Measuring the Economic Importance of an Industry: An Application to the Austrian Agricultural Sector

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Abstract

Recent developments have underlined the relevance of concepts for measuring the economic importance of an industry within the framework of input-output analysis. Standard multipliers, i.e. column sums of the Leontief inverse, can correctly be applied only to final demand (or elements and changes therof). When output, not final demand, is given exogenously, a modelling option is the concept of output-to-output multiplier developed by Miller and Blair (1985, chapter 9), which applies, however, only to the situation when the output in exactly one sector is fixed. In this paper we present a general model that can take account of (1) exogenously given output in more than one sector and (2) endogenized final demand induced by the generated income from wages and salaries. One formulation of the model is in closed form and builds on the mixed model of Miller and Blair (1985, chapter 9.3). Two other formulations are shown to be mathematically equivalent and disclose additional views of the model. The analysis of the Austrian agricultural sector forms an ideal application area for the model as it can be defined as the compound of three sectors of the Austrian input-output table. Further extensions of the model consider also exogenous

capital formation. Thus, the total economic importance of the Austrian agricultural sector for the economy can be calculated as the sum of the direct, indirect and income-induced effects of its production activities and of its capital formation, without double counting of effects.

1 Introduction

Measuring the economic importance of an industry is a recurring theme in input-output analysis and many practical applications have been conducted with precisely that subject. Several approaches to treat the problem have been developed, some of them very recently.

An approach that is sometimes applied by practitionners is to multiply the production of an industry or of a collection of industries with the traditional Leontief multipliers. However, this approach is generally viewed as qestionnable because it amounts to treating production as if it were final demand and thus entails double counting of effects.

Notwithstanding the availability of a whole spectrum of approaches, including hypothetical extraction methods (Miller and Lahr, 2001; Cai and Leung, 2004) and so called net multipliers (Oosterhaven and Stelder, 2002), we want to stress in this paper the appropriateness of the output-to-output multiplier and the mixed variables exogenous/endogenous model (Miller and Blair, 1985, chapter 9).

Many applications of mixed exogenous/endogenous variables models are intended for the analysis of supply constraints, e.g. quotas. This might be an important application field. However, we think that the model is appropriate whenever the production of a sector is to be analysed for its direct, indirect and induced impacts on the (rest of the) economy. The research question of measuring the importance of a sector or industry[1] provides enough justification to view the production of the sector as exogenous.

The classical output-to-output multiplier, as defined by Miller and Blair (1985), gives the production in the whole economy that is directly and indirectly necessary to sustain the production of one unit in a given sector. It can be derived from the mixed exogenous/endogenous variables model. As will be shown later, the concept

[1]In this paper the notions of sector and industry are used synonymously.

of the classical output-to-output multiplier cannot be applied when the production in more than one sector is exogenously given. In this situation the mixed exogenous/endogenous variables model must be used. Based on the model solution it is possible to derive a multiplier.

With this general approach we put the model before the multiplier, in contrast to a common misconception that confuses the multiplier with the model (West, 1999). Thus the model is used to calculate the impact of a certain economic impulse, and then the multiplier is calculated as

$$\frac{\text{impact}}{\text{impulse}}. \tag{1}$$

However, this approach is sometimes not available, as often the economic impulse is composed of different individual impulses that cannot be summed up. This is the case when the importance of an industry is viewed as the combined impact of its production activity and its gross capital formation. Gross capital formation is a component of final demand. Clearly, production and final demand cannot be summed up. Thus, in this case we cannot form a multiplier in the spirit of (1).

In the following section we will present the models. Though the models are not new, a new focus is adopted. In particular, a general formulation of the mixed exogenous/endogenous variables model takes account of (i) exogenously given output in more than one sector and (ii) endogenized final demand induced by the generated income from wages and salaries (income-induced effects), i.e. closing the model with respect to households. One formulation builds on Miller and Blair (1985, chapter 9.3). Two other formulations are mathematically equivalent (this is shown in the Appendix) and disclose some aspects of the economic interpretation of the model.

In the third section the analysis of the Austrian agricultural sector forms an ideal application area for the model as it can be defined as the compound of three sectors of the Austrian input-output table. The total economic importance of the Austrian agricultural sector for the economy can be calculated as the sum of the direct, indirect and income-induced effects of its production activities and of its capital formation.

The fourth section recapitulates the main properties of the modeling approach, in particular the avoidance of double counting of effects.

2 The models

Though the models presented in this section are not new, several of their aspects have not been treated by previous research. Our focus is on combining the mixed exogenous/endogenous variables model with the extension for endogenizing private consumption. We will develop different model formulations that are mathematically equivalent but disclose different economic interpretations.

