An Empirical Investigation of Paradoxes
(Reswitching and Reverse Capital Deepening)
in Capital Theory

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z.han@iklc.co.kr
schefold@wiwi.uni-frankfurt.de

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Zonghie Han

Korea Land Corporation, Land Research Institute,

Bundang-Ku, Sungnam, Kyungki, 463-755

South-Korea

and

Bertram Schefold

Fachbereich Wirtschaftswissenschaften

Johann Wolfgang Goethe-Universität, Frankfurt, Germany

Summary This paper examines the empirical relevance of the capital controversy between the proponents of the classical and of the neoclassical paradigm in economics. Aggregate capital at the macroeconomic level is regarded as the sum of capital goods employed, measured in terms of normal prices. Hence the price model of Sraffa (1960) and the dual models of the price and quantity systems of von Neumann (1937) become the basis of the investigation. The capital controversy is concerned with consequence of the choice of the cost minimizing technique in the production system for the relationship between distribution and the value of capital. *Theoretical* examples are easily constructed which contradict the fundamental neoclassical hypothesis of an inverse relationship between the rate of profit and the intensity of capital, differentiating between aggregate and sectoral intensity. This paper for the first time presents *empirical* examples. To this end, the quantity system of the von Neumann model is here used to model spectra of techniques. 32 input-output tables from 9 countries of the OECD input-output data base serve as data. One input-output table represents one technique (production system). A spectrum of technique, or book of blueprints, consists of two input-output tables from the OECD data base. A technique is chosen by selecting one out of two activities in each of the 36 sectors (each book of blueprints consists of 2^{36} techniques). The linear programming of the quantity system yields $496 = 32 \cdot \frac{31}{2}$ envelopes which result from the choice of technique. Among these, one envelope is found which involves reswitching. Reverse substitution of labour or reverse capital deepening are observed in about 3.65 % of tested cases; they involve at least two switchpoints. The existence of at least three switchpoints between two wage curves can be inferred in 0.73 % of all tested cases.

This seems to confirm the empirical relevance of capital deepening which has been controversial for almost 40 years. From the neoclassical point of view, the presence of these phenomena is "perverse" and a serious reason to question the validity of the theory.[1] But even if they were to be assumed away because they are not very frequent, they are evidence of an underlying cause, the changes of relative prices and of the value of capital with distribution, which render the construction of a surrogate production function problematic.

1. Paradoxes of capital theory

The classical paradigm in economics has influenced modern economic theory since Adam Smith and, although it was pushed into the background in the course of the so-called "marginalist revolution" at the end of the nineteenth century, it has reached its elaborate, formally self-contained form in the twentieth century. Its modernization and its resuscitation has been carried out by many economists, but it was primarily stimulated by Sraffa (1960). The ensuing debates between classical and neoclassical positions centred mainly on capital theory (the so-called "capital controversy" or "Cambridge-Cambridge controversy").

Central issues in the capital controversy were the phenomena of "reverse capital deepening" and of "reswitching", both at the theoretical and at the empirical level. Both may result from the choice of techniques in self-reproducing linear production systems as in Sraffa (1960). "Capital" here is the commodities used as means of production and valued at normal prices, according to the formula

[1] We should like to thank participants of the international Input-Output Conference at Brussels (September 2004) and two anonymous referees of the CJE for helpful comments.

$$(1+r)\mathbf{Ap}+w\mathbf{l}=\mathbf{p}, \tag{1}$$

where **A** is a productive indecomposable input matrix, **p** the price vector, **l** the labour (input) vector, *r* the rate of interest (here identical with the rate of profit) and *w* the wage rate.

From the equation system (1), we can derive the wage frontier, i.e. the wage rate (and prices) expressed in terms of some numéraire. Each wage frontier or wage curve represents a technique. If more than one technique are available, the problem of technical choice arises. The techniques on the envelope of the wage curves are most profitable according to several criteria (Schefold, 1997, pp. 30-33). The wage curves will be monotonically falling.

1.1. The Surrogate Production Function and Reswitching

Suppose a spectrum of techniques $\mathbf{A}^{(i)},\mathbf{l}^{(i)}; i=1,...,s;$ is given, with wage curves $w^{(i)}(r)$. Each technique produces net output $\mathbf{d}$ at activity levels $\mathbf{q}^{(i)}$: $\mathbf{d}=\mathbf{q}^{(i)}(\mathbf{I}-\mathbf{A}^{(i)})$, and $\mathbf{d}$ is also the standard of prices. If the wage curves could all be linear as in figure 1, their slopes would measure the intensity of capital k_i in a stationary state, since net output per head $y^{(i)}=w^{(i)}+rk_i$, hence $k_i=(y^{(i)}-w^{(i)})/r$, and the wage curves which became eligible successively at higher r would show a uniform diminution of the corresponding intensities of capital. This construction (surrogate production function, proposed by Samuelson 1962) would allow a rigorous construction of neoclassical production functions, as s tended to infinity (Schefold, 1989, pp. 297-298), if the assumptions were tenable.

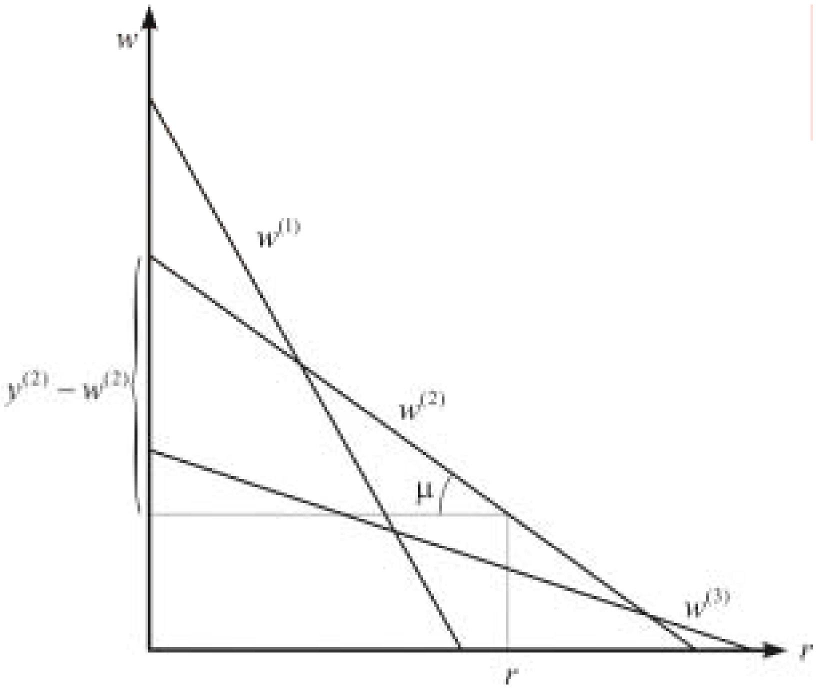

Figure 1: Surrogate production function. The tangent of μ is equal to the intensity of capital of technique 2.

However, simple numerical examples suffice to show that the wage curves are not straight lines – as a matter of fact, straight wage curves are flukes, see Schefold (1976).

Reswitching occurs, if a same technique is chosen in two intervals of the rate of profit, while some other technique(s) are chosen in an interval in between (figure 2). Reswitching contradicts the neoclassical postulate that techniques with lower intensities of capital become eligible at higher rates of profit (for the measurement of the intensity of capital along the wage curve see fig. 2). This critique is relevant not only for the aggregate versions of neoclassical theory but also for intertemporal general equilibrium: reswitching and related phenomena do not contradict the existence of intertemporal equilibria with production, but lead to questioning their stability (Schefold, 2000).

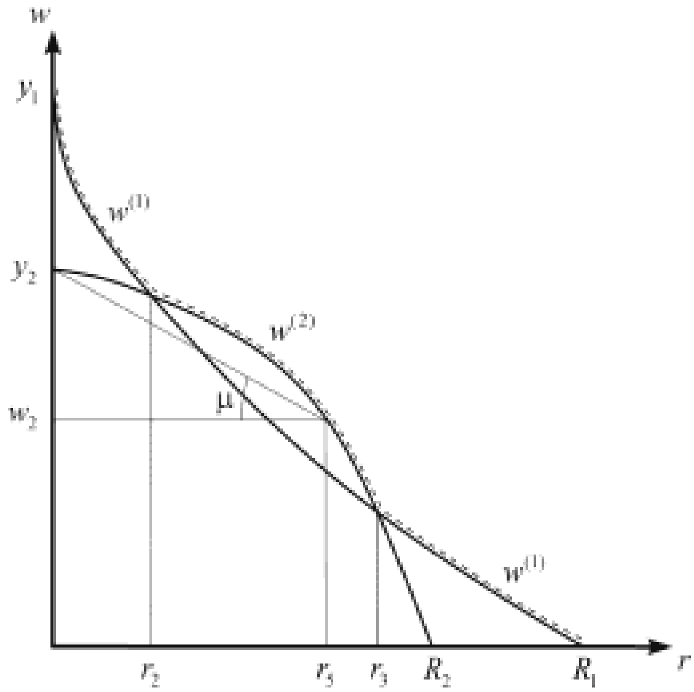

Figure 2: Reswitching; one technique, $w^{(1)}$, returns at r_3 after being dominated by $w^{(2)}$ between r_2 and r_3. The tangent of μ is equal to the intensity of capital k_2 of technique 2 at r_5 in the stationary state, for $y_2 = w_2 + r_5 k_2$. The intensity of capital rises discontinuously as r rises from below r_3 to above r_3.

1.2. Reverse capital deepening

A variety of phenomena results if there are several nonlinear wage curves. The nonlinearity makes it possible for wage frontiers to intersect more than once, so that we get the possibility of multiple switching. A debate on the impossibility of the so-called "surrogate production function" was carried on in the *Quarterly Journal of Economics*, 1966, in a Symposium on paradoxes of capital theory.

According to the neoclassical doctrine[2], the substitution should always 'deepen' the intensity of capital, whenever the wage rate *w* increases relatively to the rate of profit r. Reverse capital occurs, if the exact opposite takes place: A rise in the rate of profit r leads to the adoption of a more capital intensive technique. In this case, it is again not generally possible to order "efficient" techniques in such as way that *k* becomes a monotonically falling function of the rate of profit. Figure 3 demonstrates that the intensity of capital may rise at r_3 in passing from technique α to β although no

[2] Samuelson (1962) emphasized the stylized character of the argument by speaking of a "parable".

reswitching takes place on the envelope (the intersection of α and β at r_1 is not 'visible' on the envelope because of the dominance of technique γ up to $r_2 > r_1$).

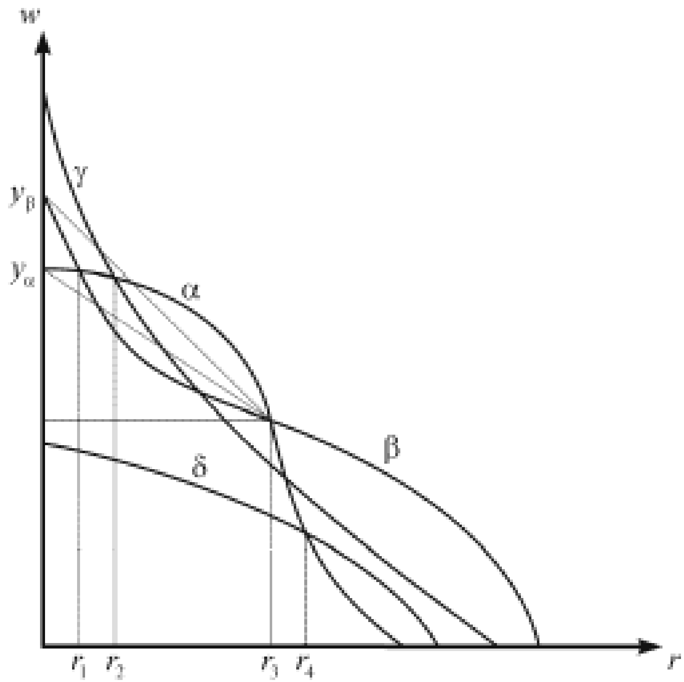

Figure 3: Reverse capital deepening.

