The Fallacy of Using US-Type Input-Output Tables

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Abstract. US input-output tables are of the type where intermediate inputs record the sum of imported and domestically produced goods. In modeling exercises this implies that imports are required to be specified exogenously. This has two consequences, which seriously restricts the usefulness of US-type tables. First, the multipliers can only be interpreted under the highly implausible assumption that the (changes in) imports are zero. Second, the results will be strongly overestimated in an empirical analysis, unless perfect foresight exists.

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1. Introduction

Input-output tables and tables of intermediate deliveries in social accounting matrices, are widely used data sources for a variety of studies. Roughly speaking we may distinguish between two groups. First, studies that are directly based on the industry data. For example, industry-level analyses of production techniques and productivity growth (such as TFP and MFP, and their connection to information technology, see e.g. Jorgenson, 2001; Stiroh, 2002; Jorgenson *et al.*, 2003) fall into this group. Second, studies that take full account of the interdependencies in the structure of production. To obtain the results, these studies typically employ the so-called Leontief inverse matrix, which is obtained from the matrix of input coefficients that is derived from an input-output table. An example that, after fifty years, still triggers new research is the Leontief paradox and the calculation of the factor contents of trade (see e.g. Davis *et al.*, 1997; Helpman, 1999; Leamer, 2000; Davis and Weinstein, 2001; Hakura, 2001; Wolff, 2004).

Also input-output tables can be divided into two categories, depending on how the competitive imports are dealt with.[1] On the one hand, the competitive imports may be included into the intermediate deliveries, which then measure the purchases of product *i* (both domestically produced and imported) by industry *j*. On the other hand, the competitive imports may be given in a separate matrix, which allows for a distinction between imported inputs and domestically produced inputs. The US is one of the countries that compiles input-output tables where the competitive imports are included in the intermediate deliveries. The non-competitive imports are for both types of tables usually added as a separate row.

It is clear that a separate import matrix provides more detail. But this additional detail is not always necessary. For many studies in the first group mentioned above, the distinction between imported and domestically produced inputs plays no role. For example, in analyzing total factor productivity growth, one is primarily interested in the technical aspects of production and it suffices to have access to the sum of imported and domestically produced intermediate deliveries.

However, also many input-output studies and models that fall in the second group are based on US-type tables. It is a common belief that this can be done correctly. In this note we will express serious doubts with respect to the validity of using US-type tables for input-output studies. We will show that applying a US-type table implies the adoption of an extremely strong and implausible assumption, and yields results that are biased.

2. The Models

2.1. Models based on tables with a separate import matrix

Our starting point is an input-output table in money terms (say dollars) as in Table 1, where the competitive imports are recorded separately. The elements d_{ij} of matrix **D** give the domestic intermediate deliveries from industry *i* to industry *j*, and the elements m_{ij} of matrix **M** denote the imports from a foreign industry *i* to industry *j*.[2] The vector **f** with typical element f_i gives the domestic deliveries of industry *i* for domestic final demand purposes (such as private consumption, private investment, government consumption and investment, and changes in stocks) and its gross exports. The row vector $\mathbf{v}'$ with typical element v_j includes the value added items (such as labor payments, capital depreciation, operating surplus, and indirect taxes minus subsidies) and the non-competitive imports. Without loss of generality, we have assumed that there are no imported final demands (for example imports

[1] See the guidelines for setting up a system of national accounts in United Nations (1993).

[2] Matrices are given in bold, capital letters; vectors in bold, lower case letters; and scalars in italicized, lower case letters. Vectors are columns by definition, row vectors are obtained by transposition, indicated by a prime.

for private consumption), neither are there any value added items in the final demand column. The vector **x** gives the domestic gross output for each industry.

INSERT TABLE 1

The matrix $\mathbf{A}^D$ of domestic input coefficients is obtained as $a_{ij}^D = d_{ij} / x_j$, which gives the domestic intermediate deliveries per unit of gross output. The matrix with import coefficients $\mathbf{A}^M$ is derived as $a_{ij}^M = m_{ij} / x_j$, indicating the imports from industry *i* per unit of gross output in industry *j*. The material balance equations yield $\mathbf{x} = \mathbf{Ds} + \mathbf{f}$, where **s** indicates the summation vector consisting of ones. Using the definition of $\mathbf{A}^D$, these balance equations can be rewritten as $\mathbf{x} = \mathbf{A}^D\mathbf{x} + \mathbf{f}$.

