Appendix 2

Input-output Based Multifactor Productivity: An Introduction

Simon Zheng

Australian Bureau of Statistics

simon.zheng@abs.gov.au

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A.1 Introduction

The purpose of this appendix is to introduce the Multifactor Productivity (MFP) measures based on the Input-Output (I/O) accounting framework. The underlying concept of MFP used in this particular approach is largely a conventional one. However, there are notable departures from the standard MFP formulation due to the structure and detailed information on sectoral inputs and outputs available from the I/O tables. When this MFP concept is applied to the I/O framework, it is able to generate some new interpretations and insights to enrich our understanding of productivity dynamics among industries within the economic system.

The I/O based approach provides a unified framework under which the industry MFP growth (and indeed the aggregate MFP growth) can be estimated. In addition, this approach is capable of shedding light on many theoretical as well as empirical issues on economic growth and productivity analysis. One distinctive feature is that with some modifications to the standard Divisia productivity indices, this approach is able to capture the effects of productivity changes in intermediate inputs on the productivity of all the industries in the economy. In fact, these modifications are originated directly from the argument on the inadequacy of the conventional MFP concept as a measure of technical progress. They highlight the longstanding contentious issue of whether and by how much capital should be treated as a reproduced input, rather than entirely as a primary input, exogenous to the economic system.

The next section discusses the theoretical origin which motivates this empirical methodology. Section 3 presents the formulae for the different types of the I/O based MFP indices. Some concluding remarks are given in the last section.

A.2 Theory and concepts

The theoretical development underlying the I/O based approach of estimating MFP growth is due to Rymes (1971, 1972 and 1983). It was initially presented in the context of Cambridge controversy over the measurement of capital and aggregate production function which was the focus of the debate at that time. The main argument put forward by Rymes is that the Hick neutral (or the Neoclassical measure of) technical progress is not adequate when capital is (as it should be, based on empirical observation) treated as a reproduced input in the economic system. He presents his analysis in one-sector as well as multi-sector models to contrast the performance of the conventional measure with that of the new measure of MFP (also called total factor productivity, TFP). He attributes this new TFP measure to the concepts which have already contained in the work by Harrod, Robinson and Read. Hence Rymes (1983) calls it the HRR measure of TFP[1].

What this new measure is essentially different from the conventional MFP is that the new productivity growth term is to be subtracted from the reproduced inputs (ie capital

[1] Rymes (1983) cautions that this concept is not to be confused with the one of labour augmenting (Harrod neutral) technical change. However, in steady state equilibrium, the growth rate of Rymes' HRR measure is exactly equal to the rate of per capita output growth (see the algebraic illustration in the next

as in Rymes' models, but can be any other intermediate inputs) to capture the effects of improved technical progress on the amount of inputs required in the production of outputs. Due to this effect, the growth rate of the HRR measure will be greater than the growth rate of the conventional TFP. Also, the productivity growth term based on the HRR measure has to be solved to obtain the reduced form.

We can illustrate these points using a simple aggregate (one-sector) production model. Let $\dot{X} = (1/X)(dX/dt)$ be the growth rate (proportionate rate of change) in any variable. *T* is the conventional TFP and *H* is Rymes' HRR measure of technical change. From the standard formula for TFP , we have

$$\dot{T} = \dot{Q} - \alpha\dot{L} - (1-\alpha)\dot{K} \qquad (1)$$

where Q is the flow of output, L and K are labour and gross capital services and α is labour input share.

Rymes' HRR measure is defined as

$$\dot{H} = \dot{Q} - \alpha\dot{L} - (1-\alpha)(\dot{K} - \dot{H}) \qquad (2)$$

This implies that

$\dot{H} = \alpha\dot{T}$; Also, in steady state, we have $\dot{H} = \dot{Q}/\dot{L}$

As α <1, then $\dot{H} > \dot{T}$ always holds. The latter equality says that in steady state, the growth rate of the HRR measure of technical progress is equal to the growth rate of per capita output (income), a result same as that from the Solow-Swan growth model with labour augmenting technical change.

This new formulation of TFP concept can be extended to multi-sector models, which will generate more results as Rymes has done in his analysis. Ultimately, it can be applied to disaggregated Sraffa-Leontief world, where each industry uses intermediate

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few paragraphs), which is the standard result from the Solow-Swan growth model where technical change is labour augmenting (Harrod neutral).

inputs other than capital and labour in its production. Of course, the algebra involved is more complex than that in the one-sector model.. Apart from this, there are some additional interpretations associated with these indices which are particularly relevant within the I/O framework (See the next section for detail).

This empirical application of the HRR concept of technical change was later exploited and developed by Cas and Rymes (1991) in their pioneer work on the I/O based industry MFP measures for Canada. Their work is further extended by Durand (1993, and 1996) to form the basis of Statistics Canada's I/O based MFP accounts in providing a unified framework of estimating industry MFP growth.

Conventional MFP formulation

MFP (or TFP) growth is conventionally defined as the difference between the weighted rate of growth of the outputs and the weighted rate of growth of the inputs. It can be written as in the following formula,

$$\tau = \sum c\dot{v} - \sum \omega\dot{x} \qquad (3)$$

where τ denotes Divisia index of productivity growth, the v's are the outputs of the production process in continuous time percentage rate of change (time derivative of the logarithm is denoted as dotted symbols) weighted by their value shares c; and the x's are the inputs also in rate of change and weighted by their cost share ω. This formula can be derived using the concept of an aggregate production possibility frontier with the assumptions of competitive equilibrium and constant return to scale under Hicks neutral technical progress[2]. Equation (3) implies that productivity growth is the growth of outputs not accounted for by the growth of inputs. This is the growth accounting approach which evaluates MFP growth residually[3].

[2] See Durand (1993) for a derivation using matrix notations.

[3] As a gross generalisation, the empirical productivity analysis based on economic theory can be broadly classified into four major methods — 1) econometric estimation of cost or production functions; 2) growth accounting approach such as MFP indices 3) data envelopment analysis and 4) stochastic frontiers. Methods 1 and 4 involve econometric estimation of parametric function, while methods 2 and 3 do not. Thus these two groups may also be terms 'parametric' and 'non-parametric' methods. See Coelli, Rao and Battese (1998) for an introduction to these methods.

