Abstract

The flexibility of relative prices stands out as one of the major contributions of Applied General Equilibrium Models (AGE) over the traditional analysis derived from input-output tables (IOT) and social accounting matrices (SAM). But are the price functions coherent with the competitive equilibrium they are supposed to describe? Even if we assume that the prices embedded in the original SAM reflect a full competitive equilibrium, this may be not the case after a wage increase or a tariff reduction (to give a couple of typical “impacts”). The new set of prices do ensure the equality between supply of and demand for commodities and factors, but not the equality of the rate of return across industries, as competition requires. Any coherent AGE model should take such long-term equilibrium condition into account and introduce the required mechanisms moving the economy towards it.
1. Introduction.

The flexibility of relative prices stands out as one of the major contributions of Applied General Equilibrium Models (AGE) over the traditional analysis derived from input-output tables (IOT) and social accounting matrices (SAM). All these models compete in analysing the economic impact associated to different shocks. The original input-output multipliers based on Leontief’s own work (Leontief, 1941, 1966; Dietzenbacher & Lahr, 2004), assumed fixed coefficients of production, fixed consumption propensities and fixed prices. So did the SAM multipliers derived by Pyat and Round (1979, 1985 and Pyat 1991), after properly explaining the redistribution process leading to disposable income of institutions. AGE models started in the eighties and have became an alternative in the nineties (Scarf & Shoven, 1984; Kehoe & Kehoe, 1994; Ginsbrugh & Keyzer, 2002; Kenow, Srinivasan & Whalley, 2004). They claim to offer a superior tool of analysis on a variety of grounds: (1) Flexible coefficients of production and consumption, embedded in the Cobb-Douglas functions, which allow for the substitution among factors of production and consumption goods. The precise combination of factors and commodities depends, precisely, on their relative prices. (2) Flexible prices, which depend not only on supply shocks (changes in wages, taxes, tariffs and so on), but also on demand shocks (changes in the composition of final demand) and demographic shocks (changes in labour supply).

In this paper we are going to analyse the formation of prices in multisectorial models. We shall focus on the supply side that signals the long run equilibrium condition in competitive markets. Competition forces firms to use efficiently the best techniques available and to adjust prices to the costs of production, in which a “normal” profit is included. Competition shapes prices in such a way that they allow a uniform rate of profit on the capital invested in the different industries. What matters is neither the absolute level of profits nor the share of profits, but the rate of profit. Sectoral levels and shares of profits have to differ so that, in each industry, the ratio “profits / value of capital invested” is the same. This convergence has to be seen as a dynamic process in a long run perspective. A change in the level and composition of demand, to give an example, may raise prices and profits of the commodities experiencing excesses in demand. But higher profitability will attract new capital and spur production until it absorbs the excess in demand. At this moment, market prices will return to their long-run equilibrium yielding a uniform rate of profit. Similar dynamics could be derived when extra profits arise from a technological shock or from an autonomous increase in costs affecting industries unevenly.  

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1 The uniformity of profit rate refers to representative firms of the different industries. Whenever we take our data from IOT we’re bound to refer to the industrial average. Within an industry the rates of profits of particular firms may diverge at a given moment. Successful innovative firms are supposed to enjoy extra-profits for the time they have patent rights. Old-fashioned firms may obtain profits below average until they modernize, go bankrupt or quit the industry. Only monopoly or oligopoly power may grant extra profits for good. Such power should not be exaggerated, however. Even in industries ruled by a handful of big firms, we appreciate that prices are stuck to the full cost of production.
A couple of remarks should be made to complete this introduction. The first one is to emphasize that the condition of a “uniform profit rate” is a common trait of the major economic schools in Economics. The distinction between market prices (those which match supply and demand at a given moment), and natural or production prices (those yielding a uniform rate of profit and playing as centres of gravity) lies in the core of classical political economy, from Cantillon to Smith, and from Ricardo to Marx and Sraffa. The long-run equilibrium condition is also present in neoclassical general equilibrium from Walras to Arrow and Debreu; and in neoclassical partial equilibrium analysis from Marshall to modern industrial economics. Despite its declared Walrasian inspiration (Walras, 1889), the long-run equilibrium condition is ignored by AGE models.