The mixed exogenous/endogenous variables model (from now on we will use the shorter but inprecise "mixed variables model") is covered by Miller and Blair (1985). These authors have popularized the model though it seems to have been around earlier (e.g., Johnson and Kulshreshtha, 1982).

We first present the classical Leontief model and the mixed variables model. Then we will generalize these models for endogenizing private consumption. For the ease of exposition we will sometimes assume an economy of $n = 4$ sectors. In the mixed variables model we assume that $k = 2$ sectors have exogenously fixed production.

In the model descriptions we assume an economy without imports. Though the distinction between imported and domestically produced commodities is of high theoretical and practical importance we leave this issue aside and discuss it only later in the application study (Section 4). Another discussion that is postponed to Section 4 concerns the distinction between commodities and sectors/industries, i.e. the so called make-use system. Thus, in this section we make no such distinction and assume that our data are arranged so that every sector produces only one commodity and every commodity is produced by only one sector.

2.1 The classical Leontief model

The classical Leontief model assumes (like most input-output models) fixed technical coefficients a_{ij}. Thus, whenever a quantity x_j of commodity j is produced $a_{ij}x_j$ units of commodity i are needed as inputs. When x_j is produced, what is not needed of it as input for the production of other commodities (intermediate use) is delivered to final demand:

$$x_j - \sum_{i=1}^{n} a_{ji}x_i = y_j, \tag{2}$$

where y_j is final demand for good j. The above relationship, applied to every commodity j, can be written in matrix notation as

$$x - Ax = y, \tag{3}$$

where x and y are n-vectors of production and final demand and A is the technical coefficient matrix.

Model (3) can be used to determine the vector of final demand, that can be satisfied on the basis of a certain vector of production. In more typical situations a certain final demand is given and the necessary production is to be determined. Thus, the model is solved for x,

$$x = (I - A)^{-1}y = Ly, \tag{4}$$

where L is the Leontief inverse matrix. The ijth element of L gives the production in the ith sector that is directly or indirectly required for the satisfaction of one unit of final demand for the jth commodity. It is also correct to say that it gives the production in the ith sector *caused* by one unit of final demand for the jth commodity.

By linearity of the model it can be used to calculate the effects on production caused by any change or component of final demand. This is sometimes expressed as

$$\Delta x = L\Delta y. \tag{5}$$

Typical components or changes of final demand to be analysed by the model are exports or changes in exports. In the application study we will use the model for the analysis of the effects of the gross capital formation of the agricultural sector on the economy.

From this model a multiplier can be derived by forming the ratio

$$\frac{e'L\Delta y}{e'\Delta y}, \tag{6}$$

where e is the vector consisting of n ones ("summation vector"). We will refer to this as the output multiplier, or, more precisely, as the final demand to output multiplier. It gives the amount of production that is triggered in the economy by an average unit of a certain final demand impulse Δy. Note that this multiplier conforms to the general concept of a multiplier as suggested by Equation (1).

In the case that Δy is the ith unit vector (or a multiple of it) the output multiplier reduces to the ith element of $e'L$. Therefore the elements of $e'L$ are often referred to as output multipliers, too. For the sake of completeness it should be added that some authors, e.g. Miller and Blair (1985) call the elements of L multipliers.

The effects of Δy on other economic quantities like value added and employment are calculated by the model

$$\Delta v = \hat{a}_v L \Delta y, \tag{7}$$

where v is a placeholder for value added or employment, a_v is the vector of direct value added or employment coefficients and the symbol $\hat{\cdot}$ denotes diagonalisation of a vector. The corresponding multiplier (value added multiplier, employment multiplier) is given by the ratio

$$\frac{a_v' L \Delta y}{e' \Delta y}. \tag{8}$$

It gives the amount of value added or employment that is caused by an average unit of the final demand impulse Δy. Again, in the case that Δy is the ith unit vector this reduces to the ith element of $a_v' L$, which is therefore often referred to as the value added or employment multiplier.

2.2 The mixed variables model

By the logic behind input-output analysis stimulating impulses for the economy can emanate not only from final demand impulses but also from production impulses. The production of a firm or a sector entails demand for intermediate goods which cause further production and so on. When the production of a certain sector or of a group of sectors is considered as an economic impulse for the rest of the economy this amounts to asking for the economic importance of those sectors and implies exogenizing their production.