This formulation of productivity growth is what Rymes called the traditional measure of MFP (Rymes 1972, 1983). It attempts to capture the disembodied technical change. It is called 'disembodied' because the technical change is not physically tied to any specific factor of production; rather, it affects inputs proportionally. This form of technical change is also called 'Hicks-neutral' and 'output augmenting' when it raises maximum output that can be produced with a given level of primary and intermediate inputs without changing the relationship between different inputs.

However, there are many restrictive assumptions (such as perfect competition, constant return to scale and no technical inefficiency associated with production) and various measurement issues involved in the empirical applications of this MFP concept at both the micro and macro level productivity analysis. Thus in practice, the productivity estimates based on this MFP formulation often reflect the combined effects of disembodied technical change, economies of scale, efficiency change, variations in capacity utilisation and measurement errors[4]. In the empirical implementation, the productivity index in equation (3) is approximated discretely by the chained Törnqvist index number formula:

$$\ln(T_t) = \sum \overline{c}_t \ln(v_t / v_{t-1}) - \sum \overline{\omega}_t \ln(x_t / x_{t-1}) \qquad (4)$$

where the bars over the shares indicate averages over year t and year t-1.

Empirical measures of MFP based on different concepts of inputs and outputs

One of the important issues in the productivity analysis is to determine which concepts of inputs and outputs should be used in the empirical implementation of Equation (4). This is especially so in its application within the I/O framework, as it is relatively easy to derive a set of different measures of outputs and inputs using the data from the I/O tables. The following table lists the different MFP measures arising from the use of different concepts of outputs and inputs.

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[4] For a detailed discussion on the interpretation of MFP and other productivity measures, see OECD (2001). Lipsey and Carlaw (2001) provides an assessment and sceptical view on the ability of MFP/TFP

as a measure of technological change. For a biographical account of the development of the TFP/MFP concept, see Hulten (2001).

Table 1: Empirical MFP measures under different concepts of output and inputs

Concept of inputs	Concept of outputs	
	Gross output	***Value added***
Combined capital and labour	KL-MFP based on gross output,	KL-MFP based on value added; also called value added MFP
Combined capital, labour, energy, materials and services	KLEMS-MFP; also called gross output MFP	

As can be seen, when the outputs used are based on gross output concept while inputs include capital and labour, the resulting MFP measure is often referred to as the KL-MFP. Note, however, this measure is not consistent, as the gross output also includes other inputs such as intermediate inputs which are excluded in the input part of this productivity measure. It is used only when there is no data available to obtain the next two (consistent) measures. The KLEMS-MFP is an empirical MFP measure for which the gross output is used as the output concept while inputs include all primary and intermediate inputs classified according to capital, labour, energy, materials and services. This measure will be called the gross output MFP when it is applied in the I/O framework in the next section. The third one in the table is the KL-MFP, which is a popular measure of MFP in the empirical literature, because the value added output and primary inputs data are regularly published by the national statistics agencies. Indeed, the aggregate MFP estimates published by the Australian Bureau of Statistics are based on this measure. It is also called the value added MFP in the I/O based productivity formulations.

Based on the I/O framework, the concepts of industry outputs and inputs can be further modified to net out the intra-industry flows of goods and services. The resulting productivity formulation is called the intra-industry MFP index. As mentioned above, the MFP indices have some additional interpretations when they are applied within the I/O framework. This is discussed in the next section.

A3 I/O based productivity indices and their interpretations

The formulations of the I/O based productivity measures using compact matrix notations have been worked out by Durand and presented in several of his papers (1993, 1996), where the conceptual issues and some new interpretations are highlighted with special reference to the I/O framework[5]. We attempt instead, in this appendix to 'dissect' these compact matrices in order to gain a deeper understanding of the mechanics of these measures and more importantly, to assist the empirical implementation. The latter, of course, is the major purpose of our study. Many measurement issues will only be touched upon here and the majority of which will be left for another paper where the empirical productivity results are presented.

Equation (3) in Section 2 has defined the conventional MFP measure. This formula has to be further modified to incorporate the information from the I/O tables. The advantage of such an approach is that the industry as well as aggregate MFP growth can now be obtained within this unified framework. The particular set of I/O tables suitable for this purpose is based on the rectangular I/O framework. In this framework, supply–use tables/matrices, final demand and value added matrices are the major building blocks. Accordingly, the I/O based MFP indices are often expressed in compact matrix forms.

Defining a few matrices from the I/O framework

The matrices which are the major building blocks of the rectangular I/O framework can be presented in the following figure[6].

[5] The matrix productivity formulae in this appendix are drawn on Durand (1993, 1996).

[6] Under the rectangular I/O framework, the I/O data are organised by industry and by commodity. It contrasts with the square I/O tables where the data often includes only industry dimension for both the rows as well as the columns.

Figure 1: The rectangular Input-Output accounting framework*

	C = no. of commodity	I = no. of industry	F = no. of final demand category
C = no. commo.		$\mathbf{U}_{C \times I}$	$\mathbf{E}_{C \times F}$
I = no. ind.	$\mathbf{V}_{I \times C}$		
l = primary inputs		$\mathbf{Y}_{l \times I}$	

* In the Australian I/O framework, the supply table contains matrix **V'** while the use table contains matrices **U**, **E** and **Y**. These tables are integrated with the Australian national accounts. For productivity analysis, both current and constant prices supply-use tables are required.

