Demand Shocks and Output Structures in Some OECD Countries: An Input-Output Euclidean Distance Multipliers Approach

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ABSTRACT. The impacts of final demand changes (in consumption, investment or exports) on the sectoral output growth potential of an economy are traditionally measured using the so-called output multipliers (the elements of Leontief inverse), with the important limitation of imposing unitary final demand shocks with a fixed (predetermined) structure. In fact, this structure usually changes over time, and it is an interesting exercise to quantify the growth effects of a stimulus of certain intensity for several possible structures of final demand. One way to do so is by means of the so-called singular decomposition method. In this paper a suggestive alternative is proposed, namely by solving an appropriate optimization problem that originates a new kind of output multipliers, the so-called Input-Output Euclidean Distance Multipliers. An empirical application of this method is made using the inter-industry tables of several OECD countries in different years.

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1. INTRODUCTION

When studying the structure of a national or regional economy according to the Leontief model hypothesis, a central role is attributed to final demand multipliers, i.e. the elements of the Leontief inverse used to measure the impacts of change(s) in one (or several) component(s) of final demand on output, value added or employment.

However, the use of this kind of multiplier, dating back to Rasmussen (1956), suffers from one important drawback, namely that it is limited to particular changes in final demand, such as a unitary shock in each sector and zero elsewhere in the case of backward multipliers, and a unitary shock in all sectors at once in the case of forward multipliers. This limitation, pointed out by Skolka (1986), reduces the usefulness of the Rasmussen multipliers.

It can even be argued that the use of traditional multipliers leads to an inadequate invasion of macroeconomic concepts into the territory of a genuine multisectoral analysis. Let us consider, for instance, a unit increase in total final demand. From a macroeconomic point of view, it is by definition irrelevant to know in advance how this monetary unit is distributed among sectors, because these sectors are not individually considered. But from a multisectoral point of view, it is crucial to know if this unit is, for example, directed entirely to one particular sector or otherwise distributed evenly among all the sectors.

In the first case, the new situation (after the increase in final demand) is far more different from the initial one than in the second case. This difference does not exist in an aggregate macroeconomic analysis. In a disaggregated intersectoral analysis, however, it should not be ignored.

For this kind of comparison between different situations, the traditional Leontief/Rasmussen multipliers are inappropriate, because they are unable to compare the impacts of changes in final demand on output (value added, employment), giving rise to new vectors equidistant from the initial vector.

One interesting approach to this problem is the work of Ciaschini (1989; 1993; 2002), based on the so-called singular value decomposition method.

In this paper, a different and easier approach is adopted. By solving an appropriately designed optimization problem, two important advantages are obtained. Firstly, the final demand structure subsequent to a final demand shock is not fixed in advance, thereby overcoming an important limitation of traditional linkage measures. Secondly, the maximum output impact can be decomposed into two significant effects: a homothetic scale effect, depending on the magnitude of the positive shock applied to a pre-existing final demand structure, and a structure effect, resulting from output maximizing changes in sectoral final demand.

This method, explained and formalized in section 2, gives rise to a new kind of multipliers, that can be termed *Euclidean distance multipliers* and may prove to be helpful in measuring interindustry linkages and choosing key sectors in a national or regional economy.

An empirical application of the method is made here for eight OECD (EU) countries with available input-output for two recent periods (section 3). The paper concludes with a summary of the main results (section 4).

2. INTERSECTORAL EUCLIDEAN DISTANCE MULTIPLIERS

Context of analysis

Consider the solution of the standard Leontief model **x = L y,** where **x** and **y** are vectors of output and final demand and **L** is the Leontief inverse (for a detailed presentation of this model, see Miller and Blair, 1985).

When this solution is used for studying the potentialities for growth of an economy in response to final demand shocks, at least three problems can be considered.

