$  must be satisfied.<sup>11</sup> This specific assumption means that the domestic and foreign good are not (or not seen as) perfect substitutes. In fact, the formulation used here implies a Cobb-Douglas utility function of the form<sup>12</sup>

$$U^s = \prod_{i,r} (q_i^r)^{\alpha_i^{sr}}$$

### 2.3.2 Investment flows

Investors have to make two decisions: First, in which country and sector to invest, and second where to buy these investment goods. The decisions for these two questions are guided by different questions: The first one is motivated, where the highest per unit rents (and profits) can be gathered, the second where the goods for investment can be bought relatively cheaply. These goods has then to be transported to the country where they should be invested. (In this case also transport costs may be considered or neglected by assumption as above for consumption goods.) But, for simplicity, we assume in this paper, that if an investor wants to invest in a certain country, the goods for investment are also bought in this country (the money is spent abroad). Thus the sum of rents and profits in a country  $s$  in sector  $i$   $[(1 - \kappa_{s_i}^s) s_i^s + r_i^s] q_i^s$  is then distributed accordingly to demand from rents and a similar specification as for consumption demand can be used. We assume that the nominal share spent abroad is

$$\beta_i^{sr} = \beta_i^r \nu_i^{sr}$$

where  $\sum_{i,r} \beta_i^{sr} = 1$  denotes the nominal share of investment expenditure of country  $s$  in country  $r$  in sector  $i$ . For simplicity we assume again that the nominal share of rents and profits which are spent abroad is constant  $\nu_i^{sr}$ .<sup>13</sup> The distribution of investment expenditures across sectors is modeled as before:

$$\dot{\beta}_i^r = \delta_{\beta_i^*}^r (\beta_i^r - \beta_i^{r*})$$

---

<sup>11</sup>In the two country case this means that

$$\alpha_i^{ss} = 1 - \alpha_i^{sr}$$

<sup>12</sup>Of course here again more flexible functions could be used which e.g. allow for home-bias effects, other price elasticities between foreign and domestic goods, etc. Further one could also introduce a specification that these nominal shares evolve gradually in the case of a sudden trade liberalisation.

<sup>13</sup>In a more advanced specification this share could also be dependent e.g. on (sector specific) rents across countries.

where

$$\beta_i^{r*} = \frac{p_i^r q_i^r}{\sum_i p_i^r q_i^r}$$

Note the difference between the nominal shares for consumption and the nominal shares for investment expenditures. In the former case the nominal share of consumption of country  $s$  in country  $r$ ,  $\alpha_i^{sr} = \alpha_i^s \mu_i^{sr}$ , depends on the consumption structure prevailing in country  $s$ ,  $\alpha_i^s$ , whereas the nominal share of investment of country  $s$  in country  $r$ ,  $\beta_i^{sr} = \beta_i^r \nu_i^{sr}$ , depends on the investment structure in country  $r$ ,  $\beta_i^r$ . From this follows that the output structure of an economy is also influenced by the consumption structure in another economy (if the nominal shares are different). On the other hand, the structure of output does not depend on the decisions in the economy abroad; only the overall growth rates are influenced by the investment flows across economies. Further we assume that intermediate investments are not traded.

### 2.3.3 Quantity dynamics

Given this assumptions on the consumption and investment behaviour in the international setting the demand for products in country  $r$  can then be written as:

$$(q_i^D)^r = \sum_j a_{ij}^r q_j^r + \sum_s \sum_j \beta_i^{sr} \frac{(1 - \kappa_{sj}^s) s_j^s + r_j^s}{p_i^r} q_j^s + \sum_s \sum_j \sum_z \alpha_i^{sr} \frac{w_j^{z,s}}{p_i^r} a_{ij}^{z,s} q_j^s \quad (2.10)$$

where again the investment in a particular sector has to be constrained with

$$(q_i^I)^r = \max \left( 0, \beta_i^r \sum_s \sum_j \frac{(1 - \kappa_{sj}^s) s_j^s + r_j^s}{p_i^r} q_j^s \right)$$

and the growth rate of a particular economy is then

$$g^r = \min \left( \frac{(q_i^I)^r}{q_i^r} \right)$$

The supply adjusts to demand differential equation is then

$$\dot{q}_i^r = \delta_{q_i}^r \left[ (1 + g^r) \sum_j \tilde{a}_{ij}^r \left( (q_j^I)^r + (q_j^C)^r \right) \right] \quad (2.11)$$

### 2.3.4 Learning processes

The international linkage of economic integration is that countries can learn from each other, meaning that technologically backward countries are catching-up with more advanced countries. For this different approaches could be regarded. The simplest modelling strategy, which is used in this paper, is that countries are catching-up to the leading country (or the productivity frontier). Different paths of catching-up processes were investigated in Landesmann and Stehrer (2000b) and should not be repeated here. In the simulations above we assume that countries lying farther behind have relatively higher productivity growth rates (Gershenkron's 'advantage

of backwardness' which is applied here on the industrial level; see also Landesmann and Stehrer (2000b) for theoretical and empirical discussion).

The specific equations for the catching-up processes are similar to the closed economy case:

$$\dot{a}_{li}^r = g_{a_{li}}^r (a_{li}^r - \bar{a}_{li}^L) \quad (2.12)$$

where  $\bar{a}_{li}^L$  denotes the labour input coefficient of the technological frontier of the productivity leader. In a more sophisticated setting, the speed of catching-up could also depend on the country-wide or industry-specific skill-structure (relative to other countries), exogenously given learning parameters, the structure and volume of imports and exports and especially the flows of international investments.

### 2.3.5 International effects on prices

The last effect of international trade is that goods prices  $p_i^r$  may equalise in the long run ('law of one price'). In this modeling set-up a long-term equilibrium could exist with persistent differences in prices, as the production structure may change to the equilibrium structure in each country and there is no effect on prices via excess supply or demand. Here we assume an exogenous trend for price equalisation. This alters the system of differential equations for prices which becomes

$$\dot{p}_i^r = \delta_{p_i}^r [(1 + \pi)c_i^r - p_i^r] + \delta_{\bar{p}_i}^r \frac{p_i^r - \bar{p}_i}{p_i^r} \quad (2.13)$$

where

$$\bar{p}_i = \frac{\sum_r q_i^r p_i^r}{\sum_r q_i^r}$$

is a weighted average of the prices in the world market. The parameter  $\delta_{\bar{p}_i}^r$  thus reflects the pricing behaviour of firms in the international context which is not specifically formulated. Further the consumer price index must be defined for the international case. Given the assumptions on the consumption structure  $P^{C,r}$  is given by:

$$(P^C)^r = \sum_r \sum_i \alpha_i^{rs} p_i^s$$

### 2.3.6 The balance of payment

The goods demanded from country  $r$  in country  $s$  in a particular sector  $i$  are

$$m_i^{rs} = \alpha_i^{rs} \sum_j \sum_z \frac{w_j^{z,r}}{p_i^s} a_{lj}^{z,r} q_j^r$$

which is import demand of country  $r$  from country  $s$ . The exports of country  $r$  to country  $s$  are then denoted by

$$x_i^{rs} (= m_i^{sr})$$

The trade balance of country  $r$  with country  $s$ ,  $b^{rs}$ , is then defined as the value of exports minus the value of imports of country  $r$ :

$$b^{rs} = \sum_i (x_i^{rs} p_i^r - m_i^{rs} p_i^s) = \sum_i (m_i^{sr} p_i^r - m_i^{rs} p_i^s)$$

and thus denotes the value of net exports. The quantities demanded for investment from country  $r$  in country  $s$  are

$$n_i^{rs} = \beta_i^{rs} \sum_j \frac{(1 - \kappa_{sj}^r) s_j^r + r_j^r}{p_i^s} q_j^r$$

which denotes a capital flow from country  $r$  to country  $s$ . Investment flows from country  $s$  to country  $r$  are then denoted by

$$y_i^{rs} (= n_i^{rs})$$

Similarly one can specify the value flows for investments as

$$k^{rs} = \sum_i (y_i^{rs} p_i^r - n_i^{rs} p_i^s) = \sum_i (y_i^{rs} p_i^r - n_i^{rs} p_i^s)$$

The balance of payments (including trade and capital flows) is then the sum of the trade and the capital account

$$BoP = b^{rs} + k^{rs}$$

### 3 Simulation Studies

In this section we present some preliminary simulation studies.<sup>14</sup> First, we present a simulation for a closed economy and analyse the effects of sector biased technical progress in a model with homogenous labour. The second set of simulations then analyses skill-biased technical progress in a closed economy. In the third part, we introduce trade with a second country and show simulations of trade and international technology spillovers.

#### 3.1 Technological progress in a closed economy

##### 3.1.1 Assumptions

The first simulation shows the effect of exogenous technological progress in a closed economy. For simplicity we assume the following particular parameters. The technology matrix is

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 0.40 & 0.10 \\ 0.10 & 0.40 \end{pmatrix}$$

This means that the technology matrix is symmetric<sup>15</sup> and half of the output of period  $t$  must be reinvested to ensure the reproduction of the system. Labour productivity is equal in both sectors

$$a_i = 1$$

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<sup>14</sup>The model was written and simulated in DMC (Medio, 1992). A Runge-Kutta algorithm is used for numerical simulations.

<sup>15</sup>This rather specific assumption facilitates the interpretation of the dynamics below but does not restrict the generality of the model.

Further wages are also equal and set to

$$w_i = 1$$

Given that the mark-up<sup>16</sup> is  $\pi = 0.0$  this gives equilibrium prices

$$p_i = 2$$

The implication of this assumption is that the economy does not grow over time, as the economy is on the maximum level of consumption and thus the economy exactly reproduces itself. Investment and thus output growth in the economy only occur if there is a deviation of unit costs to prices, meaning that real wages are below their maximum value. In the simulation below, growth can thus only occur due to (exogenous) technological progress (modeled as reduction in labour input coefficients). Further we assume that the nominal shares for investment and consumption are  $\alpha_i = 0.5$  and  $\beta_i = 0.5$ . The long run equilibrium output structure is then  $\frac{q_1}{q_2} = 1$ . I assume that the starting values are  $q_i = 1$ . In equilibrium thus  $0.5q_i$  has to be reinvested for reproduction of the system. The wage demand is - given the parameters and prices -  $0.5q_i$  in each sector.

The parameters and starting values used in the simulation below are summarised in Tables 3.1 and 3.2, respectively. The simulation run we present below starts from this long-term

Parameter	Values	
	Sector specific	
	Sector 1	Sector 2
$a_{ii}$	0.400	0.400
$a_{ij}$	0.100	0.100
$\pi_i$	0.000	0.000
$\delta_{p_i}$	0.250	0.250
$\delta_{q_i}$	0.250	0.250
$\kappa_{s_i}$	0.100	0.100
$\delta_{\beta_i^*}$	1.000	1.000
$\alpha_i$	0.500	0.500
$\bar{a}_{li}$	0.500	0.500
$g_{a_{li}}$	-0.025	0.000
	Economy wide	
$\delta_{L^S}(L^S \leq L^D)$	1.000	
$\delta_{L^S}(L^S > L^D)$	0.000	
$\kappa_u$	-0.100	
$\kappa_w$	-0.100	

Table 3.1: Parameter values used in simulations

equilibrium, but in sector 1 occurs exogenous technological progress. As mentioned above this is implemented as

$$\dot{a}_{l1} = g_{a_{l1}}(a_{l1} - 0.5)$$

Thus the labour input coefficients diminishes to a level of  $\bar{a}_{l1} = 0.5$  with a decreasing growth rate, as  $\dot{a}_{l1} \rightarrow 0$  for  $a_{l1} \rightarrow \bar{a}_{l1}$ . Figure 3.1 shows the time trajectories of the labour input coefficients. The labour input coefficient is gradually falling in sector 1 to a level of  $a_{l1} = 0.5$

<sup>16</sup>This special assumption on the value of the long-term mark-up should be interpreted as modeling the evolution of the economy along a long-term growth path and the deviations from it in the case of exogenous shocks (technological progress or trade liberalisation).

