On the consistency of commodity-based technology in the Make-Use model: an economic-circuit approach

Louis de Mesnard
LATEC (UMR CNRS 5118)
Faculty of Economics and Management - University of Dijon
2 Bd Gabriel, B.P. 26611,
F-21066 Dijon Cedex,
FRANCE
E-mail: *louis.de-mesnard@u-bourgogne.fr*

ABSTRACT. Most national accounting systems are based on the Make-Use model. Two hypotheses are traditionally made featuring either industry-based (IBT) or commodity-based (CBT) technologies. IBT corresponds to a consistent demand-driven model: its solution can be explained as a circuit or in probabilistic terms, even in the rectangular case. CBT obliges to compute an inverse matrix which is impossible when rectangular, fails to indicate how a commodity is distributed throughout industries and precludes interpretation of CBT as a circuit or in probabilistic terms. The CBT model should be reconstructed as a supply-driven model to recover its coherence as a circuit even in the rectangular case.

JEL classification. C67, D57.

KEYWORDS. Input-output, SNA, Make-Use, Supply-Use, circuit.

1 Introduction

Most national accounting systems around the world are based on the input-output model developed by Stone (1961) and adopted by the United Nations and OECD in a system called the *System of National Accounts* (United Nations, Department of Economic and Social Affairs, 1968, 1993, 1999; Blades, 1989; van Bochove and Bloem, 1987; Vanoli, 1994; Lawson, 1997). SNA brings many improvements when it is compared to the former square industry-by-industry model. However, the rectangular nature of the model allows choosing more than one way to build the model. For rectangular models as those handled by the SNA, two main hypotheses can be considered: the industry-based technology hypothesis and the commodity-based technology hypothesis. What they mean exactly will be recalled later, but at this moment, the reader just has to understand that they are normally alternative. The majority of the countries follow the SNA and the commodity-based hypothesis, but the United States use the industry-based technology [1]. It is well known that the commodity-based technology has a major drawback: it generates some negative coefficients that are always annoying [2]. For

---

[1] The differences essentially lie into the choice of the technology assumption but also on the treatment of imports (Jackson, 1998). However, it is not the aim of this paper to discuss further how the US approach differs from the SNA approach. See also (Kuboniwa, Matsue and Arita, 1986).

[2] Moreover, the commodity-based technology always needs square matrices (to allows computing direct or inverse matrices), that is the same number of industries and commodities, what can be reached by aggregating industries or commodities; making the model to be square doesn't prevent negative terms. The industry-based technology doesn't need square matrices to compute direct or inverse matrices, except in one case (when computing the commodity-by-industry inverse matrix) and it does not generate negative terms (Miller and Blair, 1985, p. 171). The industry based hypothesis has also some drawbacks in an axiomatic viewpoint as it violates some of the axioms listed by (Kop Jansen and ten Raa, 1990) but it is

ten Raa (1988), this is a major reason to abandon the commodity-based hypothesis. So, as the United Nations and OECD begin to elaborate a new set of tables for all countries after a long interruption, it is time to look again at the validity of the commodity-based model: the aim of this paper is to show that the commodity-based hypothesis could be reinterpreted in such a way that negative terms are removed, what will bring it at the same level than the industry-based hypothesis. In this paper, an original approach is chosen: the economic circuit. After recalling that the economic interpretation of the more simple square models (Leontief's and Ghosh's) can be given in terms of closed economic circuit, it will be explained that for the commodity-based model the interpretation in terms of closed economic circuit fails, even when the model is square [3]. So, a new design for this model will be proposed: the commodity-based model can be explained as a supply-driven model, to restore the closed economic circuit.

2 The economic circuit in the traditional Leontief and Ghosh models

Although it is long established, the traditional square model of input-output economics needs to be recalled here to show that both its versions, the Leontief one ("demand driven") and the Ghosh one ("supply driven"), can be -- and must be -- economically interpreted in terms of economic circuit or in probabilistic terms. Viewing the model as an economic circuit is not complicated or strange, it is only the idea to guarantee that the model can be developed such that it is possible to pass from the direct effects (read into the matrix of coefficients) to the

---

not the aim of this paper to discuss this point.

[3] The reader must not confuse between the Leontief or Ghosh models and the square models: even if the Leontief or Ghosh models are square, Make-Use models can also be square when the number of commodities is equal to the number of sectors.

total effects (read into the inverse matrix). Even if there is a link between this approach and graph theory, it is not really a circuit in the sense of graph theory from one vertex to itself via many others, as could be $i \rightarrow j \rightarrow k \rightarrow i$ [4] but a global circuit from the aggregate of sectors to the aggregate of sectors (that comprise all the arcs from any sector to any sector). The economic circuit interpretation will be developed in what follows and the reader will understand why, when the economic circuit is impossible to close correctly, the model becomes an empty exercise, economically meaningless.

2.1 The Leontief model as economic circuit

Denote x_j as the output of sector j, f_i as the final demand of commodity i, v_j as the value added of sector j; z_{ij} indicates how much of commodity i is bought by sector j, that is the flow from i to j. All quantities are computed in units of money. Matrix $\mathbf{Z}$ is homogenous by rows and columns. This can be arranged in the following accounting matrix:

$$\begin{array}{ccc} \begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{bmatrix} & \begin{matrix} f_1 \\ f_2 \\ f_3 \end{matrix} & \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} \\ \begin{matrix} v_1 & v_2 & v_3 \end{matrix} & & \\ \begin{matrix} x_1 & x_2 & x_3 \end{matrix} & & \end{array}$$

The central equation of traditional input-output economics (Leontief, 1936) is:

$$(1) \qquad \mathbf{x} = \mathbf{A}\,\mathbf{x} + \mathbf{f}$$

where $a_{ij} = \frac{z_{ij}}{x_j}$ is the technical coefficient, $\mathbf{A} = \mathbf{Z}\,\langle \mathbf{x} \rangle^{-1}$ denoting the matrix of technical coefficients. The model can be resolved simply as:

---

4 In this field, a pioneer work was those of Lantner (1974). See also (de Mesnard, 1992) for an asynchronous version of this economic circuit.

(2) $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{f}$

In the Leontief model, the quantity $(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \ldots$ can be interpreted as the sum of, respectively, the effect of final demand of any commodity *j* (say, cars), the direct effect of the intermediate demand between any couple of sectors *j* and *i* (say, cars that need steel), the indirect effect of the intermediate demand between any pairs of sectors *j* and *i* via any sector *l* (say, cars that need steel and steel that needs energy), etc.