In the above figure, **V** is the supply matrix (also called make matrix) recording industries' gross outputs broken down by commodity; **U**, the use matrix recording the commodity inputs purchased by industries; **E**, the final demand matrix containing the final demand for the commodities in different categories; **Y,** the value added matrix recording the value of the income earned by the primary inputs (i.e. capital and labour). Denoting the commodity price matrix by $\boldsymbol{p}$ and assuming that the commodity price is the same across industries[7], we then have the following additional matrices based on the above structure of the rectangular I/O framework:

The gross output vector by industry $\boldsymbol{g}$ is given by

$$\boldsymbol{g} = \boldsymbol{Vp} \quad (5)$$

In a less compact format, it can be written as

[7] This assumption is used to simplify the exposition. In the Australian supply-use tables, **V** is valued at basic price while **U**, **E** and **Y** are valued at purchase price. See the section on empirical implementation for details on the valuation issue.

$$\begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_I \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} & \cdots & V_{1C} \\ V_{21} & V_{22} & \cdots & V_{2C} \\ \vdots & \vdots & \vdots & \vdots \\ V_{I1} & V_{I1} & \cdots & V_{IC} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ \vdots \\ p_C \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{C} V_{1j} p_j \\ \sum_{j=1}^{C} V_{2j} p_j \\ \vdots \\ \sum_{j=1}^{C} v_{Ij} p_j \end{pmatrix} \tag{6}$$

The gross output (vector) by commodity is given by

$$\hat{\boldsymbol{P}}\boldsymbol{v} = \hat{\boldsymbol{P}}\boldsymbol{V}'\boldsymbol{i}\,; \qquad (\boldsymbol{v} = \boldsymbol{V}'\boldsymbol{i})^{8} \tag{7}$$

The market share matrix $\boldsymbol{D}$ (I×C) is

$$\boldsymbol{D} = \boldsymbol{V}\hat{\boldsymbol{P}}(\hat{\boldsymbol{P}}\hat{\boldsymbol{v}})^{-1} = \boldsymbol{V}\hat{\boldsymbol{v}}^{-1} = \begin{pmatrix} \frac{V_{11}}{\sum_{i=1}^{I} V_{i1}} & \frac{V_{12}}{\sum_{i=1}^{I} V_{i2}} & \cdots & \cdots & \frac{V_{1C}}{\sum_{i=1}^{I} V_{iC}} \\ \frac{V_{21}}{\sum_{i=1}^{I} V_{i1}} & \frac{V_{22}}{\sum_{i=1}^{I} V_{i2}} & \cdots & \cdots & \frac{V_{2C}}{\sum_{i=1}^{I} V_{iC}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{V_{I1}}{\sum_{i=1}^{I} V_{i1}} & \frac{V_{I2}}{\sum_{i=1}^{I} V_{i2}} & \cdots & \cdots & \frac{V_{IC}}{\sum_{i=1}^{I} V_{iC}} \end{pmatrix} \tag{8}$$

Thus its d_{ij} element of the $\boldsymbol{D}$ matrix shows the share of the *i*th industry in the total output of *j*th commodity.

The intermediate input expenditure shares (or technical parameters) matrix $\boldsymbol{B}$ (C×I) is

[8] The vector with the symbol ^ on top is a diagonal matrix with elements on the diagonal coming from the elements of the vector in the corresponding row. ***i*** is a unit (summation) vector of appropriate dimension.

$$\boldsymbol{B} = \hat{\boldsymbol{p}}\boldsymbol{U}\hat{\boldsymbol{g}}^{-1} = \begin{pmatrix} \frac{U_{11}p_1}{g_1} & \frac{U_{12}p_1}{g_2} & \cdots & \frac{U_{1I}p_1}{g_I} \\ \frac{U_{21}p_2}{g_1} & \frac{U_{22}p_2}{g_2} & \cdots & \frac{U_{2I}p_2}{g_I} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \frac{U_{C1}p_C}{g_1} & \frac{U_{C2}p_C}{g_2} & \cdots & \frac{U_{CI}p_C}{g_I} \end{pmatrix} \tag{9}$$

The b_{ji} element of $\boldsymbol{B}$ matrix shows the share of expenditures on the *j*th intermediate commodity input in the value of total output of the i*th* industry.

The output value shares (product mix) matrix $\boldsymbol{C}$ (I×C) is given by

$$\boldsymbol{C} = \hat{\boldsymbol{g}}^{-1}\boldsymbol{V}\hat{\boldsymbol{p}} = \begin{pmatrix} \frac{V_{11}p_1}{g_1} & \frac{V_{12}p_2}{g_1} & \cdots & \cdots & \frac{V_{1C}p_C}{g_1} \\ \frac{V_{21}p_1}{g_2} & \frac{V_{22}p_2}{g_2} & \cdots & \cdots & \frac{V_{2C}p_C}{g_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{V_{I1}p_1}{g_I} & \frac{V_{I2}p_2}{g_I} & \cdots & \cdots & \frac{V_{IC}p_C}{g_I} \end{pmatrix} \tag{10}$$

The c_{ij} element of matrix $\boldsymbol{C}$ indicates the share of the j*th* commodity output in the value of total output of the i*th* industry (Recall that $g_i = \sum_{j=1}^{C} V_{ij}p_j$).

The value added matrix Y (l×I) is defined as

$$\boldsymbol{Y} = \begin{bmatrix} \hat{\boldsymbol{w}}\boldsymbol{L} \\ \hat{\boldsymbol{r}}\boldsymbol{K} \end{bmatrix} = \begin{bmatrix} w_1L_{11} & w_1L_{12} & \cdots & w_1L_{1I} \\ \vdots & \vdots & \vdots & \vdots \\ w_qL_{q1} & w_qL_{q2} & \cdots & w_qL_{qI} \\ r_1K_{11} & r_1K_{12} & \cdots & r_1K_{1I} \\ \vdots & \vdots & \vdots & \vdots \\ r_sK_{s1} & r_sK_{s2} & \cdots & r_sK_{sI} \end{bmatrix} \tag{11}$$

where $\boldsymbol{w}$ (q×1) and $\boldsymbol{r}$ (s×1, q + s = l) are vectors of wage rates and price of capital services by different type of labour and capital respectively. $\boldsymbol{L}$ (q×I) and $\boldsymbol{K}$ (s×I) are matrices of labour and capital by type and industry, respectively.