Let x_1 and x_2 be the exogenously given production in the first $k = 2$ sectors of an economy of $n = 4$ sectors. For the other sectors final demand, y_3 and y_4, is given exogenously. Equation (1) must hold for every sector j, thus we can write

$$\begin{bmatrix} 1-a_{11} & -a_{12} & -a_{13} & -a_{14} \\ -a_{21} & 1-a_{22} & -a_{23} & -a_{24} \\ -a_{31} & -a_{32} & 1-a_{33} & -a_{34} \\ -a_{41} & -a_{42} & -a_{43} & 1-a_{44} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}. \quad (9)$$

The same equations can be expressed by

$$\begin{bmatrix} -1 & 0 & -a_{13} & -a_{14} \\ 0 & -1 & -a_{23} & -a_{24} \\ 0 & 0 & 1-a_{33} & -a_{34} \\ 0 & 0 & -a_{43} & 1-a_{44} \end{bmatrix} \times \begin{bmatrix} y_1 \\ y_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} a_{11}-1 & a_{12} & 0 & 0 \\ a_{21} & a_{22}-1 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 \\ a_{41} & a_{42} & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ y_3 \\ y_4 \end{bmatrix}. \quad (10)$$

Note that the first two columns of the matrices in this equation have changed places and signs and that all the exogenously given variables are now on the left hand side. The solution for the endogenously given variables is immediate. Let M be the matrix on the left hand side of equation (10) and N the matrix on the right hand side. Then, the solution of the model is given as

$$x_M = M^{-1} N y_M, \quad (11)$$

where $x_M = (y_1, y_2, x_3, x_4)'$ and $y_M = (x_1, x_2, y_3, y_4)'$. The subscript with x_M and y_M stands for "mixed".

The elements of $M^{-1}N$ define the effects of one unit of x_1, x_2, y_3, y_4 on y_1, y_2, x_3, x_4. The lower-left 2×2 submatrix of $M^{-1}N$ gives the effects that one unit of exogenous production in sectors 1 and 2 has on the production of sector 3 and 4. The colum sums of this submatrix can be viewed as multipliers. However, one has to be very careful with this kind of multipliers, because they are model specific, i.e. they are conditional on sector 1 and 2 having exogenous production and sector 3 and 4 having endogenous production (we will come back to that shortly).

Usually, the model is applied to a situation where the final demand for sectors 3 and 4 is set to zero. Thus, one asks only for the effects emanating from the production of sector 1 and 2. In this situation it is possible to adopt the general viewpoint of Equation (1) and calculate the corresponding output-multiplier (more precisely the output-to-output multiplier) as

$$\frac{x_1 + x_2 + x_3 + x_4}{x_1 + x_2}, \tag{12}$$

where x_3 and x_4 are taken from the solution of model (11). This multiplier gives the production in the whole economy required to sustain one average unit of the production in the compound of sectors 1 and 2.

It has been shown (Miller and Blair, 1985) that in the case when there is only one sector with exogenous production, say sector j, the multiplier, $(e'x)/x_j$, derived from the solution of the corresponding mixed variables model, is the classical output-to-output multiplier, which is defined as

$$(e'L_{\cdot j})/a_{jj}, \tag{13}$$

where $L_{\cdot j}$ denotes the jth column of the Leontief matrix. This multiplier gives the production that must take place in the economy as a whole when sector j produces one unit of its output. Again it should be emphasized that this multiplier is model specific, i.e. conditional on all other sectors having endogenous production.

It would be a mistake to use the output-to-output multipliers outside the context of their respective models. For example, if the production in sectors 1 and 2 is exogenous one could erroneously think that multiplying the production of sector 1 and 2 with the corresponding classical output-to-output multipliers and then summing up would yield the impact of the production of the compound of sectors 1 and 2 on the production in the whole economy. This procedure would be inconsistent in its assumptions about what is exogenous and what is endogenous. It would lead to doublecounting of the effects that the production of sector 1 has on the production of sector 2 and *vice versa*.

As said before, in application studies the exogenous final demand usually is set to zero in all respective sectors. There are, however, applications of the mixed variables model, where the exogenous final demand vector is not set to zero. One application is when impacts of the production and of the gross capital formation of one or more sectors are to be determined simultaneously in one model.

Let Y_{CF} be the matrix of intersectoral flows of gross capital formation. Its typical element, y_{ij}^{CF}, defines the quantity of commodity i that is bought by sector j for the formation of fixed gross capital. Let y_{CF} be the sum of those columns of Y_{CF} that correspond to the sectors of interest. In our simple 4-sector economy the first $k = 2$ sectors are the sectors of interest. The first two elements of y_{CF} will be non-zero when sectors 1 and 2 buy commodities from themselves and from each other for the purpose of capital formation. This reflects the fact that the capital formation of sectors 1 and 2 provides an economic stimulus not only for the other sectors but also for sectors 1 and 2. Consequently, it would be accounted for in a classical Leontief model.