There is a 'rapid succession of switches' (Sraffa, 1960, p. 85) along the envelope, for if there are e.g. 10.000 commodities in an economy and each can be produced by at least 2 methods, there will be $2^{10000} > 10^{3000}$, hence an 'avalanche' of wage curves and of switches between them. Yet it can be shown for single product systems that the methods generically are replaced only one at time at each switchpoint: at a switchpoint at some $\bar{r}$ on the envelope, such as r_2 or r_3 in fig. 3, only one method is employed in each industry of the two systems which coexist at equal prices at $\bar{r}$, and in only one of the industries two methods will coexist (Schefold, 1989, pp. 178-181; Schefold, 1997, pp. 124-126). The same must be true at any other intersection of the *same* two wage curves, even if it is below the envelope: If wage curves α and β intersect at r_3 on the envelope of fig. 3, the corresponding systems differ only with regard to the use of the methods used in one industry, and this will be true also for the intersection of α and β at r_1 below the envelope, where prices of α and β again will be equal. But no such statement can be made regarding other intersections of wage curves below the envelope, such as the one between γ and δ at r_4: in such a case, several, even all methods may be different in the industries of the corresponding two systems. We call a change of technique *piecemeal*, if, as on the envelope of single product systems, methods generically change only one by one. Whether the actual change of technique is piecemeal is also an empirical question examined below.

Capital theory in single product systems is concerned with the question whether the piecemeal change of techniques, as the rate of profit rises, is associated with a (at switchpoints) discontinuous monotonic fall of the intensity of capital. Reswitching and reverse capital deepening prove that this monotonicity is not a theoretical necessity. Reverse capital deepening is a necessary consequence of reswitching, but reswitching is not necessary for reverse capital deepening – a second intersection of the wage curves which necessarily exists in the case of reverse capital deepening at a lower rate of profit (such as the intersection at r_1 in fig. 3) then is below the envelope. Clearly, reverse capital deepening is the relevant phenomenon in the capital controversy since it is more general, and one feels intuitively that it must be more frequent.[3] Yet in the past reswitching seems to have interested economists more than reverse capital deepening, so that the impression has been created that reswitching is crucial for the capital controversy. For example Stiglitz (1973) formulated conditions under which reswitching is ruled out; Krelle (1976) tried to test the empirical relevance of reswitching with German data and found no instance of reswitching. So it was argued that the neoclassical doctrine could be defended after all.

1.3 Reverse substitution of labour[4]

Reverse capital deepening is a counterintuitive (from the neoclassical point of view) change of the ratio of *aggregate* capital to *total* labour employed in the economy as a whole at the switchpoint, where, generically, one method changes in one industry. We shall also be interested in what happens at the sectoral level as *r* rises and *w* falls. Suppose that a change occurs in the first industry where method 1 with inputs $(\mathbf{a}_1, l_1)$ is replaced by method $(\mathbf{a}_0, l_0)$ at rate of profit $\bar{r}$ and at prices $\mathbf{p}(\bar{r})$ and wage rate $w(\bar{r})$. Since prices, including that of the first commodity, are the same for both techniques at $\bar{r}$, we must have in any commodity standard

$$(1+\bar{r})\mathbf{a}_1\mathbf{p} + wl_1 = (1+\bar{r})\mathbf{a}_0\mathbf{p} + wl_0 .$$

As *w* falls and *r* is raised, passing through $\bar{r}$, the neoclassical hypothesis is that more labour will be used, hence $l_1 < l_0$ and that therefore $\mathbf{a}_1\mathbf{p}_1 > \mathbf{a}_0\mathbf{p}_0$ at $\bar{r}$. But it turns out that the opposite might also happen: the amount of labour is reduced so that the amount of capital rises in the sector, with $\mathbf{a}_1\mathbf{p}_1 < \mathbf{a}_0\mathbf{p}_0$, and the sectoral intensity of capital paradoxically increases. We call this a reversed substitution of labour.

It might seem that the aggregate and the sectoral intensity of capital must change in the same direction at a switchpoint, but this conjecture is not generally true for systems with three or more sectors; the phenomena turn out to be independent. The aggregate intensity of capital is $k_i = (y^{(i)} - w^{(i)})/r = \mathbf{q}^{(i)}\mathbf{A}^{(i)}\mathbf{p}/\mathbf{q}^{(i)}\mathbf{l}^{(i)}$, with $\mathbf{q}^{(i)} = \mathbf{d}(\mathbf{I} - \mathbf{A}^{(i)})^{-1}$, and the sectoral intensity is $\chi_i = \mathbf{a}_i\mathbf{p}/l_i; i = 0,1$. Since the rows of $(\mathbf{A}^{(1)}, \mathbf{l}^{(1)})$ and $(\mathbf{A}^{(0)}, \mathbf{l}^{(0)})$ coincide except in the first industry and since $\mathbf{p}$ is the same for both at the switchpoint, it is not obvious that k_i and χ_i can move in different directions. The neoclassical hypothesis is $k_1 > k_0, \chi_1 > \chi_0$, but $k_1 < k_0$ and $\chi_1 < \chi_0$, and all combinations, are also possible.

To see this, assume that all techniques are regular in the sense of Schefold (1989 [1971]) and that technical change is piecemeal as defined in section 1.2 of this paper; both properties are generic. Let $\hat{\mathbf{p}}(r)$ be the vector of prices in terms of the wage rate of the original system employing method $(\mathbf{a}_1, l_1)$ in the first of *n* industries, $n \geq 2$. There results a wage curve $w^{(1)}$ between $r = 0$ and $r = R$.

We have seen above (figures 2 and 3) how the aggregate intensity k_i can be read off the wage curves. The comparison of the sectoral intensities becomes possible by extending the wage curves to negative values of *r*. We get from (1), using

[3] It is important to note that reverse capital deepening at switchpoints is independent of the numéraire and is not related to Wicksell effects, since the comparison is made at a given wage rate.

[4] Sections 1.3 and 1.4 may be skipped in a first reading of this paper.

$\mathbf{d}(\mathbf{l}^{(0)}-\mathbf{l}^{(1)})=d_1(l_0-l_1)$, that $\mathbf{p}(-1)=w(-1)\mathbf{l}$ and from $\mathbf{dp}=1$ that $w^{(0)}(-1)>w^{(1)}(-1)$ if and only if $l_0<l_1$, and this is equivalent to $\chi_0>\chi_1$.

We therefore get the neoclassical case at the aggregate level and k_1 falls to $k_0<k_1$ as the rate of profit passes rising through a switchpoint $\bar{r}$ on the envelope, if $w^{(1)}$ dominates $w^{(0)}$ below $\bar{r}$ and if $w^{(1)}$ and $w^{(0)}$ have no switchpoints between $r=0$ and $r=\bar{r}$.

We get the neoclassical case at the sectoral level and χ_1 falls to $\chi_0<\chi_1$ as the rate of profit passes rising through a switchpoint on the envelope $\bar{r}$, if $w^{(1)}$ dominates $w^{(0)}$ below $\bar{r}$ and if $w^{(1)}$ and $w^{(0)}$ have no switchpoints between $r=-1$ and $r=\bar{r}$.

Reswitching (and reverse capital deepening) occurs, other things being as in the neoclassical case, if there is one switchpoint between $r=0$ and $\bar{r}$.

A reverse substitution of labour takes place if there is one switchpoint between $r=-1$ and $r=0$, and none between $r=0$ and $r=\bar{r}$. This was discussed as an "employment opportunity reversal" in Schefold (2000). It is equivalent to a paradoxical rise of the sectoral intensity of capital.

Logically, we are confronted with four possibilities at any switchpoint $\bar{r}$, for which we use shorter denominations, meaning "aggregate capital" by "capital" and the sectoral phenomenon when we speak of labour:

(a) capital intensity-reducing, labour-increasing ($k_0<k_1, \chi_0<\chi_1$),
(b) capital intensity-reducing, labour-reducing ($k_0<k_1, \chi_0>\chi_1$),
(c) capital intensity-increasing, labour-increasing ($k_0>k_1, \chi_0<\chi_1$),
(d) capital intensity-increasing, labour-reducing ($k_0>k_1, \chi_0>\chi_1$).

All four possibilities can be verified empirically, as we shall see in section 4.1; we here have to show that they are more than just flukes.

To simplify the argument, assume $\mathbf{a}_1>\mathbf{0}$. Define the column vector $\tilde{\mathbf{p}}(r)=[(1+r)\hat{\mathbf{p}}^{\mathrm{T}},1]^{\mathrm{T}}$ and the set

$$M(r)=\{(\mathbf{a}_0,l_0),\mathbf{a}_0\geq\mathbf{0},l_0>0\,|\,(\mathbf{a}_0,l_0)\tilde{\mathbf{p}}(r)=\hat{p}_1(r)\}.$$

The simplex $M(\bar{r})$ is the set of all conceivable alternative methods for the first industry such that wage curve $w^{(0)}$ has a switchpoint with $w^{(1)}$ at $\bar{r}$. Obviously, $(\mathbf{a}_1,l_1)\in M(r)$ for all r, and there will be a full open neighbourhood U_ε of $(\mathbf{a}_1,l_1)$ in $M(r)$ such that each $(\mathbf{a}_0,l_0)\in U_\varepsilon$ will be a method with $\mathbf{a}_0>0$ and such that the corresponding technique ($\mathbf{A}^0,\mathbf{l}^0$) will be productive.

We have (as is well-known) at most n switchpoints of $w^{(1)}$ and $w^{(0)}$, for if we had switchpoints at $n+1$ distinct rates of profit[5] $0\leq r_1<...<r_{n+1}<R$, the corresponding method would have to be in $M(r_1)\cap...\cap M(r_{n+1})$, but then $(\mathbf{a}_0,l_0)=(\mathbf{a}_1,l_1)$, since regularity implies that $\tilde{\mathbf{p}}(r_1),...,\tilde{\mathbf{p}}(r_{n+1})$ are linearly independent. On the other hand, there will not be fewer than n switchpoints, if we let r vary in the entire complex plane, since the polynomial $\varphi(r)=[\hat{p}_1(r)-(\mathbf{a}_0,l_0)\tilde{\mathbf{p}}(r)]\det\left(\mathbf{I}-(1+r)\mathbf{A}^{(1)}\right)$ is of degree n and has n complex roots which we assume to be generic, i.e. simple. We use them to show how essentially all the examples for the four cases distinguished above may be constructed.

[5] We rule out, here and below, that some $r_i=(1/\lambda)-1$, where λ is any of the eigenvalues of $\mathbf{A}^{(1)}$.

Let $\bar{r}$ are be the rate of profit, $0 < \bar{r} < R$, at which the switch between $w^{(1)}$ (dominant at $r < \bar{r}$) and $w^{(0)}$ (dominant at $r > \bar{r}$) is to take place. Complications arise from additional switchpoints. The following construction exhausts all possibilities, flukes apart.[6] Let s, t, u be nonnegative numbers with $s + t + u = n - 1$. Let $r_1, ..., r_{n-1}$ be complex numbers. Without loss of generality, let them be ordered so that the first are real with $-1 < r_1 < ... < r_s < 0$, $0 < r_{s+1} < ... < r_{s+t} < \bar{r} < R$. Let $r_{s+t+1}, ..., r_{n-1}$ be outside the closed interval $[-1, \bar{r}]$. We have

case (a), if and only if s even, t even,
case (b), if and only if s odd, t even,
case (c), if and only if s odd, t odd,
case (d), if and only if s even, t odd.

Each of these cases can be constructed by choosing an appropriate $(\mathbf{a}_0, l_0)$ in the straight line given by $N = M(r_1) \cap ... \cap M(r_{n-1}) \cap M(\bar{r})$ which contains $(\mathbf{a}_1, l_1)$ and therefore, in a neighbourhood of $(\mathbf{a}_1, l_1)$, an infinity of techniques $(\mathbf{a}_0, l_0) > 0$ which yield a system ($\mathbf{A}^0, \mathbf{l}^0$), capable of reproduction, with the prescribed switchpoints of $w^{(1)}$ and $w^{(0)}$ at the real rates of profit $r_1, ..., r_{s+t}, \bar{r}$ and with "switchpoints" of $w^{(1)}$ and $w^{(0)}$, considered as complex meromorphic functions, at u points $r_{t+1}, ..., r_{n-1}$. The set from which $(\mathbf{a}_0, l_0)$ may be chosen for each case now turns out to be ($n+1$)-dimensional since N is one-dimensional and each of the n rates of profit $r_1, ..., r_{n-1}, \bar{r}$ may be varied a little without changing the case and without violating non-negativity and reproduction conditions, because the system is regular. As a result, each case has been shown to be generic.[7]

But the cases differ with regard to the minimum number of dimensions required to make it possible. Case (a) is possible with $n = 1$ and $s = t = 0$; it is illustrated by the familiar corn models. Cases (b) and (d) can occur with $n = 2$, and $(s, t) = (1, 0)$ and $(s, t) = (0, 1)$ respectively. Case (c) requires $(s, t) \geqq (1, 1)$; there must be at least three switchpoints (including $\bar{r}$), hence $n \geq 3$.[8] Case (d) looks most paradoxical from a

[6] Flukes are switchpoints at $r = 0$ or $r = -1$ or at some $r = (1/\lambda) - 1$ where λ is an eigenvalue of $\mathbf{A}^{(1)}$.