Our second introductory remark is to stress the inner similarity in the supply-side treatment of prices by AGE and traditional input-output models. We have just referred to fixed-price multipliers derived from IOT and SAM. They show the impact on income and employment of changes in autonomous demand, assuming the constancy of prices. Such a statement is compatible, however, with the existence of a price system embedded in the data. It can be discovered by reading the columns of an IOT, which corresponds to the vertical production block of a SAM on which AGE models are built.

These two remarks justify the attention we are going to pay to the Sraffian system of prices of production (section 3) and input-output prices (section 4). Section 5 shows the supply-side formation of prices in AGE models. Section 6 considers the influence of demand on prices and searches for the mechanisms bringing prices back to their long-run equilibrium position. The remaining sections (2 and 7) are reserved to set up the model of analysis and to summarize our conclusions.

2. The simplest model to analyse the issues at stake.

The complexity of AGE models is enormous. They start from a complete SAM, they introduce functional relationships to explain their main parts, and they calibrate the parameters of these functions assuming that the actual data reflect equilibrium positions. Our paper is interested in a specific point: to highlight the supply side conditions for long-run equilibrium prices. For this purpose we only need to consider the production block of a SAM, i.e. the columns of the IOT.

To simplify the analysis as much as possible, we shall introduce other simplifying assumptions.

1) A competitive economy. Competition in the loose “classical sense” which only requires free mobility of capital towards the industries yielding the highest rates of return.

2) A private and closed economy. Our institutional agents are, therefore, families and firms.

3) Single production. There are $n$ homogenous industries, each one producing one single commodity.
4) **Circulating and heterogeneous capital.** In each industries capital consists in a basket of goods, that we identify with those filling intermediate consumption. Our main conclusions would stand up in economies with fixed and/or homogenous capital. The important point is that capital consists of "produced means of production".

5) **Heterogeneous labour.** We consider \( f \) types of labour whose combination differs from one industry to another. (a) Non-qualified labour; (b) qualified labour; ... (f) managers... The salary perceived by group (a) is considered the "base wage" \( (w) \). Other groups get a multiple of it (although for some applications these multiples will be set equal to one).

6) **Fixed coefficients of production.** They imply non-substitution between inputs and factors, and constant returns to scale. The hypothesis will be relaxed when we describe neoclassical AGE models. Technology is expressed by means of the following matrices, where \( n \) refers to the industries and \( f \) to the types of labour.

(a) A square matrix of technological coefficients, which results from dividing the intermediate consumptions of each industry \( j \) \( (X_{ij}) \) by its output \( (q_j) \), everything expressed in monetary units. \(^2\)

\[
[A]_{nn} = [X]_{nn} \cdot [q]_{nn}^{-1}
\]

National accounts do not provide the information required to separate prices and quantities. \( q_j \) is not the physical output of industry \( j \), but the money value of output. \( a_{ij} \) is the cost of the commodity \( i \) used in the production of a monetary unit of \( j \).

(b) A rectangular matrix of labour coefficients. It results from dividing the actual employment of each industry \( (L_j) \) by the industrial money-output \( (q_j) \). \(^3\)

\[
[L]_{fn} = [L]_{fn} \cdot [q]_{nn}^{-1}
\]

### 3. Prices of production in the classical-Sraffian tradition

Sraffa (1960) set the system of equations leading to what classical economists considered the "natural prices enforced by competition". F. Hahn (1982) read critically Sraffa’s price system to conclude that it is a coherent solution of a particular neoclassical general equilibrium model, the case with fixed coefficients of production. Although there are theoretical differences between the classical and neoclassical approaches, at this moment we are interested in emphasizing the common methodological ground. \(^4\)

The “price of production” of any commodity (to use Sraffa’s own words) is the result of adding up the following “unit costs”. (1) Cost of intermediate inputs,

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\(^2\) Industrial output will be always represented by a column vector “q”, although in some moments may appear as a diagonal matrix (angular brackets). Relative prices will always be represented by a row vector.

\(^3\) Satellite accounts of IOT and SAM provide information, in “physical” terms, about the number or workers and time of work of different types of labour in different industries. This is the content of \( L \).