The primary input value shares matrix $\boldsymbol{H}$ (l×I), the labour input value share matrix $\boldsymbol{H}_L$ (q×I) and the capital input value share matrix $\boldsymbol{H}_K$ (s×I) are defined as

$$\boldsymbol{H} = \boldsymbol{Y}\hat{\boldsymbol{g}}^{-1} = \begin{bmatrix} \hat{\boldsymbol{w}}\boldsymbol{L} \\ \hat{\boldsymbol{r}}\boldsymbol{K} \end{bmatrix} \hat{\boldsymbol{g}}^{-1} = \begin{bmatrix} \hat{\boldsymbol{w}}\boldsymbol{L}\hat{\boldsymbol{g}}^{-1} \\ \hat{\boldsymbol{r}}\boldsymbol{K}\hat{\boldsymbol{g}}^{-1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_L \\ \boldsymbol{H}_K \end{bmatrix} \tag{12}$$

Thus the labour input value share matrix is

$$\boldsymbol{H}_L = \hat{\boldsymbol{w}}\boldsymbol{L}\hat{\boldsymbol{g}}^{-1} = \begin{pmatrix} \frac{w_1 L_{11}}{g_1} & \frac{w_1 L_{12}}{g_2} & \cdots & \frac{w_1 L_{1I}}{g_I} \\ \frac{w_2 L_{21}}{g_1} & \frac{w_2 L_{22}}{g_2} & \cdots & \frac{w_2 L_{2I}}{g_I} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{w_q L_{q1}}{g_1} & \frac{w_q L_{q2}}{g_2} & \cdots & \frac{w_q L_{qI}}{g_I} \end{pmatrix} \tag{13}$$

Similarly, we have the capital input value share matrix as

$$\boldsymbol{H}_K = \hat{\boldsymbol{r}}\boldsymbol{K}\hat{\boldsymbol{g}}^{-1} = \begin{pmatrix} \frac{r_1 K_{11}}{g_1} & \frac{r_1 K_{12}}{g_2} & \cdots & \frac{r_1 K_{1I}}{g_I} \\ \frac{r_2 K_{21}}{g_1} & \frac{r_2 K_{22}}{g_2} & \cdots & \frac{r_2 K_{2I}}{g_I} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{r_s K_{s1}}{g_1} & \frac{r_s K_{s2}}{g_2} & \cdots & \frac{r_s K_{sI}}{g_I} \end{pmatrix} \tag{14}$$

For example, the $h_K(n,i)$ element of the matrix $\boldsymbol{H}_K$ shows the share of the n*th* type of capital input in the value of total output of the i*th* industry. Similar interpretation applies to the element of the matrix $\boldsymbol{H}_L$.

Gross output industry MFP index

The above matrices and vectors based on the I/O framework are also the major building blocks for constructing the productivity indices. Using the definition of MFP growth formula in equation (3), the matrix expression of industrial MFP growth based on *gross output* is given by

$$\boldsymbol{\tau}_g = \boldsymbol{C} \varnothing \dot{\boldsymbol{V}} - \boldsymbol{B}' \varnothing \dot{\boldsymbol{U}}' - \boldsymbol{H}_L' \varnothing \dot{\boldsymbol{L}}' - \boldsymbol{H}_K' \varnothing \dot{\boldsymbol{K}}' \tag{15}$$

where $\boldsymbol{\tau}$ represents the productivity growth vector with subscript g denoting the output over which it is specified. Thus $\boldsymbol{\tau}_g$ denotes the productivity growth by industry based on *gross output*. The operator $\varnothing$ is defined as $\boldsymbol{X} \varnothing \boldsymbol{Y} = (\boldsymbol{X} \cdot \boldsymbol{Y})\boldsymbol{i}$, where the dot indicates an element by element matrix product.

Equation (15) can be expanded using the matrices that have been defined previously, as follows,

$$\begin{pmatrix} \tau_g^1 \\ \tau_g^2 \\ \vdots \\ \tau_g^I \end{pmatrix} = \left(\begin{pmatrix} \frac{V_{11}p_1}{g_1} & \frac{V_{12}p_2}{g_1} & \cdots & \cdots & \frac{V_{1C}p_C}{g_1} \\ \frac{V_{21}p_1}{g_2} & \frac{V_{22}p_2}{g_2} & \cdots & \cdots & \frac{V_{2C}p_C}{g_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{V_{I1}p_1}{g_I} & \frac{V_{I2}p_2}{g_I} & \cdots & \cdots & \frac{V_{IC}p_C}{g_I} \end{pmatrix} \bullet \begin{pmatrix} \frac{d\ln V_{11}}{dt} & \frac{d\ln V_{12}}{dt} & \cdots & \cdots & \frac{d\ln V_{1C}}{dt} \\ \frac{d\ln V_{21}}{dt} & \frac{d\ln V_{22}}{dt} & \cdots & \cdots & \frac{d\ln V_{2C}}{dt} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{d\ln V_{I1}}{dt} & \frac{d\ln V_{I2}}{dt} & \cdots & \cdots & \frac{d\ln V_{IC}}{dt} \end{pmatrix} \right) \begin{pmatrix} 1 \\ 1 \\ \vdots \\ \vdots \\ 1 \end{pmatrix}_{(C\times 1)}$$

$$- \left(\begin{pmatrix} \frac{U_{11}p_1}{g_1} & \frac{U_{21}p_2}{g_1} & \cdots & \cdots & \frac{U_{C1}p_C}{g_1} \\ \frac{U_{12}p_1}{g_2} & \frac{U_{22}p_2}{g_2} & \cdots & \cdots & \frac{U_{C2}p_C}{g_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{U_{1I}p_1}{g_I} & \frac{U_{2I}p_2}{g_I} & \cdots & \cdots & \frac{U_{CI}p_C}{g_I} \end{pmatrix} \bullet \begin{pmatrix} \frac{d\ln U_{11}}{dt} & \frac{d\ln U_{21}}{dt} & \cdots & \cdots & \frac{d\ln U_{C1}}{dt} \\ \frac{d\ln U_{12}}{dt} & \frac{d\ln U_{22}}{dt} & \cdots & \cdots & \frac{d\ln U_{C2}}{dt} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{d\ln U_{1I}}{dt} & \frac{d\ln U_{2I}}{dt} & \cdots & \cdots & \frac{d\ln U_{CI}}{dt} \end{pmatrix} \right) \begin{pmatrix} 1 \\ 1 \\ \vdots \\ \vdots \\ 1 \end{pmatrix}_{(C\times 1)}$$