[7] The u rates of profit outside the interval $[-1, \bar{r}]$ in the complex plane are here introduced to ensure that no additional switchpoints in $[-1, \bar{r}]$ disturb the construction. But these degrees of freedom might also be used to gain more realism and to overcome the assumption $\mathbf{a}_1 > \mathbf{0}$. As one of us remarked on an earlier occasion (Schefold 1997, p. 61), cherries and elephants are not inputs in the production of steel. If the first sector produces steel and, without loss of generality, only the first m components of $\mathbf{a}_1$ are positive, potential alternative techniques $\mathbf{a}_0$ will probably also show zeroes in most of the last $n - m$ components which represent the "cherry-dimensions". If we postulate that some of the last $n - m$ components of $\mathbf{a}_0$ vanish, the choice of technique becomes more restricted and examples of multiple switches become more difficult to find. This consideration incidentally also illustrates that it would be a mistake to calculate a-priori-probabilities of reswitching on the assumption that the coefficients of the input-matrix are random. For that would imply a positive input-matrix, while the distribution of zeroes is an important characteristic of any large input-output matrix (cf. Schefold 1997, p. 127).

[8] For $n = 2$, it can also be shown directly that the transition form methods with $k_0 < k_1$ to methods with $k_0 > k_1$ implies the simultaneous change from $l_0 > l_1$ and $\chi_0 < \chi_1$ to $l_0 < l_1$ and $\chi_0 > \chi_1$ so that case (c) is impossible for $n = 2$; the sets of corresponding methods are on the line $M(\bar{r}) \cap M(r_1); 0 < r_1 < \bar{r} < R$; and they are separated by $(\mathbf{a}_1, l_1)$.

neoclassical point of view, but (c) is analytically the most paradoxical, since the aggregate intensity of capital rises although the sectoral intensity falls.[9]

We may also say that the aggregate and the sectoral intensity of capital move together in two-sector models. Since capital theory has mainly been studied on the basis of two-sector models, the independence of reverse capital deepening and reverse substitution of labour have hardly been noticed.

All these phenomena may remain hidden below the envelope of the wage curves if some other wage curve dominates $w^{(1)}$ and $w^{(0)}$ between $r=-1$ and $r=\bar{r}$ so that of the switchpoints $r_1,...,r_{s+t},\bar{r}$ only $\bar{r}$ appears on the envelope, as is known from reverse capital deepening. However, theorists – who always like stark abstractions – have tended to focus on reswitching.

1.4 Wicksell effects and returns of processes

Even individual wage curves can be in conflict with the neoclassical postulate of an inverse relationship between the rate of profit and the intensity of capital. The intensity of capital rises with the rate of profit along wage curve $w^{(2)}$ in figure 2. This is a perverse Wicksell effect. Indeed, any non-zero curvature of the wage curves is a problem for aggregation and the construction of a production function, as will be seen in section 4.3 below.

Wage curves are in general not straight because relative prices change with distribution and deviate from labour values. These movements of prices can be calculated directly and all the paradoxes of capital discussed so far are evidence of them. Another such paradox is the return of a process which is used in one sector at one level of the rate of profit and again at a second level, but not in-between. This is reswitching, if no other changes of technique are involved, but if other methods also change in other industries, while both switches at r_1 and r_2 are on the envelope, we have a return of a process. Such returns of processes confirm that individual methods cannot necessarily be ordered as more or less capital-intensive according to the level of the rate of profit. They are evidence of changes of relative prices.

Schefold (2000) discusses the strong changes of relative prices associated with reswitching which is a special case of a return of a process, as we saw above. We represent a mere return of a process $(\mathbf{a}_0, l_0)$ in the first industry as follows. We suppose that $(\mathbf{a}_0, l_0)$ is used to the left of r_1 and to the right of r_2; $r_1 < r_2$; the associated wage rates being w_1, w_2; $w_1 > w_2$. The economy switches to the use of $(\mathbf{a}_1, l_1)$ at r_1 and back to the use of $(\mathbf{a}_0, l_0)$ at r_2. In-between, there may be other switches in other industries. (If there are none, we have reswitching. We may have reverse capital deepening, if the other methods used at r_1 and r_2 are not all equal, while there is a second crossing of the wage curves switching at r_2 below the envelope at some r_3, $r_1 < r_3 < r_2$). At the switchpoints r_1, r_2, we get

$$(1+r_1)\mathbf{a}_0\mathbf{p}(r_1)+w_1 l_0=(1+r_1)\mathbf{a}_1\mathbf{p}(r_1)+w_1 l_1,$$
$$(1+r_2)\mathbf{a}_0\mathbf{p}(r_2)+w_2 l_0=(1+r_2)\mathbf{a}_1\mathbf{p}(r_2)+w_2 l_1,$$

[9] One of us argued in 1978 that wage curves are not linear but do not deviate much from straight lines (see Schefold 1997, p. 278). This renders case (b) more likely on a priori grounds than (d), since the interval $[-1,0)]$ may be expected to be longer than $[0,R]$, and more likely than (c) which requires at least one additional switchpoint. But this argument could also be used to justify the labour theory of value as a good approximation to the theory of prices.

where $\mathbf{p}(r_1), \mathbf{p}(r_2)$ are prices at the switchpoints, expressed in a commodity numéraire. We obtain, after forming differences,

$$\frac{1+r_2}{1+r_1}\frac{w_1}{w_2} = \frac{(\mathbf{a}_0 - \mathbf{a}_1)\mathbf{p}(r_1)}{(\mathbf{a}_0 - \mathbf{a}_1)\mathbf{p}(r_2)}.$$

Since both factors are greater than one on the left hand side, prices cannot be constant and equal to labour values on the right hand side. Intuitively: if prices are equal to values, technical change depends only on relative factor prices. Since these vary monotonously, each method becomes eligible only once and cannot return. Strong changes of relative commodity prices must compensate for a change of distribution to make the method eligible again.

2. Models of empirical investigation of the paradoxes

Joan Robinson argued that reswitching — and indeed, looking for empirical examples — was unimportant: "Nothing could be more idle than to get up an argument about whether reswitching is likely to be found in reality. Even if there was such a thing as a pseudo-production function, there would be no movement along it to pass over switchpoints, and furthermore, in reality, there is no such thing as a pseudo-production function" (Robinson 1979, p. 82). Against this extreme Keynesian position, followers of Sraffa and neoclassical economists agree that the comparison of long run positions may be useful for theoretical and applied analyses of processes of economic change such as growth with technical progress; their disagreement concerns the generality of the neoclassical theory of distribution (Petri 2004). The theoretical possibility of reswitching suffices to show that a theory of distribution, based on the postulate of a diminishing marginal productivity of capital, cannot be universally valid; its explanatory power in special cases remains to be assessed.

Schefold (1976) showed that the mathematical probability of reswitching is larger than zero.[10] But even neoclassical economists who accepted the logical possibility of the reswitching phenomenon and its theoretical consequence have raised doubts as to its empirical relevance, see Bruno, Burmeister and Sheshinski (1966); Samuelson (1966); Ferguson (1969), but for reasons different from those which motivated Joan Robinson: They asserted their faith in the applicability of the principle of the diminishing marginal product of capital for all *practical* purposes. A recent survey of the aggregation problem for neoclassical production functions is Felipe and Fisher (2003).

Some economists have tried to test the positive probability of reswitching: Sekerka, Kyn and Hejl (1970; Czechoslovakia), Krelle (1976; Germany), Ochoa (1987; USA), Hamilton (1986; Korea), Özol (1984; USA), Cekota (1988; Canada), Petrovic (1991; Yugoslavia) and Silva (1991; Brazil) derived wage frontiers from input-output tables and reported that no indication of reswitching was found. Mark Blaug argues for this reason in an online discussion at the HES network (History of Economics Society; www.eh.net/HE/HisEcSoc) in July 2001: "…any attempt to bypass the reswitching conundrum by purely theoretical arguments must obviously fail. The only argument, as I have endlessly but fruitlessly contended, is an empirical one: no one has ever shown that reswitching actually occurs in any even quasi-realistic model. The analogy with positively sloped demand curves is perfect!"[11] It seems therefore to be appropriate to

[10] For the more recent discussion on the likelihood of reswitching, see section 4.3 below.
[11] www.eh.net/lists/archives/hes/jun-2001/0025.php.

clarify the empirical relevance of the paradoxes (including reswitching), although some economists argue that the empirical relevance of the capital controversy should be separated from the logical and inner consistency of the economic theory which the capital controversy questions, see Helmedag (1986); Kalmbach and Kurz (1986).

2.1. Derivation of the envelope

The authors of the empirical investigations thought that they had investigated the choice of technique on the relevant "envelopes". However, they did not succeed in deriving those envelopes correctly so that their conclusion missed the target. The following simple example with two sectors will clarify this.

We suppose that two input-output tables are available for the technical choice, namely 1980 and 1990. Each input-output table has only two sectors, I and II. Let the symbol [*i*, *j*] represent a wage frontier which engages the production process for the sector I from input-output table *i* and the production process for the sector II from input-output table *j*. The principle followed in the conventional studies referred to above corresponds here to the construction of the envelopes from two wage frontiers, namely [1980,1980] and [1990,1990] (see figure 4). The older technique is still profitable at low wages, as traditional theories predict.

However the real envelope consists of *four* wage frontiers, that is [1980,1980], [1980,1990], [1990,1980] and [1990,1990]. For if the analysis is meant to represent a choice of technique (if there was no choice there would only be one wage curve), taking place in 1990, the composition of the technology of 1980 can still be remembered and could be used in 1990. But if this is true for the methods employed in both sectors, it must also be true for those methods individually. The decision to construct a modern motor-car today according to the standard of 2000 is a decision not to construct a horse-drawn carriage according to the standard of 1880 (which is still known). So the envelope looks as in figure 5. And it will be seen that the example of figure 5, with reverse capital deepening taking place in the transition from technique [1990,1980] to technique [1980,1980], is not fanciful. Attempts to test the paradoxes on the basis of an "envelope" as in figure 4 are inconclusive.

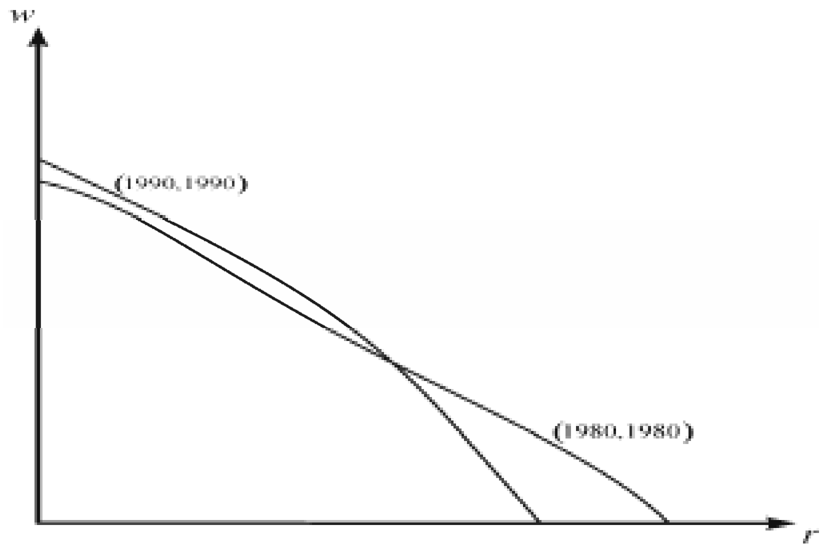

Figure 4: Conventional "envelope".