Under the assumption that the domestic input coefficient matrix $\mathbf{A}^D$ is constant, it can be calculated which output vector ($\tilde{\mathbf{x}}$) is required to satisfy an exogenously specified vector of final demands ($\tilde{\mathbf{f}}$). The solution is given by

$$\tilde{\mathbf{x}} = (\mathbf{I} - \mathbf{A}^D)^{-1}\tilde{\mathbf{f}} = \mathbf{L}^D\tilde{\mathbf{f}} \tag{1}$$

The matrix $\mathbf{L}^D \equiv (\mathbf{I} - \mathbf{A}^D)^{-1}$ is known as the Leontief inverse or multiplier matrix. Its interpretation is easily obtained by taking $\tilde{\mathbf{f}} = \mathbf{u}_\mathbf{j}$, i.e. the *j*th unit vector (with a one in position *j* and zeroes elsewhere). It then follows that l_{ij}^D denotes the (additional) domestic production in dollars by industry *i* that is required to satisfy one (extra) dollar of final demand of product *j*.

Once the gross domestic outputs are known, also the imports can be computed under the assumption that the matrix $\mathbf{A}^M$ is fixed. In the same way as we obtained the interpretation of the Leontief inverse, it immediately follows that element (*i*, *j*) of the matrix $\mathbf{A}^M(\mathbf{I}-\mathbf{A}^D)^{-1}$ $=\mathbf{A}^M\mathbf{L}^D$ gives the (additional) imports of product *i*, required to satisfy one (extra) dollar of final demand for product *j*.

2.2. *Models based on US-type input-output tables*

Table 2 describes the US-type of input-output table. It should be stressed that in this case the matrices **D** and **M** are not available separately, only their sum is known. One may now proceed in the same way as we did in the previous subsection. That is, input coefficients are defined as $a_{ij} = (d_{ij}+m_{ij})/x_j = a_{ij}^D + a_{ij}^M$ and from the balance equations it follows that $\mathbf{x}=\mathbf{Ax}+\mathbf{f}-\mathbf{m}$. Assuming that the input coefficients are fixed, the domestic gross outputs ($\tilde{\mathbf{x}}$) that satisfy a new, exogenously specified vector $\tilde{\mathbf{f}}-\tilde{\mathbf{m}}$ are given by

$$\tilde{\mathbf{x}}=(\mathbf{I}-\mathbf{A})^{-1}(\tilde{\mathbf{f}}-\tilde{\mathbf{m}})=\mathbf{L}(\tilde{\mathbf{f}}-\tilde{\mathbf{m}}) \tag{2}$$

In the literature, the same interpretation as for $\mathbf{L}^D$ is also given to the Leontief inverse **L** (and to the multipliers l_{ij}).[3]

INSERT TABLE 2

At first sight it may seem that the model in (2) is less restrictive. Instead of assuming that both $\mathbf{A}^D$ and $\mathbf{A}^M$ are constant, only **A** is required to be constant, which is a reasonable approximation for the short-run. It suggests that substitution of imported and domestically produced intermediate inputs is now allowed for. In the next section, however, we will show

[3] See e.g. introductory texts such as Dervis *et al.* (1982) or Miller and Blair (1985).

that the model in (2) is based on an unrealistically strong and highly implausible assumption. Applying the model requires that the imports be specified exogenously and the interpretation of the multipliers in **L** is only valid if the (extra) imports are assumed zero! In other words, applying the model requires perfect foresight and interpreting the multipliers assumes that all imported inputs are substituted by domestically produced inputs.

3. The Results

3.1. The interpretation of multipliers

Under the usual mathematical conditions, it can be shown that each multiplier in (2) is larger than in (1), unless **M** = 0.[4] That is, $l_{ij} > l_{ij}^D$. The multiplier l_{ij}^D gives the (extra) domestic output in industry *i*, due to one dollar (extra) consumption, for example. Surprisingly, it has become common practice in the literature to attach the same interpretation to l_{ij} although numerically it is larger. Equation (2) shows that also the imports must be specified exogenously. The multipliers l_{ij} can indeed be given the same interpretation, if and only if $\tilde{\mathbf{m}}$ (respectively $\Delta\tilde{\mathbf{m}}$) is assumed to be zero. The multiplier l_{ij} in (2) thus gives the (extra) domestic output in industry *i* due to one dollar (extra) final demand, provided there are no (extra) imports. This additional assumption clearly is heroic and grossly reduces the applicability of the multipliers l_{ij}.