$$- \left(\begin{pmatrix} \frac{w_1L_{11}}{g_1} & \frac{w_2L_{21}}{g_1} & \cdots & \frac{w_qL_{q1}}{g_1} \\ \frac{w_1L_{12}}{g_2} & \frac{w_2L_{22}}{g_2} & \cdots & \frac{w_qL_{q2}}{g_2} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{w_1L_{1I}}{g_I} & \frac{w_2L_{2I}}{g_I} & \cdots & \frac{w_qL_{qI}}{g_I} \end{pmatrix} \bullet \begin{pmatrix} \frac{d\ln L_{11}}{dt} & \frac{d\ln L_{21}}{dt} & \cdots & \frac{d\ln L_{q1}}{dt} \\ \frac{d\ln L_{12}}{dt} & \frac{d\ln L_{22}}{dt} & \cdots & \frac{d\ln L_{q2}}{dt} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{d\ln L_{1I}}{dt} & \frac{d\ln L_{2I}}{dt} & \cdots & \frac{d\ln L_{qI}}{dt} \end{pmatrix} \right) \begin{pmatrix} 1 \\ 1 \\ \vdots \\ \vdots \\ 1 \end{pmatrix}_{(q\times 1)}$$

$$- \left(\begin{pmatrix} \frac{r_1K_{11}}{g_1} & \frac{r_2K_{21}}{g_1} & \cdots & \frac{r_sK_{s1}}{g_1} \\ \frac{r_1K_{12}}{g_2} & \frac{r_2K_{22}}{g_2} & \cdots & \frac{r_sK_{s2}}{g_2} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{r_1K_{1I}}{g_I} & \frac{r_2K_{2I}}{g_I} & \cdots & \frac{r_sK_{rI}}{g_I} \end{pmatrix} \bullet \begin{pmatrix} \frac{d\ln K_{11}}{dt} & \frac{d\ln K_{21}}{dt} & \cdots & \frac{d\ln K_{s1}}{dt} \\ \frac{d\ln K_{12}}{dt} & \frac{d\ln K_{22}}{dt} & \cdots & \frac{d\ln K_{s2}}{dt} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{d\ln K_{1I}}{dt} & \frac{d\ln K_{2I}}{dt} & \cdots & \frac{d\ln K_{sI}}{dt} \end{pmatrix} \right) \begin{pmatrix} 1 \\ 1 \\ \vdots \\ \vdots \\ 1 \end{pmatrix}_{(s\times 1)} \tag{16}$$

Therefore the i*th* industry's MFP growth based on gross output is

$$\tau_g^i = \sum_{j=1}^{C} \left(\frac{V_{ij}p_j}{g_i} \right) \left(\frac{d\ln V_{ij}}{dt} \right) - \sum_{j=1}^{C} \left(\frac{U_{ji}p_j}{g_i} \right) \left(\frac{d\ln U_{ji}}{dt} \right) - \sum_{m=1}^{q} \left(\frac{w_mL_{mi}}{g_i} \right) \left(\frac{d\ln L_{mi}}{dt} \right)$$
$$- \sum_{n=1}^{s} \left(\frac{r_nK_{ni}}{g_i} \right) \left(\frac{d\ln K_{ni}}{dt} \right) \tag{17}$$

Equation (17) says that the index for the i*th* industry's productivity growth based on gross output is the difference between the weighted rate of growth of all the commodity outputs by this industry and the weighted rate of growth of all types of inputs, both intermediate and primary used in its production. The weights are the appropriate outputs and inputs value shares in the gross output of the i*th* industry. The empirical application of this index can be approximated discretely by the Törnqvist index number formula as shown in equation (4).

As mentioned before, there are additional interpretations associated with the productivity indices within the I/O framework. To see this, one has to introduce the concept of *vertical integration*.

Vertical integration, intra-industry and inter-industry MFP indices

Vertical integration is traditionally an useful concept for understanding the interconnectedness of different production units in a production system depicted by I/O framework. It is formalised by Pasinetti (1981) in his description of the economic system. Under this system, all production processes are considered as vertically integrated, in the sense that all their inputs are reduced to inputs of labour and to services from stocks of capital goods.

It turns out that this notion of vertical integration is also useful in the interpretations of the relationship between the different industry productivity indices under the I/O framework. Such interpretations are attributable to Durand (1993, 1996) in his work on the I/O based industry productivity estimation.

The industry MFP index based on gross output in equations (15) and (17) (or KLEMS-MFP measures contained in Table 1) can be considered either as non-integrated measure of productivity or as a measure of productivity at the establishment level of vertical integration; that is, within establishment flows of goods and services are netted out; only the inputs coming into the establishment from other establishments and the output going out of the establishment are accounted for.

A variant of the gross output productivity index can be derived by using the measure based on gross output net of intra-industry sales. Sales of establishments to other establishments belonging to the same industry are netted out on both the output and input side. It is as if an industry's establishments were merged or vertically integrated together into a single large establishment that buys all its inputs and sells all its output outside the industry. This alternative industry productivity index is called the *intra-industry* MFP index. It is given by the following equation in relation to the gross output industry index,

$$\tau^{i}_{gn} = \left(\frac{g^{i}}{g^{i}_{n}} \right) \times \tau^{i}_{g} \qquad (18)$$

where g^{i}_{n} is the i*th* industry nominal gross output net of intra-industry sales; and $g^{i} = g_{i}$, the nominal gross output by the *ith* industry (to be consistent the use of g^{i}_{n}).