The first one is to find, for a new situation, the largest increase in production resulting from a unitary increase in final demand, supposing that, in this new situation, no sector will decrease its final demand in relation to the initial level. This problem is easily solved using the Rasmussen multipliers. The unitary increase in final demand should be allocated to sector i in such a way that the Rasmussen multiplier $\sum_j l_{ji}$ is maximum (l_{ji} is the generic element of the matrix **L**).

The second problem is to find the largest increase in production resulting from a unitary increase in final demand, assuming that the final demand for each sector can vary and supposing that, in the new situation, this variation will not lead to a negative final demand for that sector (a negative final demand for a given sector has no meaning, with the possible exception of the existence of large stocks for that sector in the initial situation – a case that we rule out). This problem is again easily solved. All of the final demand (the total value of final demand in the initial situation plus one additional monetary unit) should be allocated to sector i of the largest $S_j\ l_{ji},$ while for the other sectors final demand should be zero.

These two problems are easily solved, but both are of limited interest because of their lack of realism, which is, of course, more pronounced in the case of the second problem. For the first problem, the macroeconomic bias is clear. It is assumed that it is possible to increase the final demand of any sector by one monetary unit and at the same time keep final demand constant for the other sectors, an assumption that a genuine multisectoral analysis cannot accept.

This is why it is worth considering a third (alternative) problem, namely to find the variations of the vector of final demand within the vicinity of a given initial vector that will maximize (or minimize) the distance of the resulting vector of production in the new situation in relation to the initial production vector.

One important characteristic of this third problem is the use of the Euclidean distance between vectors to measure the variations in relation to the initial situation. A vector resulting from concentrating all of the increase in final demand in one sector is at a greater distance from the original final demand vector than a vector that results from evenly distributing an increase in final demand of the same magnitude, which means that the Euclidean distance effectively distinguishes between two situations that must be treated as different. So, a genuinely multisectoral analysis should focus on the comparison between final demand variations that give rise to new vectors located at the *same distance* from the original vector. In the same way, the output impact of these final demand variations should be measured by the Euclidean distances between the new and the original output vectors.

Methodology

In studying the structure of a national (or regional) economy, let us suppose that we have to find the vector that maximizes the total output attainable in the next period.

Formally, let us call the initial final demand vector $\mathbf{y}^s$ and the corresponding output vector $\mathbf{x}^s$, given by the input-output relation $\mathbf{x}^s = \mathbf{L}\mathbf{y}^s$. Given a neighborhood *ß* of $\mathbf{y}^s$, $V(\mathbf{y}^s, ß)$, the objective is to find the vector $\mathbf{y}^* \in V$ such that the distance between $\mathbf{x}^*(\mathbf{y}^*)$ and $\mathbf{x}^s$ is maximum.

Note that this is not a case of calculating the output growth resulting from a unitary increase in final demand. This problem is easily dealt with by using traditional multipliers. In this case, what we want is to find, from among all the vectors at a certain distance of $\mathbf{y}^s$, the vector that maximizes the variation of the resulting output vector in relation to the initial vector, $\mathbf{x}^s$.

Let us consider, for the sake of simplicity, that *ß* = 1. In this case, a vector at a unitary distance of $\mathbf{y}^s$ is not necessarily a final demand vector in which the sum total of all its elements exceeds the sum total of all the elements of the initial vector by exactly one monetary unit. This is only true when all of the (unitary) increase in final demand is concentrated in one sector. In general, and excluding this particular case, it is a vector that represents a monetary expenditure that is more than one unit higher than the total expenditure of vector $\mathbf{y}^s$.

Particularly in studies of economic growth it is much more interesting to consider the output impacts of final demand vectors at a given distance from an initial vector than merely considering the output growth of unitary increases in final demand.