How equation (1) is obtained? There are two possibilities. The first possibility is purely mathematical (Leontief, 1985): from the accounting identity $\mathbf{Z}\,\mathbf{s} + \mathbf{f} = \mathbf{x}$, where $\mathbf{s}$ is the sum vector, substituting $\mathbf{A}$ into it, Leontief obtains directly (1); this manner is right but does not call any economic argument. The second possibility consists into giving to equation (1) an economic interpretation: this one comes naturally in terms of economic circuit. Consider a surge of final demand: the initial increase of final demand of commodity *j* generates an equal increase in the output of sector *j*: $\Delta f_j^{(0)} \to \Delta x_j^{(0)} = \Delta f_j^{(0)}$. At its turn this generates an increase in the need of input *i*: $a_{ij}\,\Delta x_j^{(0)}$. So, the total increase of the output of sector *i* is: $\Delta x_i^{(1)} = \sum_{j=1}^{n} a_{ij}\,\Delta x_j^{(0)}$. This continues at steps 2, ..., etc., and at step *k*: $\Delta x_i^{(k)} = \sum_{j=1}^{n} a_{ij}\,\Delta x_j^{(k-1)}$ that is $\Delta\mathbf{x}^{(k)} = \mathbf{A}\,\Delta\mathbf{x}^{(k-1)}$ and the economic circuit is closed. Equation (1) can be retrieved by integration. The solution of the model is found by computing $\Delta\mathbf{x}^{(k)} = \mathbf{A}^k\,\Delta\mathbf{x}^{(0)} = \mathbf{A}^k\,\Delta\mathbf{f}$, thus the total increase of output is given by $\Delta\mathbf{x} = \sum_{k=1}^{n} \Delta\mathbf{x}^{(k)} = \left(\sum_{k=1}^{n} \mathbf{A}^k\right)\Delta\mathbf{f} \underset{k\to\infty}{\longrightarrow} (\mathbf{I} - \mathbf{A})^{-1}\,\Delta\mathbf{f}$; equation (2) is retrieved by integration. So, this interpretation describes a circular process: the production by a sector generates a demand for some intermediate commodities, itself described by the technical coefficients, which in turn generates production by the relevant sectors (remembering that the bijective sector-product correspondence is assumed) and the model can be described as *demand-driven*. This is well known, but not exactly contained in the first possibility above. The economic circuit interpretation is closer to Sraffa's "production of commodities by means of commodities" (Sraffa, 1960). The interpretation in terms of economic circuit is elementary, even if it is generally overshadowed. Anyway, if the interpretation as a closed economic circuit fails, the model loses any economic meaning: it remains possible to find (1) by the first possibility (direct substitution of the economic coefficients into the accounting identity) but the economic interpretation of the model falls: it is not the case of the Leontief model, but it could be for others, as it will be shown.

Figure 1 about here

2.2 The Ghosh model as economic circuit

The alternative version of the model is *supply-driven* (Ghosh, 1958) [5]. Allocation coefficients $b_{ij} = \frac{z_{ij}}{x_i}$ are assumed to be stable. The central equation of the model is:

(3) $\mathbf{x}' \mathbf{B} + \mathbf{v}' = \mathbf{x}'$

which solves as:

(4) $\mathbf{x}' = \mathbf{v}' (\mathbf{I} - \mathbf{B})^{-1}$

This could be also interpreted as an economic circuit. The initial increase $\Delta v_i^{(0)}$ of the value added of an industry *i* generates an equal increase in the output of this industry, $\Delta x_i^{(0)} = \Delta v_i^{(0)}$; this generates an increase in the supply of sector *j*: $b_{ij}\ \Delta x_i^{(0)}$. So the total increase in the output of sector *j* is $\Delta x_j^{(1)} = \sum_{i=1}^{n} b_{ij}\ \Delta x_i^{(0)}$, that is at step *k*: $\Delta x_j^{(k)} = \sum_{i=1}^{n} b_{ij}\ \Delta x_i^{(k-1)}$, or in matrix terms,

5 I do not discuss here about its plausibility even if the Ghosh model is often seen as less plausible (Bon, 1986; Oosterhaven, 1988, 1989, 1996; Miller, 1989; Gruver, 1989; Rose and Allison, 1989) except as a price model (Dietzenbacher, 1997); but the model must be recalled for the clarity of the exposé.

$\Delta\mathbf{x}^{(k)\prime} = \Delta\mathbf{x}^{(k-1)\prime}\,\mathbf{B}$, and (3) is retrieved in derivative terms. The model solves as: $\Delta\mathbf{x}^{(k)\prime} = \Delta\mathbf{x}^{(0)\prime}\,\mathbf{B}^k = \Delta\mathbf{v}'\,\mathbf{B}^k$ and the increase in total output becomes $\Delta\mathbf{x}' = \sum_k \Delta\mathbf{x}^{(k)\prime} = \Delta\mathbf{v}'\left(\sum_k \mathbf{B}^k\right) = \Delta\mathbf{v}'\,(\mathbf{I}-\mathbf{B})^{-1}$: equation (4) can be retrieved by integration. The model is just as coherent as the demand-driven one, but one must remember also that the Leontief and the Ghosh models are incompatible. As $\mathbf{B} = \hat{\mathbf{x}}^{-1}\,\mathbf{A}\,\hat{\mathbf{x}}$, allocation coefficients cannot be stable if technical coefficients are also stable: a star denoting aggregates after a change ($\mathbf{x}$ changing into $\mathbf{x}^*$), if $\mathbf{A}$ is stable, $\mathbf{A}^* = \mathbf{A}$, then $\mathbf{B}^* = \hat{\mathbf{x}}^{*\,-1}\,\mathbf{A}\,\hat{\mathbf{x}}^* \neq \mathbf{B}$.

Figure 2 about here

3 Remind: the Make-Use models

In the rectangular models as the SNA, two rectangular homogenous [6] matrices are considered. The Use matrix, denoted $\mathbf{U}$, with industries as columns and commodities as rows and with final demand as a supplementary column and value added as a supplementary row, indicates how much of each commodity each industry buys in order to produce. For example, for two industries and three products:

$$\begin{array}{cc}
\begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \\ u_{31} & u_{32} \end{bmatrix} & \begin{array}{cc} e_1 & q_1 \\ e_2 & q_2 \\ e_3 & q_3 \end{array} \\
\begin{array}{cc} w_1 & w_2 \\ x_1 & x_2 \end{array} &
\end{array}$$

where x_i is the output of industry i, w_j is the value added of industry j, q_i is the total production of commodity i, and e_i is the amount of commodity i sold to final demand. The

[6] For sake of simplicity, only the case of data in currency units is studied here: matrices are homogenous by rows and columns, while they are not when data are given in physical units.

Make matrix, denoted **V**, with industries as rows and commodities as columns, indicates how much of each commodity an industry produces. For example:

$$\begin{array}{cc} \begin{bmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \end{bmatrix} & \begin{matrix} x_1 \\ x_2 \end{matrix} \\ \begin{matrix} q_1 & q_2 & q_3 \end{matrix} & \end{array}$$

Four accounting identities are given: $x_i = \sum_{j=1}^{m} v_{ij}$ for all i, $x_j = \sum_{i=1}^{m} u_{ij} + w_j$ for all j and $q_i = \sum_{j=1}^{n} u_{ij} + e_i$ for all i, $q_i = \sum_{i=1}^{n} v_{ij}$ for all j, that is:

(5) $\quad \mathbf{x} = \mathbf{V}\ \mathbf{s}$

(6) $\quad \mathbf{x} = \mathbf{U}'\ \mathbf{s} + \mathbf{w}$

(7) $\quad \mathbf{q} = \mathbf{U}\ \mathbf{s} + \mathbf{e}$

(8) $\quad \mathbf{q} = \mathbf{V}'\ \mathbf{s}$

Technical coefficients are defined as: $a_{ij}^{u} = \dfrac{u_{ij}}{x_j}$, or,

(9) $\quad \mathbf{A}^{u} = \mathbf{U}\ \hat{\mathbf{x}}^{-1}$

Two alternative hypotheses are posited about how the matrix **V** must be read, the industry-based technology and the commodity-based technology, each generating two alternative models. It is possible to set out the complete solution of these models: each hypothesis generates two balance-accounting identities (commodities-by-commodities and industries-by-commodities) and four total-requirement matrices (commodities - by - commodities, commodities - by - industries, industries - by - industries and industries - by - commodities). See Miller and Blair (1985, pp. 159-...) or Aidenoff (1970) [7].