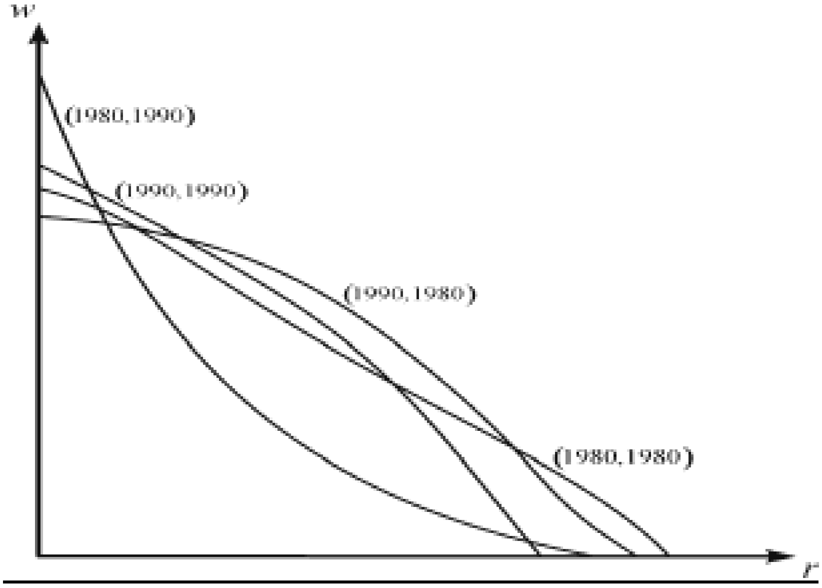

Figure 5: Real envelope.

2.2. Von Neumann model

The way to derive the envelope from the wage frontiers as in figure 5 has a practical limit, however, if the available input-output tables have many sectors. We must construct 2^n wage frontiers for the case that two input-output tables have n sectors respectively. If we have three input-output tables available for the technical choice, the number of wage frontiers to be formed rises to 3^n.

We therefore use linear programming as an alternative method to construct the envelope; one can show that the same envelope results as according to Sraffa's method (1). This procedure can also be related to the model of von Neumann (1937). Several studies show the formal equivalence of Sraffa's model and that by von Neumann for the case of single product systems; for production systems with fixed capital and for joint production, see Steedman (1976); Schefold (1980); Salvadori (1982); Kurz and Salvadori (1995).

Consider the following programme and its dual:

$$Min \quad \mathbf{ql} \quad S.T. \; \mathbf{q}\left(\mathbf{B}-(1+r)\mathbf{A}\right) \geqq \mathbf{d}, \; \mathbf{q} \geqq \mathbf{o}, \qquad (2)$$

$$Max \quad \mathbf{dp} \quad S.T. \; \left(\mathbf{B}-(1+r)\mathbf{A}\right)\mathbf{p} \leqq \mathbf{l}, \; \mathbf{p} \geqq \mathbf{o}, \qquad (3)$$

where the rows of $(\mathbf{A},\mathbf{B},\mathbf{l})$ denote the methods available in the book of blueprints or spectrum of techniques, where the goods are in the columns of **A** (inputs) and **B** (outputs) and where joint production is admitted. According to the usual economic interpretation, the solution describes a steady state golden rule path, at a rate of profit equal to r equal to the rate of growth, where a net output vector **d** is produced or overproduced (overproduced goods receive zero prices) and where the normal rate of profit r is not exceeded (less profitable activities are not used). It can then be shown under simple and general assumptions (especially regarding the productivity of the system) that solutions exist for all r between zero and a finite maximum (Schefold, 1978). Prices are here prices in terms of the wage rate ($w=1$), hence, with $\mathbf{dp}=\mathbf{ql}$ by virtue of duality, $1/\mathbf{ql}$ is the real wage; under suitable regularity assumptions, $1/\mathbf{ql}$ will fall continuously and strictly monotonically from $r=0$ to a maximal $r=R$. In essence, one then finds, except in a number of critical points, that the solution to the linear programme is 'square' (the number of positive prices in the solution is equal to the number of activities used), and the solution is a superposition of 'square' solutions in the critical points (Schefold, 1997, p. 128). The solutions can therefore be regarded as subsystems ($\bar{\mathbf{A}}, \bar{\mathbf{B}}, \bar{\mathbf{l}}$) (so-called truncations) of $\mathbf{A}, \mathbf{B}, \mathbf{l}$. The commodities in the subsystem have prices forming a truncated price vector $\bar{\mathbf{p}}$ with

$$(1+r)\bar{\mathbf{A}}\bar{\mathbf{p}} + \bar{\mathbf{l}} = \bar{\mathbf{p}}, \qquad (4)$$

and if the corresponding truncated vector $\bar{\mathbf{d}}$ of $\mathbf{d}$ is the standard of the real wage of the truncation,

$$\bar{w}(r) = 1/\bar{\mathbf{d}}\bar{\mathbf{p}} \qquad (5)$$

is the wage curve of the truncation. The point is that the envelope of the wage curves (5) of the truncations of ($\mathbf{A}, \mathbf{B}, \mathbf{l}$) is the solution to the programme (2) and (3).

In our case, with single production, the rows of $\mathbf{B}$ consist of unit vectors. The envelope is calculated using (2), formally the minimization of the labour requirement to produce net output at rate of growth r. But the economic interpretation is based on the theoretical result embodied in (4): except at critical points (where wage curves intersect on the envelope), the solution yields a (square) single product system with positive prices at rate of profit r. In the case of joint production, one cannot be sure that a truncation which appears with positive prices on the envelope at r will still have positive prices at some $r' \neq r$, and the solution will crucially depend on the composition of $\mathbf{d}$. But, with single product systems, well-known theorems ensure that the solution does not depend on the composition of $\mathbf{d}$, and prices will remain positive, if r is lowered to zero or raised to the maximum rate of profit of the system (all input matrices are here assumed to be indecomposable).

To summarise: We assume a spectrum of techniques such that each technique is a productive indecomposable Sraffa system with a wage curve that is monotonically falling. The most profitable technique is chosen at each rate of profit by adopting the system with the highest real wage, hence it is found on the envelope of all the wage curves resulting from the spectrum. Exactly the same envelope is also obtained by solving (2). Hence there is a formal equivalence between a 'Sraffa-approach' and a 'von Neumann-approach'(von Neumann, 1937); their conceptual difference (Schefold, 1980) does not concern us here.

3. Methodical problems in empirical modelling

3.1. Alternative techniques at various times and places

Strictly speaking, the available spectrum should be defined for a given time and place. Kurz and Salvadori (1995, p. 450) reject the recent empirical studies (national comparisons of the input-output tables) because they refer to different times. But all methods in a book of blueprints (except for those actually realised) are removed to some extent from actual realisation, yet their costs are compared with those of actual methods. The prospects for using future 'potential' methods (like the costs of extracting oil from oil sands) influence present decisions (like where to extract oil, i.e. at what cost). Methods used in the past are probably better known than those conceived for the future. Therefore one starts out from the assumption that the time series of the input-output tables of a national economy represent also alternative techniques: in a backward-looking perspective. What is used today, can be used tomorrow, and what was used yesterday, might be used again, and, as far as technical knowledge is concerned, more easily in most cases than what we plan for the future.[12]

One expects productivity and wages to rise with time. Hence one might expect that older methods would be profitable today if wages were lower, hence one might expect old techniques to appear still at high rates of profit on the envelope. In particular one expects that old machines could still be used if wages were lower (repair work cheaper). It has been proved (in analogy with reswitching) that this hypothesis is not generally correct in fixed capital models (Schefold, 1997, pp. 229-231). But the very consideration of the problem shows that the comparison of present with past techniques is, contrary to Kurz and Salvadori, a standard procedure.

This paper proposes to compare techniques internationally, too. The comparison of techniques in different time periods and that of technologies from different countries are analogous. The observed technology (input-output table) of a national economy represents the realized technology for the country at the given distribution (w and r), in a given economic condition, defined by international economic opportunities and national institutions. In another economic and historical constellation, another technique would be chosen, and the techniques of other countries offer such alternatives. The assumption that the technique in use of country A is in the book of blueprints of country B is easy to defend from a theoretical point of view: This paper attempts to prove that the paradoxes of capital are empirically relevant and that the empirical applicability of neoclassical theory therefore is in question. Neoclassical authors often adopt the assumption in empirical work that technology is internationally transferable (between developed countries), and they formalize the assumption by assuming that the production function (i.e. the book of blueprints ordered according to neoclassical principles) is the same for all countries. Does this assumption not mean, *at least*, that single methods actually realized in one country could be regarded as potential methods in another?

It is true that the transfer of methods of production does not always take place where one might expect it, given similar opportunities and institutions. In the discussion about the so-called ‘intra-industry trade’, various cases have been analyzed in which two countries, despite identical books of blueprints, realize different techniques: because of incomplete competition in connection with the so-called QWERTY theory or theory of ‘strategic complementarity’, or because of location problems in connection with increasing returns to scale, so that slightly different goods are produced and traded within a product group (or in a ‘homogeneous’ sector) by two economies, see Krugman (1987). The theory of ‘bounded rationality’ could also contribute to the explanation why one technology, which is often neither significantly technically better nor lower in price, is realized for no rational reasons instead of many other technical alternatives (see for example Simon, 1982). Countries are thus more dissimilar than standard theory assumes. This explains why countries have (in the same period) different realised techniques expressed in input-output tables. Nevertheless, the hypothesis that distribution and prices govern the choice of techniques and therefore also the international transfer of techniques is shared not only by neoclassical economists. Hence the assumption is made, and the task is to ascertain whether the presumed relation between the prices of factors (rate of profit) and their employment (intensity of capital) is empirically valid. We now ask,

[12] Leontief once attempted to compare national technologies based on estimates of present and of future input requirements, using reports of the impacts of automation on employment in the U.S. However, the paper was not published so that it cannot be discussed here. (We are grateful to Thijs ten Raa for making a copy of a draft available to us.) The theoretical problem of neoclassical theory concerns the choice among *existing* techniques in response to distribution, not technical progress.

which alternative technique were to be chosen, if one country had different social and economic conditions, owing to another possible "path-dependence".

Some might still object against our procedure that the observed phenomena could be attributed to other causes than we propose. Imperfect markets could affect the data or could render the assumed international transferability of techniques illusory. But, although we try to make our empirical analysis as rigorous as possible, it does not have to be more rigorous than the neoclassical analyses which we wish to question. One might call this the 'argument of sufficient rigour in comparison' which we implicitly already have invoked above to justify the assumption that methods of production, as represented in input-output tables, must be transferable, if production functions are assumed to be valid in comparisons across countries. The argument of sufficient rigour in comparison defines minimum standards to which this paper must conform (higher standards are always desirable as a matter of principle and will be fulfilled where possible).

3.2. Data

Even though input-output tables have been set up all over the world since Leontief (1953), they were not solidly comparable internationally till recently, in particular because the industry structures of the respective countries are not identical. Moreover, each country adopted a different method of compilation, and some efforts to unify at least the criteria for compilation were not successful, in spite of the importance of input-output statistics both in national accounts and in economic analysis. Really comparable input-output tables were not published until the late 1990s, when the OECD input-output table databank[13] or European input-output tables from the EUROSTAT became available.

We use 32 OECD input-output tables with 36 sectors from 9 countries, referring to the periods from 1968 till 1990. This databank is suited both for national comparisons in time series and for international comparisons. All tables have 36 sectors from ISIC (International Standard Industrial Classification), version 2. This establishes a satisfactory base for the comparison of the 32 input-output tables even though the OECD data in its current form are subject to certain inconsistencies: Some sectors in some countries have not been compiled; basic price indices for compilation are different from country to country; the entries for the public goods sector (sector 34) are zero because of U.S. conventions. An improved version of the OECD databank (with ISIC version 3) has been announced.

3.3. Unit of the input-output table: monetary or physical?

A technique is specified in a book of blueprints in physical terms, but in practice and as a rule one finds only monetary input-output tables in official statistics, that is the input-output table expressed in the currency of the country concerned. Labour is represented by wage costs per unit of output in each sector. This is particularly problematic if one is looking for the price system (1) or (3). Pasinetti (1977, p. 60) suggests to derive physical coefficients from the input-output table by dividing the monetary coefficients by the price index. Petrovic (1991) shows theoretically that the form of an original (physical) wage curve is not influenced, if one introduces monetary coefficients.

[13] http://www.oecd.org/document/6/0,2340,en_2825_495684_2672966_1_1_1_1,00.html. More detailed descriptions of each country's input-output tables are given in the *Country Notes* ; http://www.oecd.org/dataoecd/48/30/2673439.pdf.

The price indices are not reported in the OECD input-output table databank so that Pasinetti's proposal cannot be realized. The problem is not one of general inflation. The unit costs of a process in terms of commodities and labour are obtained for the tables as a result of the division of the cost items by the money value of output in each industry so that the influence of the general level of inflation cancels out. There remain the influences of relative prices and of the average wage rate, however. To eliminate those, one would have to divide by the price indices for the individual commodities and for the wage rate, but these are not available, hence Petrovic's suggestion is followed and the monetary coefficients are used as if they were physical. The same method has been adopted in other empirical investigations, e.g. Krelle (1976), Özol (1984), Hamilton (1986), Cekota (1988). This conventional method is justifiable, since distribution and relative prices change little in the real course of development at a constant rate of profit in accordance with the stylised facts so that the monetary coefficients do reflect the physical structure (for the influence of technical progress see below).