Variable	Values	
	Sector specific	
	Sector 1	Sector 2
$a_{i1}$	1.000	1.000
$w_i$	1.000	1.000
$L_i^D$	1.000	1.000
$\omega_i$	1.000	1.000
$p_i$	2.000	2.000
$c_i$	2.000	2.000
$r_i$	0.000	0.000
$s_i$	0.000	0.000
$q_i^I$	0.000	0.000
$q_i^C$	0.500	0.500
$q_i$	1.000	1.000
	Economy wide	
$L^D$	2.000	
$L^S$	2.000	
$u$	0.000	
$g$	0.000	
$P^C$	2.000	

Table 3.2: Starting values used in simulations

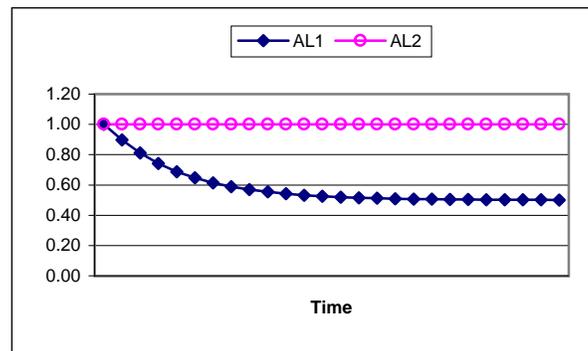


Figure 3.1: Labour input coefficients

but is staying constant in sector 2. Thus technical progress is sectorally biased.

The falling labour input coefficient  $a_{l1}$  has an impact on labour unit costs in industry 1 and thus an effect on costs and prices in sector 1, and via the input-output matrix  $\mathbf{A}$  also on sector 2. The impact of wage rate movements on costs and prices are discussed below. Figure 3.2 presents the resulting time trajectories for prices and unit labour costs. Unit labour costs

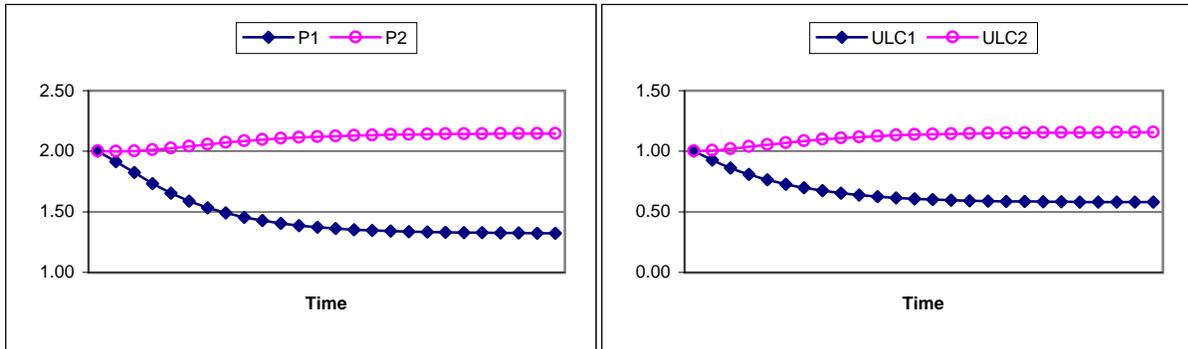


Figure 3.2: Prices and unit labour costs

in sector 1 are falling to a level of less than 0.5. In sector 2 unit labour costs are falling less and even rising in the later periods. This rise is mainly due to the equalisation of wages across sectors. This dynamics result in changes in relative prices. Price  $p_1$  is falling much faster and to a lower level than the price in sector 2,  $p_2$ , meaning that the goods of sector 1 are becoming relatively cheaper.

But, as mentioned above, prices do not adjust immediately to unit costs. In the transition unit rents are larger than zero, which are partly distributed to wages in the respective sector ( $\kappa_{s_i} > 0$ ), and on the other hand are reinvested according to the formulas given above. The evolution of the rents can be seen in Figure 3.3. In sector 1 rents are rising rapidly due to the

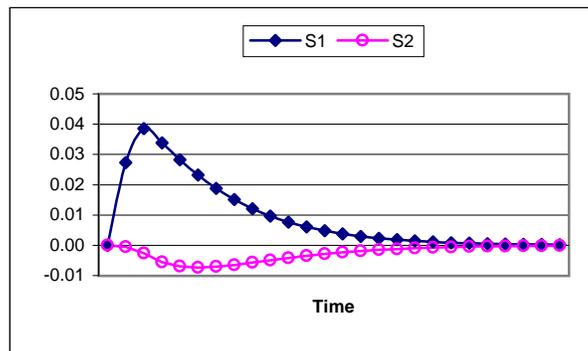


Figure 3.3: Rents

effects of technological progress and sluggish adjustment of prices and wage rates, which are discussed below. In sector 2 transitory rents even become negative. The reason for this is that wages in sector 2 are rising because of the wage equalisation across sectors ( $\kappa_w < 0$ ). This raises wage rates in sector 2 and thus unit labour costs, which together with sluggish price adjustment

leads to negative transitory rents.

The rents have an impact on sectoral wages (together with the unemployment rate and the wage equalisation). Further, the consumer price index  $P^C$  together with the nominal wage rates are important indicators for welfare improvements. The evolution of wage rates and the consumer price index can be seen in Figure 3.4. Wage rates in sector 1 are increasing due to the

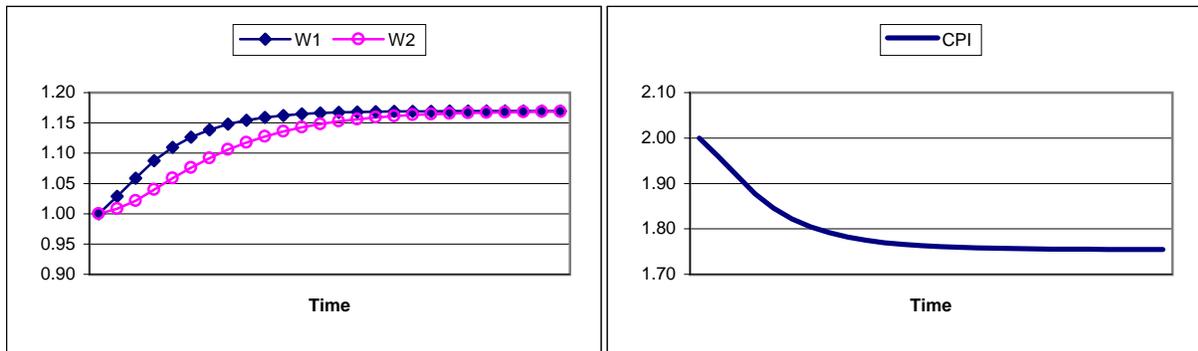


Figure 3.4: Wages and consumer price index

relatively high transitory rents. Wage rates in sector 2 are also increasing but more slowly (as there are no or even slightly negative transitory rents). Finally, the unemployment rate has an impact on wage rates in both sectors. As wage rates in both sectors are effected symmetrically this does not have an impact on relative wages, but delays adjustment of wage rates, keeps the transitory rents higher and thus raising investment and the overall growth rate of the economy.

The dynamics of sectoral and aggregate labour demand is shown in Figure 3.5. Labour

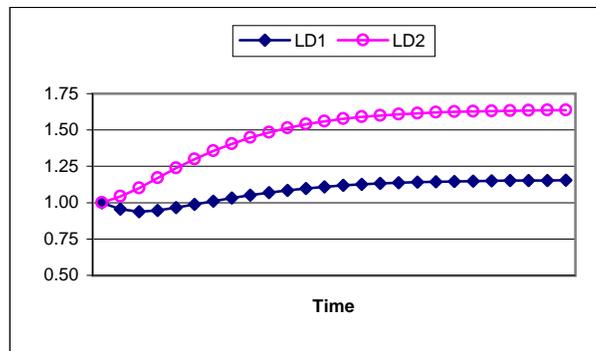


Figure 3.5: Labour demand

demand in sector 1 is slightly falling in the first phase, but then starts rising due to the overall growth of the economy and the falling speed of technological progress. Labour demand in sector 2 is rising. This results in a higher level of overall employment and an increase in relative employment in sector 2.

The dynamics of labour demand results from the dynamics of labour input coefficients and output. On the other hand, the dynamics of output and the output structure is determined by consumer behaviour and investors behaviour: Consumers demand relatively more of good

1, which has become relatively cheaper (substitution effect). The evolution of output and the overall growth rate can be seen in Figure 3.6. Output is rising faster and achieves a higher level

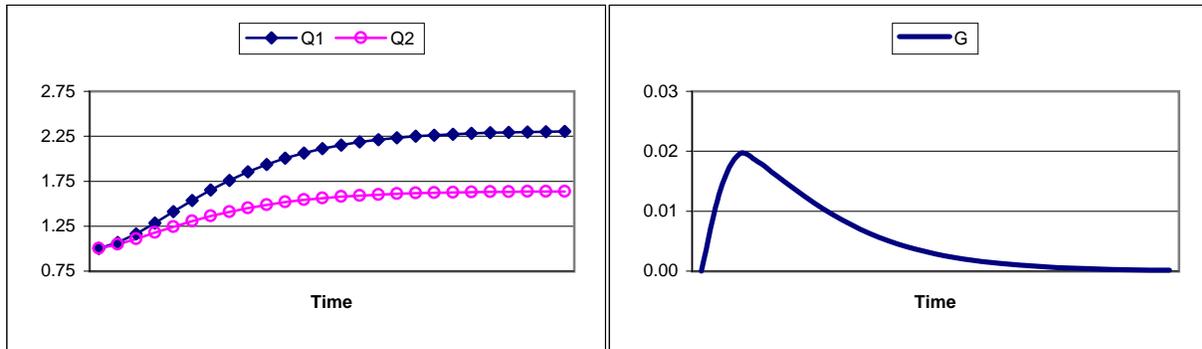


Figure 3.6: Output

in sector 1 in the new equilibrium than output in sector 2. The reason for this is the substitution effect in the demand for the goods: as goods of sector 1 are becoming relatively cheaper, demand is rising relatively more and thus output grows faster. In this simulation, the effect of the falling labour input coefficients on labour demand is stronger than the effect of output growth and demand substitution in the initial phase leading to a temporary job-loss in sector 1.

## 3.2 Skill biased technological progress in a closed economy

### 3.2.1 Assumptions

The second simulation captures the effects of skill labour-biased technical progress, which is seen to be the main source of the worsened unemployment position of unskilled workers in the recent debate. The simulation is started at the values given in 3.3. As one can see from the starting values of the labour input coefficients, sector 1 is the skill-intensive sector. As wage rates for skilled workers are relatively higher than the wage rates for unskilled workers, this leads to a relative higher price of good 1 and less demand relative to good 2. As the technology matrix is symmetric, the relative output of good 1 is smaller than 1, due the substitution effect in the formulation of consumers demand. The parameter values are given in Table 3.4. The most important assumption here is that the technological progress is biased against the unskilled workers and identical for both sectors. The input coefficient  $a_{li}$ ,  $i = 1, 2$ , is falling from 2.5 to a level of 2 with a rate of  $g_{a_{li}} = -0.015$ . The labour input for the skilled workers are constant. This special assumption means that technological progress effects both sectors in the same way. Differences thus arises only from the variation in relative skill-intensities and the wage dynamics in both sectors.