\(^4\) Von Neumann’s general equilibrium model (1945) shares the same methodological principles as Sraffa’s price system.
p·A; (2) labour costs, w·l; (3) unit profit: r times the value of capital, represented here by the intermediate goods that have been advanced at the beginning of the production process (p·A).

\[ p = pA + w·l + r(p·A) \]

We count n equations (one for each industry-commodity) and n+2 unknowns, the n commodity prices, plus r and w. (Technology is taken as given). Scalar r stands for the rate of profit, which is assumed to be uniform across industries. w stands for the basic wage. It would be a scalar if there was only one type of labour, or the different groups earned the same salary. Normally it will be a diagonal matrix, whose elements are explained elsewhere (f.i. w₂=1.5·w₁).

The existence of two degrees of freedom allows the researcher to solve the price system in different ways. Classical economists took as given the real wage (at the subsistence level) and determined relative prices (in term of a numeraire whose price is set equal to one). Sraffa preferred to take as given the rate of profit (proxied by the interest rate, a monetary phenomenon) in order to determine relative prices and the real wage. Our choice (given the applications of the model we have in mind) would be as follow:

1. We take as “historically” given the rate of profit. Abstracting from temporal deviations of the degree of capacity utilization, the rate of profit has been quite stable through the years and decades. This fact denotes that usually firms are able to pass into prices all increases in nominal wages (and other autonomous costs).

2. We take as given (but movable) the nominal base wage (w) and the wage structure. In yearly agreements, employer’s organizations and trade unions fix this wage. They can also modify (although this isn’t normal) the wage structure.

3. We determine relative prices: \( p = [p₁, p₂… pₙ] \). After dividing these prices by w we obtain “labour commanded prices”, i.e the amount of base labour that can be purchased by selling one unit of \( q₁, q₂ … qₙ \). This procedure allows us to differentiate nominal price changes (inflation) from real changes.

In his theoretical model, Sraffa assumes there are enough information to separate prices from quantities. He obtains relative prices in terms of standard physical units: one car, one ton of wheat, one barrel of petrol, and so on. If this information is unavailable, “technical coefficients” become “value shares” and relative prices equal one. What has changed is the physical unit of measurement. Instead of saying that the price of a car is 4 monetary units, we state that the price of 0,25 cars equals one. The new units of measurement are actually unknown. It doesn’t matter, because we are only interested on the price change of such undefined units.

Equation [1] makes it clear that a uniform change in the unit labour cost (all types of wages rising in the same proportions) would cause a proportional increase in all prices, i.e. a pure inflation process. A change in the wages of a particular type of labour or in the wages of a particular industry will cause disproportional changes in prices. In both cases the rate of profit is kept
constant. There would be no problem in allowing changes in $r$ or even simulating the effects on relative prices of a change in $r$. The important point is that from the supply side, and as a long-run tendency, equilibrium prices convey a uniform rate of profit on the capital invested in each industry.

4. Input-output prices in Leontief’s tradition.

Input-Output tables and most of their applications were triggered by Leontief’s own work. The analysis of prices and price changes is not an exception (Leontief, 1966, chapter 3)\(^5\). Leontief’s “dual system of prices” or “input-output prices” are generally expressed as:

\[
p = pA + v = [1,1,\ldots,1]
\]

\[
p = v[I - A]^{-1} = (\omega + \beta)[I - A]^{-1} = [1,1,\ldots,1]
\]

Each column $j$ of $A$ accounts for the share of inputs 1, 2, … $n$ of the value of commodity $j$ whose price (by construction) equals one. $v$ is a “$(f+1)\times n$” rectangular matrix. Each column $j$ expresses the share in the value of commodity $j$ of the $f$ types of wages. Whether there are many types of labour or just one, let’s refer to them as $\omega$. To this we append an additional row $(f+1)$, which informs about the share of profits in the value of the $n$ commodities. This row vector will be labelled $\beta$.