3.4. The closedness of the price system and international trade

Sraffa's price system (1) assumes a closed national economy, but it may be expanded to represent a 'small' open economy (see Kurz and Salvadori, 1995). If a national economy depends on trade, the prices of its products depend on imports. The compilation of import coefficients therefore is more complicated than that of exports, compilation-technically as well as price-theoretically. Imports are reported in the OECD input-output table databank in the category "value added" and exports in the category "total final demand". In addition, matrices of imported intermediate inputs are compiled, but not for all countries. Procedures used to construct the import matrix data vary between countries, but every country in the OECD database more or less makes use of the 'import proportionality' assumption in the construction of their import matrices. It is assumed that each industry uses an import of a particular product in proportion to its total use of that product. For example: If the motor vehicles industry uses steel in its production processes and 10 per cent of all steel is imported, it is assumed that 10 per cent of the steel used by the motor vehicle industry is imported.

If there was no competition for imported goods in domestic markets and if countries imported to a great extent from each other, the compilation of comparable input-output tables would be impossible. But if imported commodities have domestic competitors in inland markets, the distortion of the price system by imported goods is reduced, and, in spite of imports, the input-output tables are internationally comparable and so are the products of sectors in different countries. The argument of sufficient rigour in comparison holds: if production functions are thought to be transferable internationally, despite the fact that some countries are primarily agrarian, others more industrial, still others specialized in services, it is legitimate for our purpose to assume homogeneous outputs of sectors for OECD countries. The theory of 'intra-industry trade' focuses on such constellations. The practical conclusion is that imports are priced on the basis of the pricing of domestically produced goods.

3.5. Rank of input matrix A

The technique represented by input matrix **A** with 36 sectors i.e. system (1) as obtained from (2), theoretically should consist of 36 different homogeneous sectors, representing 36 different production processes for 36 different 'commodities' (each in effect being an aggregate). Formally, if one thinks of an indecomposable system, the rank of matrix **A** should be 36; the input vectors to the production processes should be linearly independent. However, the maximal rank of matrix **A** is 34 in the OECD input-output table databank; sector 34 '*Producers of government services*', devoted to public goods,[14] is added to item '*Government consumption*' in '*Final demand*' and is subsequently set to 0. Sector 36 expresses merely '*Statistical discrepancy*' and does not represent a production process. Sometimes, sector 36 is simply set to 0.[15] Moreover, some countries do not show full entries for all of the first 33 sectors so that the rank of the technique matrix is different from country to country.[16] The calculations inevitably reflect these deficiencies: the prices of 'goods' pertaining to sectors, where all entries vanish, are set equal to zero and the tables below exhibit only the first 'significant' 33 sectors. The relevant calculations of the frequency of the occurrence of reverse capital deepening and the example found of reswitching take these 33 sectors into account. However, for completeness, the results of calculations using 36 sectors are also provided.

3.6. Technical progress and growth

Rapid technical progress can make national input-output tables of one country, compiled at different dates, incomparable within 20 years, because of the problem of new goods: there was no laptop or cell phone 20 years ago. The comparability between countries suffers if the growth rates are very different. Nevertheless, the chosen 9 OECD countries in the input-output table databank may be considered as relatively homogeneous "industrialized countries" concerning technical development and growth rate (Australia, Canada, Denmark, France, Germany (West), Italy, Japan, UK, USA). Here, the argument of sufficient rigour in comparison may be involved once more: The production function is based on the aggregation of all commodities into one, and neoclassical theory shows how productivity rises in the production of this composite commodity; it does not analyse (at least not in the versions based on the Solow-model), how progress affects the differentiations of products. Moreover, Vaccara (1970) shows that the macroeconomic influence of technical progress on the whole economy is expressed in a transformation of the input-output coefficients which is slow and gradual. Gains in labour productivity then remain visible, because compensating increases of real wages were moderate in the period under consideration.

3.7. Joint production and fixed capital

Joint production systems are generalisations from single product systems. Joint production, which is found to be a ubiquitous phenomenon in real economies, changes some properties of wage curves and their envelope in the theory (see Schefold, 1989). The input-output tables used here represent single product systems; they are based on statistical procedures that eliminate joint production, primarily through aggregation. The

[14] The formation of prices of public goods must differ from that of private goods. The modern classical theory (Sraffa and von Neumann) is agnostic with regard to the pricing of public goods. Samuelson (1954) analysed the price determination of public goods within the neoclassical framework.

[15] For this reason the sectors 34 and 36 are used to construct the "correction items" together with sector 35, *other producers*, the product of which is not homogeneous between countries. Hence, the sectors 34,35 and 36 go into the solution of linear programming as correction items or for statistical correction, but they are not to be interpreted as "production processes" that are subjected to a choice of the technique. On the other hand, if we got rid of sectors 34,35 and 36, the magnitude of the surplus and hence the maximum rate of profit would be distorted. This interpretation (that sectors 34, 35, 36 are relevant for the magnitude of the surplus but that they do not represent processes) is supported by the fact that some countries show other inputs but no labour input for sector 35.

[16] The production processes (inputs and outputs) represented by $(\mathbf{A},\mathbf{B})$ are linearly independent, however, since $rk(\mathbf{A},\mathbf{B}) = rk(\mathbf{A},\mathbf{B}-\mathbf{A})$ and $\det(\mathbf{B}-\mathbf{A}) \neq \mathbf{0}$.

use of input-output tables that include joint production for the derivation of wage curves may become possible in the future.

Fixed capital is regarded as a special case of joint production system in Sraffa (1960), while Leontief (1953) introduces fixed capital as "stocks" in his input-output model in order to analyze the dynamic implications. The advantage of the joint production approach to fixed capital is based on the possibility of treating the old machine leaving a production process as a different good from the one entering it, so that depreciation can be determined simultaneously with prices. This approach is also found in von Neumann (1937)[17], because he remarks that capital goods should appear both in the input and in the output matrix and should be considered as different goods at each stage of their utilization.

One can show that fixed capital systems (where other forms of joint production and the trade of used machines is excluded) behave very much like single product systems (see Schefold, 1989), above all, in so far as only the prices of old machines may turn negative. Negative prices signal an inefficiency; old machines with negative prices can be eliminated by means of "truncation" (see Nuti, 1973).

Fixed capital could be taken account of in the empirical model by introducing stocks. Suitable data for depreciation and a capital stock matrix (not only a vector) would be necessary. Some empirical studies integrated fixed capital, see Krelle (1976); Petrovic (1991); Ochoa (1987) on the basis of national data for depreciation and capital stocks. For the purpose of international comparison, we would need data according to unified criteria compiled for depreciation and capital stock matrices for 33 sectors of the countries considered according to the second version of ISIC. Unfortunately, these data are not available in the OECD input-output table databank, and it probably will remain unavailable in the near future. So we have here to be content with single product models. The introduction of capital stock matrices or – better still – a representation of fixed capital as a joint product would constitute probably the most important single extension of our analysis in future researches. The neglect of fixed capital (inevitable, given the data) implies maximum rates of profit that are unplausibly high.

4. Results

4.1. Empirical procedure, presentation of tables and main results

The choice of technique in (2) is carried out by comparing techniques in pairs from the OECD input-output databank.[18] Each sector then has two production processes available. Thus one can represent the 'hypothetical' choice of technique in the sense of the capital controversy on the basis of actual, existing techniques.

The maximum range of the rate of profit, where the choice of technique takes place, should be determined by the maximum rate of profit R of the envelope. This is one of 2^{36} R's from 2^{36} possible techniques. Since it would be too cumbersome to determine this R, the empirical investigation is carried out only up to the smallest maximum rate of profit of all pairs of input-output table techniques. This never exceeds the maximum rate

[17] Von Neumann (1937, pp. 453-463) repudiates the other representation, sometimes erroneously ascribed to him, of wear and tear as a diminution of the physical stock of fixed capital when he writes (our translation): "Wear and tear of a capital good is to be described by introducing its various stages of wearing down and by ascribing a special P_i [price] to each."

[18] MATLAB 5.1 was used for computing the linear programming. The correctness of computing was tested by EXCEL SOLVER (in EXCEL 2002) for important results.

of profit of the envelope, so we merely explore a part of the whole range of technical choice and reduce the chance of encountering cases of reverse capital deepening.

Tables 1, 3 and 4 show how activity levels $\mathbf{q}$ as derived from (2) change with r, a positive entry at some r indicating that the process is used, a zero indicating that it is not used. 33 sectors, each with two processes, are shown (for two countries or for one country at two different dates). The spectrum of techniques, consisting of 66 processes for 33 industries, shows (in table 1) the 33 processes employed by Germany above and the 33 processes employed by Canada below. The assumption then is made that either a German or a Canadian process is used in any industry. The rates of profit in table 1 increase horizontally in steps of 0.01 (1%) so that a column is assigned to each value of the rate of profit. The result of the choice of technique for a given rate of profit, in other words the selected technique, is positioned in a column, so that each column represents a solution vector $\mathbf{q}$ of model (2). Rows represent the use of production processes for sectors at different rates of profit. If in table 1 the German production process is chosen (or dominant) for sector 1 in the range of the rate of interest 0.62-0.63, the corresponding cells for process 1 have entries of positive real numbers, namely the intensity levels or gross products, while the Canadian production process 1 is not chosen and the cells have entries of zero. If the entry for industry i at the rate of profit r is not positive for Germany, it must be positive for Canada and *vice versa*, for each industry uses either the German or the Canadian process i at any given r. A change of the chosen technique implies a switch point on the envelope. However, not the exact location of the switch but only an interval within which the switch point must lie is reported. For example, a switch point for sector 10 in table 1 lies between the rates of profit 0.64 and 0.65.

All pairs of the input-output tables are considered. Each pair is a book of blueprints and yields an envelope. In all, we get $\binom{32}{2}$= 496 envelopes.

The change of a chosen production process on an envelope is indicated by the change of a cell with a positive real number into a cell with a zero on the row considered. Reswitching means that a change of a chosen production process takes place twice on one row (that is we have two switchpoints for a sector) *and* that there is no lasting change of method in any other sector between these two switchpoints.

Since switchpoints below the envelope (such as that at r_1 in fig. 3) were not observable, reverse capital deepening could only be analysed by comparing intensities of capital. If the individual wage curves were available, reverse capital deepening could also simply be observed by asking whether, in passing from one technique given by wage curve $w^{(1)}$ to a wage curve $w^{(2)}$ at a rate of profit $\bar{r}$, one had $w^{(1)}(0) > w^{(2)}(0)$ – this would be a normal switch such as the one at r_2 in fig. 2 – or $w^{(1)}(0) < w^{(2)}(0)$: this could be reverse capital deepening such as the switch at r_3 in fig. 3 or reswitching such as the switch at r_3 in fig. 2. However, the programme only calculates the envelopes. Hence reverse capital deepening between two techniques $(\mathbf{A}^{(1)}, \mathbf{l}^{(1)})$ and $(\mathbf{A}^{(2)}, \mathbf{l}^{(2)})$ had more laboriously to be determined by means of the formula $k_i = \mathbf{q}^{(i)}\mathbf{A}^{(i)}\mathbf{p}^{(i)} / \mathbf{q}^{(i)}\mathbf{l}^{(i)}$, where $\mathbf{q}^{(i)}$ resulted from the following modification of linear programme (2):

$$Min \quad \mathbf{q}^{(i)}\mathbf{l}^{(i)} \quad S.T. \quad \mathbf{q}^{(i)}(\mathbf{B}^{(i)} - \mathbf{A}^{(i)}) \geqq \mathbf{d}, \ \mathbf{q}^{(i)} \geqq \mathbf{0}. \qquad (6)$$

The prices $\mathbf{p}$ at the switchpoint in the formula for $k^{(i)}$ are common to both techniques according to (3), where $(\mathbf{A}^{(i)}, \mathbf{l}^{(i)})$ is the technique employed, $i = 1, 2$. The rates of growth,