[7] There is also a mixed hypothesis (ten Raa, Chakraborty and Small, 1984). In (Kop Jansen and ten Raa, 1990), other types of hypotheses are listed, while the axiomatic of the rectangular model is developed; see also (ten Raa, 1995, pp. 87-100). About the link between

3.1 Industry-based technology

Following Aidenoff (1970) or Miller and Blair (1985, p. 165-166) [8], the total output q_j of a commodity j is supplied by industries i in fixed proportions, i.e., the *commodity-output proportion* $d_{ij} = \frac{v_{ij}}{q_j}$ is fixed (termed as *technology based on industries*), that is:

(10) $\mathbf{D} = \mathbf{V}\,\hat{\mathbf{q}}^{-1}$

In other words the input structure of an industry does not depend on the products that it produces. This hypothesis corresponds simply to a fixed market share of all industries, which may be realistic in the short run, and for Miller and Blair (1985, p. 166), it is also suitable for by-products (products whose production is linked to the main production, such as cars and automobile parts) [9].

The commodity-by-commodity identity is found by substituting (9) in (7), that is $\mathbf{q} = \mathbf{A}^u\,\mathbf{x} + \mathbf{e}$, then by substituting (10) in (5), that is $\mathbf{x} = \mathbf{D}\,\mathbf{q}$, what follows:

(11) $\mathbf{q} = \mathbf{A}^u\,\mathbf{D}\,\mathbf{q} + \mathbf{e} \Leftrightarrow \mathbf{q} = \mathbf{A}^I\,\mathbf{q} + \mathbf{e}$

by denoting $\mathbf{A}^I = \mathbf{A}^u\,\mathbf{D}$ the matrix of direct commodity requirements. Note that $\mathbf{A}^I = [\mathbf{U}\,\hat{\mathbf{x}}^{-1}]\,[\mathbf{V}\,\hat{\mathbf{q}}^{-1}] = \mathbf{U}\,\langle \mathbf{V}\,\mathbf{s}\rangle^{-1}\,\mathbf{V}\,\langle \mathbf{V}'\,\mathbf{s}\rangle^{-1}$. By defining final demand in terms of industries' output, $\mathbf{f} = \mathbf{D}\,\mathbf{e}$, and by premultiplying (11) by $\mathbf{D}$, the industry-by-industry identity is $\mathbf{x} = \mathbf{D}\,\mathbf{A}^u\,\mathbf{x} + \mathbf{f}$, which could be denoted $\mathbf{x} = \widetilde{\mathbf{A}}\,\mathbf{x} + \mathbf{f}$, with $\widetilde{\mathbf{A}} = \mathbf{D}\,\mathbf{A}^u$. And there is no

---

interregional models and rectangular model, see (Oosterhaven, 1984).

[8] Here I follow the most common presentation of the model. Sometimes, the hypotheses are presented in a reverse way, invoking the input structure instead of the output structure.

[9] Even if the true by-product model is different: all secondary products are by-products and are considered as negative inputs in the mixed-technology model (ten Raa, Chakraborty and Small, 1984, p. 88)

requirement for **U** and **V** to be square for industry-based technology to compute direct matrices (but in one case over four, when computing the commodity-by-industry inverse matrix, **D** must be square; see Miller and Blair 1985, p. 171).

3.2 Commodity-based technology

Again following Aidenoff (1970) or Miller and Blair (1985, p. 165), the total output x_i of any industry i is composed of commodities j in fixed proportions, i.e., the *industry-output proportion* $c_{ij} = \frac{v_{ij}}{x_i}$ is fixed (termed as *technology based on commodities*), and the input structure of a commodity does not depend on the industry that actually produces the commodity:

(12) $\mathbf{C} = \hat{\mathbf{x}}^{-1}\,\mathbf{V}$

For Miller and Blair (1985, p. 166) this hypothesis is applicable to subsidiary products (secondary products -- that are primary for other sectors -- produced with the same technology as the primary product of the industry, such as automobiles and buses). The System of National Accounts of 1993 prescribes to use the commodity-based technology.

The commodity-by-commodity identity is found from (12) giving $\hat{\mathbf{x}} = \mathbf{V}\,\mathbf{C}^{-1}$, and by premultiplying by $\mathbf{s}'$, $\mathbf{x}' = \mathbf{s}'\,\mathbf{V}\,\mathbf{C}^{-1}$, that is, by (8), $\mathbf{x}' = \mathbf{q}'\,\mathbf{C}^{-1}$, i.e.,

(13) $\mathbf{x} = \mathbf{C}'^{\,-1}\,\mathbf{q}$

Once again, substituting (9) in (7) gives $\mathbf{q} = \mathbf{A}^u\,\mathbf{x} + \mathbf{e}$ and finally from (13) $\mathbf{q} = \mathbf{A}^u\,\mathbf{C}'^{\,-1}\,\mathbf{q} + \mathbf{e}$. This can be denoted $\mathbf{q} = \mathbf{A}^C\,\mathbf{q} + \mathbf{e}$, where $\mathbf{A}^C = \mathbf{U}\,\hat{\mathbf{x}}^{-1}\,[\mathbf{V}'\,\hat{\mathbf{x}}^{-1}]^{-1}$ $= \mathbf{U}\,\hat{\mathbf{x}}^{-1}\left[\hat{\mathbf{x}}\,(\mathbf{V}')^{-1}\right] = \mathbf{U}\,(\mathbf{V}')^{-1}$ is the direct requirement matrix. By premultiplying this expression by $\mathbf{C}'^{\,-1}$ after redefining final demand in terms of industry output, $\mathbf{f} = \mathbf{C}'^{\,-1}\,\mathbf{e}$, the industry-by-industry identity is $\mathbf{x} = \mathbf{C}'^{-1}\mathbf{A}^u\,\mathbf{x} + \mathbf{f}$. Note that with commodity-based technology, most of the above formulae require the number of commodities to be equal to the number of industries when the inverse of **C** has to be computed: Make and Use matrices must be square even to compute direct matrices, which is a very restricting condition. While it is possible to generate the balance-accounting identities of industry-based technology without computing the inverse of **D**, the same is not true of commodity-based technology without computing the inverse of **C**. ■

4 Economic circuits and Make-Use models

As for the Leontief model, the Make-Use model can be interpreted in terms of economic circuit. Here, it is not a complete economic circuit in the traditional sense, from industries to consumers and conversely, but only a more limited one, between industries. Everything is dependent on the plausibility of the circular process as described by the alternative hypotheses: either the process is plausible and the solution of the model is economically meaningful or it is not.