Suppose that we want to study the impact upon the distance from the initial output vector $\mathbf{x}^s$ to the vector $\mathbf{x}^*$ of a change in final demand from $\mathbf{y}^s$ to $\mathbf{y}^*$, in which:

$$S\ (y_j - y^*_j)^2 = ß^2$$

It is a case of maximizing (with *ß* equal to *1*, according to our hypothesis):

$$(\mathbf{x} - \mathbf{x}^s)' (\mathbf{x} - \mathbf{x}^s),\ \text{(the prime means transpose)}$$

subject to:

$$(\mathbf{y} - \mathbf{y}^s)' (\mathbf{y} - \mathbf{y}^s) = 1$$

As $\mathbf{x}^s = \mathbf{L}\,\mathbf{y}^s$, the corresponding *Lagrangean* is:

$$(\mathbf{y} - \mathbf{y}^s)' \mathbf{L'L} (\mathbf{y} - \mathbf{y}^s) - ?[(\mathbf{y} - \mathbf{y}^s)' (\mathbf{y} - \mathbf{y}^s)]$$

After differentiating and equalizing to zero:

(1) $$\mathbf{L'L} (\mathbf{y} - \mathbf{y}^s) = ?(\mathbf{y} - \mathbf{y}^s)$$

Since **L'L** is symmetric, all its eigenvalues are real. Since it a case of maximizing a definite positive quadratic form, all the eigenvalues are positive.

Furthermore, multiplying both members of **(1)** by $(\mathbf{y} - \mathbf{y}^s)'$ and considering only vectors **y** such as $(\mathbf{y} - \mathbf{y}^s)' (\mathbf{y} - \mathbf{y}^s) = 1$, we have:

$$(\mathbf{y} - \mathbf{y}^s)' \mathbf{L'L} (\mathbf{y} - \mathbf{y}^s) = ?$$

and so the maximum distance between **x** and $\mathbf{x}^s$ is obtained for the greatest value of *?*, i.e. for the greatest eigenvalue, and the minimum distance for the smallest one.

An economy is more variable in terms of its final demand structures, the greater the amplitude of variation of the distance between **x** and $\mathbf{x}^s$ in response to a unitary final demand shock.

A demand management economic policy may focus on maximizing output and employment, and in this case it will try to attain the vector **y*** that maximizes the distance between **x** and $\mathbf{x}^s$. An economic policy that focuses on an inflation target will generally try to attain a vector **y** that minimizes this distance.

The amplitude of variation attainable for the distance between **x** and $\mathbf{x}^s$ can be measured by the difference $s(\mathbf{L'L}) = (?_{max} - ?_{min})$, i.e. the *spread* of **L'L**, and it is certainly an important property of each technological structure **A** (the input coefficients matrix) and its corresponding Leontief inverse, $\mathbf{L} = (\mathbf{I}-\mathbf{A})^{-1}$.

An important property of technological structures

Some linear algebra results can be used to further advance research into this property of technological structures.

It is known (Marcus *et al*, 1992, p.167) that:

$$2 \max c_{ij} = s(\mathbf{L'L}) < [2||\mathbf{L'L}||^2 - 2/n\,(tr\, \mathbf{L'L})^2]^{1/2}$$

in which by c_{ij} $(i?j)$ we mean the off-main diagonal elements of **L'L**, and in which the norm is Euclidean, i.e. with any **N**, $||\mathbf{N}|| = (?\, n_{ij}^2)^{1/2}$.

It is easy to see that $tr\, \mathbf{L'L} = ||\mathbf{L}||^2$.

Furthermore, because of the properties of the general norm and Euclidean norm:

$$||\mathbf{L'L}|| = ||\mathbf{L}||.||\mathbf{L'}|| = ||\mathbf{L}||^2$$

so that,

$$2 \max c_{ij} = s(\mathbf{L'L}) < (2-2/n)^{1/2}\ ||\mathbf{L}||^2 \sim \text{v}2\ ||\mathbf{L}||^2$$

This demonstrates the importance, for this analysis, of the maximum value of the off-main diagonal values of **L'L** and of the summation of the square elements of **L**.

An increase in the value of **L** elements (i.e. the elements of **A**) necessarily leads to an increase in the elements of **L'L,** since **L** is a matrix of positive elements. If the increase is sufficiently intense, this implies that there will be an increase in the amplitude of the possible output variations in response to a unitary final demand change. With a "fuller" technological structure, the management of final demand is more important than it is with a less "full" one.