Choice of technique				1.Switch	Sector 10; Rubber & plastic products					2.Switch
Sector	rate of profit	0.62	0.63	0.64	0.65	0.66	0.67	0.68	0.69	0.7
1		3.511252	3.601584	3.696017	3.810631	3.704383	3.795216	3.88984	3.988488	4.066091
2		0	0	0	0	0	0	0	0	0
3		3.411198	3.486491	3.565194	3.633069	3.70899	3.788962	3.872275	3.959135	4.064183
4		0	0	0	0	0	0	0	0	0
5		0	0	0	0	0	0	0	0	0
6		0	0	0	0	0	0	0	0	0
7		0	0	0	0	0	0	0	0	0
8		0.554215	0.55541	0.556657	0.5553	0.546193	0.547076	0.547994	0.548949	0.552075
9		3.158615	3.239866	3.324937	3.371977	2.9816	3.051095	3.123568	3.199201	3.297651
10		0	0	0	**2.461271**	**2.186287**	**2.226775**	**2.268783**	**2.3124**	0
11	G	0	0	0	0	0	0	0	0	0
12	E	0	0	0	0	0	0	0	0	0
13	R	0	0	0	0	0	0	0	0	0
14	M	3.076424	3.144464	3.215347	3.25355	2.893329	2.95122	3.011278	3.07363	3.151229
15	A	3.184765	3.234528	3.286298	3.422733	3.113789	3.157558	3.202894	3.249891	3.198739
16	N	0	0	0	0	0	0	0	0	0
17	Y	3.158076	3.213187	3.270484	3.375319	3.139367	3.189939	3.242309	3.296581	3.292294
18		2.058542	2.084339	2.110893	2.136093	2.033621	2.056954	2.080878	2.105419	2.126095
19		0	0	0	0	0	0	0	0	0
20		0.551174	0.552475	0.553837	0.55418	0.541444	0.542317	0.543226	0.544173	0.545542
21	1	2.331731	2.360263	2.389925	2.463191	2.287789	2.312614	2.338305	2.364911	2.340048
22	9	0	0	0	0	0	0	0	0	0
23	9	1.564474	1.574143	1.584166	1.595284	1.541126	1.549397	1.557934	1.56675	1.571923
24	0	0	0	0	0	0	0	0	0	0
25		0	0	0	0	0	0	0	0	0
26		0	0	0	0	0	0	0	0	0
27		6.200891	6.354484	6.514797	6.657964	5.738784	5.867369	6.001025	6.140056	6.256037
28		0	0	0	0	0	0	0	0	0
29		0	0	0	0	0	0	0	0	0
30		0	0	0	0	0	0	0	0	0
31		5.318301	5.463244	5.614688	5.772254	5.255204	5.3882	5.526625	5.670806	5.787988
32		9.932538	10.22308	10.52674	11.15212	10.87115	11.16954	11.48022	11.80393	11.80613
33		0	0	0	0	0	0	0	0	0
1		0	0	0	0	0	0	0	0	0
2		5.245089	5.393609	5.548844	5.658972	5.254697	5.393671	5.538376	5.68916	5.871257
3		0	0	0	0	0	0	0	0	0
4		3.808719	3.870855	3.935368	3.945223	3.87852	3.939331	4.002285	4.067502	4.186273
5		1.770759	1.801723	1.834078	1.865942	1.697768	1.723779	1.750836	1.779003	1.800342
6		3.832672	3.938094	4.048318	4.086323	2.839649	2.906814	2.976725	3.049545	3.136526
7		6.571829	6.760763	6.958363	6.770677	6.512286	6.689964	6.875177	7.068386	7.640276
8		0	0	0	0	0	0	0	0	0
9		0	0	0	0	0	0	0	0	0
10		2.206526	2.251295	2.297937	**0**	**0**	**0**	**0**	**0**	2.228868
11	C	1.272612	1.301387	1.331436	1.389109	1.312353	1.340399	1.369579	1.399963	1.400124
12	A	2.078662	2.129778	2.183028	2.250776	2.08236	2.129939	2.17931	2.230575	2.259947
13	N	2.840994	2.898212	2.957728	3.032196	2.886322	2.941116	2.997904	3.056802	3.094542
14	A	0	0	0	0	0	0	0	0	0
15	D	0	0	0	0	0	0	0	0	0
16	A	4.943005	5.001767	5.06224	5.137328	5.036	5.094013	5.153565	5.214726	5.25594
17		0	0	0	0	0	0	0	0	0
18		0	0	0	0	0	0	0	0	0
19		1.055151	1.057048	1.059024	1.060507	1.043887	1.045223	1.046605	1.048036	1.04918
20		0	0	0	0	0	0	0	0	0
21	1	0	0	0	0	0	0	0	0	0
22	9	2.288165	2.309762	2.332069	2.35816	2.303948	2.323619	2.343833	2.364617	2.379196
23	9	0	0	0	0	0	0	0	0	0
24	0	1.892703	1.912211	1.932512	1.9883	1.876426	1.894006	1.912226	1.931125	1.90998
25		3.272317	3.359152	3.449918	3.544854	3.333773	3.416314	3.502262	3.591822	3.670842
26		3.103484	3.176322	3.252545	3.337998	2.942481	3.005642	3.071487	3.140179	3.183061
27		0	0	0	0	0	0	0	0	0
28		1.824918	1.855779	1.888038	1.925589	1.47105	1.489169	1.508025	1.527665	1.520757
29		4.87675	5.015503	5.160622	5.254907	2.896054	2.96438	3.035519	3.109642	3.123614
30		1.705468	1.739015	1.774036	1.799489	1.686151	1.716402	1.747851	1.78057	1.818227
31		0	0	0	0	0	0	0	0	0
32		0	0	0	0	0	0	0	0	0
33		2.476134	2.526854	2.579882	2.615222	2.162596	2.200417	2.2398	2.280841	2.318687

Table 1; Reswitching at comparison of German technique 1990 and Canadian technique 1990.

Vector d: Total final demand. Explanation of tables: see section 4.1.

used to calculate $\mathbf{q}^{(i)}$, were thus assumed to be zero. The comparison of stationary systems at different rates of profit corresponds to the usual procedure in most theoretical expositions of the matter. It is here pragmatically justified by the fact that the actual rates of growth are on average much lower than the rates of profit at which prices are compared.

All envelopes were searched to find examples of reswitching. As a result, one case of reswitching[19] (see table 1) was observed; for details of an earlier investigation, see Han (2003). This is an important result, but only in view of the challenge posed to the proponents of the Cambridge critique to provide an example of reswitching. Now there is one. The relevant problem, however, concerns capital deepening (see table 2 as one example).

Before we come to that, we show that there are further 'microeconomic' puzzles. It is clear that returns of processes (section 1.4) are more frequent than reswitching, i.e. it happens not rarely that a process is used in two separate intervals of the rate of profit I_1 and I_3, separated by another interval I_2 where it is not used. This is no reswitching, however, unless all other processes in I_1 and I_3 are also the same. Mere returns of processes (i.e. returns of processes not associated with reswitching or reverse capital deepening) are not necessarily in conflict with the macroeconomic neoclassical hypothesis of an inverse relationship between the intensity of capital and the ratio of the rate of profit to the wage rate. They do, however, question the meaning of a microeconomic ordering of processes according to capital intensity or the generality of the identification of certain techniques as more labour intensive, more apt for use at low wages, etc., and they are evidence of strong changes of relative prices. In fact, it is often observed that returns of processes are connected with reverse capital deepening within the scope of our investigation (but it can easily be seen that either phenomenon can also occur without the other).

Apart from reverse capital deepening, defined as an increase of the intensity of capital, i.e. of $k_i = \mathbf{q}^{(i)}\mathbf{A}^{(i)}\mathbf{l}^{(i)} / \mathbf{q}^{(i)}\mathbf{l}^{(i)}$, at a switch point, as w is lowered, there is also the simpler possibility that the lowering of w leads (against neoclassical intuition) to the introduction of a process with a labour coefficient lower than that of the process being replaced at the switch. We introduced this possibility in section 1.3 as a reversed substitution of labour; we here call this a labour-reducing switch. We then have the four possibilities discussed in section 1.3, and we add in brackets the frequency of the case observed (in per cent). The sample consists of all 4389 switchpoints of 496 envelopes:[20]

(a) capital intensity-reducing, labour-increasing: 4229 cases (96.35%),
(b) capital intensity-reducing, labour-reducing: 100 cases (2.28%),
(c) capital intensity-increasing, labour-increasing: 32 cases (0.73%),
(d) capital intensity-increasing, labour-reducing: 28 cases (0.64%).

(a) is what is to be expected in the neoclassical perspective. (d) in comparison with (c) shows that reverse capital deepening seems primarily to be associated with the

[19] We observed a "switch point" for the row (sector) 35 between two switchpoints in row (sector) 10. However, sector 35 is to be treated as a correction sector and not as a production process (see footnote 15).

[20] Excepted were those "points" where technical change seems not to be piecemeal because more than one method seems to change (in more than one sector). This mistaken perception arises because the calculation of the wage curve does not proceed in sufficiently small steps so that two switchpoints appear as one. To recalculate the wage curve at all such "points" would have been too laborious. However, an example of such a recalculation, using smaller intervals to separate the switch points, is given in tables 3 and 4; see also section 4.2 below.

counterintuitive 'microeconomic' phenomenon of the introduction of labour-saving process at a lower wage. (b) is, like mere returns of processes, a puzzle for neoclassical theory at the sectoral level, not a macroeconomic one. (b) occurs more frequently than (d) or (c), as was to be expected on the basis of the discussion in 1.3.

The decisive question of reverse capital deepening was examined by analyzing all 4389 switchpoints of 496 envelopes. 535 of these switchpoints occurred in national comparisons. National comparisons yielded 12 cases of reverse capital deepening, using 33 sectors (11, if 36 sectors were considered), i.e. more than 2 %. International comparisons yielded 48 cases of reverse capital deepening with 33 sectors, i.e. more than 1 % (14 cases with 36 sectors). The wage curves in the national comparisons are perhaps closer together and therefore more intertwined which results in a higher frequency (Schefold 1997, p. 279). The null hypothesis that the likelihood of reverse capital deepening is smaller than 1 % (0.2 %) in the consideration of 33 (36) sectors can be rejected; these results are statistically significant at the 1 % (5 %) level (test statistic Z= 2.4436 (5.4896)). An example of reverse capital deepening is shown in table 2: As r rises from 0.79 to 0.8, the intensity of capital rises from 6.38 to 6.39.

	r=0.79			r=0.80	Price	Intensity	Intensity
Sector	qAp	Ap	Ap	qAp	r=0.80	r=0.79	r=0.80
1	124.409512	3.39978963	3.39978963	124.450482	6.22574163	2.2120903	2.18961439
2		0.65057542	0.65057542		1.24890315	4.29128456	4.31043456
3		4.07409209	4.07409209		7.49204031	2.24845694	2.2331955
4		**2.90616122**	**2.92384541**		5.54366083	2.07827704	2.05308517
5		2.65931538	2.65931538		5.05846406	1.71026133	1.71427139
6	ql	1.85287065	1.85287065	ql	3.6764876	3.23159979	3.23254269
7	19.4976156	2.41463742	2.41463742	19.4638427	4.51233004	3.81367807	3.83816707
8		1.7587887	1.7587887		3.41520007	1.13746336	1.1373233
9		1.24191125	1.24191125		2.25585311	2.38659749	2.40995018
10		2.34043125	2.34043125		4.51362699	2.06842255	2.05591105
11	qAp/ql	1.57852791	1.57852791	qAp/ql	3.11333871	1.63603837	1.63861377
12	6.38075519	2.05566519	2.05566519	6.39393179	3.94757158	3.3756466	3.38173454
13		2.1359719	2.1359719		3.98514508	2.44380899	2.44443406
14		2.06724319	2.06724319		4.01980804	1.83575223	1.83862068
15		1.89258385	1.89258385		3.76094786	2.42588539	2.41594769
16		1.80396689	1.80396689		3.46685448	1.18125565	1.18147372
17		1.93903663	1.93903663		3.82482727	1.71301475	1.7121987
18		1.77146491	1.77146491		3.53539619	2.07648103	2.07560683
19		1.70843016	1.70843016		3.55050636	1.23035897	1.23116861
20		1.74361758	1.74361758		3.58416382	1.14164211	1.1426045
21		2.37295726	2.37295726		4.57135526	1.38019956	1.3818246
22		2.23295285	2.23295285		4.38912505	1.33335319	1.33375946
23		1.67550552	1.67550552		3.40083659	1.14936258	1.1486589
24		2.18641696	2.18641696		4.22256976	1.13344327	1.13486851
25		1.20914656	1.20914656		2.37504477	2.50394029	2.50598539
26		1.92021924	1.92021924		3.67162485	1.72074164	1.72287681
27		1.09321189	1.09321189		2.37428061	3.14702603	3.16072626
28		1.88616258	1.88616258		3.82773551	1.07065706	1.07532622
29		1.37335372	1.37335372		2.83791478	3.59900135	3.65255143
30		0.71335781	0.71335781		1.76709073	1.79325716	1.79450206
31		0.94813535	0.94813535		1.95212644	1.90019234	1.88801339
32		0.4469039	0.4469039		1.2665855	2.88809141	2.86911847
33		0.66428134	0.66428134		1.63707148	1.85644773	1.91089254

Table 2: Reverse capital deepening (vector *d*: (1,…,1)); United Kingdom 1979 and 1984.