### 3.2.2 Simulation results

We shall now discuss the most interesting features of this factor biased technical progress. Figure 3.7 shows the trajectories of the labour input coefficients. These are falling in both sectors to a level of  $a_{li}^u = 2.000$ . The resulting dynamics of prices and relative prices can be seen in Figure 3.8. Unit labour costs and prices are falling in both sector. But the relative price of good 1

Variable	Values			
	Sector and skill specific			
	Sector 1		Sector 2	
	Skilled	Unskilled	Skilled	Unskilled
$a_{li}^z$	1.000	2.500	0.500	2.500
$w_i^z$	1.000	0.200	1.000	0.200
$L_i^{D^z}$	1.000	2.500	0.614	3.068
	Sector specific			
	Sector 1		Sector 2	
$\omega_i$	1.500		1.000	
$p_i$	2.857		2.143	
$c_i$	2.857		2.143	
$r_i$	0.000		0.000	
$s_i$	0.000		0.000	
$q_i^I$	0.000		0.000	
$q_i^C$	0.477		0.636	
$q_i$	1.000		1.227	
	Economy wide, skill specific			
	Skilled		Unskilled	
$L^{D^z}$	1.614		5.568	
$L^{S^z}$	1.614		5.568	
$u^z$	0.000		0.000	
	Economy wide			
$g$	0.000			
$p^C$	2.500			

Table 3.3: Starting values used in simulations

Parameter	Values			
	Sector specific			
	Sector 1		Sector 2	
$a_{ii}$	0.400		0.400	
$a_{ij}$	0.100		0.100	
$\pi_i$	0.000		0.000	
$\delta p_i$	0.100		0.100	
$\delta q_i$	0.500		0.500	
$\kappa s_i$	0.100		0.100	
$\delta \beta_i^*$	0.100		0.100	
$\alpha_i$	0.500		0.500	
	Sector and skill specific			
	Sector 1		Sector 2	
	Skilled	Unskilled	Skilled	Unskilled
$\bar{a}_{li}^z$	1.000	2.000	0.500	2.000
$g a_{li}^z$	0.000	-0.015	0.000	-0.015
	Economy wide, skill specific			
	Skilled		Unskilled	
$\delta_{L_{S^z}}(L^{S^z} < L^{D^z})$	1.000		1.000	
$\delta_{L_{S^z}}(L^{S^z} > L^{D^z})$	0.000		0.000	
$\kappa u^z$	0.075		0.075	
$\kappa w^z$	0.010		0.010	

Table 3.4: Parameter values used in simulations

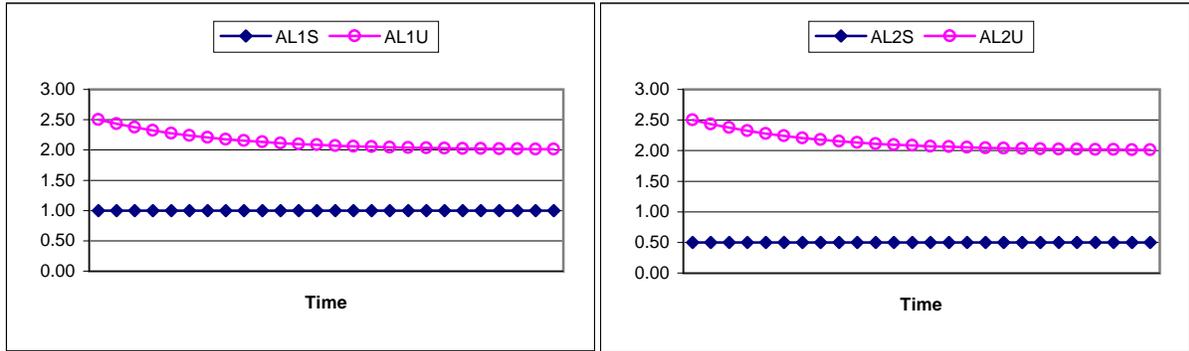


Figure 3.7: Labour input coefficients

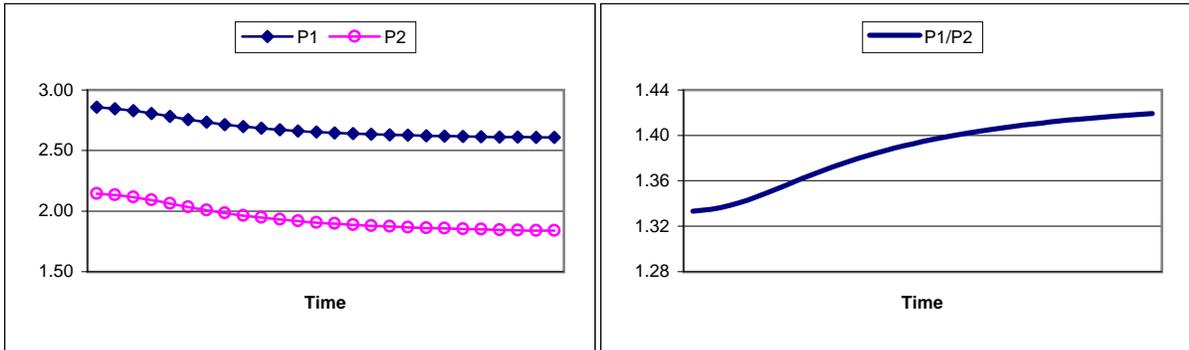


Figure 3.8: Prices and unit labour costs

(the skill intensive good),  $p_1/p_2$  is rising for two reasons: First, the share of unskilled workers in total unit labour costs is higher in sector 2 and thus unskilled labour-biased technical progress has a larger impact in this sector. Second, wages of unskilled workers are more under pressure, as there is unemployment for unskilled workers, but not for skilled ones, as we shall see below. This leads to a wage dynamics which is also in favour of the skilled workers, thus enhancing the effects on relative unit labour costs.

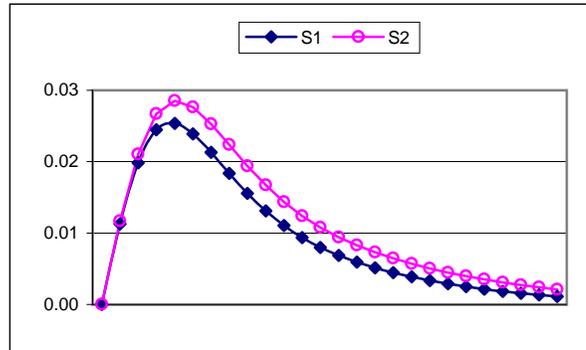


Figure 3.9: Rents

Rents (Figure 3.9) are rising and then falling gradually to a zero level due to the falling labour input coefficients and sluggish adjustment of prices and wage rates. Rents in sector 2, which is the unskill-intensive sector, are being higher than in sector 1.

The outcome on the labour market, which is now divided into skilled and unskilled workers, are shown in Figures 3.10 and 3.11, which present the trajectories for wages and labour demand. Wage rates for skilled workers (Panel A in Figure 3.10) are increasing in both sectors, first

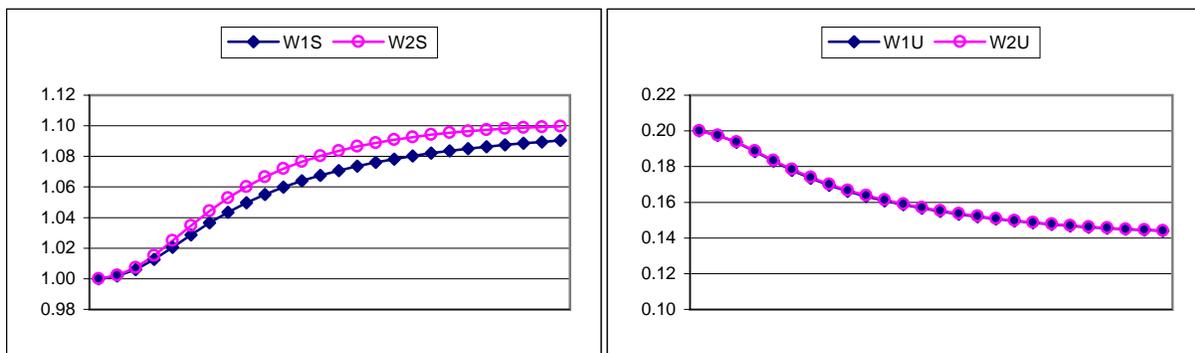


Figure 3.10: Wage rates

because there are positive rents, which are partly distributed to workers, and second, labour demand is rising for skilled workers (as will be discussed below). The shortage of skilled labourers have a slightly positive impact on the wage rates for skilled workers. The labour market situation for unskilled workers is worsening. As one can see (Panel B in Figure 3.10) the wage rates of unskilled workers are falling to a lower level. Although there are positive rents, which are also distributed to the unskilled workers, this effect is - given the parameter values - smaller than the

effect of the unemployment rate, which arises due the unskilled labour-biased technical progress.

Figure 3.11 shows the time trajectories for employment levels in both sectors and for both types of workers, the skill-specific unemployment rates and the relative labour demand. First,

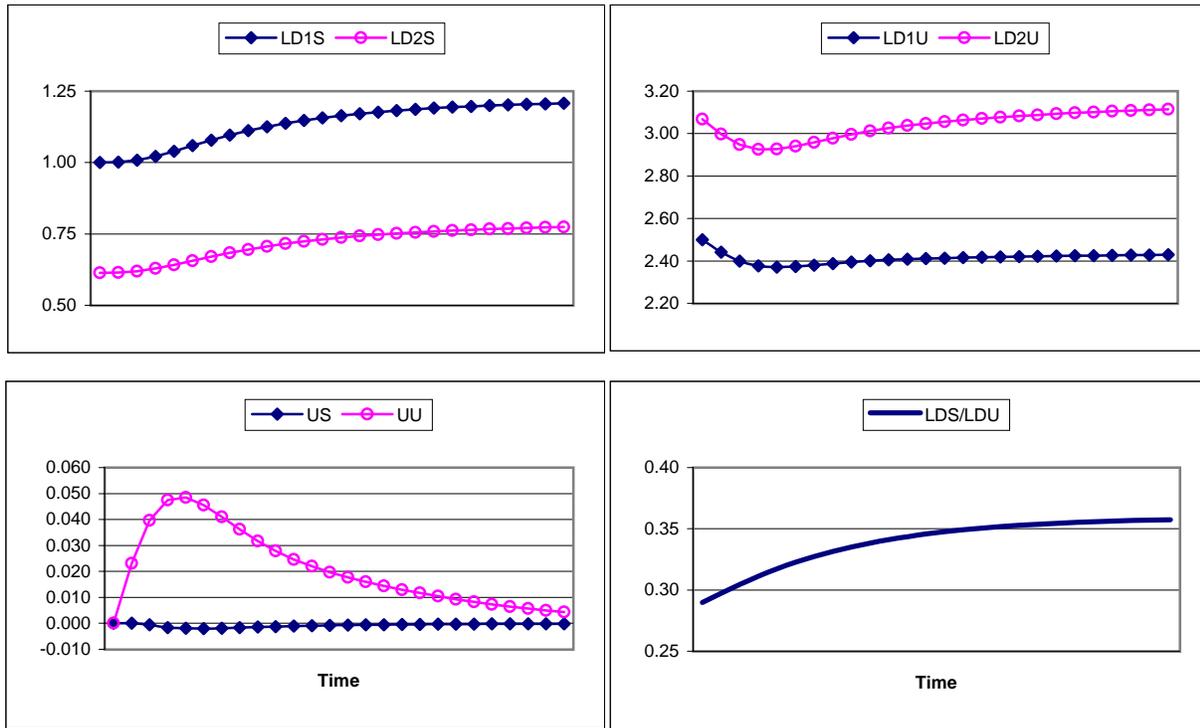


Figure 3.11: Labour demand

labour demand for the skilled workers is rising in both sectors. As we shall see below the growth rate of the economy becomes positive and thus raises the demand for skilled workers, together with the fact that for this group does not occur any labour-saving technical progress. Labour demand for skilled workers is rising relatively more in sector 2 than in sector 1, as demand shifts to good 2. Further, demand for unskilled workers is first falling (the effect of the technical progress is stronger the effect of the overall growth of the economy), but then starts rising, as the technological progress is becoming slower and thus the overall growth rate of the economy leads to a positive growth rate of the demand for unskilled workers. Further the overall growth of the economy due to investments is too low, to boost demand for unskilled workers enough. Thus in this model wage adjustment does not cure the labour market situation for unskilled workers.<sup>17</sup> Overall, relative demand for skilled workers increases mainly due to the unskilled labour-biased technical progress. Thus in this simulation the effect of the demand shift to the unskill-intensive good is not large enough to counteract the effect of the technical progress.

This shift in the output structure is shown in Figure 3.12, which shows the effects on output and the dynamics of the growth rate. Output in sector 2 is rising faster and thus increasing relative to the output in sector 1, as the price of good 2 becomes relatively cheaper, which leads

<sup>17</sup>It should be noted, however, that in this model, first, we do not assume factor substitution, thus increasing demand for unskilled workers, as they become relatively cheaper, and second, there are no workers leaving the labour market.