Now that the additional assumption has been made explicit, it also is clear why l_{ij} should numerically be larger than l_{ij}^D. Whereas in (1), extra final consumption leads to extra domestic output and therefore extra imports, the imports are not allowed to increase in model

[4] See, for example, Takayama (1985) for a detailed, yet concise, overview of mathematical properties.

(2). The imported inputs in (1) have to be produced domestically when using (2), so that satisfying the same extra consumption requires more domestic output in (2) than in (1).

The extra domestic output ($\Delta\mathbf{x}$) due to one extra dollar of final demand *j*, is in model (1) given by

$$\Delta\mathbf{x} = [\mathbf{I} + \mathbf{A}^D + (\mathbf{A}^D)^2 + (\mathbf{A}^D)^3 + ...]\mathbf{u}_\mathbf{j} = (\mathbf{I} - \mathbf{A}^D)^{-1}\mathbf{u}_\mathbf{j} = \mathbf{L}^D\mathbf{u}_\mathbf{j} \tag{3}$$

That is, $\mathbf{u}_\mathbf{j}$ has to be produced, plus the domestic inputs ($\mathbf{A}^D\mathbf{u}_\mathbf{j}$) required to produce $\mathbf{u}_\mathbf{j}$, plus the domestic inputs $(\mathbf{A}^D)^2\mathbf{u}_\mathbf{j}$ required for the production of inputs $\mathbf{A}^D\mathbf{u}_\mathbf{j}$, and so forth. This yields the *j*th column of the Leontief inverse $\mathbf{L}^D$. The extra imports are given by

$$\Delta\mathbf{m} = [\mathbf{A}^M + \mathbf{A}^M\mathbf{A}^D + \mathbf{A}^M(\mathbf{A}^D)^2 + ...]\mathbf{u}_\mathbf{j} = \mathbf{A}^M(\mathbf{I} - \mathbf{A}^D)^{-1}\mathbf{u}_\mathbf{j} = \mathbf{A}^M\mathbf{L}^D\mathbf{u}_\mathbf{j} \tag{4}$$

To obtain the extra domestic output in model (2), however, it does not suffice to add (3) and (4). Because all imports are domestically produced in model (2), we should also take the input requirements for the imports into consideration. This yields

$$[\mathbf{I} + (\mathbf{A}^D + \mathbf{A}^M) + (\mathbf{A}^D + \mathbf{A}^M)^2 + ...]\mathbf{u}_\mathbf{j} = [\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + ...]\mathbf{u}_\mathbf{j} = \mathbf{L}\mathbf{u}_\mathbf{j}$$

It is clear that model (2) incorporates an extreme kind of substitution. Model (1) assumes that each additional unit of output requires fixed amounts of domestically produced inputs and fixed amounts of imported inputs. Model (2) instead assumes that *all* additional imported inputs are substituted by domestically produced intermediate inputs.

3.2. Overestimation

Consider the effects of a change $\Delta\mathbf{f}$ in the final demands. According to the model in (1), this yields an extra output given by $\Delta\mathbf{x} = \mathbf{L}^{D}(\Delta\mathbf{f})$. The extra imports are given by $\Delta\mathbf{m} = \mathbf{A}^{M}\mathbf{L}^{D}(\Delta\mathbf{f})$.

Using the model in (2), the same extra domestic output is obtained only if the change in imports is appropriately specified exogenously, namely $\mathbf{A}^{M}\mathbf{L}^{D}(\Delta\mathbf{f})$. This can be seen as follows. Let the exogenously specified change in imports be denoted by $\Delta\tilde{\mathbf{m}}$. The model (2) yields the same result as model (1) if and only if $(\mathbf{I}-\mathbf{A})^{-1}(\Delta\mathbf{f}-\Delta\tilde{\mathbf{m}})$ equals $(\mathbf{I}-\mathbf{A}^{D})^{-1}(\Delta\mathbf{f})$. Using $\mathbf{A} = \mathbf{A}^{D} + \mathbf{A}^{M}$, we have $(\mathbf{I}-\mathbf{A}^{D}-\mathbf{A}^{M})^{-1}(\Delta\mathbf{f}-\Delta\tilde{\mathbf{m}}) = (\mathbf{I}-\mathbf{A}^{D})^{-1}(\Delta\mathbf{f})$. Premultiplying both sides by $(\mathbf{I}-\mathbf{A}^{D}-\mathbf{A}^{M})$ yields