The process of vertical integration can be extended beyond the intra-industry level to include also inter-industry sales. The establishments of an industry may be integrated with their upstream suppliers which may themselves be integrated upstream with their own suppliers and so on. Under full vertical integration, the output of an industry becomes a function of the direct use of the industry's own primary inputs and the indirect use of the primary inputs of all their upstream suppliers. The resulting productivity measure is called the *inter-industry* productivity index because it is obtained by taking into account inter-industry transactions and the productivity changes in producing the intermediate inputs. This index is given by

$$\boldsymbol{\tau}_{gi} = \boldsymbol{C} \varnothing \dot{\boldsymbol{V}} - \left[\boldsymbol{B}' \varnothing \left(\dot{\boldsymbol{U}}' - (\boldsymbol{i} \otimes \boldsymbol{\tau}'_{gi} \boldsymbol{D}) \right) \right] - \boldsymbol{H}'_{L} \varnothing \dot{\boldsymbol{L}}' - \boldsymbol{H}'_{K} \varnothing \dot{\boldsymbol{K}}' \qquad (19)[9]$$

where $\otimes$ represents the Kronecker matrix product and the subscript *gi* under $\boldsymbol{\tau}$ denotes that this measure of MFP growth is gross output based with inter-industry as its level of integration.

[9] This formula is from equation (6) in Durand (1996). There is a minor mistake for the same formula in his 1993 (preliminary) paper.

Comparing equation (19) with equation (15), it can be seen that the only difference between the two indices is in the second term on the right hand side (RHS) of the equations. In equation (19), the rate of growth of intermediate inputs is deflated by the productivity growth of its original industries. This reflects the productivity change of the economic system to produce outputs which are also used as intermediate inputs. Thus the intermediate inputs must be 'reduced' by productivity change[10]. If industries' productivity gains are positive, this implies that $\boldsymbol{\tau_{gi}}$ is greater than $\boldsymbol{\tau_{g}}$. We can see these relationships more clearly by expanding equation (19) as we have done to equation (15). Because only the second terms on the right hand side in the two equations are different, we only need to expand this term for equation (19) by its components. We have,

[10] This deflation can also be applied to capital and other primary inputs if they are treated as re-produced inputs by the system as the intermediated inputs are. Part of capital stock was treated as a re-produced input in Cas and Rymes' (1991) formulation of the new measure. This relates to the concept of 'waiting' associated with the measurement of capital (Rymes 1972, 1983). See also Durand (1996) for an analysis of capital in a dynamic I/O framework.

$$(\boldsymbol{i}\otimes\boldsymbol{\tau}_{gi}^{'}\boldsymbol{D})=\begin{pmatrix}1\\1\\\vdots\\1\end{pmatrix}_{(I\times 1)}\otimes\begin{pmatrix}\tau_{gi}^{1} & \tau_{gi}^{2} & \cdots & \tau_{gi}^{I}\end{pmatrix}\begin{pmatrix}\frac{V_{11}}{\sum_{i=1}^{I}V_{i1}} & \frac{V_{12}}{\sum_{i=1}^{I}V_{i2}} & \cdots & \cdots & \frac{V_{1C}}{\sum_{i=1}^{I}V_{iC}}\\ \frac{V_{21}}{\sum_{i=1}^{I}V_{i1}} & \frac{V_{22}}{\sum_{i=1}^{I}V_{i2}} & \cdots & \cdots & \frac{V_{2C}}{\sum_{i=1}^{I}V_{iC}}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ \frac{V_{I1}}{\sum_{i=1}^{I}V_{i1}} & \frac{V_{I2}}{\sum_{i=1}^{I}V_{i2}} & \cdots & \cdots & \frac{V_{IC}}{\sum_{i=1}^{I}V_{iC}}\end{pmatrix}$$

$$=\begin{pmatrix}1\\1\\\vdots\\1\end{pmatrix}_{(I\times 1)}\otimes\begin{pmatrix}\frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i1}}{\sum_{i=1}^{I}V_{i1}} & \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i2}}{\sum_{i=1}^{I}V_{i2}} & \cdots & \cdots & \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{iC}}{\sum_{i=1}^{I}V_{iC}}\end{pmatrix}_{(1\times C)}$$

$$=\begin{pmatrix}\frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i1}}{\sum_{i=1}^{I}V_{i1}} & \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i2}}{\sum_{i=1}^{I}V_{i2}} & \cdots & \cdots & \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{iC}}{\sum_{i=1}^{I}V_{iC}}\\ \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i1}}{\sum_{i=1}^{I}V_{i1}} & \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i2}}{\sum_{i=1}^{I}V_{i2}} & \cdots & \cdots & \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{iC}}{\sum_{i=1}^{I}V_{iC}}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i1}}{\sum_{i=1}^{I}V_{i1}} & \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i2}}{\sum_{i=1}^{I}V_{i2}} & \cdots & \cdots & \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{iC}}{\sum_{i=1}^{I}V_{iC}}\end{pmatrix} \quad (20)$$

It follows that

$$\left(\dot{\boldsymbol{U}}' - (\boldsymbol{i} \otimes \boldsymbol{\tau}_{gi}'\boldsymbol{D})\right) = \begin{pmatrix} \frac{d\ln U_{11}}{dt} & \frac{d\ln U_{21}}{dt} & \cdots & \cdots & \frac{d\ln U_{C1}}{dt} \\ \frac{d\ln U_{12}}{dt} & \frac{d\ln U_{22}}{dt} & \cdots & \cdots & \frac{d\ln U_{C2}}{dt} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{d\ln U_{1I}}{dt} & \frac{d\ln U_{2I}}{dt} & \cdots & \cdots & \frac{d\ln U_{CI}}{dt} \end{pmatrix}$$

$$- \begin{pmatrix} \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i1}}{\sum_{i=1}^{I}V_{i1}} & \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i2}}{\sum_{i=1}^{I}V_{i2}} & \cdots & \cdots & \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{iC}}{\sum_{i=1}^{I}V_{iC}} \\ \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i1}}{\sum_{i=1}^{I}V_{i1}} & \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i2}}{\sum_{i=1}^{I}V_{i2}} & \cdots & \cdots & \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{iC}}{\sum_{i=1}^{I}V_{iC}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i1}}{\sum_{i=1}^{I}V_{i1}} & \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i2}}{\sum_{i=1}^{I}V_{i2}} & \cdots & \cdots & \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{iC}}{\sum_{i=1}^{I}V_{iC}} \end{pmatrix}$$