Imagine that all the elements of $\omega$ (or just one of them) rise by 10%. How will prices move? To answer this question we only have to substitute the whole row $\omega$ by $\omega(1+0.1)$ (or the single cell $\omega_j$ by $\omega_j(1+0.1)$). Prices will rise enough to pay for higher wages and for more expensive inputs. Sectoral profits will not change, however. In the base year (this is an example) $\beta_j$ amounted to 0.2 out of $p_j=1$. After the shock, $p_j$ rises to 1.05, while $\beta_j$ continues to be 0.2. Entrepreneurs seem to be concerned with a “level” of profits, independently of the general level of prices. It seems more plausible to assume that they will try to pass wage increases into prices in such a way that profit shares remain constant. This would lead to a pure inflationary process, in which wages, prices and profits increase \textit{pari passu}.

In order to capture this inflationary effect, the share of profits (now a diagonal matrix) should appear in the inverted matrix that plays the role of a multiplier. It is mathematically correct because all the elements in $\beta$ are shares of a price equal to one. Since, by construction, $p_j=1$ we can write $\beta_j = \beta_j/p_j$. The “multiplicand” should be reserved for autonomous costs. In our model only wages are autonomous. In an open economy we should include the prices of imported intermediate goods. Actually they contain four autonomous elements: the international prices, the exchange rate, tariffs and taxes on imports. The new structure of prices would be as follows.

\[^5\] The “state of the art” can be found in Pulido & Fontela, 1993, chapter 3.3. Original and more detailed treatments in Bródy (1965), Aukrust (1970) and Sekerka, Kyn & Hejl (1970). The last one is quite interesting for our purposes since it contemplates also Sraffa’s prices of production.
Equation [3] would represent properly a pure inflationary process triggered by a general increase in wages (the only autonomous cost). Prices and profits will increase by the same percentage. The sectoral profit shares will remain constant and so will happen with the rate of profit. Notice, however, that the last one is bound to change and diverge in the general case, i.e. when the increase in autonomous costs differs from one sector to another. Equation [3] warrants the maintenance of the profit shares after any cost-shock. But to ensure a uniform profit rate (this is the true equilibrium condition), profit shares should change to neutralize movements in relative prices. Consider the unit profits of industry 1, as they appear in expression [4]. The left hand side corresponds to prices of production which include the condition of uniform rate of profit; the right-hand side corresponds to input-output prices. In the base year, physical units have been defined in such a way that all prices equal one and both sides of [4] amount to the same.

\[
[p = (\omega) \left[ I - A - \beta \right]^{-1}]
\]

Simple mathematical inspection makes it clear that changes in relative prices will wreck the initial equivalence since the operative prices on the left hand side are different from those on the right hand side.

To understand the confusions in input-output prices we should go back to the origins of input-output economics. Leontief (1941) studied the quantity system from the demand side and summarized it in the expression: \[q = (I - A)^{-1} \cdot y\]. Changes in the quantities demanded for final consumption and/or investment (both represented in the column vector \(y\)) will increase total quantities \((q)\) by a multiple. Whether the increase is proportional or not, it doesn’t matter. The result is supposed to represent always a new equilibrium since the structure of final output is not fixed by any rule. Consider now the value-added system from where the price equations [2] are derived: \[q = V \cdot (I - A)^{-1}\]. The structure of the row vector of value added \((V)\) is not longer arbitrary. Profits have to be such that the ratio “profits / value of capital invested” is uniform across sectors. Proportional changes in the value added component of all industries will bring about a general increase in nominal output, more concretely, in the price component implicit in “\(q\)”. If it is a pure inflationary process, relative prices will remain constant at the initial (equilibrium) values. Equation [2] may capture this inflationary effect, but only when we simulate a general and proportional increase in wages and profits. An autonomous rise in wages (only wages but all types of wages) may also result in a pure inflationary process compatible with the original relative prices. The condition is that firms are able to pass into prices the cost-push, as it will happen using equation [3]. In the general case (in which only certain types of autonomous costs do change), prices will rise but not proportionally. Then neither [2] nor [3] warrant that the new relative prices

\[r(p_i, \sum_{i=1}^{q} a_{ii}) = \beta_i \cdot p_i\]
be in equilibrium. Only equation [1] leads to full equilibrium prices with a uniform rate of profit.
5. Supply-side prices in neoclassical AGE models.