4.1 The closed economic circuit under the industry-based technology hypothesis

The interpretation in terms of economic circuit works well for the industry-based technology. Consider a variation of final demand $\Delta e_j^{(0)}$ for commodity *j*. It is an equal need for commodity *j*: $\Delta q_j^{(0)} = \Delta e_j^{(0)}$ which generates an increase in the production of industry *i*: $d_{ij}\,\Delta q_j^{(0)}$; so, in total, industry *i* has to produce: $\Delta x_i^{(1)} = \sum_{j=1}^{m} d_{ij}\,\Delta q_j^{(0)}$. Then, the additional production of industry *i* generates the need for intermediate goods, which is for commodity *l*: $a_{il}^u\,\Delta x_i^{(1)}$. The total intermediate demand for commodity l is: $\Delta q_l^{(1)} = \sum_{i=1}^{n} a_{il}^{u} \, \Delta x_i^{(1)}$. And the economic circuit is closed and begins again with this demand for commodity l. At step k, one has: $\Delta x_i^{(k)} = \sum_{j=1}^{m} d_{ij} \, \Delta q_j^{(k-1)}$, that is $\Delta \mathbf{x}^{(k)} = \mathbf{D} \, \Delta \mathbf{q}^{(k-1)}$, and $\Delta q_l^{(k)} = \sum_{i=1}^{n} a_{il}^{u} \, \Delta x_i^{(k)}$, that is $\Delta \mathbf{q}^{(k)} = \mathbf{A}^u \, \Delta \mathbf{x}^{(k)}$. Finally, $\Delta \mathbf{q}^{(k)} = \mathbf{A}^u \, \mathbf{D} \, \Delta \mathbf{q}^{(k-1)}$ and the model of (11) is recovered: both possibilities -- the mathematical one and the economic one -- explained for the Leontief model hold. In graphical terms, the economic circuit is as in figure 3:

Figure 3 about here

Obviously, the process could begin with demand made on an industry instead of demand for a commodity.

4.2 The broken economic circuit under the commodity-based technology hypothesis

With commodity-based technology, industries demand commodities by means of technical coefficients, but these commodities are assumed to be produced by industries in accordance with the industry output proportions, c_{ij}. If we are to translate this in terms of economic circuit, the process could begin with a final demand for commodity j: $\Delta e_j^{(0)} \rightarrow \Delta q_j^{(0)} = \Delta e_j^{(0)}$. There are two cases.

1) If the number of industries is not the same than the number of commodities, which is the general case, then obviously the inverse of **C** cannot be computed [10] but the difficulties are not only a question of being able to inverse or not a matrix. When the inverse of **C** cannot be

[10] It is not a matter of computing pseudo-inverses or other artifices of computation.

computed, it is obvious to say that **C** does **not** correctly indicate which industry will produce this commodity: **C** indicates how commodities are produced by each industry, and not what industry has to produce any commodity. To understand what happens, the reader might consider the following rectangular example:

$$\mathbf{C} = \begin{array}{c} \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.1 & 0.6 & 0.3 \end{bmatrix} \\ \text{Commodities} \end{array} \begin{array}{c} 1 \\ 1 \end{array} \quad \text{Industries}$$

From, $\Delta q_3 = 1000$ for example, one cannot determine how much x_1 or x_2 will be increased (even if **C** is square). No information is available in **C** to determine whether it is industry 1 or industry 2, or both, that will increase their output. One could decide that it is a particular industry i_0 that has to produce this commodity *j*, that is $\Delta q_j^{(0)} \rightarrow \Delta x_{i_0}^{(1)} = \dfrac{\Delta q_j^{(0)}}{c_{i_0 j}}$, but it is arbitrary.

Figure 4 about here

2) When the number of commodities (miraculously) equals the number of industries [11], it is mathematically true that $\mathbf{x} = \mathbf{C}'^{\,-1}\,\mathbf{q} \Leftrightarrow \mathbf{q} = \mathbf{C}'\,\mathbf{x}$, this does not mean that **x** is determined by

[11] Is is a trickery to advocate that, as the inverse of **C** must be computed, the number of industries must be equal to the number of commodities. On the other hand, why to develop a rectangular model if it is to transform it into a square model by aggregation, even if it is with two matrices, just to be able to use it? Actually, there is no reason to have the number of industries and commodities. The only justification is: the name of an industry comes from the name of its main production. But it is not a good one because many industries can have the same main production, e.g. cars, while the can have not the same secondary productions; an aggregation of all of them will change the picture. By example, Fiat produces mainly cars but secondarily airplanes, while BMW also produces mainly cars but bikes as secondary

$\mathbf{C}$, i.e., $\Delta\mathbf{q} \rightarrow \Delta\mathbf{x} = \mathbf{C}'^{\,-1}\,\Delta\mathbf{q}$, only the contrary remains true: $\Delta\mathbf{x} \rightarrow \Delta\mathbf{q} = \mathbf{C}'\,\Delta\mathbf{x}$. Care is required with the meaning of the equals sign: in this last expression, "=" does not mean that the right side "equals" the left side but that the left side implies the right side [12]. To explain this, consider the following square example:

$$\mathbf{C} = \begin{array}{c} \left[\begin{array}{ccc} 0.5 & 0.3 & 0.2 \\ 0.1 & .06 & 0.3 \\ 0.1 & 0.2 & 0.7 \end{array}\right] \\ \text{Commodities} \end{array} \begin{array}{c} 1 \\ 1 \\ 1 \\ \; \end{array} \quad \text{Industries}$$

Computing the inverse of $\mathbf{C}$ is mathematically correct:

$$\mathbf{C}^{-1} = \begin{array}{c} \left[\begin{array}{ccc} 2.25 & -1.0625 & -0.1875 \\ -0.25 & 2.0625 & -0.8125 \\ -0.25 & -0.4375 & 1.6875 \end{array}\right] \\ \text{Industries} \end{array} \begin{array}{c} 1 \\ 1 \\ 1 \\ \; \end{array} \quad \text{Commodities}$$

This would indicate that, say, $\Delta q_3 = 1000$, generates $\Delta x_1 = -250$, $\Delta x_2 = -437.5$ and $\Delta x_3 = 1687.5$. $\mathbf{C}^{-1}$ seems to be able to indicate what industry has to produce each commodity but the apparition of negative terms "from nothing" proves that an illegal operation has been done. These negative terms have long been misunderstood: many authors have tried to eliminate them or to test if they are due to errors of measurement (see ten Raa and van der Ploeg (1989), Steenge (1990) with the introduction of a transition matrix between $\mathbf{A}^u$ and $\mathbf{C}$, or Almon (2000)). Ten Raa (1988) argues that negative terms are not due to errors in the

---

production; aggregating both leads to an industry that produces mainly cars, plus airplanes and bikes as secondary product. It is better to aggregate the less that it is possible; at least, the justification of a square model must not be mathematical (computing the inverse of a matrix) but economic. Note that ten Raa (1995, pp. 97-8) has discussed how coefficients can be found in the rectangular case.