$$= \begin{pmatrix} \left(\frac{d\ln U_{11}}{dt} - \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i1}}{\sum_{i=1}^{I}V_{i1}}\right) & \left(\frac{d\ln U_{21}}{dt} - \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i2}}{\sum_{i=1}^{I}V_{i2}}\right) & \cdots & \cdots & \left(\frac{d\ln U_{C1}}{dt} - \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{iC}}{\sum_{i=1}^{I}V_{iC}}\right) \\ \left(\frac{d\ln U_{12}}{dt} - \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i1}}{\sum_{i=1}^{I}V_{i1}}\right) & \left(\frac{d\ln U_{22}}{dt} - \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i2}}{\sum_{i=1}^{I}V_{i2}}\right) & \cdots & \cdots & \left(\frac{d\ln U_{C2}}{dt} - \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{iC}}{\sum_{i=1}^{I}V_{iC}}\right) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \left(\frac{d\ln U_{1I}}{dt} - \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i1}}{\sum_{i=1}^{I}V_{i1}}\right) & \left(\frac{d\ln U_{2I}}{dt} - \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i2}}{\sum_{i=1}^{I}V_{i2}}\right) & \cdots & \cdots & \left(\frac{d\ln U_{CI}}{dt} - \frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{iC}}{\sum_{i=1}^{I}V_{iC}}\right) \end{pmatrix} \quad (21)$$

Now the second term on the RHS of equation (19) is expanded into

$$\boldsymbol{B}'\varnothing\left(\dot{\boldsymbol{U}}'-(\boldsymbol{i}\otimes\boldsymbol{\tau}_{gi}^{'}\boldsymbol{D})\right)=\left[\boldsymbol{B}'\bullet\left(\dot{\boldsymbol{U}}'-(\boldsymbol{i}\otimes\boldsymbol{\tau}_{gi}^{'}\boldsymbol{D})\right)\right]\boldsymbol{i}$$

$$=\begin{pmatrix} \frac{U_{11}p_1}{g_1} & \frac{U_{21}p_2}{g_1} & \cdots & \cdots & \frac{U_{C1}p_C}{g_1} \\ \frac{U_{12}p_1}{g_2} & \frac{U_{22}p_2}{g_2} & \cdots & \cdots & \frac{U_{C2}p_C}{g_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{U_{1I}p_1}{g_I} & \frac{U_{2I}p_2}{g_I} & \cdots & \cdots & \frac{U_{CI}p_C}{g_I} \end{pmatrix}\bullet$$

$$\left(\begin{matrix} \left(\frac{d\ln U_{11}}{dt}-\frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i1}}{\sum_{i=1}^{I}V_{i1}}\right) & \left(\frac{d\ln U_{21}}{dt}-\frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i2}}{\sum_{i=1}^{I}V_{i2}}\right) & \cdots & \cdots & \left(\frac{d\ln U_{C1}}{dt}-\frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{iC}}{\sum_{i=1}^{I}V_{iC}}\right) \\ \left(\frac{d\ln U_{12}}{dt}-\frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i1}}{\sum_{i=1}^{I}V_{i1}}\right) & \left(\frac{d\ln U_{22}}{dt}-\frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i2}}{\sum_{i=1}^{I}V_{i2}}\right) & \cdots & \cdots & \left(\frac{d\ln U_{C2}}{dt}-\frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{iC}}{\sum_{i=1}^{I}V_{iC}}\right) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \left(\frac{d\ln U_{1I}}{dt}-\frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i1}}{\sum_{i=1}^{I}V_{i1}}\right) & \left(\frac{d\ln U_{2I}}{dt}-\frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{i2}}{\sum_{i=1}^{I}V_{i2}}\right) & \cdots & \cdots & \left(\frac{d\ln U_{CI}}{dt}-\frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{iC}}{\sum_{i=1}^{I}V_{iC}}\right) \end{matrix}\right)\begin{pmatrix}1\\1\\\vdots\\\vdots\\1\end{pmatrix}_{(C\times 1)} \tag{22}$$

Using these expanded matrices, the i*th* industry's MFP growth based on the inter-industry productivity concept can be expressed as,

$$\tau_{gi}^{i}=\sum_{j=1}^{C}\left(\frac{V_{ij}p_j}{g_i}\right)\left(\frac{d\ln V_{ij}}{dt}\right)-\sum_{j=1}^{C}\left(\frac{U_{ji}p_j}{g_i}\right)\left(\frac{d\ln U_{ji}}{dt}-\frac{\sum_{i=1}^{I}\tau_{gi}^{i}V_{ij}}{\sum_{i=1}^{I}V_{ij}}\right)-\sum_{m=1}^{q}\left(\frac{w_mL_{mi}}{g_i}\right)\left(\frac{d\ln L_{mi}}{dt}\right)$$
$$-\sum_{n=1}^{s}\left(\frac{r_nK_{ni}}{g_i}\right)\left(\frac{d\ln K_{ni}}{dt}\right) \tag{23}$$

In general, vertical integration increases the value of productivity measures. This is due to the fact that intermediate inputs are replaced by primary inputs (inter-industry productivity index), which most often increase at a lower rate, or that they are simply eliminated on both sides of the productivity equation (intra-industry indices). This property can also be explained by the effects of productivity changes in the intermediate inputs on the industry productivity indices. The productivity indices based on gross output do not take into account the effects of productivity changes on the production of intermediate inputs. On the other hand, both intra-industry and inter-industry productivity indices do take into account these effects with the difference only in the degree of consideration; the inter-industry MFP index is the result of full vertical integration. Thus the following inequalities hold.

$$\tau^{i}_{gi} > \tau^{i}_{gn} > \tau^{i}_{g} \tag{24}$$

Industry value-added productivity and the relationships between the MFP indices

The last I/O based MFP index to be discussed in this appendix is the industry value-added productivity index. It is well known that there is a direct relation between the gross output and the value-added productivity measure (Bruno 1978). Indeed, this relation also carries over to the productivity indices under the I/O framework. In matrix terms, it is given by

$$\boldsymbol{\tau}_{y} = \hat{\boldsymbol{y}}^{-1}\hat{\boldsymbol{g}}\boldsymbol{\tau}_{g} \tag{25}$$

where $\boldsymbol{y}$ is a vector of industry value added.