As an example, consider the case of an economy with just two sectors, in which, for the sake of simplicity, there are only identical inputs:

$$\mathbf{A} = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

Table 1 summarizes some possible values for *a* and *b* and the corresponding values for the *spread,* in which it is clear that this increases when the values of *a* and *b* increase.

Table 1: *Spread* of matrix A for different values of *a* and *b*

		b									
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
a	0	0	0.41	0.87	1.45	2.27	3.56	5.86	10.77	24.69	99.72
	0.1	0	0.56	1.21	2.08	3.41	5.74	10.67	24.61	99.65	
	0.2	0	0.81	1.78	3.17	5.56	10.52	24.49	99.56		
	0.3	0	1.22	2.77	5.25	10.28	24.31	99.41			
	0.4	0	1.96	4.69	9.88	24.00	99.17				
	0.5	0	3.47	9.07	23.44	98.77					
	0.6	0	7.11	22.22	97.96						
	0.7	0	18.75	96.00							
	0.8	0	88.89								
	0.9	0									

Homothetic scale and structure effects

As we saw previously, there are two vectors of final demand variations that result in maximum output movement: the vector in which all the final demand components increase and the other vector that is symmetric to this. If we are interested in the vector of increasing output, we will consider the vector $?\mathbf{y}^*$, in which all the components are positive. The corresponding output vector, $?\mathbf{x}^*$, is $\mathbf{L}\ ?\mathbf{y}^*$, and this variation can be decomposed into two components: a scale effect and a structure effect.

Without structural changes, we would have a proportional increase in all sectors

$$\Delta\mathbf{x}^* = d_0\mathbf{x}$$

However, in general, we do not observe this proportional change. On the contrary, $\Delta\mathbf{x}^*$ is a result of the combination of economic expansion in keeping with the existing structure and economic development as given by structural changes in the economy (an identical decomposition can be made for the "optimal" impulse vector of final demand, $\Delta\mathbf{y}^*$).

Formally

$$\Delta\mathbf{x}^* = \mathbf{SC} + \mathbf{ST}$$

where **SC** and **ST** are the *scale vector* and the *structural change vector*. Defining d such that

$$d = \min\left\{\frac{\Delta x_1^*}{x_1}, \frac{\Delta x_2^*}{x_2}, \cdots, \frac{\Delta x_n^*}{x_n}\right\}$$

we have for the scale vector,

$$\mathbf{SC} = d\mathbf{x}$$

The vector **ST** is then obtained by

$$\mathbf{ST} = ?\mathbf{x}^* - \mathbf{SC}$$

Our measures for the scale and the structure effects are then the Euclidean norms of **SC** and **ST**, respectively.

In the empirical application, we present the values for the length of $\Delta\mathbf{x}^*$, **SC** and **ST**, in order to compare the effects produced in terms of scale and structural change with the overall effect.

3. AN APPLICATION TO EIGHT OECD COUNTRIES

Considering different countries and two different periods, two interesting issues are the following: a) in what extent did the different economies develop according to the "optimal" output vector? b) Is that the case that the economies with an observed evolution close to the "optimal" pattern where also those with higher growth rates?

In this section we analyse eight OECD (EU) countries for which IO domestic production tables were available for two different years, 1995 and 2000 (1999 in the case of Portugal). Table 2 shows a group of results, some of them related with subject a).

Regarding to the impact values of a unitary distance variation in final demand we see that, although with some oscillations, the values are not very different from country to country, mainly in relation to the value of minimum ?. Nevertheless, significant differences in the spread are already observed.

As for the effects of scale and structural change, besides some differences among countries, we observe, above all, the importance of structural changes connected to the "optimum" vector. In terms of evolution in time, a slight increase of the first effect and a decrease in the second effect are observed in all the countries considered here.