[12] In computer science, often the programming languages make this distinction between "=" ("equal") and ":=" ("put this value into that variable").

data but to the model and he concludes that the commodity-based model must be abandoned. The inverse matrix $\mathbf{C}^{-1}$ has necessarily many negative terms. As $\mathbf{C}$ in not negative by hypothesis, and as $\mathbf{C}\,\mathbf{C}^{-1} = \mathbf{I}$, then for the off-diagonal terms of $\mathbf{I}$, the following formula holds: $\sum_k c_{ik}\ \sigma_{kj} = 0$ for all *i*, *j*, where is the $\{i, j\}$ term of $\mathbf{C}^{-1}$. So at least: $\exists k / \sigma_{kj} < 0$ for all *j*, i.e., there is one negative term by column of $\mathbf{C}^{-1}$, that is by industry. But as one could have written $\mathbf{C}^{-1}\ \mathbf{C} = \mathbf{I}$, there is also at least one negative by row of $\mathbf{C}^{-1}$, that is by commodity: finally, the inverse of $\mathbf{C}$ has at least one negative term by row and columns, that is by industry and commodity. In the above example, all off-diagonal terms are negative.

First, in the valuated-graphs theory, a negative coefficient corresponds to an arrow pointing in the reverse way. This is the case with many (all?) of the off-diagonal terms of $\mathbf{C}^{-1}$. So, returning to graph theory, in $\mathbf{C}^{-1}$, the negative coefficients $\{i,\ j\}$ (commodity *i* and industry *j*) must point in the reverse direction, not $i \to j$ (commodity $\to$ industry) but $j \to i$ (industry $\to$ commodity). They must not describe the industry structure of commodities (what industries are producing each commodity) but the commodity structure of industries (what commodities each industry is producing) what brings back to the beginning of our story.

Second, if $|\mathbf{C}| \neq 0$, then:

$$(14) \qquad \mathbf{I} + (\mathbf{I} - \mathbf{C}) + (\mathbf{I} - \mathbf{C})^2 + (\mathbf{I} - \mathbf{C})^3 + \ldots (\mathbf{I} - \mathbf{C})^k \underset{k \to \infty}{\longrightarrow} \mathbf{C}^{-1}$$

This equation indicates that the link between a coefficient of $\mathbf{C}$ and the corresponding coefficient of $\mathbf{C}^{-1}$ is very complicated. Obviously each coefficient of $\mathbf{C}^{-1}$ depends on all coefficients of $\mathbf{C}$ and is not the simple inverse of each individual coefficient. Equation (14) also suggests an interpretation of the inverse of $\mathbf{C}$ in terms of internal economic circuit: the matrix $\mathbf{C}^{-1}$ corresponds to the sum of many terms, $\mathbf{I}$, $\mathbf{I} - \mathbf{C}$, $(\mathbf{I} - \mathbf{C})^2$, etc., each corresponding to many economic circuits, of length equal to 0, 1 (from *i* to *j*), 2 (from *i* to *j* through any *l*), etc., respectively. What is the interpretation of such economic circuits, internal to matrix **C**, without any role of $\mathbf{A}^u$? Who knows! They could be neglected if $\mathbf{C}^{-1}$ could be given before **C** but it is not the case: the coefficients c_{ij} have been defined before the coefficients of $\mathbf{C}^{-1}$ and (14) holds. So, computing the inverse of **C** is a valid matrix operation whenever $n = m$, but this is economically meaningless [13]. To summarize, the paradox with the commodity-based technology model is that its matrix computing is correct but its economic-circuit interpretation is not: the first possibility explained for the Leontief model holds but not the second.

Figure 5 about here

4.3 Commodity-based technology and push-process

As it is necessary for the economic circuit to enter matrix **C** by the industries, it is possible to reverse the economic circuit, converting the model into a supply-driven one. Replace the technical coefficient matrix $\mathbf{A}^u$ by a matrix of allocation coefficients, $b^u_{ij} = \dfrac{u_{ij}}{q_i}$, that is:

(15) $\mathbf{B}^u = \hat{\mathbf{q}}^{-1}\,\mathbf{U}$

From (8) and (12), it follows:

(16) $\mathbf{q} = \mathbf{C}'\,\mathbf{x}$

From (15), we obtain

(17) $\hat{\mathbf{q}}\,\mathbf{B}^u = \mathbf{U}$

[13] It is not the case of $(\mathbf{I} - \mathbf{A})^{-1}$ in the Leontief model.

and substituting this in (6) gives $\mathbf{x} = \mathbf{B}^{u\prime}\ \hat{\mathbf{q}}\ \mathbf{s} + \mathbf{w} = \mathbf{B}^{u\prime}\ \mathbf{q} + \mathbf{w}$; so, the equation of the model is obtained by substituting (16) in this last equation:

(18) $\quad \mathbf{x} = \mathbf{B}^{u\prime}\ \mathbf{C}'\ \mathbf{x} + \mathbf{w}$

which could be denoted $\mathbf{x} = \mathbf{B}'\ \mathbf{x} + \mathbf{w}$, with $\mathbf{B} = \mathbf{C}\ \mathbf{B}^u = \hat{\mathbf{x}}^{-1}\ \mathbf{V}\ \hat{\mathbf{q}}^{-1}\ \mathbf{U} = \langle \mathbf{V}\ \mathbf{s}\rangle^{-1}\ \mathbf{V}\ \langle \mathbf{V}'\ \mathbf{s}\rangle^{-1}\ \mathbf{U}$. This could be transformed into a commodity-commodity equation by premultiplying (18) by $\mathbf{C}'$ and using (16), that is: $\mathbf{q} = \mathbf{C}'\ \mathbf{B}^{u\prime}\ \mathbf{q} + \boldsymbol{\varpi}$, where $\boldsymbol{\varpi} = \mathbf{C}'\ \mathbf{w}$ is the value added by commodity, or $\mathbf{q} = \tilde{\mathbf{B}}'\ \mathbf{q} + \boldsymbol{\varpi}$, with $\tilde{\mathbf{B}} = \mathbf{B}^u\ \mathbf{C}$.

A supply of a commodity *j* generates an output from all industries as indicated by $\mathbf{B}^u$, in the Ghoshian way, then the industries sell commodities in the proportions indicated by the coefficients c_{ij}. In terms of economic circuit, the initial increase $\Delta v_i^{(0)}$ in the value added of an industry *i* generates an equal increase in the output of this industry: $\Delta x_i^{(0)} = \Delta v_i^{(0)}$. By matrix **C**, this generates an increase in the supply of all commodities: $c_{ij}\ \Delta x_i^{(0)}$, that is, all told, the increase in the supply of commodity *j* is: $\Delta q_j^{(1)} = \sum_{i=1}^{n} c_{ij}\ \Delta x_i^{(0)}$. This supplementary supply of a commodity *j* induces an increase in the output of all industries *l* following $\mathbf{B}^u$: $\Delta q_j^{(1)} \rightarrow b_{jl}^u\ \Delta q_j^{(1)}$, so, in total, industry *l* increases its output of $\Delta x_l^{(1)} = \sum_{j=1}^{m} b_{jl}^u\ \Delta q_j^{(1)}$ and the economic circuit is closed.