But when, in the context of the mixed variables model, the production in sectors 1 and 2 is exogenous, the impact of the gross capital formation of sectors 1 and 2 on the production of sectors 1 and 2 must not be counted since it is already contained in the exogenous production. Therefore the first two elements of y_{CF} must be discarded and the vector of exogenous variables to be used in the model is given as $y_M = (x_1, x_2, y_3^{CF}, y_4^{CF})'$, where y_3^{CF} and y_4^{CF} are the third and fourth element of y_{CF}.

Note that in this application of the mixed variables model a multiplier in the spirit of Equation (1) cannot be formed, since the impulse to be analysed for its economic impact is composed of production impulses and final demand impulses, which must not be summed.

2.3 The one-sided mixed variables model

An alternative formulation of the mixed variables model is the one-sided mixed variables model. To our knowledge this model formulation has not been proposed before. In this model only the vector on the right hand side is mixed. The vector on the left hand side is the vector of production and is not mixed.

The one-sided model is given as

$$x = (I - \hat{A})^{-1} y_M = \hat{L} y_M, \tag{14}$$

where $\hat{A}$ is a modification of the technical input coefficient matrix where the elements in the rows corresponding to the sectors with exogenous production are are replaced by zeros. Thus in the case of the 4-sector economy with the first two sectors exogenous it is

$$\hat{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \tag{15}$$

This modification makes sure that there is no stimulation of the sectors with exogenous production by the other sectors. Thus, the modification conforms to the notion of "exogenous", as it makes sure that the exogenous variables are not influenced within the model.

An advantage of the model is that it allows the formulation of a multiplier in a natural way:

$$\frac{x_1 + x_2 + x_3 + x_4}{x_1 + x_2} = \frac{e' \hat{L} y_M}{x_1 + x_2}, \tag{16}$$

The model is mathematically equivalent to the mixed variables model. This can be seen by inspecting the lower-left 2x2 submatrix of the modified Leontief matrix $\hat{L}$, which is identical to the corresponding submatrix of $M^{-1}N$. The equivalence is not complete, because the solution of (14) does not provide us with y_1 and y_2. These can be calculated in a second step by inserting the solution of (14) into (3).

2.4 The extended Leontief model

In the extended Leontief model, consumption expenditure of households is treated as endogenous. The economic rationale is well-known. Our formulation of the model is rather explicit with respect to the economic mechanism. Let y_{PC} denote the vector of consumption expenditure of households (private consumption). Since it is a component of final demand, y now denotes final demand without private consumption. We have

$$x - Ax - y_{PC} = y. \tag{17}$$

Housholds spend a part of their labor income for private consumption:

$$y_{PC} = hb, \tag{18}$$

where b is a scalar denoting the labor income of households and h is a vector with elements h_j that denote the proportion of labor income that is spent on the consumption of commodity j. b is also endogenous. We have

$$b = a_b x, \tag{19}$$

where a_b are the direct labor income coefficients.

Collecting these equations, the model is given in matrix notation as

$$\begin{bmatrix} (I-A) & -I & O \\ O & I & -h \\ a_b' & o' & -1 \end{bmatrix} \times \begin{bmatrix} x \\ y_{PC} \\ b \end{bmatrix} = \begin{bmatrix} y \\ o \\ 0 \end{bmatrix}, \tag{20}$$

where O is a $n \times n$ matrix of zeros and o is a n-vector of zeros.

The solution of the model can be denoted as

$$\begin{bmatrix} x \\ y_{PC} \\ b \end{bmatrix} = \bar{L} \begin{bmatrix} y \\ o \\ 0 \end{bmatrix}. \tag{21}$$

As in the case of the classical Leontief model the model can be used to determine the impacts of changes or components of y, thus

$$\begin{bmatrix} \Delta x \\ \Delta y_{PC} \\ \Delta b \end{bmatrix} = \bar{L} \begin{bmatrix} \Delta y \\ o \\ 0 \end{bmatrix}. \tag{22}$$

2.5 The extended mixed variables model

As in the description of the conventional mixed variables model we assume that the production in sector 1 and 2 and final demand in sector 3 and 4 is exogenous. The underlying equations of the model, i.e. Equations (17-19) must hold, but they are rearranged so that all exogenous variables are on the right hand side of the equation. The extended mixed variables model is given as

$$\bar{M}\bar{x}_M = \bar{N}\bar{y}_M \tag{23}$$

where $\bar{x}_M = (x_M, y_{PC}, b)'$ and $\bar{y}_M = (y_M, o', 0)'$,

$$\bar{M} = \begin{bmatrix} -1 & 0 & -a_{13} & -a_{14} & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -a_{23} & -a_{24} & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1-a_{33} & -a_{34} & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -a_{43} & 1-a_{44} & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -h_1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -h_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -h_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -h_4 \\ 0 & 0 & a_3^b & a_4^b & 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

$$\bar{N} = \begin{bmatrix} -1+a_{11} & a_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & -1+a_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -a_1^b & -a_2^b & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

and a_j^b denotes the jth element of a_b.