$$\Delta\mathbf{f}-\Delta\tilde{\mathbf{m}} = (\mathbf{I}-\mathbf{A}^{D}-\mathbf{A}^{M})(\mathbf{I}-\mathbf{A}^{D})^{-1}(\Delta\mathbf{f})$$

$$= \Delta\mathbf{f}-\mathbf{A}^{M}(\mathbf{I}-\mathbf{A}^{D})^{-1}(\Delta\mathbf{f}) = \Delta\mathbf{f}-\mathbf{A}^{M}\mathbf{L}^{D}(\Delta\mathbf{f})$$

This result has important consequences.

Despite its shortcomings, the simple Leontief framework in (1) and (2) is widely used by practitioners – and by policymakers and consulting agencies in particular – to estimate at an industry level the effects of some final demand change. A thorough analysis would preferably be based on a full fledged CGE model designed for the purpose at hand. But, because its development is time consuming and therefore costly, the input-output model is often used as an alternative to provide a quick – albeit approximate – answer. The effects on gross outputs are typically used to calculate induced effects (such as the requirement of various types of labor, imports and energy, the emission of pollutants, and generation of solid wastes).

Applying the model in (2) requires that the extra imports are specified exogenously. But how can we ever get a reliable estimate of the change in the imports when $\mathbf{A}^D$ and $\mathbf{A}^M$ themselves are unknown? In the absence of reasonably accurate import estimates, the multipliers in model (2) grossly overestimate the effects. This overestimation accumulates when the model is used in a scenario or impact analysis, including the calculation of the relative factor contents of trade.[5]

Suppose we would like to estimate the effects of a 5% increase in private consumption. Using model (2) without any further information on imports, would largely overestimate the outcome of model (1). Only if an accurate estimate of the effects on the competitive imports were available, model (2) would yield reliable results. In general, this is a very odd situation. In order to get some insight into the effects, part of these effects have to be specified exogenously first. The only valid application we can think of, falls in the category of goal programming or planning. That is, what are the effects of some change in final demand, given what the imports in the new situation should be. This might be a relevant analysis in the case where an expansion of the imports is not allowed, for example due to quota restrictions.[6] In other cases, however, is the requirement to specify the (changes in) imports exogenously, highly unrealistic and restrictive.

Another curious situation arises with respect to intra-industry trade. According to model (2), increasing both the exports and the imports of product *j* by any amount would leave the domestic production unaffected. Again, the reason for this strange outcome is that the model in (2) works under the assumption of fixed imports. A one dollar increase in the exports of product *j* (implicitly assuming no changes in the imports) has exactly the same effect as a one dollar decrease in the imports of product *j* (assuming no final demand changes).

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5 It can be shown that the relative capital (labor) intensity of both the imports and the exports is likely to be smaller for model (2) than for model (1), if the exports are capital (respectively labor) intensive in model (1).

6 See Albino *et al.* (2003) for an application in the context of a so-called enterprise input-output model.

The decrease in the imports of product *j*, implies that this one dollar has to be produced domestically (along with all its input requirements). So, the stimulus for domestic production is in model (2) exactly the same for a decrease in imports and an increase in exports (or any other final demand change) for product *j*.

As a final remark, we would like to emphasize that the exposition in this section was based on the assumption that no final demands were imported. It is easily seen that including such imports does not affect our findings. In Table 1 we would have to add a vector $\mathbf{f}^M$ in its row “imports” and column “final demands”. In Table 2, this vector $\mathbf{f}^M$ is added in the row “domestic outputs” and the column “final demand”, but at the same time subtracted in the column “imports”. The analysis and results in this section, however, remain unaltered.

4. Conclusions

In this note we have shown that applying an input-output model that is based on US-type of tables requires that imports are specified exogenously. The multipliers in this model can be given an economic interpretation under the highly unrealistic assumption that (changes in) imports are zero. Input-output models are widely used to analyze at the industry level what the effects are (for example, on domestic output, the demand for labor, energy requirements, or the emission of pollutants) of an exogenous change in final demands (such as private consumption, investments, or exports). In such cases, it is difficult to maintain that the imports could ever remain constant, while anything else changes. In particular, it seems very implausible to assume that an increase in domestic production leaves the imports unaffected.