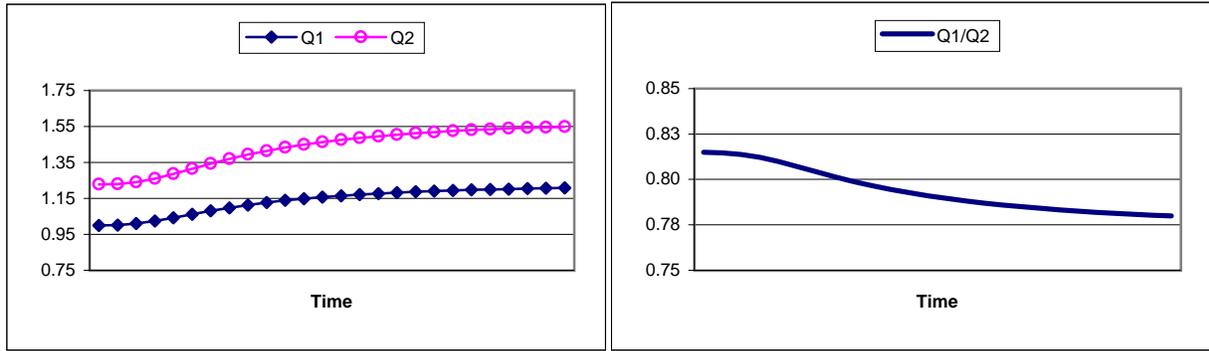


Figure 3.12: Output

to a substitution effect on the consumer side. Further the growth rate is first rising, but then gradually falling to a zero level, similarly to the simulation in Section 3.1 above. Finally, Figure 3.13 shows the trajectories of the overall growth rate  $g$  and the consumer price index  $P^C$ , which is falling to a lower level.

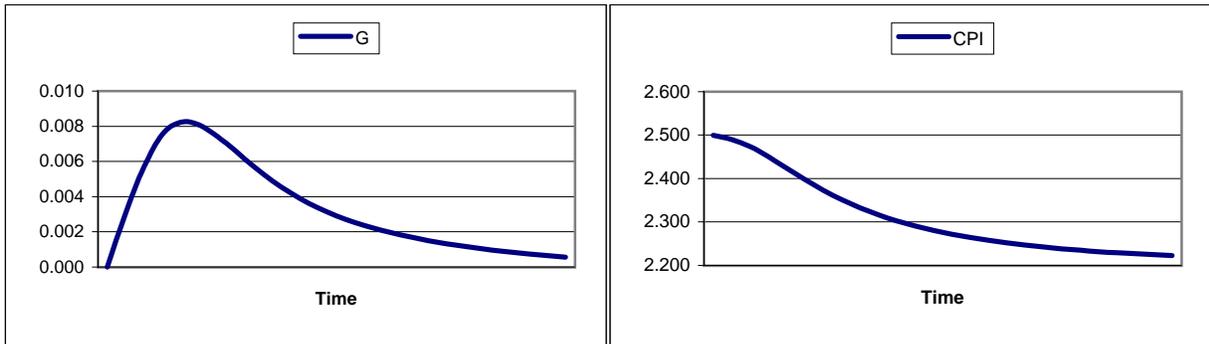


Figure 3.13: Growth rate and consumer price index

### 3.3 Effects of international trade and catching-up

In this section we now study the dynamics of two interacting economies. This is done in the following way: The more advanced country (country A) is characterised by the same parameter and starting values as in Section 3.2 above. Further the catching-up economy (country B) has also the same parameter values (with exception to one as discussed below) but different starting values which will be discussed below. The parameter of international linkages have also to be specified.

#### 3.3.1 The assumptions

Tables 3.5 and 3.6 summarise the assumptions made for both countries for parameter and starting values. As mentioned above the parameter values are the same as in Section 3.2. Specifically we assume that both countries are equal in every respect with exception of the

Parameter	Country A				Country B			
	Sector specific				Sector specific			
	Sector 1		Sector 2		Sector 1		Sector 2	
$a_{ii}$	0.400		0.400		0.400		0.400	
$a_{ij}$	0.100		0.100		0.100		0.100	
$\pi_i$	0.000		0.000		0.000		0.000	
$\delta_{p_i}$	0.100		0.100		0.250		0.250	
$\delta_{\bar{p}_i}$	0.010		0.010		0.150		0.150	
$\delta_{q_i}$	0.500		0.500		0.500		0.500	
$\kappa_{s_i}$	0.100		0.100		0.100		0.100	
$\delta_{\beta_i^*}$	1.000		1.000		1.000		1.000	
$\alpha_i$	0.500		0.500		0.500		0.500	
$\mu_i^{s^r}$	0.250		0.250		0.250		0.250	
$\nu_i^{s^r}$	0.250		0.250		0.250		0.250	
	Sector and skill specific				Sector and skill specific			
	Sector 1		Sector 2		Sector 1		Sector 2	
	Skilled	Unskilled	Skilled	Unskilled	Skilled	Unskilled	Skilled	Unskilled
$\bar{a}_{ii}^z$	1.000	2.000	0.500	2.000	1.000	2.000	0.500	2.000
$g_{a_{ii}^z}$	0.000	-0.015	0.000	-0.015	-0.015	-0.015	-0.015	-0.015
	Economy wide, skill specific				Economy wide, skill specific			
	Skilled		Unskilled		Skilled		Unskilled	
$\delta_{L_{S^z}}(L^{S^z} \leq L^{D^z})$	1.000		1.000		1.000		1.000	
$\delta_{L_{S^z}}(L^{S^z} < L^{D^z})$	0.000		0.000		0.000		0.000	
$\kappa_{uz}$	0.075		0.075		0.075		0.075	
$\kappa_{wz}$	0.010		0.010		0.010		0.010	

Table 3.5: Parameter values used in simulations

influence of the average world price  $\bar{p}_i$  on national prices. Here we assume that the less advanced country adjusts to world prices (or average prices  $\bar{p}_i$ ) quite fast with  $\delta_{\bar{p}_i}^B = 0.15$  whereas the prices of the more advanced countries are less influenced ( $\delta_{\bar{p}_i}^A = 0.01$ ).<sup>18</sup> The parameter values for the international linkages are specified with  $\mu_i^s$  for the shares of consumption expenditures spent abroad which are assumed to be equal across sectors and countries. The same assumption was made for investment expenditures  $\nu_i^{s^r}$ . There are some differences across countries and sectors with respect to the starting values. Here especially the assumptions on labour productivity and wages for the two skill-types of workers are relevant, as all other starting values are influenced by them. As above sector 1 is the more skill-intensive sector in both countries. Country *B* has lower productivity levels in both sectors,  $i = 1, 2$ . Specifically we assume that the skilled workers have equal labour productivity levels, whereas the labour input coefficient for unskilled workers in country *B* in sector 1 are three times higher and in sector 2 are two times higher than in country *A*. From this structure of labour input coefficients follows, that country *B* has a comparative advantage (in terms of productivity) in sector 2, the low-skill labour intensive sector.

Further we assume that the relative wages of skilled workers are lower in country *A* than in country *B*; the wage rates are equalised across sectors in both countries. This again leads to the effect that country *B* has a comparative advantage in sector 2.

<sup>18</sup>This assumption can be justified in the following way: First, one can observe quite rapid adjustment processes in quality levels (measured as export unit values) in advanced countries (see Stehrer and Landesmann, 1999). Second, the starting values are set in a way that both countries are of similar size. The larger value of  $\delta_{\bar{p}_i}^r$  for the less advanced country thus could also be seen as a parametrisation for different sized of countries. The less advanced country is more strongly influenced by the leader country (or the world market) than vice verse. Finally, the starting values could also be interpreted as undervaluation of the currency of country *B*, although we do not model exchange rate dynamics directly.

Variable	Country A				Country B			
	Sector and skill specific				Sector and skill specific			
	Sector 1		Sector 2		Sector 1		Sector 2	
	Skilled	Unskilled	Skilled	Unskilled	Skilled	Unskilled	Skilled	Unskilled
$a_{i1}^s$	1.000	2.500	0.500	2.500	1.000	7.500	0.500	5.000
$w_i^s$	1.000	0.200	1.000	0.200	0.600	0.100	0.600	0.100
$L_i^{D^s}$	1.000	2.500	0.614	3.068	1.000	7.500	0.650	6.501
	Sector specific				Sector specific			
	Sector 1		Sector 2		Sector 1		Sector 2	
$\omega_i$	1.500		1.000		1.350		0.800	
$p_i$	2.857		2.143		2.543		1.757	
$c_i$	2.857		2.143		2.543		1.757	
$r_i$	0.000		0.000		0.000		0.000	
$s_i$	0.000		0.000		0.000		0.000	
$q_i^I$	0.000		0.000		0.000		0.000	
$q_i^C$	0.477		0.636		0.470		0.680	
$q_i$	1.000		1.300		1.000		1.292	
	Economy wide, skill specific				Economy wide, skill specific			
	Skilled		Unskilled		Skilled		Unskilled	
$L^{D^s}$	1.614		5.568		1.650		14.001	
$L^{S^s}$	1.614		5.568		1.650		14.001	
$u^s$	0.000		0.000		0.000		0.000	
	Economy wide				Economy wide			
$g$	0.000				0.000			
$P^C$	2.500				2.000			
$b$	0.000				0.000			

Table 3.6: Starting values used in simulations

Given these assumptions the prices  $p_i^c$  can be derived. The relative price of good 1 (the skill intensive good) is lower in country *A* than in country *B*. Given the structure of consumption and investment the relative output of good 1 is relatively higher in country *A* than in country *B*. The absolute price level is lower in country *B* for both goods. Although this does not have an effect on the specialisation structures it will lead to a shift of aggregate demand to country *B* via the formulation of the expenditures abroad. Finally, this leads to the labour market outcome that in country *A* relatively more skilled workers are employed. As one can see, this structure of starting values captures Ricardian and factor endowments (or payments) characteristics, which are quite common in the literature.<sup>19</sup>

With respect to the evolution of labour productivity we assume that the less advanced country, country *B*, starts immediately with catching up to the labour productivity levels of the more advanced country, country *A*. The specific assumption of the dynamics of the labour input coefficients implies, that convergence is relatively faster in sector 1 than in sector 2.<sup>20</sup> Further there is exogenous technical progress in country *A*, which is biased against the low-skilled workers. The resulting implications for the other variables as prices, output structure, wage rates, and so on can be seen and are discussed below.

### 3.3.2 Simulation results

In the following we shall now present the following stylised scenario: Both economies, which are starting from long-term autarkic equilibria, are 'shocked' by a sudden trade liberalisation. At

<sup>19</sup>Here one has to mention that in this model this structure is not derived by the assumption of relative factor endowments but rather assumed as a 'stylized' fact.

<sup>20</sup>For empirical evidence of this pattern of catching-up see Landesmann and Stehrer (2000b).

the same time technological progress occurs as describe above.

The evolution of the labour input coefficients in both countries is drawn in Figure 3.14. The skill-biased technical progress in country A is equal in both sectors and has been discussed

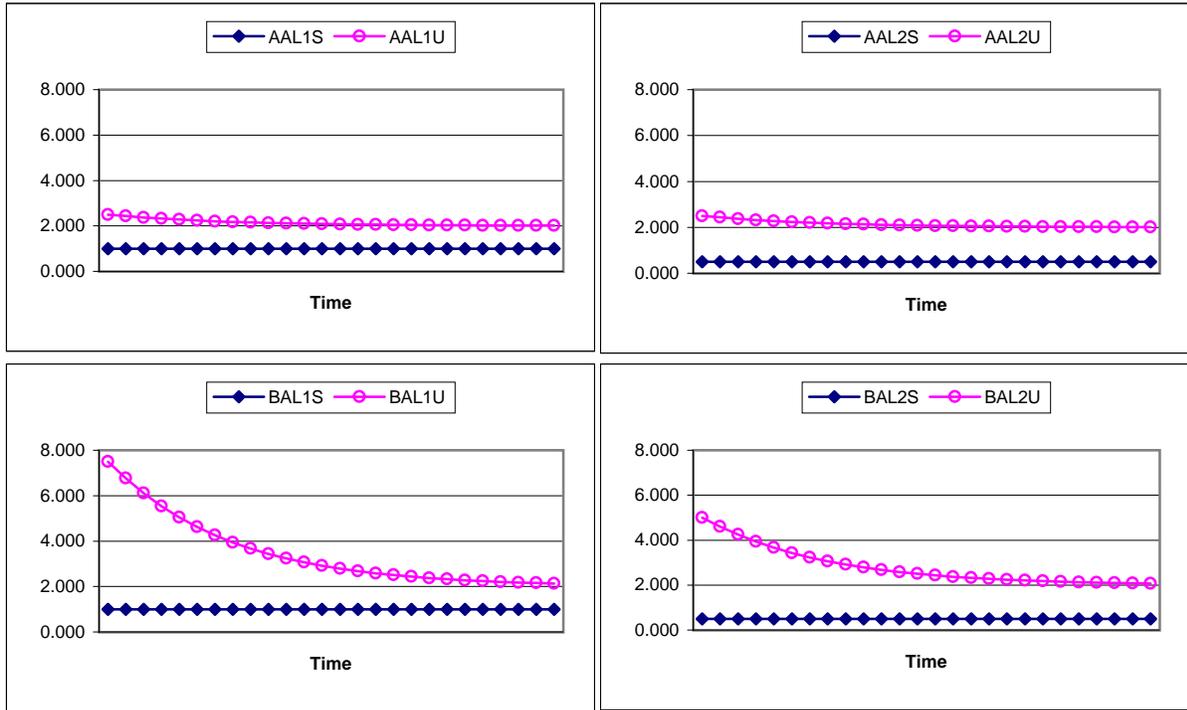


Figure 3.14: Labour input coefficients

in Section 3.2. The impact on unit labour costs is thus larger in sector 2, as this is the low-skill intensive sector. In country B technical progress (or convergence) is also biased against the low-skilled workers and is faster in sector 1, the skill-intensive sector, as the initial gap is larger than in sector 2. In this sense we have factor-biased technical change and simultaneously sector-biased technical change in country B.