The solving the model for $\bar{x}_M$ gives

$$\bar{x}_M = \bar{M}^{-1}\bar{N}\bar{y}_M = K\bar{y}_M. \tag{24}$$

As in the conventional mixed variables model model specific multipliers can be derived. When the impact to be analysed is composed of more than one sector with exogenous production and includes no final demand impulses then an overall multiplier can be formulated. However no such multiplier can be formulated when the final demand impulses are non-zero.

2.6 The extended one-sided mixed variables model

The extended one-sided mixed variables model is constructed in complete analogy to the conventional mixed variables model. Also its economic interpretation is analog, apart from endogenized private consumption. So we write down only the model formulation:

$$\begin{bmatrix} (I-\hat{A}) & -I & O \\ O & I & -h \\ a_b' & o' & -1 \end{bmatrix} \times \begin{bmatrix} x \\ y_{PC} \\ b \end{bmatrix} = \begin{bmatrix} y_M \\ o \\ 0 \end{bmatrix}, \tag{25}$$

Solving the model gives

$$\begin{bmatrix} x \\ y_{PC} \\ b \end{bmatrix} = \hat{K} \begin{bmatrix} y \\ o \\ 0 \end{bmatrix}, \tag{26}$$

where $\hat{K}$ is the inverse of the matrix on the left hand side of Equation (25).

The Appendix contains the proof for the mathematical equivalence of the extended mixed variables model and the extended one-sided mixed variables model, that is, as far as the equivalence goes. The two models are not fully equivalent. The extended onesided mixed variables model does not use the information about the intermediate use of commodities of sectors 1 and 2 by the other sectors, information which is contained in the first $k = 2$ rows of A. Consequently the solution of the model also furnishes less information than the solution of the extended mixed model as it does not tell us the final demand for commodities 1 and 2, that can be satisfied. Thus, the proof in the Appendix will only establish the identity of the relevant submatrix of K and $\hat{K}$.

2.7 The two-stage extended mixed variables model

The two-stage extended mixed variables model proceeds in two stages. First, the conventional onesided mixed variables model is applied to calculate the direct and indirect impacts of an economic impulse defined by y_M. Then, based on the solution of the first stage, x_P, the total impacts, including also those induced via endogenized consumption, are calculated.

So we assume that x_P is known. The model is given as

$$\begin{bmatrix} I & -\check{L} & o \\ O & I & -h \\ a_b' & o' & -1 \end{bmatrix} \times \begin{bmatrix} x \\ y_{PK} \\ b \end{bmatrix} = \begin{bmatrix} x_P \\ o \\ 0 \end{bmatrix}, \tag{27}$$

where $\check{L} = (I - \check{A})$ and $\check{A}$ is a modification of A, where the elements in all columns and all rows corresponding to sectors with exogenized production are set to zero. Thus, in the 4-sector economy with the first $k = 2$ sectors with exogenous production it is

$$\check{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix} \tag{28}$$

The economic rationale behind this modification is the following. The first $k = 2$ rows of $\check{A}$ must be set to zero because the production in the first two sectors is exogenous. The first $k = 2$ columns must be set to zero because the direct and indirect effects that emanate from the first $k = 2$ sectors on the other sectors have already been accounted for on the first stage of the model.

The Appendix contains a proof of the mathematical equivalence of the two-stage extended mixed variables model and the extended mixed variables model. For this purpose a matrix formulation of the model must be formulated that encompasses both stages of the model. This matrix must be identical to K.

3 Application study

The models presented in the previous sections are used to measure the economic importance of the agricultural sector for the Austrian economy. The basic data source is the Austrian make and use table for the year 2002 (Statistik Austria, 2006). In the next subsection we present the definition of the agricultural sector used in the application study. Another subsection sketches the preparation of the input-output table. The following subsections contain the results.