Capital intensity=qAp/ql.

4.2. Other results

The structure (i.e. the distribution of zero coefficients reflecting the choice of techniques) of the solution intensity vector **q** found at *r* according to (2) – not to be confused with $\mathbf{q}^{(i)}$ in formula (6) – is independent from the composition of the final demand vector **d**. This is what the non-substitution theorem suggests (the corresponding tables showing it empirically are here omitted) and this is why the choice of the first demand vector in table 1 was not discussed.[21]

For all 496 results of technical choice, no input-output table turned out to be superior for all 33 sectors throughout the considered range of the rate of profit, and no input-output table was in the whole range and for all sectors inferior (dominated).

The property of the envelope that a switch point represents the change of the production process merely in one sector,[22] therefore that two different techniques (systems) are different merely in one sector at a switch point, i.e. the property that the choice of techniques generically is 'piecemeal', can not be verified immediately at all switchpoints. At more than 100 rates of profit, the production processes seem to change in more than two sectors. But if the steps, by which the interest rate changes, are made smaller, the processes change one by one at switchpoints, and the theory is confirmed (see table 3 and table 4).

As a matter of fact, at least three switchpoints are registered on each of the 496 envelopes. On average, nearly ten switchpoints per envelope are found. In the majority of cases, as many wage curves are involved. This is in accordance with the theoretical expectation that the chosen technique should change as distribution varies (though far fewer switchpoints are found empirically in the relevant range than can exist according to the theoretical argument of the 'avalanche of switchpoints' referred to above – Schefold 1997, pp. 278-80). But we obtain some implausible results from the input-output tables of the 1960' and 1970': According to these data, old production processes often dominate the production processes of later periods. This suggests that these data were compiled incorrectly.[23] The input-output tables of 1980' and 1990' show more plausible results.

4.3. Alternative conclusions

Some economists have examined the probability of reswitching and reverse capital deepening phenomena on theoretical grounds. They predicted that these phenomena do not appear frequently, but that their probability of occurring is positive, see Schefold (1976); D'Ippolito (1987); D'Ippolito (1989); Mainwaring and Steedman (1995); Petri (2000). Zambelli (2004) uses computer experiments with random numbers to test the neoclassical aggregate production function and finds that Wicksell effects are frequent, but that the likelihood of reswitching is low. The examination in this paper supports such prognoses. Nevertheless, observed cases of reswitching and reverse capital deepening are more than flukes, hence they seem to suffice to undermine the neoclassical production and distribution theory, both in a stochastical and falsificatorial sense, if Popper's methodology[24] is accepted.

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[21] The intensities in tables are computed with the vector **d** of arithmetical means of "Total final demand" in the OECD input-output table databank.

[22] see Bruno, Burmeister and Sheshinski (1966, p.542); Schefold (1989, p.122).

[23] Inconsistencies in data collection are not the only possibility: Cheap old techniques can disappear from the book of blueprints at later times because quality improvements, which are not represented in the input-output table, lead to higher costs and because regulations change, in particular if more severe environmental standards are enacted and drive up costs.

[24] See Popper (1935); Lakatos (1970). See Lutz and Hague (1961, p. 305-306) on the empirical methodology of Sraffa.

It may be too early to speculate on what has to follow if this conclusion is, as we expect, confirmed by other studies which develop the new method proposed in this paper further. A few remarks will nevertheless be made.

The fact that the observed effects are in line with what could be expected on the basis of *a priori* considerations could mean that the discussion will continue; the results obtained so far are ambiguous and likely to lead to conflicting interpretations. More than 96 % of cases of an unequivocal fall of the intensity of capital with the wage rate will be regarded as a confirmation by many neoclassical authors, while their opponents will state that the logical problems of the theory remain unsolved and have received empirical support.

We have found that most of the 2^{33} (or 2^{36}) wage curves derived from the spectrum of two countries (or one country at two dates) never appear on a relevant part of the envelope, yet substitution exists on all envelopes; on average, about ten wage curves constitute the envelope which we calculated. There is one case of reswitching and nearly four percent of cases with reverse capital deepening or reverse substitution of labour; both can occur, but there can also be reverse capital deepening without reverse substitution of labour. In this last case, there are at least three switchpoints between two wage curves, only one of which is visible on the envelope while the existence of the others can be inferred from the properties of the visible switchpoint, using the theory of section 1.3. This result is perhaps the most spectacular. We also found that technical change is piecemeal and that returns of processes are frequent. They imply that relative prices change strongly with distribution (section 1.4) which is the key point to be examined below.

The paradoxes of capital therefore occur not only as thought experiments; the critique of capital theory is not only a matter of logic, but an aspect of the real world. The results are empirically significant. Physicists have changed their theories for smaller deviations – remember that the elliptical orbits of planets are *almost* circles and that general relativity was taught for decades with only three types of empirical observations, involving minute effects, to support it.[25]

Neoclassical economists will probably regard this analogy with physics as acceptable, provided the production function and marginal productivity are interpreted as the essential components of the simple theory, still to be used for practical purposes as long as the deviations remain small, and intertemporal general equilibrium represents the generalisation. We would like to object that the problems of capital theory re-emerge in general equilibrium theory as problems of stability[26], but we here concentrate on the problem of aggregation.

As far as the production function is concerned, different reactions are possible. One might turn to a *pragmatic approach* which we illustrate by means of an example. The comparison of the present technique with that of the past or that of other countries yields the best representation of the spectrum of *available* techniques (as opposed to the incomplete emerging blueprints of designs for the future). The spectrum implies a measure of the wage elasticity of the demand for labour, to the extent that it depends on technology. This elasticity, in conjunction with hypotheses about the ensuing changes of effective demand for goods, might be used to assess the likely increase in the demand for labour, if any, which might result from a wage reduction in a closed economy. Lowering the general level of wages (as opposed to an adjustment of relative wages) reduces the demand for finished goods from wage earners, may increase demand out of profits and perhaps increases the demand for personal services; whether demand for labour in

[25] Shift of the perihelion of the planet Mercury, deflection of the light rays near the sun, red shift.

[26] Cf. the remark in section 1.1 and the debate by Garegnani, Mandler and Schefold in the forthcoming issue of *Metroeconomica*.

production would increase because of substitution effects, given an appropriate level of demand for finished goods, could be estimated, using the spectrum of techniques. The method here proposed may thus serve not only a critical, but a positive function. The problem of "keeping capital constant" in some meaningful way would remain, however.

There also remains the still burning *theoretical question* of what the paradoxical effects imply for the theory. One might say that the empirical facts could be neglected because they are small. However, a strict analysis, based on the production function as constructed by Samuelson, would have to deal with wage curves as if they were linear and with labour as if it were homogeneous. For even only one wage curve which deviated from linearity by only a small amount might lead to reserve capital deepening through a double switch with a straight wage curve. The linearity of the wage curves, all expressed in terms of the same numéraire, which would therefore have to be assumed, then signified that relative prices remained unaffected by changes in distribution. This meant that one would analyse reality as if a simple labour theory of value held. Moreover, one would have to put aside the inconsistency that, of two techniques with prices equal to labour values, dominating the spectrum of technique at two different levels of the rate of profit, a third could be formed which dominated in-between, for which relative prices could not be equal to relative values and the wage curve could not be straight.[27] It is therefore not as easy to assume away the paradoxes of capital theory as it is to assume away non-convexities in consumption. One would have to consciously use a theory with inconsistent foundations. And not only the logical structure of the formal analysis would be inconsistent, but also its interpretation, since both marginal productivity and the theory of exploitation could be invoked to explain distribution in such a framework. "Contradictions!", Marx would murmur in his grave, and, after historical defeats, turn around for once somewhat contented.

On the other hand, we found that a full 96 % of the switchpoints are of the neoclassical type. One could therefore – less strictly – try to work with *almost* linear wage curves, according to some definition. But more assumptions would be needed. With piecemeal technical change, no second switch could be allowed to exist between any to wage curves which had a first switch on the envelope. One would have to assume that each wage curve, to the extent that it appeared on the envelope, was convex to the origin (so as to rule out perverse Wicksell effects, section 1.4). The envelope then would strictly be falling and convex to the origin. One would have to assume that the wage curves would be sufficiently numerous and intersected on the envelope at sufficiently large angles to permit a smooth approximation by a smooth 'aggregate' wage curve $w(r)$, twice continuously differentiable, so that the intensity of capital could be defined as $k = -w'(r)$ and output per head as $y = w + rk = w - rw'$; this parametric representation of the production function $y = f(k)$ could be made legitimate by observing that $k(r)$ and $y(r)$ could not vanish for $0 \le r < \infty$, since $w' < 0$. Following Schefold (1989 [1971], p. 298), one could in fact conclude that $f' = r$ and that the marginal product would be positive and falling. But one crucial hypothesis would still be needed for consistency: for any individual wage curve $w^{(i)}$ appearing on the aggregate wage curve at r, $w^{(i)}(0)$ would, strictly speaking, have to be equal to y as here defined, i.e. $y = w - rw' = w^{(i)}(0)$ had to hold, if the growth rate g equalled zero. And if g varied, it would be necessary for $w^{(i)}(r)$ to be linear between zero and r, since $y = c + gk$ and $c = w^{(i)}(g)$ and since

[27] Two techniques $\mathbf{A}^{(i)}, \mathbf{l}^{(i)}; i = 1, 2;$ exhibit uniform organic compositions of capital if each $\mathbf{l}^{(i)}$ is eigenvector of $\mathbf{A}^{(i)}$. But this condition will not be fulfilled for techniques composed of different rows of $(\mathbf{A}^{(1)}, \mathbf{l}^{(1)})$ and $(\mathbf{A}^{(2)}, \mathbf{l}^{(2)})$. Technical change is piecemeal. It is therefore impossible for two neighbouring techniques on the envelope to be both strictly linear; cf. Salvadori and Steedman (1988).

$k = -w'(r)$ and $y = w + rk$ would be constant, given r, for all g. In essence, the linearity of the individual wage curves is not only sufficient, but also necessary for the construction of the surrogate production function, if it is also used to explain growth for any g, $0 \leq g \leq r$. The same idea can also be expressed by stating that the intensity of capital of a given technique varies with the rate of growth because the activity levels with which this chosen technique is to be activated vary with g (except if the organic composition of capital is uniform). We made a concession to the neoclassical approach by simplifying in (6) and putting $g = 0$.

But even if one only attempts to construct a surrogate production function for stationary states, one must have $y = w - rw' = w^{(i)}(0)$, or in a less-than-strict approximation, one needs that this is almost the case, which, with individual wage curves supposed to be convex to the origin, again means that there cannot be much deviation from linearity. There is therefore little room for variations of the intensity of capital along any individual wage curve. But if we had asked whether k varied along individual wage curves, we would have found that k did vary in 100 % of the cases to some extent!

The absence of reverse capital deepening and reverse substitution of labour is not enough to justify marginal productivity theory at the aggregate level; these phenomena are only the most manifest, therefore the didactically most useful expressions of the fact that the amount of capital depends on prices and distribution. Returns of processes also reveal strong changes of relative prices. The exclusion of reverse capital deepening does therefore not suffice to show that the production function exists; even in the absence of reverse capital deepening, the formula $y = w - rw'$ may give the wrong value of output per head.

Let us call $w^{(i)}(0) - (w - rw')$ the declination of output per head, by analogy with the declination of a compass due to the difference between the geographical and the magnetic North Pole. Then, for neoclassical theory to work well, it is necessary that wage curves are sufficiently straight to guarantee a small declination and sufficiently curved to rule out the labour theory of value.[28] Is it possible to get out of this awkward corner? Only if the difference between prices and values is large enough even in the aggregate to prevent the mass of profits from being interpreted as redistributed surplus value and if the declination of output per head remains small all the same because aggregation smoothes the movements of relative prices. – The declination of output per head could not be observed directly in this paper, however; the existence of reverse capital deepening without reverse substitution of labour indicates that declination can be large in reality.

A rigorous fulfilment of the assumptions leads back to the perplexities of the surrogate production function itself. With the assumption fulfilled only halfway, the construction – if meaningful at all – would hardly be more than the empiricism mentioned above, dressed up as theory. There is much to be said for the historical and eclectic currents in economics which strive to be realistic and do without a unified, rigorous and comprehensive theory. But if a new form of historicism is not enough, the only way out is *to extend the theory which has lead to the discovery of the paradoxes*, i.e. to return to the old debates about value, distribution and employment and to refresh them, so as to move towards a new synthesis.