At step *k*, one has: $\Delta q_j^{(k)} = \sum_{i=1}^{n} c_{ij}\ \Delta x_i^{(k-1)}$ and $\Delta x_l^{(k)} = \sum_{j=1}^{m} b_{jl}^u\ \Delta q_j^{(k)}$, that is $\Delta \mathbf{q}^{(k)} = \mathbf{C}'\ \Delta \mathbf{x}^{(k-1)}$ and $\Delta \mathbf{x}^{(k)} = \mathbf{B}^{u\prime}\ \Delta \mathbf{q}^{(k)}$, so $\Delta \mathbf{x}^{(k)} = \mathbf{B}^{u\prime}\ \mathbf{C}'\ \Delta \mathbf{x}^{(k-1)}$. This is in conformity with the corresponding model (18): the supply-driven commodity-based-technology model is consistent [14].

---

[14] All this is irrespective of the discussion about the artificial character of a supply-driven model: it is just to demonstrate that commodity-based technology is inconsistent, hesitating between a supply-driven and a demand-driven-model.

Figure 6 about here

5 Conclusion

Most national accounting systems are based on Stone's Make-Use model. The two traditional alternative hypotheses have been explored. The first one, industry-based technology, can receive an economic explanation in terms of economic circuit even in the rectangular case and it represents a fairly conventional demand-driven model. The alternative hypothesis, commodity-based technology, is problematic because the inverse of **C**, the matrix of industry output proportions, must be computed -- which is impossible in the rectangular case --. In the square case, it generates inexplicable negative terms: this problem, that authors have tried to correct empirically, suggests internal economic circuits inside **C** or reversed circuits that lack credibility. Consequently, the commodity-based model cannot be interpreted in terms of economic circuit: the problem of the negative terms generated in the direct requirement matrix is not simply an annoyance, but leads to reject the model. However, if this demand-driven commodity-based model is converted into a supply-driven one, it recovers its coherence even in the rectangular case, as it is not required to compute an inverse matrix: it can be economically interpreted in terms of economic circuit. To summarize, the industry-based technology corresponds to a demand-driven model while the commodity-based technology model should be reconstructed as a supply-driven model, which is a completely different thing. On the other hand, the supply-driven model has long been criticized as unrealistic [15], what

[15] For a discussion, see: Bon, 1986; Oosterhaven, 1988, 1989, 1996; Miller, 1989; Gruver, 1989; Rose and Allison, 1989; Dietzenbacher, 1997.

finally means that the commodity-based hypothesis itself either generates negative terms or is unrealistic, and so must be definitively rejected.

On the basis of this paper, the promoters of the System of National Accounts, United Nations and OECD, should take the opportunity of the introduction of new tables to reflect on the foundations of SNA, even if it is to call into question long years of established practice. The industry-based model, which was adopted by the United States, should be carefully reconsidered, even it has also some drawbacks in an axiomatic viewpoint: perhaps the price to pay will be to give up the idea of respecting some of the axioms listed by Kop Jansen and ten Raa (1990).

6 Bibliographical references

AIDENOFF, A. 1970, "Input-Output Data in the United Nations System of National Accounts', in. Anne P. CARTER and Andrew BRODY, 1970, Applications of Input-Output Analysis, North Holland; reprinted in Ira SOHN, Ed., 1986, *Readings in Input-Output Analysis. Theory and Applications*. Oxford, pp. 130-50.

ALMON, C. 2000. "Product-to-product tables via product technology with no negative flows", *Economic Systems Research*, 12: 27-43.

BLADES, D. 1989. "Revision of the System of National Accounts: A note on objectives and key issues", *OECD Economic Studies*, 0, 12: 205-19.

BON, R. 1986. "Comparative Stability Analysis of Demand-Side and Supply-side Input-Output Models," *International Journal of Forecasting* 2: 231-235.

de MESNARD, Louis 1992. "The Asynchronous Leontief Model", *Economic Systems Research*, 4, 1: 25-34.

DIETZENBACHER, E. 1997. "In vindication of the Ghosh model: a reinterpretation as a price model", *Journal of Regional Science* 37: 629-651.

GIGANTES T. 1970. "The representation of technology in input-output systems", in CARTER A. P. and A. BRODY (Eds), *Contributions in input-output analysis*, Amsterdam: North-Holland: 270-290.

GHOSH, A. 1958. "Input-output approach to an allocative system", *Economica*, 25, 1: 58-64.

GRUVER, G. W. 1989. "A comment on the plausibility of supply-driven input-output models", *Journal of Regional Science*, 29, 3: 441-50.

KONIJN P.J.A. and A.E. STEENGE, 1995. "Compilation of input-output data from the national accounts", *Economic Systems Research*, 7, 1: 31-45.

KOP JANSEN P. and T. ten RAA. 1990. "The choice of model in the construction of input-output matrices", *International Economic Review*, 31: 213-227.

JACKSON Randall W. 1998. "Regionalizing national commodity-by-industry accounts", *Economic Systems Research*, 10, 3: 223-238.

KUBONIWA, M., Y. MATSUE and F. ARITA. 1986. "Derivation of U.S. Commodity-by-Commodity Input-Output Tables from SNA Use and Make Tables", *Hitotsubashi Journal of Economics*, 27, 1: 49-76.

LANTNER Roland 1974. *Théorie de la dominance économique*. DUNOD, PARIS.

LAWSON Ann M. 1997. "Benchmark input-output accounts for the U.S. economy, 1992", *Survey of Current Business*, Bureau of Economic Analysis, U.S. Department of Commerce, Washington, November 1997: 36-82.

LEONTIEF, Wassily 1936. "Quantitative Input-Output Relations in the Economic System of the United States," *Review of Economics and Statistics*, 18, 3: 105-125.

______ , 1985. "Input-output analysis", in *Encyclopedia of Materials Science and Engineering*, Pergamon Press, Oxford. reprinted in: Leontief, Wassily, 1986. *Input-Output Economics*. Oxford University Press, New-York.

MILLER, Ronald E., 1989. "Stability of Supply Coefficients and Consistency of Supply-Driven and Demand-Driven Input-Output Models: a Comment," *Environment and Planning A*, 21: 1113-1120.

MILLER, Ronald E. and Peter D. BLAIR 1985. *Input-output analysis: foundations and extensions*, Englewood Cliffs, New-Jersey: Prentice-Hall.

OOSTERHAVEN, Jan 1984. "A Family of Square and Rectangular Interregional Input-output Tables and Models", *Regional Science and Urban Economics*, 14, 565-582.

______ 1988. "On the Plausibility of the Supply-Driven Input-Output Model," *Journal of Regional Science*, 28: 203-217.

______ 1989. "The Supply-Driven Input-Output Model: A New Interpretation but Still Implausible," *Journal of Regional Science*, 29: 459-465.

______ 1996. "Leontief versus Ghoshian Price and Quantity Models", *Southern Economic Journal*, 62, 3: 750-759.

ROSE, Adam and Tim ALLISON 1989. "On the Plausibility of the Supply-Driven Input-Output Model: Empirical Evidence on Joint Stability," *Journal of Regional Science*, 29: 451-458.

SRAFFA P. 1960. *Production of Commodities by Means of Commodities. Prelude to a Critique of Economic Theory*. Cambridge University Press, Cambridge.

STEENGE A. E. 1990. "The commodity technology revisited: theoretical basis and an application to error location in the make-use framework", *Economic Modelling*, 7: 376-387.