We have compared model (1) based on tables that separate imported from domestically produced intermediate deliveries with model (2) based on US-type input-output tables that do not make such a distinction. It was shown that the outcomes obtained from model (2) overestimate the effects as obtained from model (1), which does not suffer from the implausible assumption on imports.

US-type of tables are clearly inferior. Not only do they provide less detail, we also found that the multipliers based on these tables implicitly rest on a highly unrealistic assumption and lead to biased results. One way to circumvent these problems (or at least attempt to do so) is that researchers themselves separate domestically produced from imported deliveries, on the basis of the limited available information. Like we did in the previous section, we assume again, that there are no imports for final demand purposes (i.e. $\mathbf{f}^M = 0$). Suppose the available information is as given in Table 2, then define

$$\pi_i = \frac{\Sigma_j d_{ij}}{\Sigma_j (d_{ij} + m_{ij})} = \frac{x_i - f_i}{x_i - f_i + m_i} \tag{5}$$

This is, for product *i*, the ratio of total domestically produced intermediate deliveries to the total intermediate deliveries. Note again that information is available for $d_{ij} + m_{ij}$, but not for d_{ij} separately. To estimate the domestic intermediate deliveries, the ratio π_i is applied uniformly within row *i*. That is, $\bar{d}_{ij} = \pi_i (d_{ij} + m_{ij})$, where a bar is used to indicate that it is an estimate. Then we also have $\bar{a}_{ij}^D = \pi_i a_{ij}$. Observe that it follows from (5) that $\mathbf{x} - \mathbf{f} = \hat{\boldsymbol{\pi}}(\mathbf{x} - \mathbf{f} + \mathbf{m})$, where a circumflex is used to indicate a diagonal matrix. From Table 2 it follows that $(\mathbf{x} - \mathbf{f} + \mathbf{m}) = \mathbf{Ax}$, hence $\mathbf{x} - \mathbf{f} = \hat{\boldsymbol{\pi}}\mathbf{Ax} = \bar{\mathbf{A}}^D\mathbf{x}$. That is, $\mathbf{x} = \bar{\mathbf{A}}^D\mathbf{x} + \mathbf{f}$, which corresponds to the model in (1) except that the matrix of domestic input coefficients is now estimated. Such estimation (also called domestication) techniques can be further refined and easily extended to other cases, such as the case where $\mathbf{f}^M \neq 0$.[7]

In conclusion, the best option for researchers working with US-type of input-output tables is to estimate themselves the domestic input coefficients from the limited data available. Yet, this is only a surrogate option. Given the importance of the US economy and given that it probably is the object of study that is analyzed most by economists, it is a great pity that only an outdated type of database with serious limitations is available. We thus would strongly advocate that the US government decides to compile input-output tables where imported deliveries are separated from domestic deliveries. The importance of such a distinction may not have been very large in the old days, when the first input-output tables were published, because imports played only a minor role in the US. Today, however, the US has become a major international trading partner, so that detailed data for the competitive imports have become much more important.

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[7] See for example Lahr (2001) for a discussion on the so-called re-exports (i.e. imports that are exported again), which we have left aside in this note.

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Table 1. Input-output table with a separate import matrix.

	Inputs	*Final demands* (*incl gross exports*)	*Totals*
Domestic outputs	$\mathbf{D}$	$\mathbf{f}$	$\mathbf{x}$
Imports	$\mathbf{M}$	0	$\mathbf{m}$
VA & N-C imports	$\mathbf{v}'$	0	v
Totals	$\mathbf{x}'$	f	

Table 2. US-type input-output table (i.e. imports included in the intermediate deliveries).

	Inputs	*Final demands* (*incl gross exports*)	*Imports*	*Totals*
Domestic outputs [a]	$\mathbf{D} + \mathbf{M}$	$\mathbf{f}$	$-\mathbf{m}$	$\mathbf{x}$
VA & N-C imports	$\mathbf{v}'$	0	0	v
Totals	$\mathbf{x}'$	f	$-m$	

[a] Note that this row provides the allocation of domestic outputs to which the imports are added and subtracted again.