Despite its sophistication, AGE treatment of price formation from the supply-side is similar to input-output analysis (Scarf, 1967). Initially all prices are assumed to be “one”, so that figures in the columns of the IOT reflect physical quantities in unspecified units. Technical coefficients continue to be fixed. This hypothesis allows us to express the value of sectoral output (let say, $q_1$) as $\pi$ times the quantity of any of the intermediate inputs ($X_{i1}$) or factors of production ($V_1$). ($\pi$ being input’s “productivity”, measured by the inverse of the technical coefficient in matrix $A$).

$$q_1 = X_{i1} \left( \frac{1}{a_{i1}} \right) = X_{21} \left( \frac{1}{a_{21}} \right) = ... = V_1 \left( \lambda_i \right)$$

The value-added term, “$V_1$”, includes factors of production (labour and capital) operating in a Cobb-Douglas production function. The peculiarity of this function is that it allows factor substitution and combines decreasing returns to substitution with constant returns to scale. For industry 1 we can write:

$$q_1 = K_1^\alpha \cdot L_1^{(1-\alpha)} \cdot (\lambda_i)$$

All the variables and parameters are computed by “calibration”.

- The amount of labour is identified with money wages. If the IOT says that wages in industry 1 amount to 125 million euros, it is stated that there are 125 units of labour who earn a wage of 1 million euros. After dividing by the value of output we get the row vector $l'$. The dash is meant to show the differences with our previous vector $l$. Labour was measured in an already known physical unit, let’s say “hours”. To give an example, $l_1$ would indicate the hours of labour necessary to produce an undefined amount of cars worth one million euros. The unit of measurement in $l'$ is unknown. We can only state that the undefined unit of labour required to produce “one million euros cars” gets a salary equal to one.

- The amount of capital is identified with the amount of profits indicated in the IOT. A figure of 112 would be interpreted as the existence of 112 units of capital, each one hired by a rental price equal to one monetary unit ($\rho=1$). After dividing by the money value of sectoral output we obtain a row vector of capital ($k'$). Again, the units of measurement of capital are unknown.

- $\alpha$ is the elasticity of capital which coincides with the share of profits in output. $(1-\alpha)$ is the elasticity of labour and coincides with the share of wages in output. After fixing ($\rho=1$ and $w=1$) one can compute the value of $\alpha$.

- $\lambda$ stands for total factor productivity and can be easily calibrated once we know $k'$, $l'$ and $\alpha$.

What a simple and ingenious procedure to measure the stock of capital! Is it a legitimate one? The capital controversies of the 1960 showed the impossibility of measuring the capital stock without reference to the prices of the capital goods. (For a recent reminder, see Cohen & Harcourt 2003). AGE models bypass these criticisms identifying the quantity of capital with profits and

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7 Actually, $l$ is a rectangular matrix with as many rows as types of labour. Writing a “row vector” implies either that there is only one type of labour or that all types earn the base wage.
reinterpreting the equilibrium condition as a uniform rental price of capital. The procedure might work in a single commodity economy with homogenous capital (only one good is produced, consumed and invested in, whose relative price can only be one). In the ordinary case of a multi-sector economy with heterogeneous capital, the solution simply doesn’t work. We cannot give the same price to different baskets of capital goods. In the base period the size of these baskets can be chosen so that all of them are worth one monetary unit. After a change in relative prices each basket will have a different rental price, unless we alter the physical units in which capital is measured.

The problems caused by heterogeneous capital are far reaching, as the capital controversy showed. The problem we analyse in this paper is a simpler one and will show up even in the case of homogenous capital. Do the supply-side prices envisaged by AGE models warrant a uniform rate of profit across sectors? - No. Supply price equations are similar, in this respect, to the input-output price equations in [2]. The treatment of profits either as the rental price of capital or as a share of money output doesn’t warrant a uniform rate of profit. The condition of a general profit rate has to be introduced explicitly as we did in equation [1].