3.1 The agricultural sector: overview and definition

The make-use system makes a distinction between industries and commodities, thus allowing for industries to produce more than one commodity. The availability of such information brings the necessity for the analyst to decide upon the definition of an economic sector. Should an economic sector be defined by industry or by commodity? In order to be able to apply input-output-models in a commodity by commodity framework, we chose the second alternative.[2] Thus, the agricultural sector is defined as that part of the economy that produces agricultural goods in the broad sense. Our definition comprises the following goods as "agricultural":

- Products of agriculture, hunting (ÖCPA 01)[3]
- Products of forestry, logging (ÖCPA 02)
- Fish, other fishing products (ÖCPA 05)
- Wines (ÖCPA 15.93)

The agricultural *industries* producing these goods are "Agriculture, hunting" (ÖNACE 01), "Forestry and logging" (ÖNACE 02) and "Fishing, fish farms" (ÖNACE 05). Together, these industries have 100 percent market share in agricultural goods. Besides

[2] In principle, it should also be possible to use the commodity by commodity framework of input-output-analysis in order to analyse the economic importance of an *industry*. This would imply that only a part of the production of a certain commodity is treated as exogenous, namely that part that is produced by the industry in consideration. Accounting for this requires extensions of the mixed exogenous/endogenous variables model that go beyond the present paper.

[3] ÖCPA is the Austrian version of CPA, ÖNACE is the Austrian variant of NACE

agricultural goods they also produce non-agricultural goods, e.g., hotel and restaurant services.

We define two subsectors. The production of products of agriculture and hunting, wine and of products of fish and other fishing are aggregated to form the first subsector (agricultural goods in the narrow sense), and products of forestry are the second subsector. Tab. 1 summarizes the production of agricultural and non-agricultural goods by the three agricultural industries along with the definition of the (sub)sectors used in the application study.

[Table 1 about here.]

3.2 Data preparation

From the make and use table a symmetric (i.e. commodity by commodity) input-output table was derived with the help of a modified and extended version of the algorithm of Almon (2000), see Koller (2006) for a description of the modifications and extensions. This algorithm avoids negative elements in the coefficients matrices by allowing deviations from the commodity technology assumption (CTA). It is a recommendation for the use of Almon's algorithm that the deviations from the CTA to be modeled by the algorithm should be small only. Therefore large deviations should be treated in a previous phase of the data preparation. We did so in one case, production of chemical goods by oil refineries, for which the intermediate inputs, value added and employment were estimated separately. Production of wine by the agricultural sector, another typical problem area for application of the CTA, was treated by reclassification of wine as agricultural good.[4]

Almon's algorithm was applied to the domestic and import use tables, the total use table then formed as sum of the two. Furthermore Almon's algorithm was applied to the value added and employment table. Several plausibility checks, modifications and extensions of Almon's algorithm and the RAS procedure were used to guarantee the inner consistency of the input-output table, e.g. the validity of input-output balance

[4]The reclassification required only minor changes in the make and use table. In the make table the corresponding value was reallocated to the diagonal element. The reallocation in the use table concerned the intermediate use of wine, thus the corresponding part of the row describing the intermediate use of food products and beverages was reallocated to the row describing the intermediate use of agricultural goods.

equations. See Koller (2006) for a description of the procedure in the context of the preparation of the data for the Austrian INFORUM model.

The resulting input-output table is not a pure CTA-table but makes various concessions towards a hybrid table. It should be mentioned that for this reason using this table in the context of Leontief-type models is of approximative nature since Leontief-type models by the assumption of fixed input coefficients inherently rely on the CTA.

Because this application is intended to measure the impact on the domestic economy the input-output calculations are based on the domestic input-output table and on domestic input coefficients assumed as fixed. Thus, the models described in section 2 were used substituting A_d for A, where A_d is the domestic input coefficient matrix.

Gross capital formation by the agricultural sector poses a special problem. Like the use table data on gross capital formation are available only on a commodity by industry basis. It seemed too adventurous to assign gross capital formation by the *industry* agriculture to its various production acivities, for example by (mis-)using the CTA. Therefore we attributed the whole of its gross capital formation to the production of agricultural goods. This decision possibly exaggerates the gross capital formation linked to the agricultural sector (as defined in this application study) by about 5 to 10 percent.

3.3 Impact of the gross capital formation of the agricultural sector

Since capital formation is a component of final demand determining the economic impact of the gross capital formation of the agricultural sector is an application of the classical and extended Leontief model. By the linearity of the model the impact of the overall sector is simply the sum of the impact of the two subsectors.

In 2002 the agricultural sector invested 1,256.050 millions EUR in domestic capital goods, i.e. capital good produced by the domestic economy. The agricultural sector in the narrow sense accounted for 1,157.908 millions EUR and the forestry sector for 98.142 millions EUR. Tab. 2 summarizes the impacts of this economic impulse on the Austrian economy.