From this evolution of labour productivity and relative wage dynamics, which is discussed below, results the dynamics of prices presented in Figure 3.15. The absolute price level in both sectors in country A is falling due to the effect of changes in labour input coefficients and the incomplete nominal wage rate adjustment. Further there is a small impact of the lower prices of country B on prices in country A. The relative price  $p_1/p_2$  in country A is increasing (thus the relative price of the skill-intensive good is increasing). The reasons for this (biased technical change, sectoral skill intensities, and wage dynamics) has already been discussed in Section 3.2 above and should not be repeated here. Price levels in country B are first rising due to the adjustment to the higher average world price levels ( $\delta_{\bar{p}_i}^B = 0.15$ ), which is assumed to be quite strong in the less advanced country. But then the prices start falling due to the rapid technical progress. As technical progress is faster in sector 1 (the skill-intensive sector with the higher scope for catching-up), the relative price of good 1 is declining. In the longer run the relative price of good 1 is increasing as the price levels are equalising across countries (by assumption of the price adjustment processes). This goes in hand with the increasing relative wages of skilled

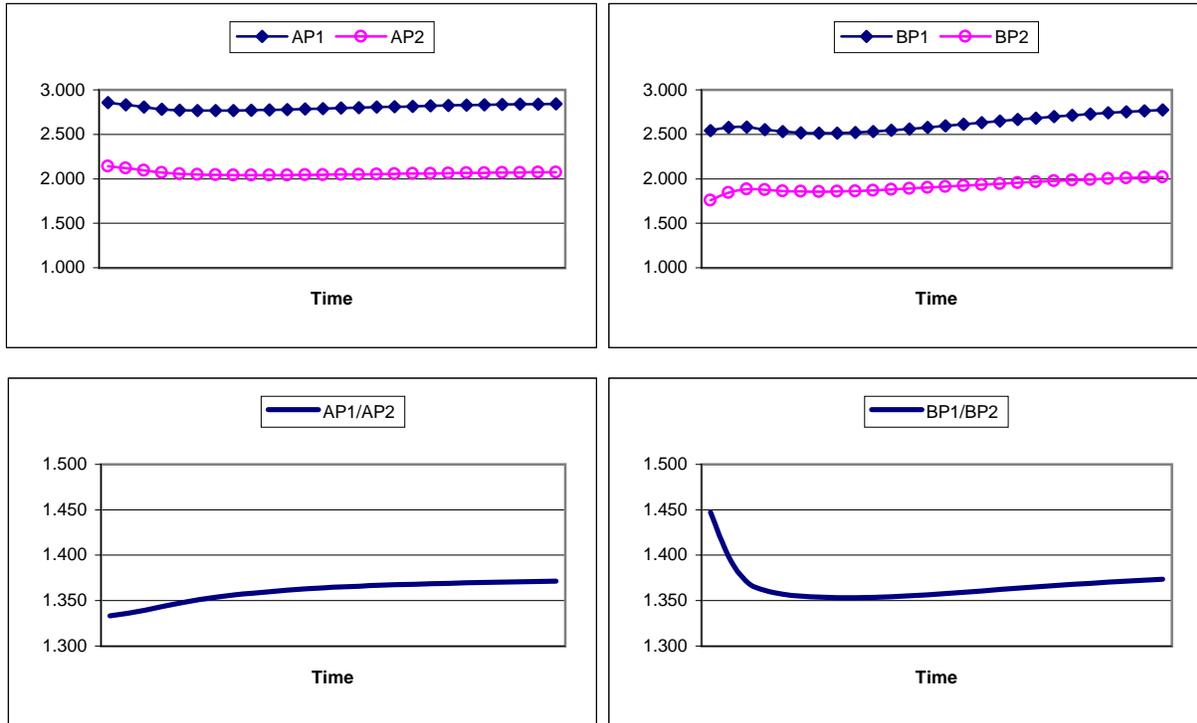


Figure 3.15: Prices

workers (which has relatively more impact on the skill-intensive sector 1) and the vanishing impact of technical progress when approaching the technological frontier.

The evolution of the consumer price index in both countries can be seen in Figure 3.16. The CPI is falling in country A in the initial phase but then starts rising due to the dynamics of the price levels in country B. In country B the CPI is first rising sharply, as nominal prices adjust quite rapidly to the price level of country A ( $\delta_{p_i}^B = 0.15$ ). Then it declines slightly and finally adjusts gradually to the CPI of country A as price levels adjust in the long run.

Before studying the labour market effects, we show the evolution of the transitory rents in Figure 3.17. Rents are larger in country B as there the technical progress is faster (because of the advantage of technology catching-up) and the impact of the average world price tends to raise price levels in country B. In this model this is a second source of capturing rents, which means a redistribution of income from workers to investors. The difference of rents across sectors is very small. One would expect higher rents in sector 1, as there is more technical progress than in sector 2. But there are two other forces which raises rents in sector 2. First, the adjustment to world prices is higher in sector 2 due to the price structure at the beginning  $\frac{p_1^A}{p_1^B} < \frac{p_2^A}{p_2^B}$  and thus raises rents in sector 2. Second, the relative wages of skilled workers are increasing which lowers rents in sector 1. Further rents are small and vanishing very soon in country A, as there is only small technical progress. Rents are becoming even slightly negative, first, because of the impact of world prices and, second, because scarcity in labour supply.

These rents have an impact on wages, which can be seen in Figure 3.18. The relative wages of skilled workers are increasing in both countries, although much more strongly in country B.

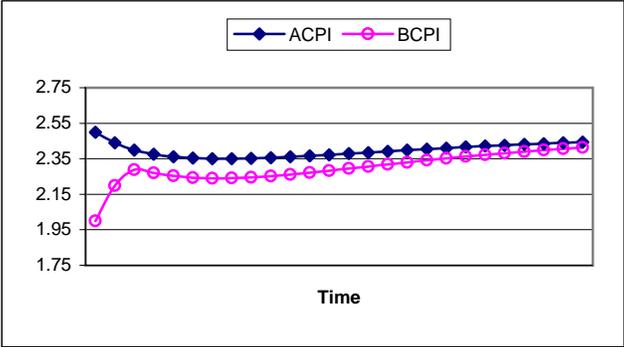


Figure 3.16: Consumer price index

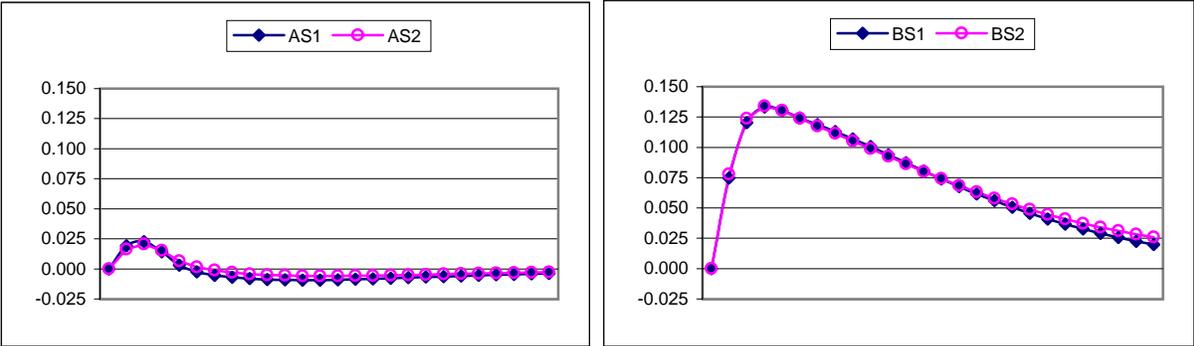


Figure 3.17: Rents

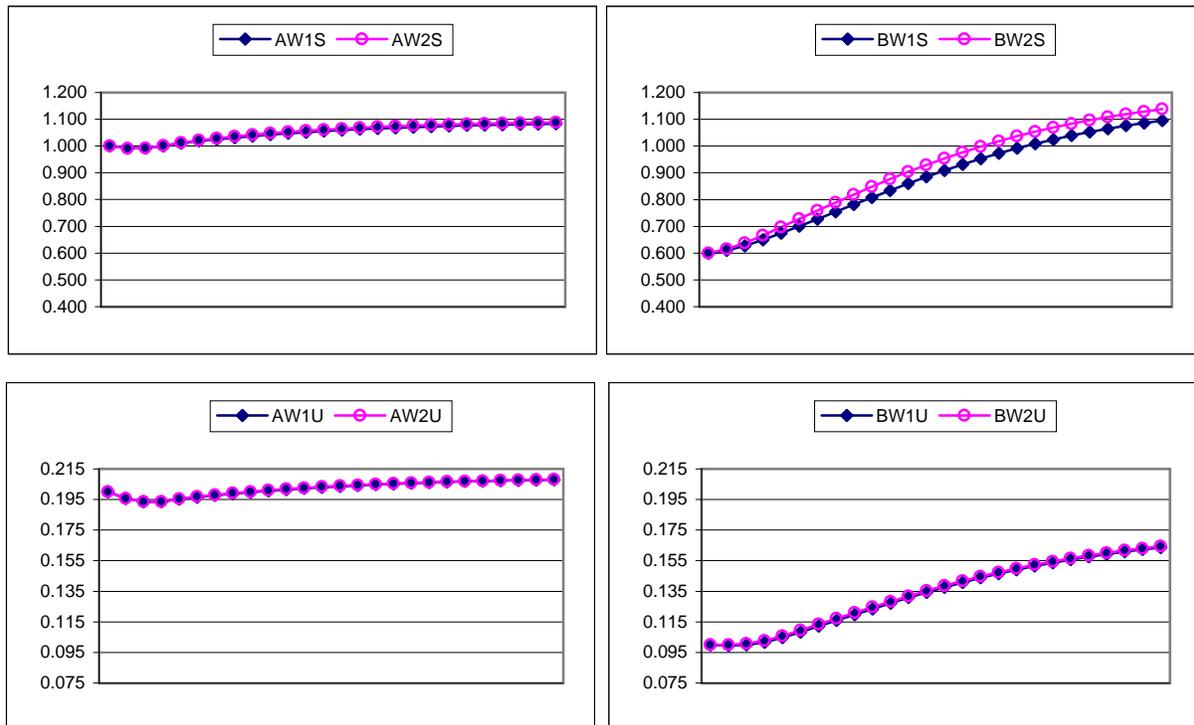


Figure 3.18: Wages

The reason for this is, that technical progress is biased against the low-skilled workers, which rises unemployment and thus depresses wages of these group of workers. In this simulation run the wage differentiation across sectors is not particularly strong as the rents are quite similar across sectors (see Figure 3.17).

The evolution of unemployment rates can be seen in Figure 3.19. The rise in unemployment rates in country *A* results from shifts of aggregate demand to country *B* as we assumed that country *B* has lower price levels in both goods. In the long run, however, the economies exhibit overall growth which then even leads to a shortage of labour supply.<sup>21</sup> There are also differences in the structure of unemployment in country *A* which is quite high for low-skilled workers. This high unemployment rates for low-skilled workers result from the biased technical change and the competition from country *B*, which is especially strong in the low-skill intensive sector 2 (at least in the initial phase). In country *B* there is only unemployment for the low-skilled workers due to the biased technical change. As there are high transitory rents, which raises the overall growth rate, the period of unemployment is relatively short in both countries. Here one has to note that the overall growth in country *A* is driven from rents in country *B*, as rents are vanishing in country *A* quite soon.