6. Demand and prices in AGE models.

Because of their generality, AGE models are better suited to analysing the whole range of requirements for equilibrium. In equilibrium, relative prices are supposed:
- To cover full costs of production. This is the supply-side equilibrium condition that is similar to the one introduced in input-output prices.
- To ensure the full employment of given endowments (labour and capital). This important macroeconomic outcome requires the substitution of factors in a process of cost minimization related to their relative prices (ratio $w/\rho$). The Cobb-Douglas production function we have just examined was meant for this purpose.
- To match supply of and demand for different commodities in such a way that households maximize utility. For this purpose they introduce Cobb-Douglas consumption functions that allow substitution among commodities when relative prices ($\rho/\rho_i$) do change.

Some economists would disagree with the idea of full-employment prices. Nevertheless everyone should acknowledge the superiority of AGE models in grasping a huge array of complex interrelationships. Price changes influence quantities (the size and structure of demand), while changes in demand may have an impact on prices. AGE models are also prepared to accommodate the short run influences of demand and the long run forces that push prices towards cost of production. After a rise in prices and profits due to excess of demand for

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8 Add to equation [4] the term “$\rho k_i$”. After a change in relative prices the value of the three parts of the expression will change.
9 As a matter of fact, AGE models warrant the full employment of given endowments in the base year. It may be compatible with massive unemployment, although it is considered “voluntary”.

certain goods, investment should increase the quantities produced until the excess of demand disappears and normal profits go back to the former equilibrium. A divergence in profits rates may also result from productivity or cost changes that are translated into prices in order to maintain a given share of profits. This could be a valid short-run equilibrium provided we consider the tendency to restore a uniform profit rate via investment flows.

Unfortunately AGE lack the mechanism that leads the economy towards its long run equilibrium position, as far as prices are concerned. They don’t have an independent investment function. Investment equals saving, which is presented as a part of consumption decisions. In an exercise of utility maximization, individuals decide the percentage of income to be spent on a variety of consumption goods, and the percentage to be saved. No explanation is provided about why and where firms decide to invest.

In a review of the performance of AGE models, Kehoe (2004) accepts their failure to predict the terrific effects of NAFTA on the growth of production employment and trade in North America. He concludes: “To capture changes in macroeconomic aggregates, the model needs to be able to capture changes in productivity” (Kehoe, 204, p. 341). We add: “…changes in productivity that generate abnormal profits and trigger huge investment flows”. The investment function is the ultimate link between prices and quantities, and between market prices and prices of production, as well. In order to be more reliable and useful, AGE models should pay attention to investment, and relate it to capacity shortages and differential profits rates.

7. Conclusions.

In studying AGE models a biblical story came to my mind. The Prophet Daniel reconstructs and interprets King Nebuchadnezzar’ dream.

“Your Majesty, in your vision you saw standing before you a giant statue, bright and shining, and terrifying to look at. Its head was made of the finest gold; its chest and arms were made of silver; its waist and hips of bronze, its legs of iron and its feet partly of iron and partly of clay. While you were looking at it, a great stone broke loose from a cliff without anyone touching it, struck the iron and clay feet of the statue and shattered them”. (Daniel, 2, 31-35)

AGE models look like an impressive giant; their dimension and completeness overshadow previous aggregate and disaggregate models. They have the iron strength provided by a general equilibrium theory. They are brilliant in many respects and shed light both in theoretical and applied economics. Their value-foundations, however, look as brittle as clay. Amazingly enough, prices governing the allocation of resources in a “general equilibrium” model, are not full-equilibrium prices. They do not warrant a uniform rate of profit across sectors, simply because such equilibrium condition is absent in the supply-side equations. Neither do they provide a valid mechanism to correct short-term deviations from equilibrium. This should be
the role of the investment function, which is conspicuously absent from AGE models. These absences weaken the foundations of these models; and render them vulnerable to any “stone”.

This failure to obtain full-equilibrium prices has two possible explanations. One should start recognizing the difficulty of neoclassical models to deal with heterogeneous capital. Notice, however, that certain problems would remain in economies with homogeneous capital. In addition, one should blame for the negligent oblivion of fundamental issues in the history of economic thought. The condition of “uniform profit rate” is neither our invention nor an obsession of a marginal school of thought. On the contrary, it is one of the few points of agreement among the major economic schools: classical political economy and neoclassical economics; Marshallian partial equilibrium and Walrasian general equilibrium, as well. How can it be absent from a declared Walrasian model?
References.


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