[Table 2 about here.]

As can be seen from the table, the capital formation of the agricultural sector (in the broad sense) in 2002 caused total production effects of about 2,371.2 million EUR in the whole economy, 439.6 millions EUR of which were generated via consumption-induced effects. The generated value added amounts to 1,205.0 millions EUR. 20,677 full-time jobs were secured via direct, indirect or consumption-induced effects emanating from the capital formation of the sector.

The last column of Tab. 2 contains the multipliers that are derived from the model. For example the production multiplier of 1.888 for the overall sector means that one million EUR value of domestic capital goods purchased by the sector that has an average composition causes 1.888 millions EUR of production in the whole economy.

3.4 Impact of the production activities of the agricultural sector

The effects of the production activities of the agricultural sectors are calculated with the mixed-variables model. When the agricultural sector in the broad sense is to be analyzed, the production in its two subsectors is given as exogenous. Otherwise the model has only one sector with exogenously fixed production. Tab. 3 summarizes the impacts of the production activities of the agricultural sector and its subsectors.

[Table 3 about here.]

As can be seen from the table, the production of agricultural goods (in the broad sense) in 2002 generated total production effects of about 10,629.7 million EUR in the whole economy, 763.5 millions EUR of which were generated via consumption-induced effects. The generated value added amounts to 5,280.2 millions EUR. 203,891 full-time jobs were secured via direct, indirect or consumption-induced effects of the production of agricultural goods.

Care has to be taken in the interpretation of the multipliers that are derived from the model. For example the employment multiplier of 27 means that one million value of production of the sector in an average composition secures 27 jobs in the whole economy.

Though the models used are linear, the effects of the overall sector are not identical to the sum of the effects of each subsector. As mentioned in section 2, the reason for this is the difference of exogeneity assumptions of the three models. For example, one can see from Tab. 2 that the sum of the total effects on jobs generated by the agricultural sector in the narrow sense and by the forestry sector is 204.840 jobs. However, the total number of jobs generated in the whole economy by the production in these two sectors is 203.891, correctly calculated. The difference, 949 jobs, are jobs that are either generated via indirect or consumption-induced effects by the agricultural sector in the narrow sense in the forestry sector or by the forestry subsector in the agricultural sector in the narrow sense. To avoid such double counting of effects is the main intention of this paper.

3.5 Impact of production activities and capital formation of the agricultural sector

The mixed variables model can be used to calculate the effects of production activities and capital formation on the whole economy. Thus, in the case where the overall agricultural sector is to be analyzed the vector of exogenous variables contains the production of agricultural goods and forestry goods in the first two elements and the demand for capital goods in the other elements of the vector. Tab. 4 summarizes the effects of production activities and capital formation of the agricultural sectors on the whole economy.

[Table 4 about here.]

In 2002 the total production caused by the production activities and gross capital formation of the agricultural sector (in the broad sense) amounts to 12,885.9 million EUR. Via direct, indirect or consumption-induced effects, the sector generated a value added in the whole economy of 6,428.1 millions EUR and secured 222,362 full-time jobs.

These numbers are without double counting. One kind of double counting is avoided by modeling the two subsectors, agricultural goods in the narrow sense and forestry goods, within the same extended mixed variables model, thus applying consistent exogeneity/endogeneity assumptions. As in the analysis presented above in Tab. 3, the relevance of (the avoidance of) this kind of double counting can be verified by comparing the sum of the effects of the individual sectors with the effects for the overall sector.

Another kind of double counting is avoided by modeling the effects of production activities and gross capital formation in one model, thus applying the same exogeneity/endogeneity assumptions. The relevance of (the avoidance of) this kind of double counting can be verified by comparing the sum of the effects of gross capital formation (see Tab. 2) and the effects of production activities (see Tab. 3) with the effects of production activities and gross capital formation (Tab. 4). Tab. 5 contains such a comparison.

[Table 5 about here.]

For example, the figure 90.379 in Tab. 5 documents the double-counting of direct and indirect effects on production that occurs when the overall effects of production activities and gross capital is, incorrectly, calculated as the sum of the effects of gross capital formation and the effects of production activities. A large part of this figure is the direct effect of gross capital formation on the agricultural sector: in 2002 the gross capital formation vector of the agricultural sector contained agricultural goods of 59.5 million EUR. When the production of the agricultural sector is treated as exogenous, this effect must not be counted as it is already contained in the exogenous production.

Finally we recall that from the results presented in Tab. 4 no overall multipliers in the spirit of Equation (1) can be derived, because production and final demand can not be added.