Labour demand for both skill types are plotted in Figure 3.20. In the long run, labour demand in both countries is rising for both skill types of workers, although there are negative short term effects. The relative employment of skilled to low-skilled workers is rising in both

<sup>21</sup>In the numerical simulation we set the adjustment parameter of labour supply to 1. Although this implies quite fast adjustment to labour demand, this adjustment is obviously too slow. As mentioned above, production is not restricted by this shortage but there is an impact on wages via the unemployment term.

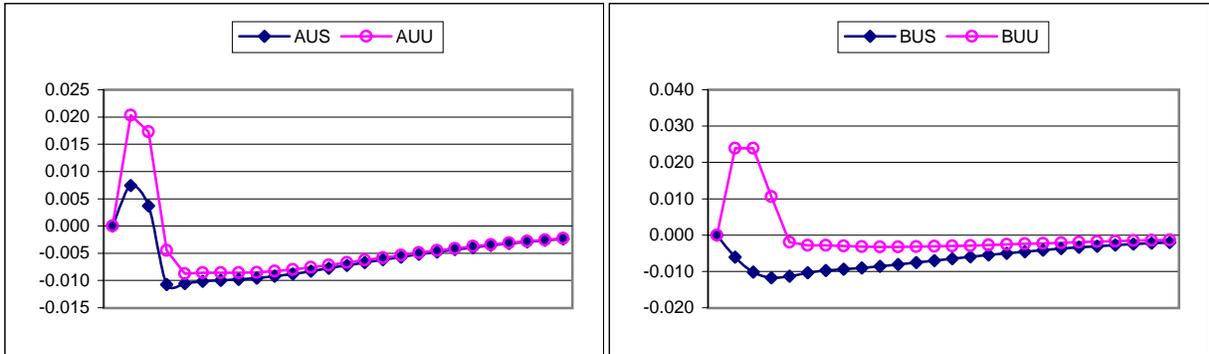


Figure 3.19: Unemployment

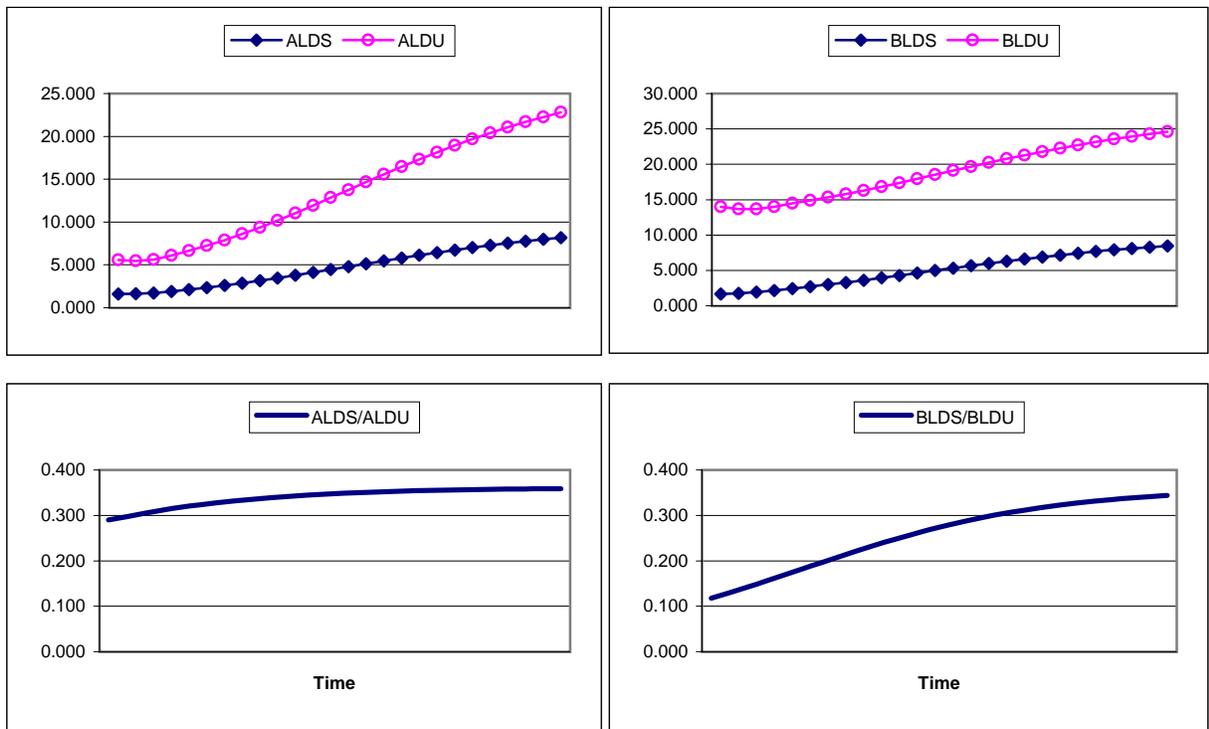


Figure 3.20: Labour demand

countries. In this model so far, this is only due to a substitution effect on the quantity (demand) side, as we leave out any kind of substitution on the production side (techniques of production).<sup>22</sup> As country *B* catches up fully to the labour productivity levels in country *A*, relative demand for skilled workers becomes equal in the long run, as although the output structure converges which is presented in see Figure 3.21. Output is growing in both economies over the long run

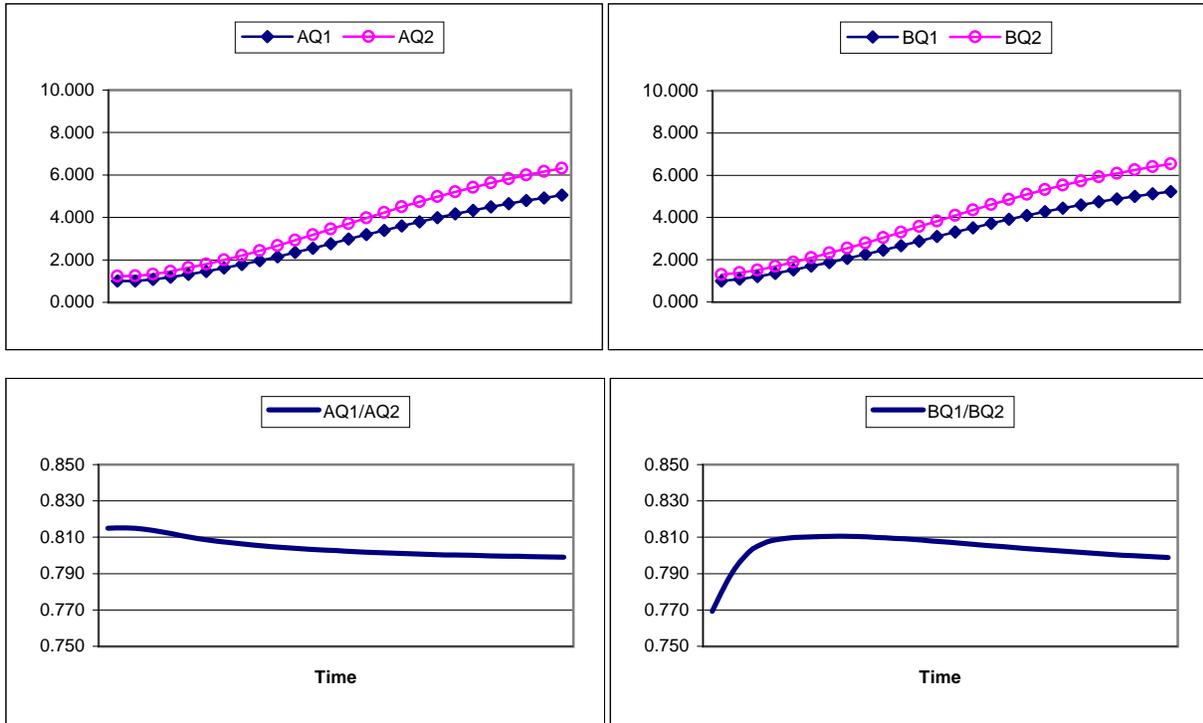


Figure 3.21: Output

due to investment out of the transitory rents. Relative output of good 1 in both countries is getting smaller, as this good is becoming relatively more expensive and thus (consumer) demand shifts to good 2. The relative output of good 1 in country *B* is even growing in the first phase, as the relative price of this good is getting smaller in this phase.

Finally we discuss the structure of trade and investment flows between the two countries. Figure 3.22 presents the volumes of trade and investment flows in quantity and value terms. Panel A shows the net exports of country *A* in both sectors. As the price level is lower in country *B* the net exports are negative in both sectors.<sup>23</sup> As country *A* has a comparative advantage in sector 1 (the skill-intensive sector) net exports are absolutely higher in sector 2. In values the net exports are equal as the Cobb-Douglas specification implies price elasticities of  $-1$ .

The capital flows are shown in the right hand panels of Figure 3.22. There are positive capital inflows in country *A*. This outcome, which is quite unrealistic, depends on our specific simple

<sup>22</sup>In fact a substitution of skilled workers due to the rising relative factor prices  $w^s/w^u$  would lower the increase in relative factor demand of skilled workers.

<sup>23</sup>Further the size of the two economies are very similar in terms of quantities and even in terms of gross domestic product.

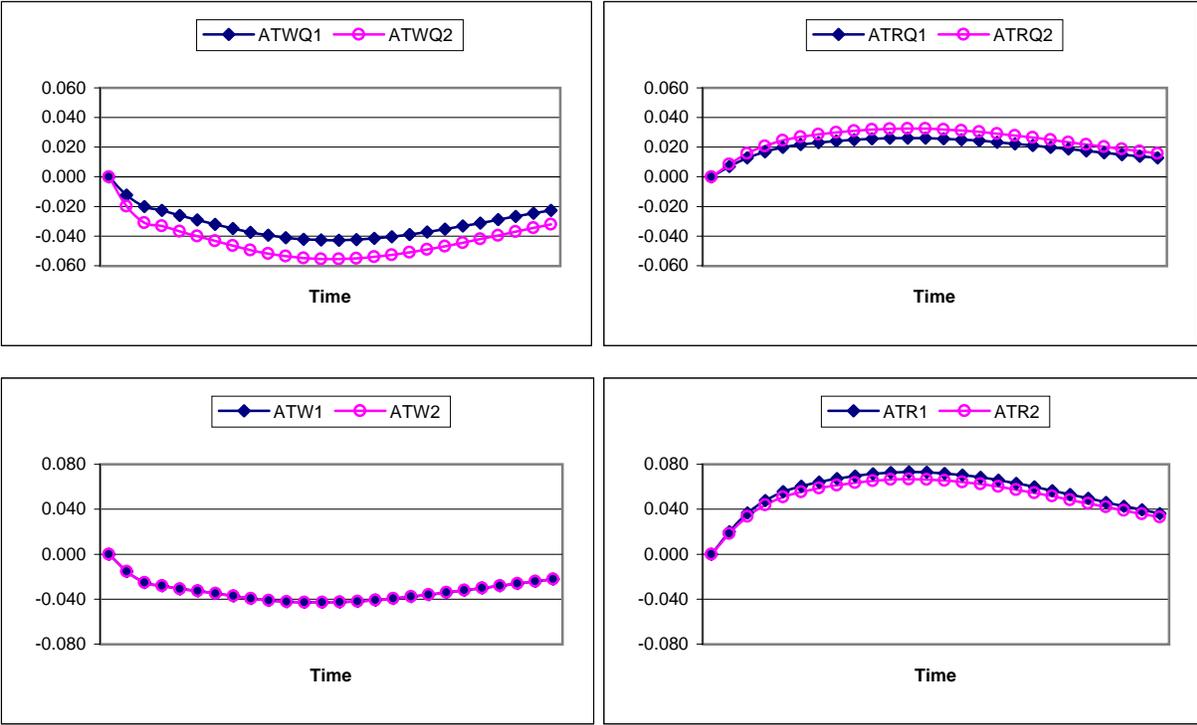


Figure 3.22: Trade and investment

assumptions of the equal nominal investment shares across countries: As rents are higher in country B than in country A the investments from country B in country A are higher. By the way, this is also the reason for the positive overall growth rate in country A.

Figure 3.23 then presents the trade and the capital account in values (Panel A) and the balance of payments in Panel B. The balance of payment is positive as the capital inflows in country A are higher than the (negative) net exports.

With these simulations which relies on quite unrealistic parameter values (especially for capital flows and the fixed exchange rate) we only want to show the properties of the model and we shall not discuss economic problems on this basis.