For the i*th* industry, it is simply

$$\tau^{i}_{y} = \left(\frac{g^{i}}{y^{i}}\right) \times \tau^{i}_{g} \tag{26}$$

Since $\left(\frac{g^{i}}{y^{i}}\right) > 1$, we have

$$\tau^{i}_{y} > \tau^{i}_{g} \tag{27}$$

Equation (27) says that the value-added MFP growth for a particular industry is systematically higher than the gross output MFP measure for the same industry.

The interpretation of the value added MFP index is somewhat more complex than that of the previous three. This is because this measure of productivity growth requires the existence of a *value-added function*, which is an equivalent representation of the technology described by a production function. A value-added function represents the maximum amount of current price value added that can be produced, given a set of primary inputs and given prices of intermediate inputs and output. As the measure of output is changed (see Table 1)[11], the productivity index based on value added naturally embeds full vertical integration. This is also why the value added productivity index is greater than the index based on gross output as shown in equation (27). Due to the close relationship between the industry value added and industry final demand, the industry value added productivity index reflects the contribution of the primary inputs of an industry to the productivity gains of the whole chain of industries producing the final goods and services.

It can be shown that there are linear relationships among the four types of productivity indices discussed above, because they are all derived with the information from the same rectangle I/O tables (see Durand 1993). However, the empirical implementation of these formulae may have to rely on the data from other sources and hence these linear relationships may not be observed in the actual estimates.

The relationship between the gross output productivity index and the value added index and that between the gross output index and the intra-industry index are evident from equations (26) and (18). Using equations (15) and (19), one can express the inter-industry productivity index as a linear transformation of the gross output MFP index as follows,

$$\boldsymbol{\tau}_{gi} = [\boldsymbol{I} - \boldsymbol{B'D'}]^{-1} \boldsymbol{\tau}_g \tag{28}$$

[11] Under the I/O accounting framework, the real value added can be derived without using the traditional double deflation technique (see Durand 1994 for details).

where $[\boldsymbol{I} - \boldsymbol{B'D'}]^{\boldsymbol{1}}$ is the transpose of the Leontief inverse. The Leontief inverse is also known as total requirement matrix or impact matrix in the standard I/O model; an element of this matrix defines the output of industry i required both directly and indirectly to deliver one dollar's worth of industry j's output to final demand.

Aggregation

Given the four types of industry MFP indices presented above, how do we use them to derive the index of MFP growth for the whole market economy? This is an issue of aggregation. Focusing on the value added industry MFP index, it is natural to use the following aggregation rule,

$$\tau = \sum \left(\frac{y^i}{\sum y^i} \right) \tau_y^i \tag{29}$$

where τ is the aggregate productivity growth for the market economy. $\left(\frac{y^i}{\sum y^i} \right)$ is the nominal value–added share for the i[th] industry, which is the weights used for aggregating the industrial value added MFP index and it sums to one. Now using the relationship between the gross output and value added productivity indices as in equation (26), the above equation can be written to

$$\tau = \sum \left(\frac{y^i}{\sum y^i} \right) \left(\frac{g^i}{y^i} \right) \tau_g^i = \sum \left(\frac{g^i}{\sum y^i} \right) \tau_g^i \tag{30}$$

where $\left(\frac{g^i}{\sum y^i} \right)$ is the current price value of gross output of industry i divided by the total market sector nominal value added. Thus to aggregate the gross output industry MFP index, the weights to be used are $\left(\frac{g^i}{\sum y^i} \right)$, which sum to more than one. This sectoral weighting scheme is known as Domar weights, which is initially proposed by Domar (1961) and formally justified by Hulten (1978).

Note that Domar's aggregation weights for productivity indices are defined on the gross output. In fact, these weights play the dual roles of aggregation as well as integration. To see this, notice that the first equality in equation (30) contains $\left(\frac{g^i}{y^i}\right)$, which is an integration factor for transforming the gross output MFP index to a value added basis, and then the value added index is aggregated by the weights $\left(\frac{y^i}{\sum y^i}\right)$, which sums to one, to arrive at the market sector index. Hence, it is clear that it is the integration factor that makes the Domar's weights summing to more than one. This integration factor captures the effects of vertical integration which underlies the different types of MFP indices within the I/O framework. Durand (1996) calls Domar's weights the aggregation-integration weights.

A4. Concluding remarks

The industry MFP indices based on the I/O framework incorporate various productivity concepts based on the economic theory and are closely connected among themselves by the level of vertical integration within the I/O framework. Estimating the industry productivity growth under this unified framework enables us to identify the effects of productivity and technological interdependence among industries which are important in the modern economy. Thus these indices entail some rich interpretations and are able to complement other partial productivity indices to present a comprehensive picture of productivity changes in the economy.

The methodology reviewed in this appendix is based on the static I/O framework. When capital is considered as a produced input instead of a primary input, the I/O system has to be dynamic to necessitate the analysis. Despite the additional interpretations associated with the I/O based productivity estimates, the underlying cause of the MFP growth is still explicitly or implicitly assumed to be the disembodied technological change which is also assumed to be exogenous to the economic system. However, there is no reason to assume that all productivity growth actually occurred and empirically captured by the MFP indices is uniformly caused by the same type of technological change. Indeed, there have been ongoing debate and research on the theory and estimation of the capital-embodied technical progress and of the interaction between human capital and productivity changes. Also, the endogenous growth theory has provided new theoretical insights into our understanding of economic and productivity growth.

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