As for the second issue, in spite of the reduced number of countries in the sample, the results suggest a negative relationship between the evolution according to the "optimum" production pattern, as determined for the initial period, and the real product growth rates observed in the period. In fact, the correlation coefficient between the two variables is negative (equal to -0.60), although not statistically significant at the 5% level (table 3).

Table 2: *Distance* multipliers

		Belgium	Denmark	Germany	Finland	Italy	Netherlands	Sweden	Portugal
1995	?max	3.14	2.74	3.36	3.38	3.32	2.59	2.91	3.13
	?min	0.75	0.83	0.86	0.89	0.86	0.88	0.88	0.79
	Spread (**L'L**)	2.38	1.91	2.50	2.49	2.46	1.70	2.03	2.34
	lim_inf	1.31	1.29	0.82	1.48	1.06	1.18	0.92	1.45
	Total effect	1.77	1.66	1.83	1.84	1.82	1.61	1.71	1.77
	Scale effect	0.37	0.37	0.55	0.37	0.42	0.50	0.50	0.36
	Struct. change effect	1.46	1.38	1.37	1.56	1.47	1.22	1.30	1.48
2000	?max	3.23	2.70	3.45	3.34	3.20	2.68	3.06	3.09
	?min	0.81	0.84	0.82	0.86	0.83	0.94	0.88	0.84
	Spread (**L'L**)	2.43	1.86	2.63	2.49	2.37	1.75	2.18	2.26
	lim_inf	1.06	1.20	0.81	1.37	0.92	0.99	0.77	1.20
	Total effect	1.80	1.64	1.86	1.83	1.79	1.64	1.75	1.76
	Scale effect	0.42	0.40	0.58	0.46	0.47	0.57	0.58	0.40
	Struct. change effect	1.43	1.33	1.36	1.47	1.39	1.17	1.26	1.44
95-00	Cos(? Y_{opt}, ? Y_{obs})	0.72	0.63	0.71	0.53	0.71	0.74	0.70	0.60
	Cos(? X_{opt}, ? X_{obs})	0.84	0.70	0.80	0.62	0.74	0.79	0.79	0.69

Table 3: *Optimal* and actual intersectoral expansion and GDP growth

	cos(? x_{opt}, ? x_{obs})	Average growth rate of real GDP (1995-2000)
Belgium	0.84	2.95
Denmark	0.70	2.89
Germany	0.80	1.99
Finland	0.62	4.61
Italy	0.74	2.06
Holland	0.79	3.62
Sweden	0.79	3.35
Portugal	0.69	5.05

4. CONCLUDING REMARKS

In this paper, we present a new kind of intersectoral output multipliers that can be used to overcome a serious limitation of the traditional Leontief/Rasmussen multipliers, namely the obligation to consider a fixed (predetermined) structure of final demand.

By solving a properly designed optimization problem, one can calculate the impact on sectoral outputs of a shock in final demand along all vectors at a certain *Euclidean distance* from the initial final demand vector.

Across the full spectrum of all possible new final demand vectors, two in particular play an important role for economic policy: the vector maximizing output growth if the objective is to promote employment; the vector minimizing output growth if the objective is to control inflation or, for example, minimize CO_2 emissions.

An important property of productive structures is the so-called *spread* of the technological matrix, the difference between the maximizing and the minimizing impacts.

In the maximizing case, an interesting exercise consists of decomposing the total impact into two effects: a homothetic scale effect, where the economy grows in accordance with the initial structure; a structure effect, shown by the change in structure that is brought about by the maximizing purpose in hand.

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APPENDIX

Table A1. Sectors used in section 3

1	Agriculture, hunting and forestry
2	Fishing and fish products
3	Mining and quarrying
4	Manufacturing
5	Electricity, gas and water
6	Construction
7	Automobile trade and repair
8	Hotels, restaurants
9	Transports and communications
10	Financial services
11	Real estate services, renting
12	Public administration, defence and social security
13	Education services
14	Health and Social Services
15	Other services