## 4 Conclusions

This paper presented a dynamic multisectoral model where trade liberalisation and (skill-biased) technical change implies changes in output and employment structures. As the results of the exogenous shocks depend on parameter values (and combinations of parameter values) the model may be used as a guideline for empirical research to discuss the relative strength of different effects and - what is the advantage of a model like this - the combination of parameter values (which are partly due to institutional settings) on the various variables.

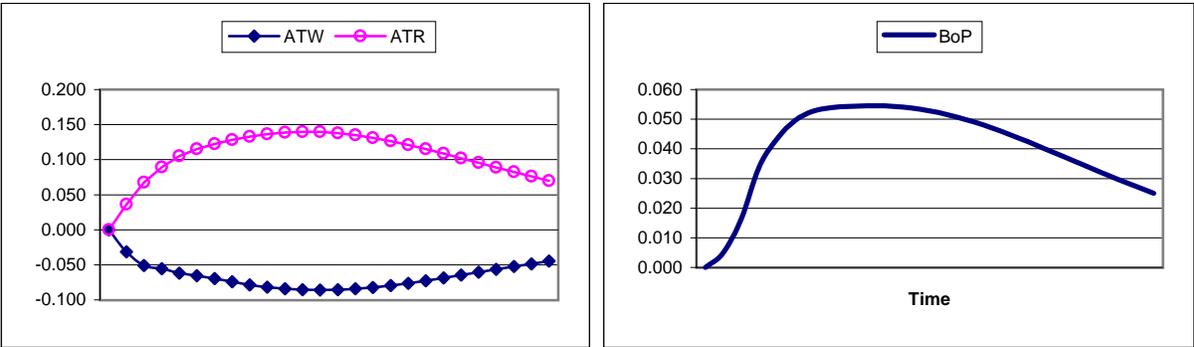


Figure 3.23: Balance of payment

## A Equilibrium, balanced and steady-state growth

In this section we shortly discuss the main properties of the model in the steady-state (balanced growth). Here we assume that the technology (input-output matrix) is given and the labour input coefficients are also fixed. Further we assume that wage rates  $w_i^z$  are set exogenously and constant. This is the case if, first, there are no transitory rents,  $\mathbf{s}' = \mathbf{0}$  (what will be the case when prices are in equilibrium), or the parameters  $\kappa_{s_i} = 0$  for all  $i$ , second, wages are equalised across sectors or  $\kappa_{w^z} = 0$ , and third, unemployment has no effect on wages, either because  $\kappa_{u^z} = 0$  or labour supply is perfectly elastic. We first discuss only the model for a closed economy and then show how the results can be applied to integrated economies.

### A.1 Closed economy

#### A.1.1 Technology

Technology is given and denoted by an input-output matrix

$$\mathbf{A} = \begin{pmatrix} a_{11} & \dots & a_{N2} \\ \vdots & \ddots & \vdots \\ a_{1N} & \dots & a_{NN} \end{pmatrix}$$

and labour productivity is given by a vector

$$\mathbf{a}_l = (a_{l1}, \dots, a_{lN})$$

#### A.1.2 Prices

The price system is modeled as a simple system of differential equations where prices adjust to unit costs.

$$\begin{aligned} \dot{\mathbf{p}}' &= (1 + \pi) (\mathbf{p}' \mathbf{A} + \boldsymbol{\omega}') - \mathbf{p}' \\ &= (1 + \pi) [\mathbf{p}' (\mathbf{A} - \mathbf{I})] + (1 + \pi) \boldsymbol{\omega}' \end{aligned} \quad (\text{A.1})$$

where  $\mathbf{p}'$  is a vector of prices,  $\mathbf{A}$  is the technology matrix, and  $\boldsymbol{\omega}' = (w_i a_{li}, \dots, w_N a_{lN})$  is a vector of unit labour costs for each industry. We assume that wages  $w_i$  and labour input coefficients  $a_{li}$  can be different across sectors. If there are more skill-types of workers the vector of unit labour costs is

$$\boldsymbol{\omega}' = \left( \sum_z w_i^z a_{li}^z, \dots, \sum_z w_N^z a_{lN}^z \right)$$

or with the assumption that skill-specific wages are equalised across sectors

$$\boldsymbol{\omega}' = \left( \sum_z w^z a_{li}^z, \dots, \sum_s w^s a_{lN}^s \right)$$

$\pi$  gives the long-run mark-up rate, which is assumed to be equal across sectors.<sup>24</sup> Setting  $\dot{\mathbf{p}}' = \mathbf{0}$  gives the equilibrium price vector

$$\mathbf{p}' = (1 + \pi) (\mathbf{p}' \mathbf{A} + \boldsymbol{\omega}')$$

---

<sup>24</sup>This assumption is not necessary from a technical point of view, although it is quite common in the literature where the equalisation of profits across sectors is assumed. Differences in the profitability of sectors in the model discussed in this paper may come from differences in transitory rents  $s_i$ .

which can be solved for

$$\mathbf{p}^{*'} = (1 + \pi)\boldsymbol{\omega}'[\mathbf{I} - (1 + \pi)\mathbf{A}]^{-1} \quad (\text{A.2})$$

It should be noted that solving the system of differential equation above directly yields the same result for exogenously given wages and the technology parameters.<sup>25</sup>

### A.1.3 Profits and rents

The per unit profits in each sector are defined as a mark-up on costs

$$\mathbf{r}' = \pi\mathbf{c}'$$

Further in disequilibrium there are rents which are defined as:

$$\mathbf{s}' = \mathbf{p}' - (1 + \pi)\mathbf{c}'$$

In equilibrium these rents are zero as one can show by inserting the equilibrium price vector:

$$\begin{aligned} \mathbf{s}' &= \mathbf{p}^{*'} - (1 + \pi)\left(\mathbf{p}^{*'}\mathbf{A} + \boldsymbol{\omega}'\right) \\ &= (1 + \pi)\boldsymbol{\omega}'[\mathbf{I} - (1 + \pi)\mathbf{A}]^{-1} - (1 + \pi)(1 + \pi)\boldsymbol{\omega}'[\mathbf{I} - (1 + \pi)\mathbf{A}]^{-1}\mathbf{A} - \\ &\quad (1 + \pi)\boldsymbol{\omega}'[\mathbf{I} - (1 + \pi)\mathbf{A}][\mathbf{I} - (1 + \pi)\mathbf{A}]^{-1} \\ &= (1 + \pi)\boldsymbol{\omega}'[\mathbf{I} - (1 + \pi)\mathbf{A}]^{-1} - (1 + \pi)(1 + \pi)\boldsymbol{\omega}'[\mathbf{I} - (1 + \pi)\mathbf{A}]^{-1}\mathbf{A} - \\ &\quad (1 + \pi)\boldsymbol{\omega}'[\mathbf{I} - (1 + \pi)\mathbf{A}]^{-1} + (1 + \pi)(1 + \pi)\boldsymbol{\omega}'[\mathbf{I} - (1 + \pi)\mathbf{A}]^{-1}\mathbf{A} \\ &= \mathbf{0} \end{aligned}$$

In the following we define a vector  $\mathbf{R}$  which adds up profits and rents

$$\mathbf{R}' = (\mathbf{r}' + \mathbf{s}') = \mathbf{p}' - \mathbf{c}'$$

For guaranteeing stability of wages we thus have to assume that  $\kappa_{s_i} = 0$  for all  $i$  if  $s_i > 0$ .

### A.1.4 The quantity system

Next we discuss the quantity system. Here we have to assume that  $\dot{\mathbf{p}} = \mathbf{0}$ . Thus, the results presented assume stable prices, although these need not be equilibrium prices.

Demand consists of three different components: First there is demand for intermediate goods used in production,  $\mathbf{A}\mathbf{q}$ , where  $\mathbf{q}$  denotes the vector of quantities. Second there is a matrix of demand from profits

$$\mathbf{D}_R\mathbf{q} = \begin{pmatrix} \beta_1 \frac{R_1}{p_1} & \cdots & \beta_1 \frac{R_N}{p_1} \\ \vdots & \ddots & \vdots \\ \beta_N \frac{R_1}{p_N} & \cdots & \beta_N \frac{R_N}{p_N} \end{pmatrix} \mathbf{q}$$

---

<sup>25</sup>For given wages and labour input coefficients this is a non-homogenous system of differential equations with a constant coefficient matrix, in general  $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{c}$ . The solution is given by  $\mathbf{x}^* = -\mathbf{A}^{-1}\mathbf{c}$ . Further the system is stable if the eigenvalues of the matrix  $(1 + \pi)[\mathbf{p}'(\mathbf{A} - \mathbf{I})]$  are negative. Given the assumptions on the technology matrix this is a stable system.

$\beta_i$  denotes the nominal share of profit expenditure in sector  $i$  with  $\sum_i \beta_i = 1$ , and  $r_i$  the per unit profit in each sector. Thus investment expenditures out of profits depend on nominal shares and (relative) prices. One has to note here, that the nominal shares  $\beta_i$  only describe the outcome of investment behaviour at the aggregate level.

The third source of demand comes from wage income. Consumption expenditures out of wages are denoted in matrix form

$$\mathbf{D}_W \mathbf{q} = \begin{pmatrix} \alpha_1 \frac{\sum_z w_1^z a_{11}^z}{p_1} & \dots & \alpha_1 \frac{\sum_z w_N^z a_{1N}^z}{p_1} \\ \vdots & \ddots & \vdots \\ \alpha_N \frac{\sum_z w_1^z a_{N1}^z}{p_N} & \dots & \alpha_N \frac{\sum_z w_N^z a_{NN}^z}{p_N} \end{pmatrix} \mathbf{q} = \begin{pmatrix} \alpha_1 \frac{\sum_z w_1^z L_1^{Dz}}{p_1} & \dots & \alpha_1 \frac{\sum_z w_N^z L_N^{Dz}}{p_1} \\ \vdots & \ddots & \vdots \\ \alpha_N \frac{\sum_z w_1^z L_1^{Dz}}{p_N} & \dots & \alpha_N \frac{\sum_z w_N^z L_N^{Dz}}{p_N} \end{pmatrix} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

$\alpha_i$  are the nominal shares in consumption with  $\sum_i \alpha_i = 1$ . The specific assumption in this formulation is that workers are maximising a Cobb-Douglas utility function, which is linear-homogenous and homothetic. This means that all workers have the same (constant) nominal shares of consumption. Again a more more general specification of the demand (e.g. dependent on real income levels and prices) could be used here. For given wage rates and prices the nominal shares  $\alpha_i^z(w_i^z, \mathbf{p})$  would then also be constant although differing across skill (and even in the case of wage differentiation sector specific) types of workers. A typical element in the matrix would then be  $\sum_z \alpha_{i,j}^z \frac{w_i^z a_{ij}^z}{p_j}$  where  $\sum_j \alpha_{i,j}^z = 1$ .

Total demand is the sum of these three components

$$\begin{aligned} \mathbf{q}^D &= \mathbf{A} \mathbf{q} + \mathbf{D}_R \mathbf{q} + \mathbf{D}_W \mathbf{q} \\ &= (\mathbf{A} + \mathbf{D}_R + \mathbf{D}_W) \mathbf{q} \end{aligned} \tag{A.3}$$

In equilibrium we must have  $\mathbf{q}^D = \mathbf{q}$  and thus the expression above must satisfy

$$(\mathbf{A} - \mathbf{I} + \mathbf{D}_R + \mathbf{D}_W) \mathbf{q} = \mathbf{0}$$

This is a linear-homogenous system which has a non-trivial solution,  $\mathbf{q} \neq \mathbf{0}$ , if

$$\det(\mathbf{A} - \mathbf{I} + \mathbf{D}_R + \mathbf{D}_W) = 0$$

The determinant of a matrix equals zero if the columns or rows are linearly dependent. Multiplying each item of a particular column in the matrix above by the relative price  $\frac{p_j}{p_n}$  and summing

over one particular column  $k$  of the matrix above gives

$$\begin{aligned}
& \sum_j a_{ji} \frac{p_j}{p_n} - \frac{p_k}{p_n} + \sum_j \beta_j \frac{R_k p_j}{p_j p_n} + \sum_j \alpha_j \frac{\omega_k p_j}{p_j p_n} = \\
& \frac{1}{p_n} \left( \sum_j p_j a_{ji} - p_k + \sum_j \beta_j R_k + \sum_j \alpha_j \omega_k \right) = \\
& \frac{1}{p_n} \left( \sum_j p_j a_{ji} - p_k + R_k \sum_j \beta_j + \omega_k \sum_j \alpha_j \right) = \\
& \frac{1}{p_n} \left( \sum_j p_j a_{ji} - p_k + R_k + \omega_k \right) = \\
& \frac{1}{p_n} \left( \sum_j p_j a_{ji} - p_k + \frac{\pi}{1+\pi} p_k + \frac{1}{1+\pi} p_k - \sum_j p_j a_{ji} \right) = \\
& \frac{1}{p_n} \left( \sum_j p_j a_{ji} - p_k + p_k - \sum_j p_j a_{ji} \right) = 0
\end{aligned}$$

This shows that there exists a non-trivial solution, i.e. an output vector  $\mathbf{q} \neq \mathbf{0}$ . Here, two important features should be mentioned: First, this condition is generally true as long as  $\sum_i \alpha_i = \sum_i \beta_i = 1$ . Thus the condition does not depend on a particular formulation of investment or consumption demand. The only necessary condition is, that the nominal shares resulting from the underlying decision structure sum to unity. Second, a solution exists also at non-equilibrium price vectors where investments out of rents then come from both profits and rents,  $r_i + s_i$ , respectively, if the condition of unity of nominal shares is satisfied.