[28] See Steedman and Tomkins (1998) for results and references to the literature on the deviation of prices from values.

Intensity vector x

Sector	r	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15
1		1.032	1.048	1.064	1.129	1.146	1.164	1.182	1.201	1.22	1.24	1.26	1.28	1.293	1.314	1.336	1.358
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		1.658	1.675	1.693	1.719	1.737	1.756	1.776	1.796	1.817	1.837	1.859	1.881	1.896	1.919	1.942	1.965
4		1.771	1.786	1.802	1.81	1.826	1.842	1.858	1.875	1.892	1.909	1.927	1.945	1.943	1.962	1.98	1.999
5		0	0	0	0.997	1.004	1.01	1.017	1.024	1.031	1.038	1.045	1.053	1.04	1.047	1.054	1.062
6		1.411	1.438	1.467	1.477	1.507	1.537	1.569	1.601	1.634	1.668	1.703	1.739	1.604	1.638	1.673	1.709
7		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8		0.438	0.438	0.438	0.439	0.439	0.439	0.44	0.44	0.44	0.441	0.441	0.442	0.442	0.442	0.443	0.443
9		1.427	1.448	1.468	1.49	1.512	1.534	1.557	1.581	1.606	1.631	1.658	1.685	1.692	1.719	1.746	1.775
10		0.99	1.001	1.012	1.023	1.034	1.046	1.058	1.07	1.082	1.095	1.108	1.121	1.075	1.088	1.101	1.114
11	G	0.696	0.706	0.716	0.727	0.737	0.748	0.759	0.771	0.783	0.795	0.808	0.82	0.819	0.83	0.842	0.855
12	E	2.873	2.962	3.055	3.085	3.184	3.286	3.393	3.505	3.622	3.744	3.872	4.006	0	0	0	0
13	R	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	M	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	N	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	Y	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18		1.004	1.011	1.019	1.022	1.03	1.038	1.046	1.054	1.062	1.07	1.079	1.087	0.948	0.955	0.962	0.969
19		1.053	1.054	1.055	1.055	1.056	1.057	1.057	1.058	1.059	1.06	1.061	1.061	0	0	0	0
20		0.397	0.398	0.399	0.399	0.4	0.401	0.402	0.403	0.404	0.405	0.406	0.407	0.406	0.407	0.409	0.41
21	1	1.402	1.408	1.415	1.424	1.431	1.439	1.446	1.454	1.462	1.469	1.478	1.486	1.514	1.523	1.532	1.541
22	9	0	0	0	0	0	0	0	0	0	0	0	0	1.506	1.512	1.517	1.522
23	8	0	0	0	0	0	0	0	0	0	0	0	0	1.263	1.266	1.269	1.272
24	6	0	0	0	0	0	0	0	0	0	0	0	0	1.134	1.138	1.143	1.148
25		1.407	1.431	1.454	1.484	1.51	1.536	1.563	1.59	1.619	1.649	1.679	1.711	1.634	1.661	1.689	1.717
26		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27		2.265	2.299	2.335	2.391	2.429	2.467	2.507	2.547	2.589	2.632	2.676	2.722	2.695	2.738	2.782	2.827
28		1.005	1.009	1.013	1.024	1.028	1.033	1.038	1.043	1.048	1.053	1.058	1.064	1.083	1.088	1.094	1.099
29		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30		0.777	0.783	0.789	0.795	0.801	0.808	0.814	0.821	0.828	0.835	0.842	0.85	0.84	0.847	0.855	0.862
31		1.424	1.444	1.465	1.51	1.532	1.556	1.58	1.604	1.629	1.656	1.682	1.71	1.771	1.798	1.825	1.853
32		2.463	2.508	2.554	2.648	2.697	2.748	2.8	2.853	2.908	2.965	3.023	3.084	3.102	3.158	3.215	3.274
33		1.229	1.244	1.26	1.248	1.264	1.28	1.297	1.315	1.333	1.351	1.37	1.389	1.316	1.334	1.353	1.372
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		1.681	1.719	1.759	1.797	1.84	1.883	1.929	1.975	2.024	2.074	2.127	2.181	1.949	1.991	2.034	2.078
3		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5		1.112	1.121	1.13	0	0	0	0	0	0	0	0	0	0	0	0	0
6		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7		2.214	2.255	2.297	2.351	2.396	2.441	2.488	2.537	2.586	2.637	2.69	2.744	2.682	2.735	2.789	2.845
8		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	U	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	K	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12		0	0	0	0	0	0	0	0	0	0	0	0	1.693	1.725	1.758	1.792
13		1.478	1.498	1.519	1.544	1.566	1.588	1.611	1.634	1.658	1.682	1.707	1.733	1.551	1.575	1.599	1.623
14		1.04	1.051	1.062	1.059	1.071	1.083	1.095	1.107	1.12	1.133	1.147	1.161	1.145	1.158	1.171	1.185
15		1.642	1.656	1.671	1.68	1.695	1.711	1.727	1.743	1.759	1.776	1.794	1.811	1.749	1.766	1.784	1.802
16		1.552	1.555	1.558	1.56	1.563	1.566	1.569	1.572	1.576	1.579	1.582	1.586	1.607	1.61	1.614	1.618
17		1.347	1.358	1.369	1.397	1.408	1.42	1.432	1.444	1.457	1.47	1.483	1.496	1.514	1.527	1.54	1.554
18	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	9	0	0	0	0	0	0	0	0	0	0	0	0	1.194	1.196	1.199	1.201
20	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22		1.49	1.495	1.501	1.506	1.512	1.518	1.523	1.529	1.535	1.541	1.547	1.553	0	0	0	0
23		1.269	1.271	1.274	1.276	1.279	1.282	1.285	1.288	1.291	1.294	1.297	1.3	0	0	0	0
24		1.065	1.068	1.072	1.11	1.115	1.119	1.124	1.129	1.133	1.138	1.143	1.149	0	0	0	0
25		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26		1.304	1.316	1.329	1.348	1.361	1.375	1.389	1.404	1.419	1.435	1.451	1.467	1.47	1.486	1.502	1.519
27		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29		1.709	1.737	1.765	1.758	1.788	1.818	1.849	1.882	1.915	1.949	1.985	2.021	1.937	1.97	2.004	2.039
30		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 3: Five switches between 0.11 and 0.12 (Germany 1986 and UK 1979) appear to coincide at one rate of profit.

Sector		0.11	0.111	0.112	0.113	0.114	0.1142	0.1144	0.1146	0.1148	0.115	0.116	0.117	0.118	0.119	0.12
1		1.28	1.283	1.291	1.293	1.295	1.2956	1.2961	1.2965	1.2909	1.285	1.288	1.29	1.292	1.295	1.293
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		1.881	1.883	1.896	1.899	1.901	1.9015	1.9019	1.9024	1.8942	1.888	1.89	1.893	1.895	1.9	1.896
4		1.945	1.947	1.955	1.957	1.959	1.9594	1.9598	1.9602	1.9344	1.936	1.938	1.94	1.942	1.938	1.943
5		1.053	1.053	1.071	1.072	1.072	1.0727	1.0728	1.073	1.0698	1.048	1.048	1.049	1.05	1.043	1.04
6		1.739	1.743	1.727	1.731	1.735	1.7353	1.736	1.7368	1.5835	1.58	1.583	1.586	1.59	1.571	1.604
7		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8		0.442	0.442	0.442	0.442	0.442	0.4418	0.4418	0.4418	0.4418	0.442	0.442	0.442	0.442	0.442	0.442
9		1.685	1.688	1.672	1.675	1.678	1.6785	1.6791	1.6796	1.6596	1.661	1.664	1.667	1.669	1.666	1.692
10	G	1.121	1.123	1.122	1.123	1.125	1.1251	1.1253	1.1256	1.1133	1.105	1.106	1.107	1.109	1.068	1.075
11	E	0.82	0.822	0.821	0.822	0.823	0.8236	0.8238	0.8241	0.8229	0.827	0.828	0.83	0.831	0.836	0.819
12	R	4.006	4.02	3.851	3.864	3.877	3.8801	3.8828	3.8855	4.1517	4.037	4.05	4.064	4.078	3.83	0
13	M	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	N	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	Y	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18		1.087	1.088	0.975	0.976	0.976	0.9764	0.9765	0.9766	0.9747	1.018	1.019	1.02	1.02	0.943	0.948
19		1.061	1.062	1.062	1.062	1.062	1.0618	1.0618	1.0618	1.0606	0	0	0	0	0	0
20	1	0.407	0.407	0.407	0.407	0.407	0.407	0.407	0.407	0.406	0.406	0.406	0.406	0.406	0.405	0.406
21	9	1.486	1.487	1.494	1.495	1.495	1.4955	1.4957	1.4959	1.4963	1.493	1.493	1.494	1.495	1.5	1.514
22	8	0	0	1.503	1.503	1.504	1.5039	1.504	1.5041	1.5028	1.503	1.503	1.504	1.504	1.504	1.506
23	6	0	0	0	0	0	0	0	0	0	0	0	0	0	1.261	1.263
24		0	0	0	0	0	0	0	0	1.1526	1.123	1.123	1.124	1.124	1.142	1.134
25		1.711	1.714	1.709	1.712	1.715	1.7157	1.7163	1.717	1.7014	1.691	1.694	1.698	1.701	1.69	1.634
26		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27		2.722	2.726	2.718	2.722	2.727	2.7279	2.7288	2.7298	2.6901	2.642	2.646	2.651	2.655	2.645	2.695
28		1.064	1.064	1.08	1.081	1.081	1.0813	1.0814	1.0815	1.0886	1.083	1.084	1.085	1.085	1.09	1.083
29		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30		0.85	0.85	0.847	0.848	0.848	0.8484	0.8486	0.8487	0.8427	0.832	0.833	0.834	0.835	0.839	0.84
31		1.71	1.713	1.762	1.765	1.768	1.7684	1.7689	1.7695	1.7902	1.755	1.758	1.761	1.764	1.799	1.771
32		3.084	3.09	3.163	3.169	3.175	3.1767	3.178	3.1792	3.2347	3.124	3.13	3.136	3.142	3.213	3.102
33		1.389	1.391	1.358	1.36	1.362	1.3625	1.3629	1.3633	1.3136	1.314	1.316	1.317	1.319	1.278	1.316
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		2.181	2.186	2.166	2.172	2.177	2.1786	2.1797	2.1808	2.1497	2.134	2.139	2.145	2.15	2.128	1.949
3		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7		2.744	2.75	2.742	2.748	2.753	2.7544	2.7555	2.7566	2.6795	2.662	2.667	2.672	2.677	2.663	2.682
8		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	U	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	K	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12		0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.693
13		1.733	1.735	1.745	1.747	1.75	1.7503	1.7508	1.7513	1.4725	1.446	1.448	1.45	1.452	1.464	1.551
14		1.161	1.162	1.195	1.196	1.198	1.198	1.1983	1.1986	1.2498	1.146	1.148	1.149	1.151	1.149	1.145
15		1.811	1.813	1.757	1.758	1.76	1.7604	1.7608	1.7611	1.7867	1.691	1.692	1.694	1.696	1.703	1.749
16		1.586	1.586	1.604	1.604	1.605	1.6046	1.6047	1.6048	1.6045	1.585	1.586	1.586	1.586	1.604	1.607
17		1.496	1.498	1.569	1.571	1.572	1.5723	1.5726	1.5729	1.5746	1.488	1.489	1.49	1.492	1.523	1.514
18	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	9	0	0	0	0	0	0	0	0	0	1.189	1.189	1.189	1.19	1.19	1.194
20	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22		1.553	1.554	0	0	0	0	0	0	0	0	0	0	0	0	0
23		1.3	1.3	1.296	1.296	1.297	1.2969	1.2969	1.297	1.2898	1.28	1.281	1.281	1.281	0	0
24		1.149	1.149	1.155	1.156	1.156	1.1562	1.1563	1.1564	0	0	0	0	0	0	0
25		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26		1.467	1.469	1.469	1.471	1.473	1.4729	1.4732	1.4736	1.4765	1.473	1.475	1.477	1.478	1.483	1.47
27		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29		2.021	2.025	2.02	2.024	2.028	2.0283	2.029	2.0298	1.9809	1.96	1.964	1.968	1.971	1.946	1.937
30		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 4: Same case as in Table 3. At a greater resolution (smaller steps of increase of r) at most one switch appears at each rate of profit (the change of technique is piecemeal).

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