STONE, R. 1961. *Input-output and national accounts*. Paris: OECD.

ten RAA, Th. 1988. "An alternative treatment of secondary products in input-output analysis: frustration.", *The Review of Economics and Statistics,* 70, 3: 535-538.

________, 1995. *Linear Analysis of Competitive Economies*. Harvester Wheatsheaf, New-York.

ten RAA, Th., D. CHAKRABORTY and J. A. SMALL, 1984. "An alternative treatment of secondary products in input-output analysis", *The Review of Economics and Statistic*s, 66, 1: 88-97.

ten RAA, Th. and R. van der PLOEG. 1989. "A statistical approach to the problem of negatives in input-output analysis", *Economic Modelling*, 6: 2-19.

UNITED NATIONS, DEPARTMENT OF ECONOMIC AND SOCIAL AFFAIRS, 1968, *A System of National Accounts (SNA)*, series F, No. 2, Rev. 3, New-York: United Nations.

________, DEPARTMENT OF ECONOMIC AND SOCIAL AFFAIRS, 1993, *System of national accounts 1993 / prepared under the auspices of the Inter-Secretariat Working Group on National Accounts*. Brussels/Luxembourg: Commission of the European Communities; Washington, DC: International Monetary Fund; Paris: OECD; New York: United Nations; Washington, DC: World Bank.

________, DEPARTMENT OF ECONOMIC AND SOCIAL AFFAIRS, 1999, *Handbook of National Accounting: Input/Output Tables - Compilation and Analysis*. New York: United Nations.

Van BOCHOVE, C.A. and BLOEM, A.M. 1987. "The Structure of the Next SNA: Review of the Basic Options", *Statistical Journal of the United Nations Economic Commission for Europ*e, 4, 4: 369-90

VANOLI, A. 1994. "Extension of national accounts: opportunities provided by the implementation of the 1993 SNA", *Statistical Journal of the United Nations Economic Commission for Europe*, 11, 3: 183-91.

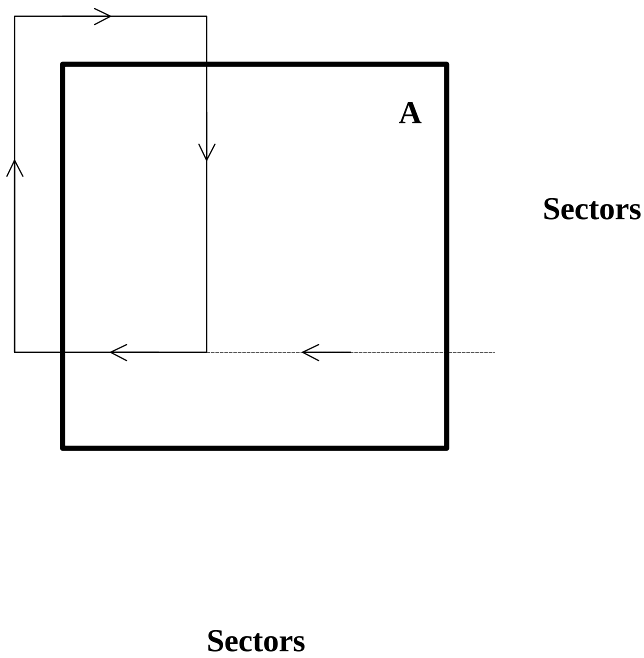

Figure 1. The economic circuit of the demand-driven Leontief model
(the entry is the final demand **f**)

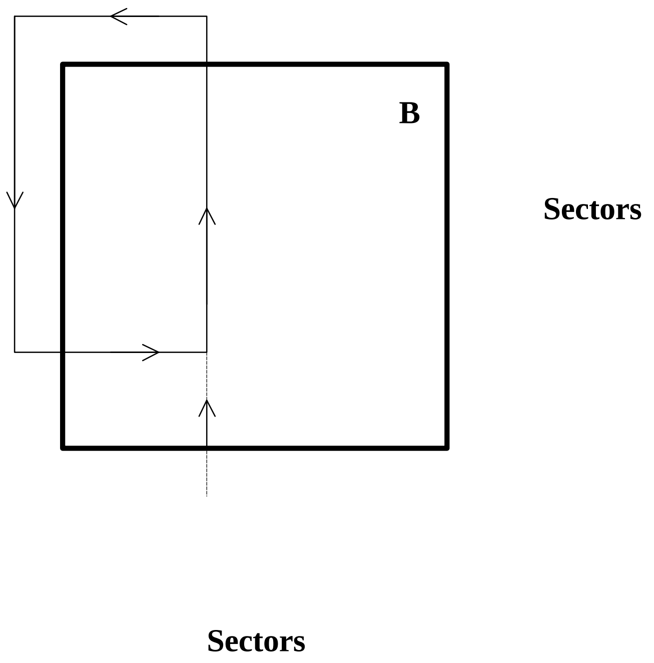

Figure 2. The economic circuit of the supply-driven Leontief model
(the entry is the value added **v**)

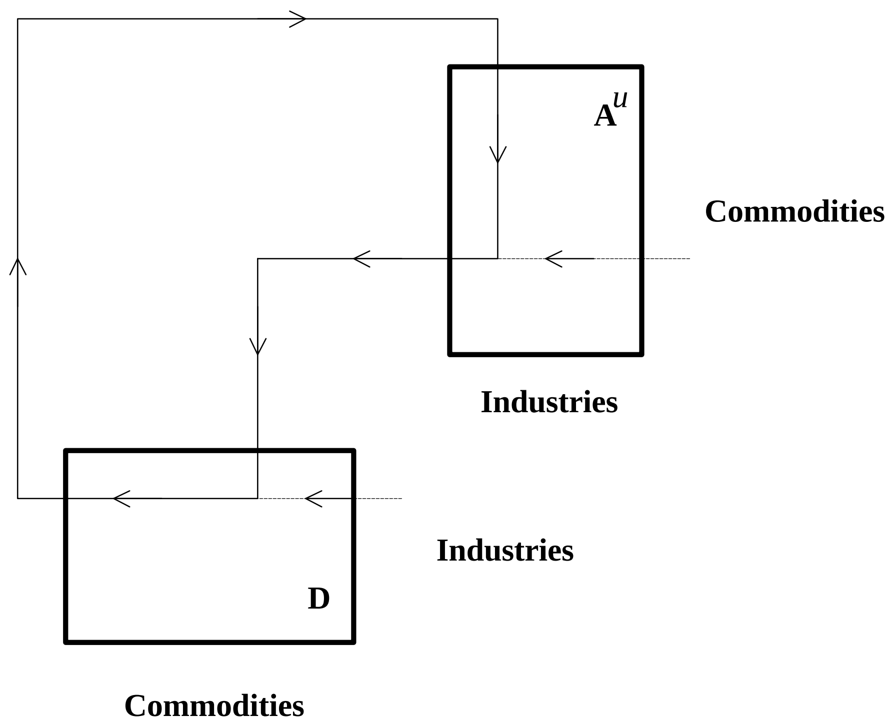

Figure 3. The economic circuit of the demand-driven industry-based model (two entries are possible: the final demand of commodities, **e**, or the final demand to industries, **f**)