The proof of existence of a non-trivial output vector which is stable (or growing at the same proportions) can be formulated in matrix notation. Premultiplying the quantity equation above with the price vector  $\mathbf{p}'$  gives

$$\mathbf{p}'(\mathbf{A} - \mathbf{I} + \mathbf{D}_R + \mathbf{D}_W) \mathbf{q} = 0$$

Rearranging gives

$$\mathbf{p}' \mathbf{D}_R \mathbf{q} - \mathbf{p}' [\mathbf{I} - (\mathbf{A} + \mathbf{D}_W)] \mathbf{q} = 0$$

Using

$$\mathbf{p}' \mathbf{D}_R = \left( \sum_i \beta_i R_1, \dots, \sum_i \beta_i R_N \right) = \mathbf{R}'$$

and

$$\mathbf{p}' \mathbf{D}_W = \left( \sum_i \alpha_i w_1 a_{i1}, \dots, \sum_i \alpha_i w_N a_{iN} \right) = \boldsymbol{\omega}'$$

and inserting gives

$$\begin{aligned}\mathbf{R}'\mathbf{q} - [\mathbf{p}' - (\mathbf{p}'\mathbf{A} + \boldsymbol{\omega}')] \mathbf{q} &= \mathbf{0} \\ \mathbf{R}'\mathbf{q} - \mathbf{R}'\mathbf{q} &= \mathbf{0}\end{aligned}$$

which again shows the existence of a non-trivial solution.

The supply of goods is modeled as a system of supply-adjusts-to-demand differential equations

$$\dot{\mathbf{q}} = (1 + g) [\mathbf{I} - \mathbf{A}]^{-1} (\mathbf{D}_R + \mathbf{D}_W) \mathbf{q} - \mathbf{q} \quad (\text{A.4})$$

Inserting for  $(\mathbf{D}_R + \mathbf{D}_W) = (\mathbf{I} - \mathbf{A})$  gives

$$\dot{\mathbf{q}} = (1 + g) [\mathbf{I} - \mathbf{A}]^{-1} [\mathbf{I} - \mathbf{A}] \mathbf{q} - \mathbf{q} = g\mathbf{q}$$

Thus the quantity system grows at a constant rate  $g$  (steady-state balanced growth path). The system is constant (in the case  $\pi_i = 0$  or  $R_i = 0$  for all  $i = 1, \dots, N$ ) or is growing at a constant rate  $g$  where

$$g = \min_i \left( \frac{q_i^I}{q_i} \right)$$

and

$$q_i^I = \beta_i \sum_j \frac{R_j q_j}{p_i}$$

where we assume the non-negativity of  $\mathbf{R}$ . As we have stated above, the optimal structure of investment is given when

$$\beta_i^* = \frac{p_i q_i}{\mathbf{p}'\mathbf{q}}$$

To show this we insert the quantities demanded for investment into the definition of the growth rate

$$g = \min_i \left( \beta_i \frac{(\mathbf{r} + \mathbf{s})\mathbf{q}}{p_i q_i} \right)$$

where  $\sum_i \beta_i = 1$ . As we want to maximise the growth rate this should be rewritten

$$g^* = \max_{\beta_i} \left[ \min \left( \beta_i \frac{(\mathbf{r} + \mathbf{s})\mathbf{q}}{p_i q_i} \right) \right]$$

This problem has the solution  $\beta_i^* = \frac{p_i q_i}{\mathbf{p}'\mathbf{q}}$  for given  $\mathbf{p}$ ,  $\mathbf{q}$ , and  $\mathbf{R}$ . Thus the structure of nominal shares must be equal to the structure of output. If this condition is not satisfied, then there would be excess investment in all but the sector with the lowest  $\frac{q_i^I}{q_i}$  and the growth rate is bounded by this sector. Inserting  $\beta_i^*$  in the formulation of the growth rate yields

$$g^* = \frac{\mathbf{R}'\mathbf{q}}{\mathbf{p}'\mathbf{q}}$$

for all  $i$ . In equilibrium (i.e. with  $\mathbf{s} = \mathbf{0}$  or at prices  $\mathbf{p}^*$ ) this can be reformulated as

$$g^* = \frac{\mathbf{r}'\mathbf{q}}{\mathbf{p}'\mathbf{q}} = \frac{\pi\mathbf{c}'\mathbf{q}}{(1+\pi)\mathbf{c}'\mathbf{q}} = \frac{\pi}{1+\pi}$$

If this condition is satisfied then the economy is growing in equilibrium exactly with  $g = \frac{\pi}{1+\pi}$  which denotes the real profit in each sector.<sup>26</sup>

## A.2 Integrated economies

### A.2.1 Prices, profits, and rents

The extension to a number of integrated economies is straightforward. Here again we discuss a quite general case, namely we assume that the prices need not be equalised across economies and thus write a world price vector

$$\mathbf{p}^W = (\mathbf{p}^1, \dots, \mathbf{p}^C)$$

The corresponding system of differential equations for prices is then similar to the one country case if one assumes  $\delta_{\bar{p}}^r = 0$ . With long-term price equalisation this would result in  $\mathbf{p}^{W'} = (\mathbf{p}', \dots, \mathbf{p}')$ . Similarly the other vectors can be written, i.e. for  $\boldsymbol{\omega}^{W'}$ ,  $\mathbf{r}^{W'}$ ,  $\mathbf{s}^{W'}$ , and  $\mathbf{R}^{W'}$ .

### A.2.2 The quantity system

To show the existence of a non-trivial output vector for integrated economies we have first to define the demand components.

For intermediate demand we have

$$\mathbf{A}^W \mathbf{q}^W = \begin{pmatrix} \mathbf{A}^1 & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^2 & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{A}^{C-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{A}^C \end{pmatrix} \mathbf{q}^W$$

as intermediate goods must be produced at home by assumption. The second component, demand out of rents, has to be rewritten as

$$\mathbf{D}_R^W \mathbf{q}^W = \begin{pmatrix} \mathbf{D}_R^{11} & \dots & \mathbf{D}_R^{C1} \\ \vdots & \ddots & \vdots \\ \mathbf{D}_R^{1C} & \dots & \mathbf{D}_R^{CC} \end{pmatrix} \mathbf{q}^W$$

A typical matrix is

$$\mathbf{D}_R^{rs} = \begin{pmatrix} \beta_1^{rs} \frac{R_1^r}{p_1^s} & \dots & \beta_1^{rs} \frac{R_N^r}{p_1^s} \\ \vdots & \ddots & \vdots \\ \beta_N^{rs} \frac{R_1^r}{p_N^s} & \dots & \beta_N^{rs} \frac{R_N^r}{p_N^s} \end{pmatrix}$$

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<sup>26</sup>This is not the maximum (von Neumann) growth path  $g^{max}$ . For this case consumption would have to be at zero level.

with  $\sum_{i,s} \beta_i^{rs} = 1$ . Similarly we have for demand out of wage income

$$\mathbf{D}_W^W \mathbf{q}^W = \begin{pmatrix} \mathbf{D}_W^{11} & \cdots & \mathbf{D}_W^{C1} \\ \vdots & \ddots & \vdots \\ \mathbf{D}_W^{1C} & \cdots & \mathbf{D}_W^{CC} \end{pmatrix} \mathbf{q}^W$$

again with typical element

$$\mathbf{D}_R^{rs} = \begin{pmatrix} \alpha_1^{rs} \frac{\sum_z w_1^z a_{11}^z}{p_1^s} & \cdots & \alpha_1^{rs} \frac{\sum_z w_1^z a_{11}^z}{p_1^s} \\ \vdots & \ddots & \vdots \\ \alpha_N^{rs} \frac{\sum_z w_1^z a_{11}^z}{p_N^s} & \cdots & \alpha_N^{rs} \frac{\sum_z w_1^z a_{11}^z}{p_N^s} \end{pmatrix}$$

where again the condition  $\sum_{i,s} \alpha_i^{rs} = 1$  has to be satisfied. With similar reasoning as above we can then show that

$$\begin{aligned} \mathbf{p}^{W'} (\mathbf{A}^W - \mathbf{I} + \mathbf{D}_R^W + \mathbf{D}_W^W) \mathbf{q}^W &= \mathbf{0} \\ - \left[ \mathbf{p}^{W'} - (\mathbf{p}^{W'} \mathbf{A}^W + \boldsymbol{\omega}^W) \right] \mathbf{q}^W + \mathbf{p}^{W'} \mathbf{D}_R^W \mathbf{q}^W &= \mathbf{0} \\ - \mathbf{R}^{W'} \mathbf{q}^W + \mathbf{R}^{W'} \mathbf{q}^W &= \mathbf{0} \end{aligned}$$

This again proves the existence of a non-trivial output vector. Finally, with the assumptions given in the paper, the growth rate of the economies, even if the output structure is optimal in the above defined way, need not be equal across economies.

This survey on analytical results is based on the simplification that we disentangled the price and the quantity system in deriving the results. Further research has to focus on the interactions between the two systems (and maybe on the labour supply side or the factor markets in a more general sense) to analyse the model with respect to existence of equilibria, uniqueness or multiplicity of equilibria and stability requirements.

## References

- Berman, E., J. Bound, and Z. Griliches (1994). Changes in the demand for skilled labour within U.S. manufacturing: Evidence from the annual survey of manufactures. *The Quarterly Journal of Economics* (5), 367–379.
- Feenstra, R. and G. Hanson (1996). Globalisation, outsourcing, and wage inequality. *NBER Working Paper* (5424).
- Kurz, H. and N. Salvadori (1997). *Theory of production: A long-period analysis*. Cambridge University Press.
- Landesmann, M. and R. Stehrer (2000a, February). Industrial specialisation, catching-up and labour market dynamics. *Metroeconomica* 51 (1), 67–101.
- Landesmann, M. and R. Stehrer (2000b, June). Potential switchovers in comparative advantage: Patterns of industrial convergence. *WIIW Working Paper* (14).
- Lawrence, R. and M. Slaughter (1993). International trade and American wages in the 1980's: Giant sucking or small hiccup? *Brookings Papers of Economic Activities*, 161–226.
- Leamer, E. (1994). Trade, wages and revolving door ideas. *NBER Working Paper* (4716).
- Leamer, E. (1996). In search of the Stolper-Samuelson effects on U.S. wages. *NBER Working Paper* (5427).
- Lewis, W. (1954). Economic development with unlimited supply of labour. *The Manchester School*.
- McCombie, J. and A. Thirlwall (1994). *Economic Growth and the Balance-of-Payments Constraint*. St. Martins Press.
- Medio, A. (1992). *Chaotic dynamics. Theory and applications to economics* (1 ed.). Cambridge University Press.
- Morishima, M. (1970). *Theory of economic growth*. Oxford University Press.
- Murphy, K. and F. Welch (1992). The structure of wages. *The Quarterly Journal of Economics* (2), 285–326.
- Pasinetti, L. (1977). *Lectures on the theory of production*. MacMillan Press London.
- Stehrer, R. and M. Landesmann (1999, October). Convergence patterns at the industrial level: The dynamics of comparative advantage. *WIIW Working Paper* (11).
- Wood, A. (1994). Give Heckscher and Ohlin a chance! *Weltwirtschaftliches Archiv* 130, 20–49.
- Wood, A. (1995). How trade hurt unskilled workers. *Journal of Economic Perspectives* 9(3), 57–80.