4 Conclusions

The conclusions will be included only in the full version of the paper.

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5 Appendix

The Appendix will be included only in the full version of the paper (in the meantime it is available from the first author upon request).

List of Tables

Table 1: Production of agricultural and non-agricultural goods by agricultural industries, 2002

ÖCPA-Code	Description	producing industry (ÖNACE)	production (millions EUR)
01	Products of agriculture, hunting	01	5,047.6
02	Products of forestry	02, 01	2,004.7
05	Fish, other fishing products	05	18.1
15.93	Wines	05	546.0
01 + 05 + 15.93	Agricultural goods in the narrow sense	01, 05	5,611.7
01 + 02 + 05 + 15.93	Agricultural goods (in the broad sense)	01, 02, 05	7,616.5
Non-agricultural goods produced by agriculture:			
15 w/o 15.93	Food products and beverages	01	115.8
45	Construction work	01	133.2
55	Hotel and restaurant services	01	114.3
	Rest of non-agricultural goods	01, 02	128.8
	Sum of non-agricultural goods	01, 02	492.1

Table 2: Impact of gross capital formation of agricultural sectors on the Austrian economy, 2002

	Direct and Indirect Effects	Consumption-induced Effects	Total Effects	Total Effects per 1 million
Agricultural sector in the narrow sense:				
Production (million EUR)	1,781.315	404.497	2,185.811	1.888
Value Added (million EUR)	893.463	218.094	1,111.557	0.960
Employment (n. of jobs)	15,379	3,770	19,148	17
Forestry sector:				
Production (million EUR)	150.311	35.082	185.393	1.889
Value Added (million EUR)	74.555	18.916	93.471	0.952
Employment (n. of jobs)	1,202	327	1,529	16
Agricultural sector in the broad sense:				
Production (million EUR)	1,931.626	439.579	2,371.205	1.888
Value Added (million EUR)	968.019	237.009	1,205.028	0.959
Employment (n. of jobs)	16,580	4,097	20,677	16

Table 3: Impact of the production activities of agricultural sectors on the Austrian economy, 2002

	Direct and Indirect Effects	Consumption-induced Effects	Total Effects	Total Effects per 1 million
Agricultural sector in the narrow sense:				
Production (million EUR)	7,699.529	634.038	8,333.567	1.485
Value Added (million EUR)	3,743.363	342.920	4,086.283	0.728
Employment (n. of jobs)	177,720	5,648	183.367	33
Forestry sector:				
Production (million EUR)	2,226.184	141,365	2,367.549	1.181
Value Added (million EUR)	1,154.172	76,217	1,230.389	0.614
Employment (n. of jobs)	20,148	1,325	21,473	11
Agricultural sector in the broad sense:				
Production (million EUR)	9,866.167	763.511	10,629.678	1.396
Value Added (million EUR)	4,867.213	413.019	5,280.232	0.693
Employment (n. of jobs)	197,090	6,801	203,891	27

Table 4: Impact of the production activities and gross capital formation of agricultural sectors on the Austrian economy, 2002

	Direct and Indirect Effects	Consumption-induced Effects	Total Effects
Agricultural sector in the narrow sense:			
Production (million EUR)	9,393.670	1,017.260	10,410.930
Value Added (million EUR)	4,593.895	550.187	5,144.082
Employment (n. of jobs)	191,325	9,061	200,386
Forestry sector:			
Production (million EUR)	2,376.260	176.258	2,552.518
Value Added (million EUR)	1,228.611	95.029	1,323.640
Employment (n. of jobs)	21,345	1,652	22,997
Agricultural sector in the broad sense:			
Production (million EUR)	11,707.414	1,178.460	12,885.874
Value Added (million EUR)	5,790.646	637.485	6,428.131
Employment (n. of jobs)	211,865	10,497	222,362

Table 5: Differences between the sum of the effects of gross capital formation and the effects of production activities on the one hand and the effects of gross capital formation and production activities on the other hand, for Austrian agricultural sectors, 2002

	Direct and Indirect Effects	Consumption-induced Effects	Total Effects
Agricultural sector in the narrow sense:			
Production (million EUR)	87.174	21.275	108.448
Value Added (million EUR)	42.931	10.827	53.758
Employment (n. of jobs)	1,774	357	2,129
Forestry sector:			
Production (million EUR)	235	189	424
Value Added (million EUR)	116	104	220
Employment (n. of jobs)	5	0	5
Agricultural sector in the broad sense:			
Production (million EUR)	90.379	24.630	115.009
Value Added (million EUR)	44.586	12.543	57.129
Employment (n. of jobs)	1,805	401	2,206