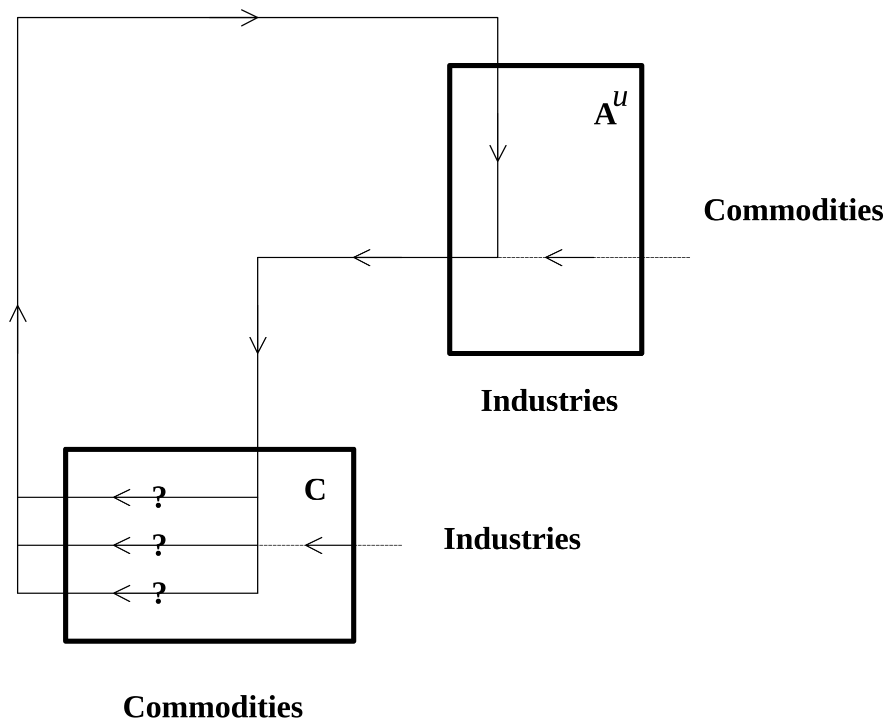

Figure 4. The undetermined circuits
of the rectangular demand-driven commodity-based model
(two entries are possible: the final demand of commodities, **e**,
or the final demand to industries, **f**)

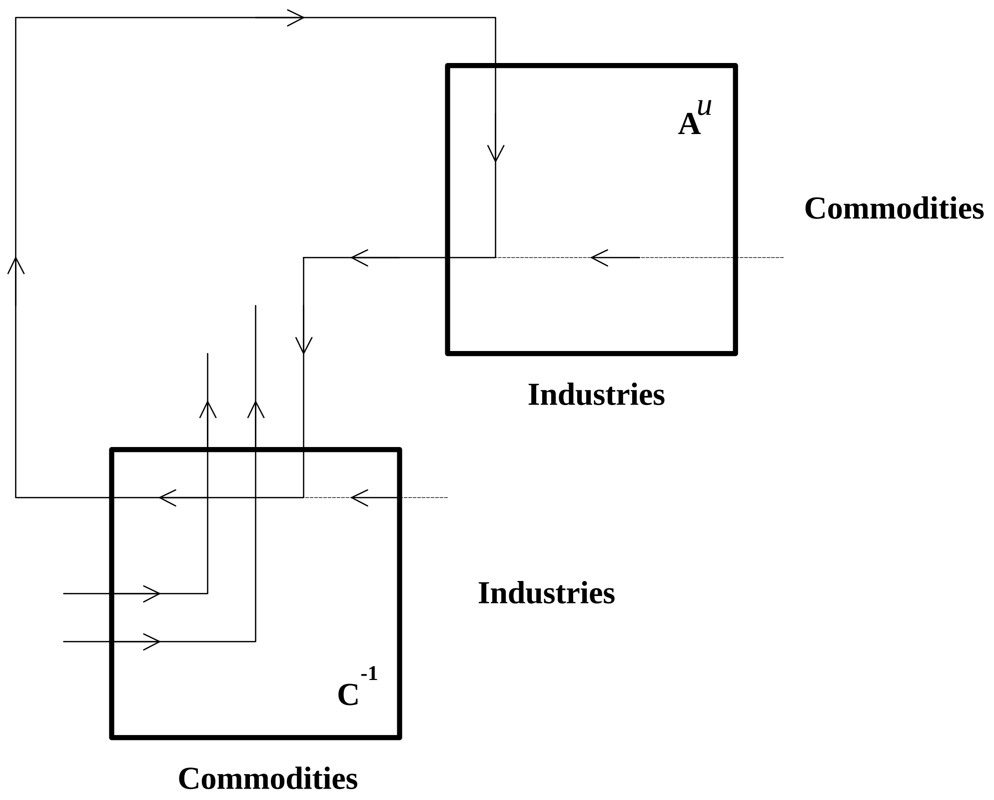

Figure 5. The reversed partial circuits
of the square demand-driven commodity-based model
(two entries are possible: the final demand of commodities, **e**,
or the final demand to industries, **f**)

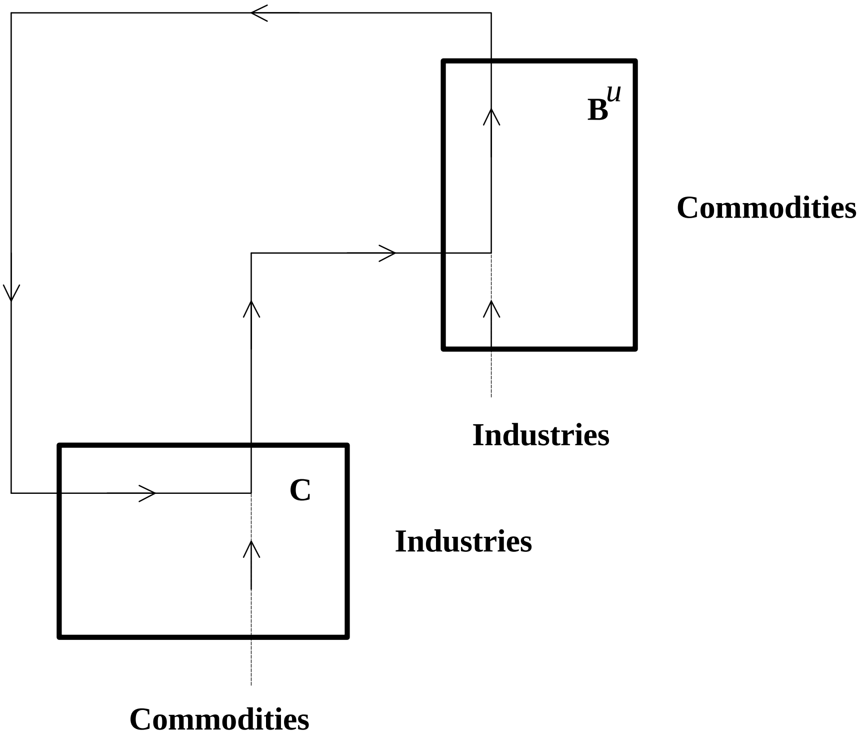

Figure 6. The economic circuit of the rectangular supply-driven commodity-based model
(two entries are possible: the value added of industries, $\mathbf{w}$,
or the value added by commodity, $\boldsymbol{\varpi}$)